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Desarrollo y Evaluación de Modelos Analíticos para Aplicaciones y Técnicas Cross-layer en Redes Vehiculares

Tesis doctoral

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Development and Evaluation of Analytical Models for Applications and Cross-layer Techniques in Vehicular Networks

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“Toda idea nueva pasa inevitablemente por tres fases: primero es ridícula, después es peligrosa, y después... ¡todos la sabían!”

Henry George Bohn

Abstract

Vehicular Ad-hoc Networks (VANETs) have been receiving a growing research interest from both the Academia and the Industry, due to the potential benefits provided by the broad range of applications that might be derived from their use. VANETs use wireless communications in which the information can be transmitted among vehicles (V2V) or among vehicles and road infrastructure (V2I). This technology paves the way for future Intelligent Transportation Systems (ITS), which integrate advanced information, communication and control technologies to bring major improvements to the existing transportation network, like vehicle traffic control or driver information systems.

One of the most promising benefits of vehicular communications is the improvement of traffic safety. Cooperative Collision Avoidance (CCA) applications are a new emerging means of reducing the number of accidents on the road by providing cars with collaborative communication capabilities, thus allowing them to better react against possible accident risks. However, to design and implement such applications, a deep understanding of the vehicle collision process is needed. The influence of different driving parameters on the collision event must be assessed at an early design stage to develop applications that can timely adapt vehicle dynamics to avoid or at least mitigate the danger. In this context, this thesis presents and evaluates a novel stochastic model that enables the computation of the average number of collisions that occur in a platoon of vehicles driving in a single-lane road. At the same time, the model allows to study the effect of different driving parameters (inter-vehicle distance, driver reaction time, braking deceleration, etc.) on the collision process.

Next, we focus on the efficiency and reliability of emergency messages propagation, which should reach all the vehicles within a certain area in a limited time. The

delivery of these geographically-addressed messages is performed by the *GeoNetworking* protocol, which uses a forwarding mechanism to route packets through intermediate nodes until reaching the destination. We assess here how *cross-layer* techniques, allowing the exchange of information between the different communication layers, can help to improve the operation of GeoNetworking by optimizing the forwarding algorithm in use. We finally provide a survey and comparative evaluation of the most relevant proposals in the context of vehicular environments, focusing on the particular cases involving the MAC (Medium Access Control) and network layers.

Resumen

Las redes ad hoc vehiculares (VANETs) comprenden un campo de investigación que está recibiendo un creciente interés tanto por parte de la industria como en el ámbito académico, debido a las ventajas que proporcionaría la gran diversidad de aplicaciones que se pueden derivar de su uso. Las VANETs utilizan comunicaciones inalámbricas en las que la información se puede transmitir entre vehículos (vehicle-to-vehicle, V2V) o entre vehículos e infraestructuras de carretera (vehicle-to-infrastructure, V2I). Estas tecnologías constituyen la clave de los futuros Sistemas de Transporte Inteligente (ITS), los cuales integran diferentes sistemas y tecnologías para mejorar distintos aspectos del transporte en general, como el control del tránsito o los sistemas de información a los conductores.

Uno de los beneficios más importantes de las comunicaciones entre vehículos es la mejora de la seguridad vial. Las aplicaciones para evitar colisiones de forma cooperativa (Cooperative Collision Avoidance, CCA) se presentan como un método novedoso para reducir el número de accidentes en la carretera, proporcionando a los vehículos capacidades de comunicación cooperativa, de tal manera que sean capaces de reaccionar coordinadamente ante posibles riesgos de accidente. Sin embargo, para diseñar e implementar este tipo de aplicaciones se necesita estudiar en profundidad el proceso de colisión de un vehículo y conocer la influencia de los diferentes parámetros de la conducción en el origen de las colisiones. Este estudio se debe llevar a cabo en una etapa previa al desarrollo de aplicaciones que pretendan adaptar a tiempo la dinámica de los vehículos para evitar las colisiones, o al menos mitigar sus efectos. En este contexto, en esta tesis doctoral se presenta y evalúa exhaustivamente un modelo estocástico novedoso que permite calcular el porcentaje medio de vehículos accidentados en una cadena de vehículos que circulan en una carretera de una sola dirección con un solo carril, permitiendo estudiar el efecto que tienen los diferentes parámetros de la conducción (distancia

intervehicular, tiempo de reacción del conductor, deceleración de frenada, etc.) en el proceso de colisión de un vehículo.

Continuamos nuestra investigación con el estudio de la eficiencia y fiabilidad en la propagación de mensajes de emergencia, que deben alcanzar a todos los vehículos en el rango de transmisión y tiene que difundirse en un tiempo acotado a un conjunto de vehículos que se encuentran dentro de un área determinada. La transmisión de estos mensajes con un destino geográfico se realiza mediante el protocolo *GeoNetworking*, que utiliza un mecanismo de retransmisión para enviar los paquetes a través de nodos intermedios hasta alcanzar el destino. En esta tesis estudiamos cómo las técnicas de *cross-layer*, que permiten el intercambio de información entre las diferentes capas de comunicación, pueden mejorar el funcionamiento del GeoNetworking optimizando el algoritmo de retransmisión utilizado. Finalmente, presentamos un trabajo de revisión y una evaluación comparativa de las técnicas de cross-layer más relevantes en el contexto de las redes vehiculares, centrándonos en particular en las técnicas relativas a las capas de control de acceso al medio y de red.

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Part I

Presentation

1 Introduction

1.1 Introduction

Nowadays, mobility and transport have become essential aspects of our society - almost everybody has a car these days. Beyond a doubt, all of us have experienced being stuck in heavy traffic, wasting our time and energy resources. The infographic in Figure 1.1, from the Nationwide Insurance [8], showcases the real cost of traffic jams in the United States. It shows that 1.9 billion gallons[1] of fuel are wasted annually in traffic jams and an average commuter is stuck in traffic 34 hours per year at a total personal expense of $713. More importantly, traffic accidents have traditionally been responsible for one of world's highest death rates. The European Commission's Directorate General for Mobility and Transport [4] provides an estimation of road fatalities in the European Union since 2001 (Figure 1.2). In the last year, even following a descending trend, near 30,000 people died on the roads of the European Union, which is equivalent to a medium town. Moreover, for every death on Europe's roads there are an estimated 4 permanently disabling injuries, such as damage to the brain or spinal cord, 8 serious injuries and 50 minor injuries.

It is for that reason that Vehicular Ad-hoc Networks (VANETs) have become an extensive area of research, standardization, and development, which is bringing a total revolution to the driving experience. For several decades, researchers and engineers from all over the world have been interested on the idea of vehicles being "inter-connected" through wireless communications. Though the initial motivation behind inter-vehicle communications was to increase safety on the roads, more recently its use has been extended to a larger variety of applications, ranging from dynamic vehicle routing to downloading on-demand video.

[1] 1 galon = 3,78541178 litres

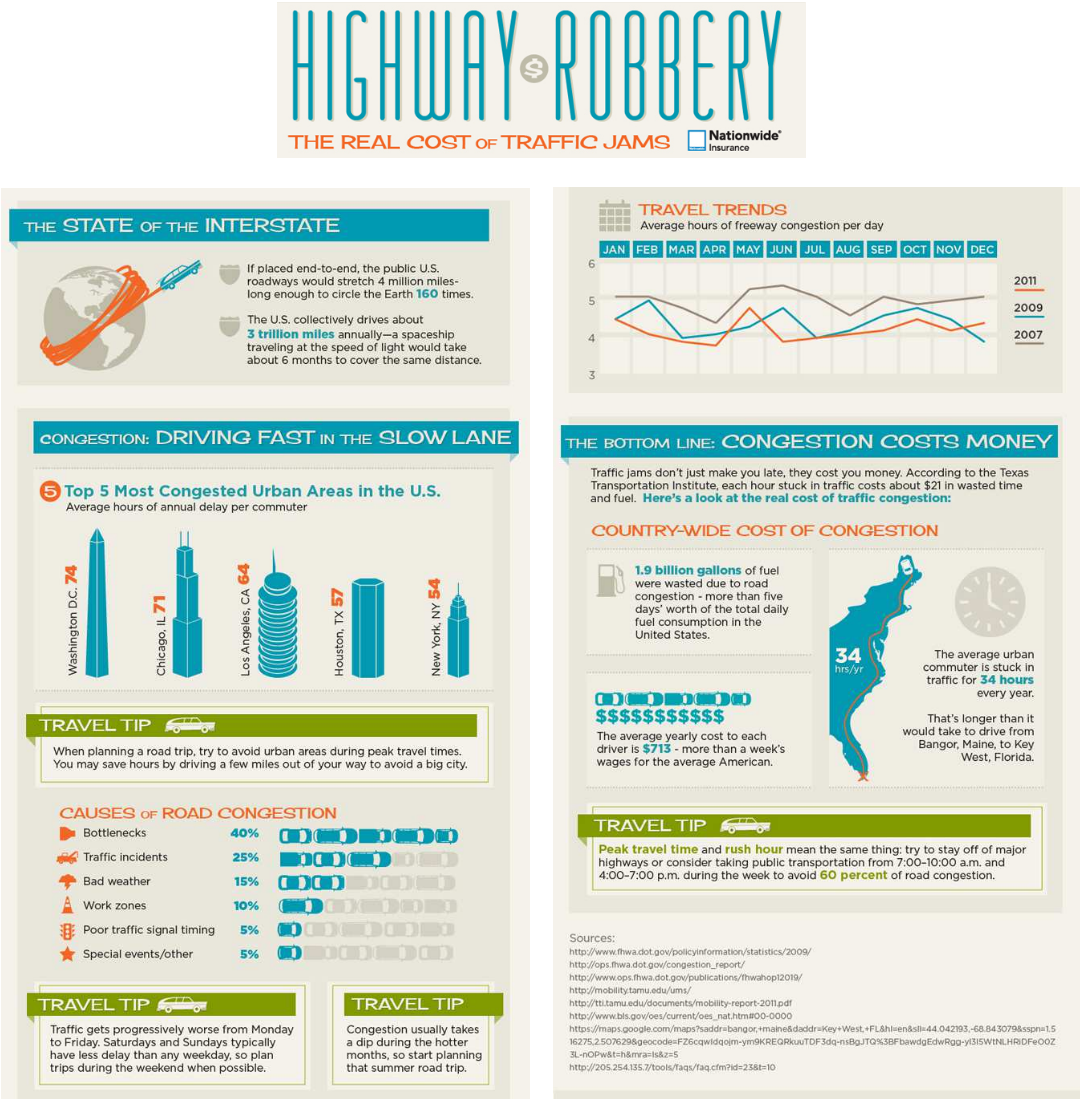


Figure 1.1: Infographic showing the real cost of traffic jams in the United States.

In this thesis we focus on the evaluation of road safety and the potential improvements brought by inter-vehicular communications. More specifically, on the one hand we study the Cooperative Collision Avoidance (CCA) applications and on the other hand we evaluate different broadcasting protocols and study their characteristics for the efficient transmission of emergency warning messages. Next, we briefly review the evolution of VANETs through last decades, and then we thoroughly explain the two research lines developed in this thesis.

Figure 1.2: Road fatalities in the EU since 2001.

1.2 Overview on Vehicular Ad-hoc Networks

The idea of inter-vehicle communications has been proposed and studied for several decades. As mentioned in [67] the basic concepts of roadway automation, i.e., the use of communication and control techniques to make road traffic safe, efficient, and environmentally friendly, were exhibited at the 1939 New York World's Fair. In his Futurama exhibit, sponsored by the General Motors Corporation, Bel Geddes sketched an automated highway system that is focused on safety, comfort, speed and economy [61], illustrating how future transportation systems may look like 20 years into the future. Later, since at least the late 1960s, actual radio-based "roadway automation" systems were developed and demonstrated.

The first major project focused on wireless vehicular connectivity was ERGS (Electronic Route Guidance System), sponsored by the U.S. Federal Highway Administration in 1970. The research was aimed at providing drivers with in-vehicle directional guidance based on the desired origin-destination trip plan [98]. In 1971, the government quickly abandoned this project owing to the expensive roadside infrastructure.

In Japan, the Comprehensive Automobile Traffic Control System (CACS) project was carried out from 1973 to 1979 by the Agency of Industrial Science and Technology of the Ministry of International Trade and Industry (MITI). The objectives of the project, as presented by Kawashima [79], were to reduce road traffic congestion and exhaust fumes, avoid traffic accidents, and enhance the public and social role of automobiles.

In the following decades, different research activities were initiated in Europe, Japan and the United States. In 1987, the European Commission funded the Eureka PROMETHEUS Project (PROgraMme for an European Traffic of Highest Efficiency and Unprecedented Safety, 1987-1995), which aimed to create concepts and solutions for a road traffic system more efficient and less detrimental to the environment and to guarantee an unprecedented degree of safety. The achievements of PROMETHEUS were the basis for most subsequent work on driverless cars. In 1997, the California Partners for Advanced Transport and Highways (PATH) demonstrated a prototype for cooperative autonomous driving at the San Diego demo. The Advanced Safety Vehicle (ASV1) programme was coordinated by the Japanese Ministry of Transport and carried out from 1991 to 1996. The aim of the programme was to develop methods and devices to improve the safety of the transportation system. Many Japanese automobile manufacturers, like Mitsubishi, Nissan, and Toyota, among others, participated in the programme and developed demonstration vehicles.

The focus then shifted from cooperative autonomous driving to cooperative driver assistance systems. Following this trend, the U.S. Department of Transportation (DOT) launched the Intelligent Vehicle Initiative (IVI) in 1997, focusing on preventing highway crashes and the fatalities and injuries they cause [69]. In Europe, the CarTalk [5] and FleetNet projects [52] investigated technologies and applications for cooperative driver assistance, aiming to improve the driver's and passengers' safety and comfort.

In 1999, the U.S. Federal Communication Commission (FCC) allocated 75 MHz bandwidth of the 5.9 GHz band to Dedicated Short-Range Communications (DSRC), which significantly impacted subsequent research projects. These studies put more emphasis on the evaluation of architecture and protocol related issues. In 2001, the Standards Committee E-17.51 of the American Society for Testing and Materials (ASTM) selected IEEE 802.11a as the underlying radio technology for DSRC. In 2004, the Institute of Electrical and Electronics Engineers (IEEE) started the work on the 802.11p amendment and Wireless Access in Vehicular Environments (WAVE), finally approved in 2010. In [67] a detailed chart summarizing project activities until 2010 can be found, including consortia such as the Car-to-Car Communications Consortium (C2C-CC) [2] sponsored by the European Union, the Vehicle Infrastructure Integration (VII) in the U.S. (which was rebranded as *IntelliDrive* and it is now named *Connected Vehicle Research* program [3]), and the Advanced Safety Vehicle Program (ASV) [1] in Japan, which is currently in its fifth phase.

Recently, there is a growing interest to foster cooperative international research in the field of ITS and to support international harmonization of standards. Coordinated research can support and accelerate the deployment and adoption of cooperative vehicle systems and prevent the development and adoption of redundant standards, providing significant cost savings. Thus, in January 2009, the U.S. DOT Research and Innovative Technology Administration (RITA) and the European Commission Directorate General for Communication Networks, Content and Technology (CONNECT) signed an Implementing Arrangement to develop coordinated research programs, specifically focusing on cooperative vehicle systems. Representatives from the Japanese Ministry of Land, Infrastructure, Transportation and Tourism participate in these groups as official observers [70].

Even though VANETs are a kind of wireless ad-hoc networks, they have some unique characteristics which make them different from MANETs and bring up many challenging research issues. Next we proceed to describe these particular characteristics and the fundamental aspects of VANETs.

1.2.1 Challenges and requirements

The technical implementation of VANETs is not as straightforward as one might think. Indeed, inter-vehicle communication networks are challenged by several issues and requirements, which exist either due to the intrinsic characteristics of the considered scenario, or due to the fact that a communication technology not specifically designed for this environment has been selected. As Hartenstein and Laberteaux outline in their book [67], the key technical and socio-economic challenges include the following issues:

- **Inherent characteristics of the radio channel.** VANET present scenarios with unfavorable characteristics for developing wireless communications, i.e., multiple reflecting objects able to degrade the strength and quality of the received signal. Additionally, owing to the mobility of the surrounding objects and/or the sender and receiver themselves, fading effects have to be taken into account.

- **Lack of an online centralized management and coordination entity.** The fair and efficient use of the available bandwidth of the wireless channel is a hard task in a totally decentralized and self-organizing network. The lack of an entity able to synchronize and manage the transmission events of the different nodes might result in a less efficient usage of the channel and in a large number of packet collisions.

- **High mobility, scalability requirements, and the wide variety of environmental conditions.** The challenges of a decentralized self-organizing network are particularly stressed by the high speeds that nodes in VANET can experience. Their high mobility presents a challenge to most iterative optimization algorithms aimed at making better use of the channel bandwidth or the use of predefined routes to forward information.

- **Security and privacy needs and concerns.** There is a challenge in balancing security and privacy needs. On the one hand, the receivers want to make sure that they can trust the source of information. On the other hand, the availability of such trust might contradict the privacy requirements of a sender.

- **Standardization versus flexibility.** Without any doubt, there is a need for standardizing communications to allow VANET to work across the various makes and brands of original equipment manufacturers (OEMs). Yet, it is likely that OEMs will want to create some product differentiation with their VANET assets. These goals are somewhat in tension.

- **Analyzing and quantifying the benefit of VANET for traffic safety and transport efficiency.** So far, relatively little work has been done to assess the impact of VANET as a new source of information on driving behavior. Clearly, the associated challenge in addressing the issue of impact assessment is the modelling of the related human factor aspects.

- **Analyzing and quantifying the cost-benefit relationship of VANET.** Because of the lack of studies on the benefits of VANET, a cost-benefit analysis can hardly be done.

- **Designing deployment/penetration strategies for this type of VANET that are not based on a single infrastructure and/or service provider.** Owing to the "network effect", there is the challenge of convincing early adopters to buy VANET equipment when they will rarely find a communication partner.

- **Embedding VANET in intelligent transportation systems architectures.** VANET will be a part of an intelligent transportation system where other elements are given by traffic-light control or variable message signs. Also public and individual transportation have to be taken into account in a joint fashion. Therefore, truly cooperative systems need to be developed.

It is not the solution of each requirement or challenge alone, but the combination of all of them, which actually poses a big challenge for the design of an optimal solution. This combination turns the field of vehicular inter-networking technologies into an interdisciplinary research area, in the cross section of communication and networking, automotive electronics, road operation and management, and information and service provisioning.

1.2.2 Applications of VANETs

VANETs can enable a wide variety of applications. This diversity makes a systematic literature review very difficult. Several surveys on inter-vehicle communication can be found in which the potential applications are classified regarding different issues. The most typical way of categorizing the upcoming VANETs applications is according to the utility offered to the passengers, as in [31, 68, 87, 102]. For instance, in [68] the applications are categorized as safety, transport efficiency and information/entertainment, while in [31] they are classified into collision warning systems, collision avoidance or vehicle automation. In [102], a complete overview of applications for VANETs is provided, which can be summarized as follows:

1. **Active safety:** considered as the typical and most desirable group of applications for VANETs with direct impact on road safety. The basic intention is to make driving safer by communication, which can mean that drivers are warned about a dangerous situation or even that the vehicle can try to avoid an accident or react appropriately if an accident cannot be avoided anymore. Active safety applications can be categorized as well, according to the danger level.

   - Dangerous road features: curve speed warning, low bridge warning, warning about violated traffic lights or stop signals.
   - Abnormal traffic and road conditions: vehicle-based road condition warning, infrastructure based road condition warning, visibility enhancer, work zone warning.
   - Danger of collision: blind spot warning, lane change warning, intersection collision warning, forward/rear collision warning, emergency electronic brake lights, rail collision warning, warning about pedestrians crossing.
   - Crash imminent: pre-crash sensing.
   - Incident occurred: post-crash warning, breakdown warning, SOS service.

2. **Public service:** vehicular networks are also intended to support the work of public service such as police or emergency recovery units.

    - Emergency response: approaching emergency vehicle warning, emergency vehicle signal preemption, emergency vehicle at scene warning.
    - Support for authorities: electronic license plate, electronic drivers license, vehicle safety inspection, stolen vehicles tracking.

3. **Improved driving:** applications that try to improve or simplify driving by means of communication.

    - Enhanced Driving: highway merge assistant, left turn assistant, cooperative adaptive cruise control, cooperative glare reduction.
    - Traffic Efficiency: notification of crash or road surface conditions to a traffic operation center, intelligent traffic flow control, enhanced route guidance and navigation, map download/update, parking spot locator service.

4. **Business/entertainment:** a large block of applications can be embraced under this category, focusing on delivering services to customers, automation of vehicle-related tasks or payment applications.

    - Vehicle Maintenance: wireless diagnostics, software update/flashing, safety recall notice, just-in-time repair notification.
    - Mobile Services: internet service provisioning, instant messaging, point-of-interest notification.
    - Enterprise solutions: fleet management, rental car processing, area access control, hazardous material cargo tracking.
    - E-Payment: toll collection, parking payment, gas payment.

On the other hand, an interesting approach is also followed by Willke *et al.* [120], which separate applications by communication paradigm, constructing the following taxonomy:

- *General Information Services*: services for which delayed or lost information does not compromise safety or render application useless.

- *Vehicle Safety Information Services*: services for which delayed information may result in compromised safety or render application useless.
- *Individual Motion Control*: applications that issue operator warnings or regulate local vehicle actuators to ensure safe and/or efficient operation.
- *Group Motion Control*: vehicle motion planning involving global optimizations or negotiations and that may or may not involve group motion regulation.

1.2.3 The IEEE 802.11p standard

This section provides a brief overview of the IEEE 802.11p standard for Wireless Access in Vehicular Environments (WAVE) [13]. This amendment extends the IEEE 802.11 [16] standard for Wireless Local Area Networks (WLANs), taking into account the particular characteristics of vehicle-to-vehicle communications: high relative velocities between nodes, short duration of connections, constant handovers and/or signal attenuation losses.

The first version of the IEEE 802.11 standard was published in 1997, defining the Medium Access Control (MAC) and several physical layer (PHY) specifications for wireless connectivity for fixed, portable and moving stations within a local area. Over the years the standard has continuously been developed, so that numerous amendments have been created in order to extend the functionality, support advanced transmission techniques and higher data rates, and operate in several frequency bands. These amendments were aggregated in one version to form the up-to-date standard IEEE 802.11-2012 [16], including the latest version of the IEEE 802.11p standard [13], which was approved by the IEEE on June 2010. It is strictly a MAC and PHY levels standard, while the IEEE 1609 standard family [15] contains the necessary procedures for the upper layers.

The original IEEE 802.11 provided two different approaches for medium access control: Point Coordination Function (PCF), that is only applicable if a central coordinating station like an access point is available, and Distributed Coordination Function (DCF). An important aspect for vehicular communications concerning safety is the prioritization of important and time-critical messages over the ones that do not directly concern safety. Therefore, IEEE 802.11p specifically adapts the Enhanced Distributed Channel Access (EDCA), originally proposed in the IEEE 802.11e amendment [11], introducing Quality of Service (QoS) support. The medium access rules defined by the DCF are replaced by the ones of EDCA, where four different Access Categories (AC) are defined. Each frame is assigned one of the four ACs by the application

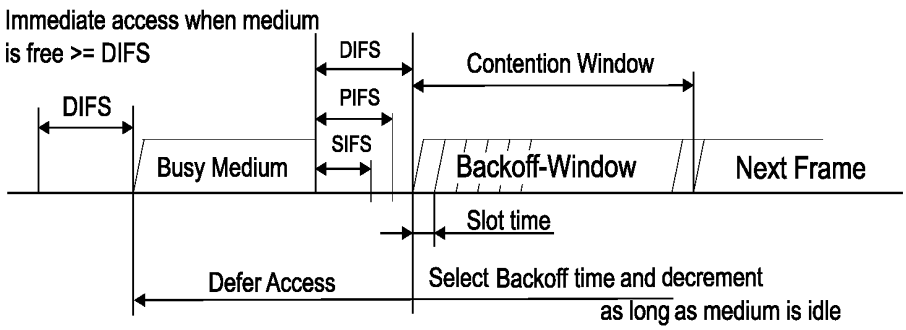


Figure 1.3: Basic access mechanism for CSMA/CA.[1]

creating the message, depending on the importance and urgency of its content. To summarize, frames corresponding to AC 0 have regular access, AC 1 is foreseen for non-prior background traffic, and ACs 2 and 3 are reserved for prioritized messages, like critical safety warnings.

For the medium access the standard employs Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA), that is, the channel is only accessed if the physical layer does not observe any ongoing activity on it and collision avoidance is provided by several additional technologies on the MAC layer. CSMA/CA uses Inter-Frame Spaces (IFSs), which are time durations that the medium has to be indicated as idle before the station may transmit. IFSs of different length for different frame types allow prioritized access. For example, important control packets such as acknowledgments are sent after a Short Inter-Frame Space (SIFS), whereas regular data packets are not transmitted before the medium was sensed idle for the duration of a Distributed IFS (DIFS), that exceeds the length of SIFS by two so-called slot times.

The basic access mechanism is illustrated in Figure 1.3. A node willing to transmit will sense the medium, and if the medium is idle for a period greater than or equal to the corresponding IFS, the node starts transmitting directly. In the case where the medium is determined busy, the station selects a random number of backoff slots within a certain range, the Contention Window (CW). The slots are counted down after the medium was sensed idle for the duration of an IFS; the countdown is interrupted whenever the medium is determined busy. When the countdown reaches zero, if the medium is sensed to be idle, the frame is transmitted immediately, but if the medium becomes busy, the node will go to backoff again. In the case of unicast packets for which no acknowledgment is received, a retransmission is scheduled after

[1]This picture was extracted from the standard [16].

a newly selected number of backoff slots under the use of an increased CW, which is known as Binary Exponential Backoff (BEB). Retransmission and CW limits are defined that restrict the number of transmission retries.

In EDCA, the Arbitration Inter-Frame Space Number (AIFSN) replaces the fixed DIFS time defined for the DCF, and determine the number of time slots during which the channel has to be sensed idle in order to enter the channel contention phase. Through the use of small AIFSNs and small contention window sizes for high priority ACs, packets that belong to this category have a higher chance to gain access to the channel earlier than packets that belong to a category that employs longer inter-frame spaces and larger contention window sizes.

For a deeper background, please refer to [67, 75, 90] or the standard itself [13].

1.3 Cooperative Collision Avoidance applications

While different factors contribute to vehicle crashes, such as vehicle mechanical problems and bad weather, driver behavior, such as tiredness, over speeding or drunken driving, is considered to be the leading causes of road accidents. The inability of drivers to react in time to emergency situations often creates a potential for chain collisions, in which an initial collision between two vehicles is followed by a series of collisions involving the following vehicles.

Without the use of communications, a driver typically relies on the tail brake light of the car immediately ahead to react to an emergency situation. In many cases, drivers cannot detect an emergency event occurring at some distance ahead, which combined with the fact that drivers usually choose to follow the vehicle ahead too closely, results in a late reaction and the inability to stop the vehicle without colliding with the preceding one. Driver reaction time (the time elapsed since the leading vehicle starts to brake and the following one perceives the change and starts to brake itself) typically ranges from 0.75 to 1.5 seconds [109], which at high speeds results in a significant distance traveled before any reaction occurs. In dense traffic, the effects of cumulative reaction times, as one vehicle after another reacts to the vehicle ahead braking, can further aggravate the situation.

With Cooperative Collision Avoidance (CCA) systems a fast dissemination of warning messages to the vehicles in the platoon enables them to promptly react in emergency situations, as illustrated in Figure 1.4. In this way the number of car accidents and the associated damage can be significantly reduced.

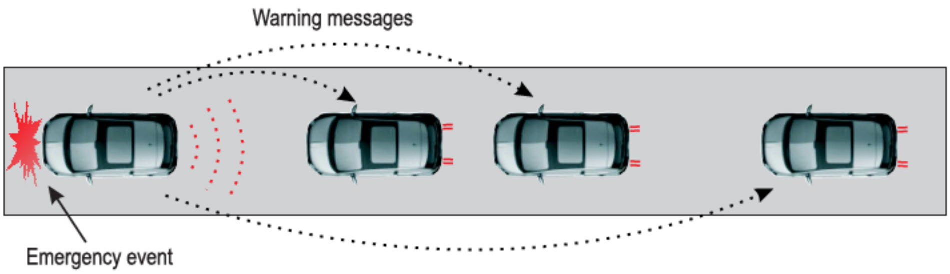


Figure 1.4: Illustration of Cooperative Collision Avoidance operation.

A reference work in the field of CCA applications is [32], in which the authors demonstrate how DSRC-based wireless communication protocols can be used for the development of such an application for enhancing highway traffic safety. They present a class of example context-aware packet forwarding protocols to demonstrate their effectiveness in designing a CCA application for intra-platoon scenarios.

In [32], the mechanism of CCA is explained depicting a three-car highway platoon example. Basically, the vehicles in the platoon are assumed to travel at constant speed, when the front car initiates an emergency deceleration as a result of an emergency event. Upon meeting the emergency event, the leading car starts sending collision warning messages to all cars behind it. Since the identities of those prospective receivers may not be known a priori, classical unicast and multicast routing will not work. Therefore, the vehicle in the emergency situation broadcasts a message first, and then all its recipients selectively forward the message based on its direction-of-arrival. To ensure a complete coverage within the platoon, the message is transmitted over multiple hops. Upon reception of a warning message, a driver reacts by decelerating, even if the brake light on the car ahead is not already lit.

As Biswas *et al.* [32] stated, the following design targets arise for a CCA system:

- Minimize the number of vehicles involved in intra-platoon chain collisions.
- Prioritize data from safety-related ITS applications over lower-priority ITS applications.
- Limit vehicle collisions in the presence of radio channel errors.

CCA applications require timely communication of safety messages between vehicles with high reliability, and the MAC protocol has a vital role to play. Therefore, many researchers have focused their efforts on the development of adequate MAC protocols in order to improve the efficiency of the emergency messages delivery, since the basic approach leads to the generation of a large number of messages, which literally flood the VANET, and the generation of redundant messages (originated from different vehicles) pertaining to the same emergency event. It is the case of [107], which proposes a vehicle clusterization mechanism based on different parameters such as speed and inter-vehicular distance. A risk-aware MAC protocol is also designed, in which an emergency level is assigned to every vehicle in the different clusters, and which is used to modulate backoff stages to keep priority differentiation for critical applications. By combining random access protocols and topology-transparent algorithm, Farnoud *et al.* [48] introduce a protocol based on constant-weight codes that is capable of ensuring reliable broadcast in highly mobile networks while maintaining low delay. In [100] the authors propose an efficient multi-hop MAC-layer broadcast protocol for emergency message dissemination in VANETs. They aim at lowering the contention delay incurred in one hop in an effort to allow significant reduction in the total broadcast delay. In addition to this, the protocol relies on control message exchange similar to request-to-send/clear-to-send handshake to get rid of the hidden terminal problem.

A different approach was adopted by Torrent-Moreno *et al.* [113], which proposed a distributed transmit power control method based on a distributed fair power adjustment for vehicular environments, to control the load of periodic messages on the channel. This proposal makes use of the principles used in [32] and further complements them with mechanisms that were aimed at reducing dissemination delay and improving reliability, particularly in high channel load conditions.

A thorough state of the art of existing broadcast schemes for VANETs can be found in [100].

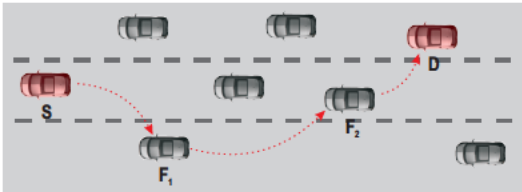
1.4 GeoNetworking

In conventional networking, an application does not care about the geographic location of the physical devices with which it intends to communicate. Nevertheless, a large number of VANET applications are likely to involve the dissemination of information in a particular geographical region. When addressing a set of nodes in a specific geographical area in an ad hoc network of mobile devices which is continuously changing, an application should be more concerned with the location rather than the identity of the physical devices for which the information is intended. In such dynamic situations, the conventional networking concept of preconfiguring a network path or set of paths from source to destination(s) and subsequently transmitting and attempting to forward packets along the path(s) will not succeed. Therefore, a different type of routing method is required in order to cope with the VANET environment. Researchers in projects such as CarTalk [5] and FleetNet [52] have introduced and proposed the use of position-based routing methods, which do not require maintenance of routes and have been proved to be particularly suitable for highly mobile networks [112]. A geo-dissemination protocol called GeoNetworking is currently being specified by the European Telecommunications Standards Institute Technical Committee (ETSI TC) on ITS, which provides mechanisms for packet forwarding in an ad hoc collection of ITS stations. In such mechanisms, nodes are addressed using not only their network addresses but also their geographical positions, supporting the communication among individual ITS stations as well as the distribution of packets in geographical areas. The current set of ETSI standards [14, 17, 18, 19, 20, 21, 23] impose GeoNetworking implementations in all ITS stations, as well as its use for communications over 5.9 GHz in Europe, including periodic transmission of safety-related messages, such as Cooperative Awareness or Decentralized Environment Notification Messages (CAM/DENM).

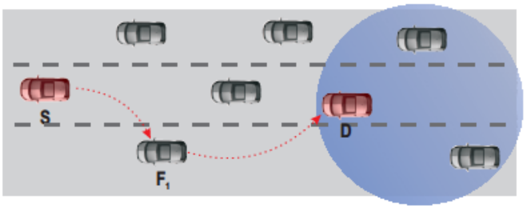
Geo-dissemination of information was primarily investigated and initial concepts were developed within the European research project GeoNet, whose final specification document [12] constitutes the basis for the development of the standard GeoNetworking protocol. In this document, the following basic transmission modes are identified.

- **GeoUnicast:** refers to the routing protocol which, based on position and movement information of involved nodes, routes data from a source to a destination node for which the exact geographical location is known (see Figure 1.5(a)). This corresponds to point-to-point scenario. GeoUnicast protocols use a forwarding mechanism to route packets through intermediate nodes till reaching the destination location.

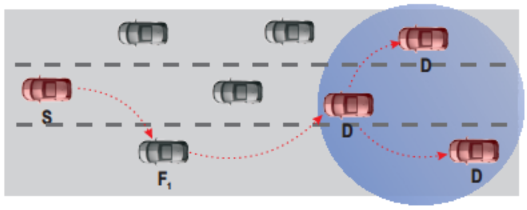
- **GeoAnycast:** refers to the routing protocol which, based on position and movement information of involved nodes, routes data from a source to any node located within a specific geographical area (see Figure 1.5(b)). As GeoUnicast, GeoAnycast targets one destination node, but not defined as destination in advance. In fact, the destination in GeoAnycast is the first node reached in a specific geographical area. Therefore, within the GeoNet project, an adapted version of the GeoUnicast protocol is used for GeoAnycast, where each node, when receiving a packet, first checks if it is located within the destination geo-area or not. If it is, then it considers itself as destination, otherwise, if it is the next forwarder, it forwards the packet towards the destination area.



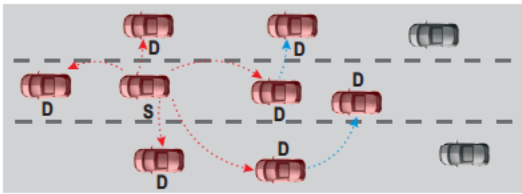
(a) GeoUnicast



(b) GeoAnycast



(c) GeoBroadcast



(d) TopoBroadcast

Figure 1.5: GeoNetworking transmission modes. The source, destination and forwarder nodes are marked with S, D and F, respectively. Geographical destination areas are depicted as blue circles.

- **GeoBroadcast:** refers to the routing protocol which, based on position and movement information of involved nodes, delivers data from a source to all nodes located within a specific geographical area (see Figure 1.5(c)). In GeoBroadcast, the source node may be located inside or outside of the targeted geo-area. If the source node belongs to the destination geo-area, then the broadcast packet should be just broadcast in this area. If the source node does not belong to the destination geo-area, then the packet should be forwarded until reaching a node which belongs to it, which takes care on broadcasting the packet to all nodes located within this area. Within GeoNet project, GeoAnycast is used to reach the first node which belongs to the broadcast geo-area, and then, a simple broadcast mechanism is used to deliver the packet to all nodes located in the destination geo-area.

- **TopoBroadcast:** refers to the routing protocol which, based on network topology information, routes data from a source node to all nodes located up to a specific distance in terms of hops (see Figure 1.5(d)). This corresponds to point-to-multipoint scenario. In GeoNet project, a basic flooding mechanism is used to disseminate all nodes up to a desired hop distance.

As basic forwarding algorithms, the standard proposes Greedy Forwarding (GF) and Contention Based Forwarding (CBF). With the former, the router uses the location information of the destination carried in the packet header and selects the neighbour with the smallest geographical distance to the destination as the next hop, thus providing the greatest progress when the packet is forwarded. If no neighbour with greater progress than the local router exists, the packet has reached a local minimum. In that case, a recovery procedure must be performed in order to make the packet progress [112].

With the CBF algorithm, the packet is broadcast and each receiver station decides whether it becomes the next hop according to its position. Upon receiving a packet, all receivers start a timer whose timeout depends on the specific position of the receiver, usually inversely proportional to the distance to the source. Upon expiration of the timer, the node re-broadcasts the packet. Before the timer expires, the node may receive a duplicate of the packet from another node with a shorter timeout, i.e. with a smaller distance to the destination. In this case, the node inspects its CBF packet buffer, stops the timer and removes the packet from it. Compared to the GF algorithm, the major advantage of CBF is that it provides an implicit reliability mechanism at the cost of larger forwarding delay and additional processing. The reliability mechanism ensures that a packet is re-forwarded by an alternative node if the theoretically optimal forwarder does not receive the packet, e.g. due to wireless link errors.

1.5 Main goals and contributions

Our main concern in this thesis is the evaluation of road safety and the potential benefits that inter-vehicular communications can entail. Of course, this research does not cover the entire matter of how vehicular networks can make roads a bit safer, but merely a few aspects of it. On the one hand, we analyze the use of CCA applications for reducing the number of accidents that take place in a platoon of vehicles driving in convoy, where a sudden emergency situation triggers a chain collision, likely involving a large number of vehicles. In order to develop properly such applications, the influence of the different driving parameters on the event of a vehicle collision must be assessed at an early design stage. Simulation is the usual choice to evaluate these systems. However, it usually requires the integration of networking and traffic simulation tools, which is not mature yet and requires further work [66]. In addition to that, available simulation tools are not directly suitable to model accidents and so cannot be effortlessly used for the design of cooperative applications. The reason is that current traffic simulation tools are based on car-following mobility models, which are specifically developed to avoid vehicle crashes and so cannot be seamlessly used to simulate accidents. Therefore, the first goal of this thesis is the development of an analytical model to be used as a numerical evaluation tool and as an alternative to simulation, specially at early stages of development.

In an initial stage, we developed a preliminary simple model to compute the average percentage of collisions in a chain of vehicles driving in a single-lane road. It should be noted that the establishment of a CCA application will be deployed gradually, equipping vehicles with the proper hardware and software so as they can communicate in an effective way within the vehicular environment. Therefore, it is highly convenient to study how the system of vehicles in a platoon will behave at different stages of technology deployment until full penetration in the market. We have developed a first approach mathematical model to calculate the average percentage of accidents in the platoon, varying the number of considered vehicles, their average speed, the average inter-vehicle spacing and the penetration ratio of the CCA technology.

Specifically when the CCA penetration ratio is taken into account, the growth in the number of operations of the analytical model is such that the sequential computation of a numerical solution is no longer feasible. Consequently, we resorted to the use of the OpenMP parallelization techniques for solving those computational cases considered as unapproachable by means of sequential procedures. Additionally, we executed our programs in the Ben-Arabi Supercomputing environment [6], taking the advantage of utilizing the fourth fastest Supercomputer in Spain. In the work presented in [92], it was shown how the parallelization techniques coordinated with supercomputing resources make the simulation process a more suitable and efficient one, allowing a thorough evaluation of the CCA application.

Then we extended the stochastic model in order to represent the collision process in a more realistic way, and we introduced variability on the model parameters to study their influence on collisions. In this second stage, we assume that all the vehicles in the platoon are equipped with vehicular communications, since this assumption removes the dependence of motion equations on the preceding vehicles and facilitates the development of a stochastic model and its solution.

In the paper published in [59] the model is described in detail, and an evaluation of the influence of each kinematic parameter on the number of accidents is provided. The most relevant conclusion obtained from this study is that a great reduction in the number of accidents can be achieved by using a CCA application able to reduce the variability, random in nature, of the different driving parameters. That is, an appropriately designed automated control system can remove the stochastic variability of the parameters by taking control of certain aspects of the driving process, such as the braking operation or reaction time.

In addition to the average number of collisions, the analytic model enables the computation of the probabilities of the different ways in which the collisions may occur: both vehicles in motion, one stopped and one in motion, etc. By assigning different degrees of severity to each collision possibility, detailed accident severity functions can be defined. On the other hand, different probability distributions for the parameters (inter-vehicle distance, velocity, driver reaction time, etc.) can be evaluated with the model, as well as different communication technologies, since the communication system has been abstracted and characterized by an appropriate message notification delay.

To finish with this research line, in the paper published in [57], we propose the use of the model as an alternative to simulation for the design and performance evaluation of CCA applications. The validity of the model for evaluating such applications is shown by comparing our results with other authors' simulation results. Then, an evaluation of different types of CCA applications in two scenarios, a freeway and an urban scenario, is provided to exemplify the use of the model at an early stage to shed relevant guidelines for the design of this kind of applications.

Moving on to the second part of this thesis, many safety-related applications in VANETs require fast and reliable emergency message dissemination through multi-hop broadcast, reaching all nodes within a certain geographical area. The delivery of these geographically-addressed messages is performed by the GeoNetworking protocol [19]. This protocol uses a forwarding mechanism to route packets through intermediate nodes until reaching the destination location (or geo-area). As basic forwarding algorithms, the standard defines Greedy Forwarding (GF) and Contention-Based Forwarding (CBF). As stated previously, CBF brings some improvements when compared to GF, like the implicit reliability mechanism it provides. However, it still has some drawbacks, such as the larger forwarding delay and additional processing. Our goal in the second part of this thesis is to investigate how cross-layer techniques allowing the exchange of information between the MAC and Network layers can help to improve the operation of GeoNetworking by optimizing the forwarding algorithm in use.

As defined by the standard, CBF is completely implemented at the network layer. However, CBF might be also implemented directly at the MAC layer, in order to optimize its operation. For instance, implementing CBF at MAC layer should result in lower latency, since forwarding delay is removed and only access delay counts. In this thesis we provide a survey and comparative evaluation of the most relevant MAC-Network cross-layer proposals in the context of vehicular networks. We focus on contention-based MAC mechanisms for wireless nodes. The majority of them are based on the CSMA/CA mechanism, whose operation and performance can be controlled by several parameters, namely: contention window size, random and deterministic carrier sense intervals as well as the probability distribution for the contention slots selection. Overall, it results in multiple degrees of freedom to optimize the medium access operation according to the most critical functionality offered by the network. We discuss both, techniques specifically addressed to vehicular networks as well as general-purpose proposals, which can be adapted to VANETs. For the evaluation, we focus on the most critical functionality, that is, the delivery of emergency messages to a particular location in multi-hop scenarios. Thereby, we intend to define a baseline scenario and a comparison as fair as possible of the performance of different proposals.

To summarize, the main contributions of this thesis are:

- The development of a stochastic model to compute the average number of collisions in a chain of vehicles where a warning collision system is in operation and its validation through Monte Carlo simulations [59].
- The use of parallelization techniques together with supercomputing resources to make the Monte Carlo simulation process a more suitable and efficient one [60, 92].
- The use of the developed stochastic model as an effective alternative to simulation for the numerical evaluation of CCA mechanisms [57].
- The illustration of the model capabilities as an assessment tool for CCA application design [57].
- The elaboration of a survey and comparative evaluation of the most relevant MAC-Network cross-layer proposals in the context of vehicular networks (Chapter 5).
- The unified formal description of the discussed techniques in terms of the form that takes both the random and/or deterministic delays of the contention mechanism, not only

qualitatively describing the operation of the contention mechanism, but also extracting a more precise mathematical description of it (Chapter 5).

- The proposal of a common framework for the analysis of the different techniques performance in the baseline scenario and its validation through simulation. Unlike other analytical models developed to this purpose, the one presented here can be used in the specific case in which each vehicle uses a different contention mechanism to access the channel (Chapter 5).

- The evaluation of the different proposals for both ideal and realistic scenarios, comparing them with the basic CBF mechanism specified by the standard (Chapter 5).

1.5.1 List of publications

The work reported in this thesis is supported by the following publications.

International Journals indexed in the Journal Citation Reports with impact factor:

- **García-Costa, C.**; Egea-López, E.; Tomás-Gabarrón, J.B.; García-Haro, J.; Haas, Z.J., "A stochastic model for chain collisions of vehicles equipped with vehicular communications", *IEEE Transactions on Intelligent Transportation Systems*, vol. 13, no. 2, pp. 503-518, June 2012. [59]

- **García-Costa, C.**; Egea-López, E.; García-Haro, J., "A Stochastic Model for Designing and Evaluation of Chain Collision Avoidance Applications", *Transportation Research Part C: Emerging Technologies*, vol. 30, pp. 126-142, May 2013. [57]

International Conference Proceedings:

- Murcia-Hernández, R.; **García-Costa, C.**; Tomás-Gabarrón, J.B.; Egea-López, E.; García Haro, J., "Parallelization of a mathematical model to evaluate a CCA application for VANETS", *Proceedings of 1st International Conference on Simulation and Modeling Methodologies, Technologies and Applications*, Noordwijkerhout, Holland, 29 July 2011. [92]

- **García-Costa, C.**; Egea-López, E.; García-Haro, J., "MAC Contention Distributions for Efficient Geo-routing in Vehicular Networks", *Proceedings of 5th International Symposium on Wireless Vehicular Communications: WIVEC 2013*, Dresden, Germany, 2-3 June 2013. [58]

- **García-Costa, C.**; Egea-López, E.; García-Haro, J., "A Stochastic Approach for Vehicle Safety Modeling in a Platoon of Vehicles Equipped with Vehicular Communications", *Proceedings of 15th International Conference on Transparent Optical Networks: ICTON 2013*, Cartagena, Spain, 23-27 June 2013. [56]

Book Chapters:

- **García-Costa, C.**; Tomás-Gabarrón, J.B.; Egea-López, E.; García-Haro, J., "Speeding Up the Evaluation of a Mathematical Model for VANETs Using OpenMP", *Simulation and Modeling Methodologies, Technologies and Applications. Advances in Intelligent Systems and Computing*, vol. 197, pp. 23-37, 2013. [60]

Spanish Journals:

- **García-Costa, C.**; Egea-López, E.; García-Haro, J., "Desarrollo de un Modelo Estocástico Para el Estudio de Accidentes en Cadena", *V Jornadas de Introducción a la Investigación de la UPCT*, pp. 105-107, Abril 2012. [55]

1.6 Methodology and schedule

Having studied Bachelor's and Master's degrees in Mathematics, the author of this thesis lacked the necessary background on the field of Telecommunications for its development. So, the first months of work was devoted to acquire this background, which would be gradually consolidated by means of constant work in this matter. To take advantage of this interdisciplinarity, the first step was to perform a literature review to identify the principal aspects of VANETs that could be analyzed and optimized by applying mathematical tools. As it could be expected, this study results in a very broad range of problems and approaches.

To be coherent with the ongoing research activities within the group, we decided to focus on the analytical modeling of chain collisions of vehicles, which was being studied by Juan

Bautista Tomás Gabarrón, who was simulating this scenario in the NCTUns 6.0/Estinet network simulator. The result of this cooperation was very satisfactory and determined the work plan for the next two years. This has been reflected in the publication of two journal articles and the presentation of other two works in international conferences.

All the experiments in this stage were led either by simulation and mathematical evaluation. For the mathematical evaluations we chose to work with Matlab, mainly because of its usability and the great support that this environment has in the scientific community for advanced mathematical evaluations. For simulation, as mentioned before, the NCTUns network simulator was used.

As we deepened into the study of road safety, another problem started to attract our attention: the promptness, efficiency and reliability of safety-related messages transmission. So, we begun to study the prioritization of relays based on the distance to the source node and the use of broadcast transmission models that do not rely on preconfigured network paths to reach the destination. Instead, each receiver participates in the next hop selection process and the forwarding decision is based on the actual position of the nodes at the time a packet is forwarded. With this idea on mind, we proposed two mechanisms to do that in an international conference paper. Then, the necessity to compare these and other existent techniques under a common framework emerged, which led to an extensive study, including a survey and a comparative evaluation. This work was sent to a relevant journal for its evaluation and possible publication.

In this stage, we also used Matlab for the mathematical evaluations. However, in this case, OMNeT++ was used as simulation tool. All the articles, as well as this thesis, have been written using the Latex environment.

To end this section, Figure 1.6 offers the reader a general vision of the main research lines addressed in this thesis, as well as the main publications accomplished during the predoctoral stage.

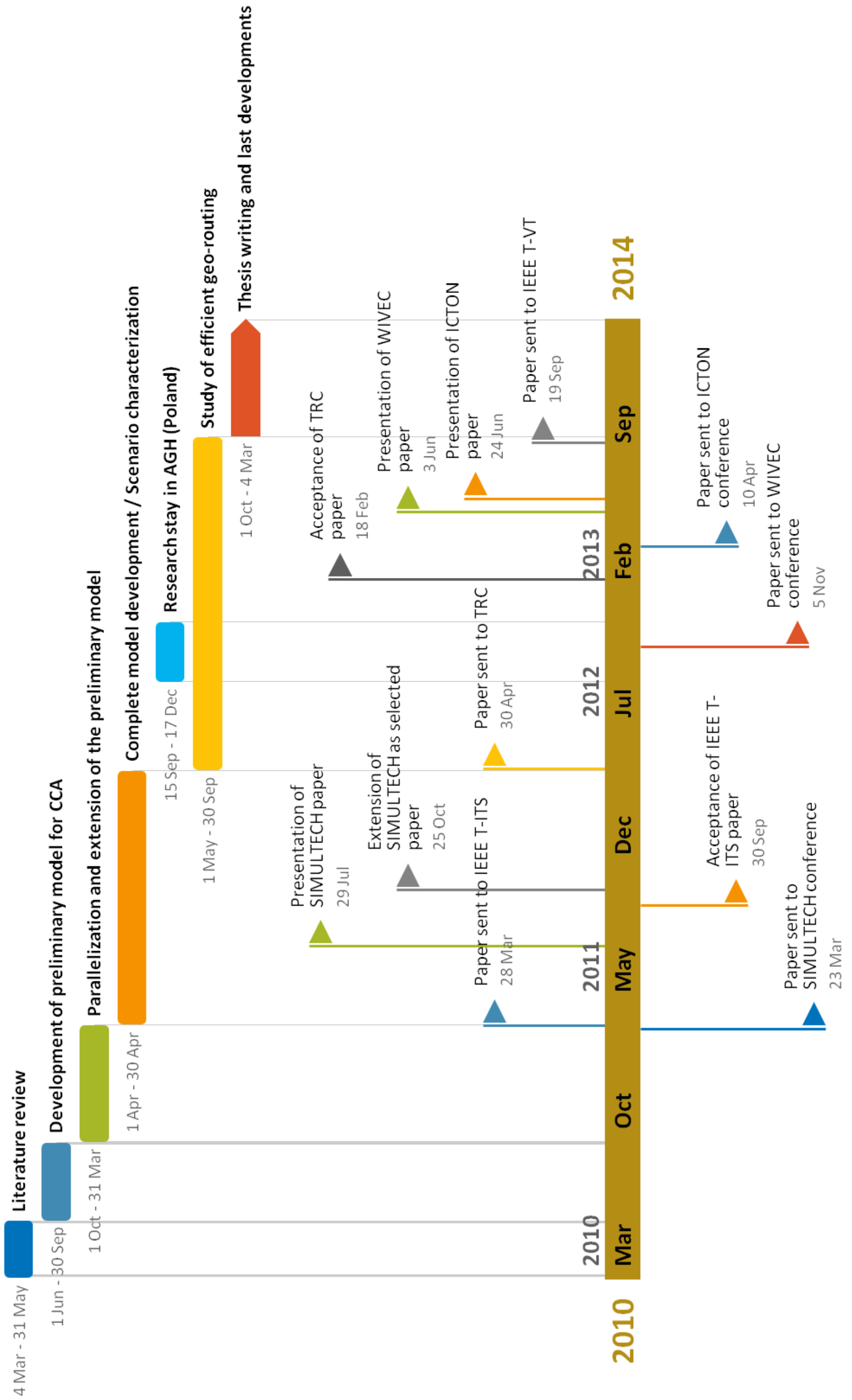


Figure 1.6: Working timeline during the predoctoral stage.

1.7 Structure of this thesis

This thesis is divided into four differentiated parts. The first part contains only the present chapter, which provides an introduction to the main objectives of this thesis as well as an overview of those aspects of VANETs that we consider essential to a complete understanding of this document. It also includes a complete list of publications (journal papers, conference proceedings, and book chapters) authored and coauthored by the Ph.D. candidate during the predoctoral stage.

Part II covers Chapters 2 and 3, focused on modeling chain collisions of vehicles. In particular, Chapter 2 thoroughly explains the process for developing a stochastic model to analyze chain collisions in a platoon of vehicles equipped with communication capabilities. This model enables the computation of the average number of collisions in the platoon as well as the probabilities of the different ways in which the collisions may occur. In addition to that, different probability distributions can be used for the main parameters of interest. Since the communication system is abstracted and characterized by an appropriate message notification delay, it allows to evaluate different communication technologies. Later in Chapter 3, we discuss the potential of the model as a numerical evaluation tool and as an alternative to simulation for the design and performance evaluation of CCA applications, specially at early stages of development. In this chapter, the suitability of the model for evaluating such applications is shown by comparing our results with other authors' simulation results. Finally, an evaluation of different types of CCA applications in two scenarios, a freeway and an urban scenario, is provided to exemplify the use of the model at an early stage to shed relevant guidelines for the design of this kind of applications, by disclosing the influence of kinematic parameters on the collision process.

In Part III we deal with the efficient geo-routing in VANETs. First, in Chapter 4, we propose two MAC contention distributions that prioritize the access to the channel based on position, while ensuring a high success probability. Moreover, these distributions are proved to scale gracefully when increasing the vehicle density. In Chapter 5, these proposals together with other approaches found in the literature are surveyed and fairly compared under a common framework.

Finally, in Part IV, Chapter 6 reports the main conclusions of this thesis and presents possible future research topics. After the general conclusions, we present two appendices regarding some additional aspects about the material presented so far. In Appendix A we describe the supporting tools for the parallelization process performed in Section 2.4, namely, the OpenMP environment and the Ben-Arabi Supercomputer arquitecture. Appendix B will describe some necessary mathematical operations to compute the distance traveled by a vehicle in case of collision, which is needed for the stochastic model developed in Chapter 2.

Part II

Modeling chain collisions of vehicles

2 A stochastic model for chain collisions of vehicles equipped with vehicular communications

2.1 Introduction

Improvement of traffic safety by cooperative vehicular applications, such as CCA, is one of the most promising technical and social benefits of VANETs [71, 108]. However, in order to design and implement such applications, a deep understanding of the vehicle collision processes is needed. The influence of the different driving parameters on the collision event must be assessed at an early design stage in order to develop applications that can timely adapt vehicle dynamics to avoid or at least mitigate the danger [117].

It should be noted that the establishment of a CCA application will be deployed gradually, equipping vehicles with the proper hardware and software so as they can communicate in an effective way within the vehicular environment. Therefore, it is highly convenient to study how the system of vehicles in a platoon will behave at different stages of technology deployment until full penetration in the market. We have developed as a first approach a mathematical model to calculate the average percentage of accidents in the platoon, varying the number of considered vehicles, their average speed, the average inter-vehicle spacing and the penetration ratio of the CCA technology.

Specifically when the CCA penetration ratio is taken into account, the growth in the number of operations of the analytical model is such that the sequential computation of a numerical solution is no longer feasible. Consequently, we resort to the use of the OpenMP parallelization techniques for solving those computational cases considered as unapproachable by means of sequential procedures. Additionally, we execute our programs in the Ben-Arabi Supercomputing environment [6], taking the advantage of utilizing the fourth fastest Supercomputer in Spain. We show how the parallelization techniques coordinated with supercomputing resources make the simulation process a more suitable and efficient one, allowing a thorough evaluation of the CCA application.

Then we have extended the stochastic model in order to represent the collision process in a more realistic way. Very detailed models of vehicle motion and collision dynamics can be found [65, 82], but the equations are completely deterministic, whereas, in reality, randomness is always present as an effect of human behaviour or noisy operation introduced by sensors or other reasons. To account for it, the usual methodology is to evaluate deterministic models by applying a Monte-Carlo or stochastic analysis over an extensive range of their parameters [63, 82, 114]. However, little effort has been devoted to develop models which are stochastic in nature, and in particular for rear-end chain collisions of vehicles. Some reasons behind it are the difficulties of evaluating all the possible ways in which a collision may occur and the complexity posed by the fact that the motion equations for those possibilities involve a dependence on the parameters of preceding vehicles. That is, the driver reacts to variations in the driving conditions of the preceding vehicle, as in a car-following approach [37, 115]. However, if vehicles use a communication system which is able to inform all the vehicles about an emergency event, those difficulties can be overcome. The key is that, in that case, it can be assumed that drivers react as soon as they receive a warning message and they start braking independently of the preceding vehicles behavior. This is in fact the goal of warning collision systems or Electronic Brake Warning (EBW) applications. This assumption removes the dependence of the motion equations on the preceding vehicles and facilitates the development of a stochastic model.

Here we take this approach. Our goal is to describe and analyze the risk of colliding for a set of moving vehicles equipped with a warning collision system when there is a sudden stop of the leading vehicle. The scenario under consideration is basically a platoon of vehicles moving along a unidimensional road in the same direction in which the leading vehicle suddenly comes to a complete stop. To consider a worst case scenario we add two strong assumptions: first, the leading vehicle stops instantly (it may also be considered that a fixed obstacle lays on the road). Second, vehicles will not be able to change their direction of movement to cope with the unexpected incident.

Our model is stochastic because all its parameters may be described by random variables. We derive the equations assuming always a random inter-vehicle spacing, in particular for an exponentially distributed spacing, though the model is valid for other distributions. When additional parameters are assumed random, the solutions have been computed numerically. Additionally, it should be observed that the model is independent of the communication technology, since the operation of the communication system is abstracted by the use of a message reception/notification delay variable. Finally, the probabilities for all the ways the collision may take place are also derived, which can be further used to evaluate the severity of accidents in higher detail, for instance, by assigning different severity weights to different types of collision. A deeper discussion on this topic is given in Chapter 3.

The main practical utility of this model lays in its ability to quickly evaluate numerically the influence of the different parameters on the collision process, without the need to resort to complex simulations in a first stage. Such an evaluation provides relevant guidelines for the design of vehicular communication systems as well as Chain Collision Avoidance (CCA) applications. As an example, it can quickly reveal for which range and distributions of the parameters the communication delay has a serious impact on the metric of interest, which can be the average number of accidents but also the probability of collision of every vehicle in the chain. Since it turns out that in some scenarios a low delay is not relevant for the outcome, a communication system could trade it off for additional reliability mechanisms. Moreover, in this chapter we set either constant or purely random parameters, but the model can be used with arbitrary parameters to evaluate more specific applications. For instance, to evaluate multi-hop communications we can set up a vector of delays with progressively increasing values. We provide examples of use in Section 2.5.3, but in any case, a careful characterization of the model parameters for the scenarios and applications is a necessary previous step.

The remainder of this chapter is organized as follows. In Section 2.2 we briefly review the related work. The derivation of the model is provided in Section 2.3. The particular characteristics of the first approach model and its parallelization are described in 2.4, while the extended model and its validation are provided in 2.5. Conclusions and future work are remarked in Section 2.6, while the necessary auxiliary material is supplied in the Appendices A and B.

2.2 Related work

Our model assumes that there is a communication system between vehicles that allows them to receive warning messages to start braking in the event of a sudden stop of the leading car. However, such a system is abstracted in the model and characterized by the use of a message reception/notification delay variable. Therefore, our model is actually independent of it and can be applied to any communication system whose operation can be abstracted by an appropriate delay variable. For instance, current VANET standards specify the use of IEEE 802.11p which is based on contention (Carrier Sense Multiple Access (CSMA)) Medium Access Control (MAC) [67]. Such a shared channel MAC technique can be abstracted in our model by a delay random variable with an appropriate probability distribution [129]. Further details on current VANET communication technologies can be found in [67].

Regarding collision models for chains of vehicles two different groups of studies can be found: 1) statistical models of the frequency of accidents occurrence and their circumstances [25, 104]; and 2) models of the collision process itself based on physical parameters [63, 82, 114]. This paper falls on the latter category and additionally assumes that an automated warning system is in place. In most of these studies, deterministic equations for the occurrence of collisions are derived and, to account for random variability, stochastic analysis or Monte-Carlo simulations over a wide range of model parameters are performed afterward to obtain an estimate of the collision probability or other metrics of interest. Our approach is different and the model shown here is directly stochastic and assumes that at least the inter-vehicle distance is a random variable, which is, in fact, a realistic assumption, as shown in [122]. We also perform Monte-Carlo simulations but, unlike the previously mentioned papers, we use them to validate our model rather than to obtain metrics of interest. Looking into these works in particular, in an early study, Glimm and Fenton [63] defined an accident cost function to evaluate the severity of vehicle collisions. The collision model used is derived for an automatically controlled[1] platoon of vehicles which advance at constant speed with a constant inter-vehicle spacing. A more recent work [114] provides a similar collision model for a four-car platoon of vehicles assuming that just one of the vehicles is equipped with an autonomous intelligent cruise control. In both cases, the collision model defines how vehicles decelerate in order to obtain a deterministic equation for the collision. Afterwards, the evaluation is done by randomizing some parameters of the model and running a Monte-Carlo simulation. In [37], authors derive necessary conditions for a chain collision, starting from a car-following model. However, they assume that all the vehicles are driving with equal initial speeds and inter-vehicle distances.

Interestingly, the proposed vehicle collision model is more general, it explicitly accounts for random inter-vehicle spacing, and can be used to assign arbitrary variables, even random

[1]Let us note that early research, which goes back to the 1960s, considered the hypothesis of achieving “automated highway systems”, where most of the driving tasks were automatically controlled.

ones, to the kinematic parameters of each vehicle as well as the warning message communication delay. Moreover, there are additional applications of our model, for instance, it can be readily used to evaluate the severity of collisions, as in [63]: since we compute the probability of collisions occurring in several manners, we could assign a severity weight to each possibility, that is, we may assign more severity to a collision when both vehicles are in motion than to other cases, for example. On the other hand, some of the results in [63] are similar to ours, for instance the sensitivity shown to the decrease in deceleration capabilities of the subsequent vehicles. In all the cases, as well as in our model, only rear-end collisions are considered. Head-on collisions are evaluated in [82], based on a very detailed analytical model of the vehicle.

Finally, in this chapter we provide examples about the kind of results that can be drawn from the proposed model which are useful for the design of CCA applications. A review on intelligent collision avoidance algorithms can be found in [117]. In particular, the influence of delay notification on different scenarios is useful to set appropriate time horizons for CCA systems based on trajectory prediction [108].

Regarding the parallelization of our stochastic model, most typical High Performance Computing (HPC) problems focus on those fields related with certain fundamental problems in several areas of science and engineering. Other typical applications are the ones related to commerce, like databases and data mining [29]. That is the reason why we consider our VANET mathematical model approximation as a non-classical issue to be solved under HPC conditions, contributing to extend the use of supercomputing to other fields of interest.

In the implementation of our mathematical model we parallelize a sparse matrix-vector multiplication. This operation is considered as a relevant computational kernel in scientific applications, which performs not optimally on modern processors because of the lack of compromise between memory and computing power and irregular memory access patterns [86]. In general, we find quite a lot of work done in the field of sparse matrix-vector multiplications using parallelization techniques [64, 83, 119]. These works study in depth the optimal performance of this operation, but in this chapter, we show that even using a simpler parallelization routine, the computation time is noticeably shortened.

Several mathematical models have been developed to study different aspects of VANETs. Most of them are related with the vehicle routing optimization [93, 123], the broadcasting methods [45, 50, 85], the mobility of vehicles [44, 66] and the communication delay time [24, 53, 95]. Other related VANET issues have been studied as well, like network connectivity

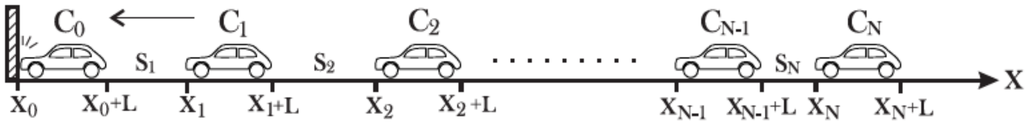


Figure 2.1: The scenario under consideration.

[80], or survivability [125]. As mentioned previously, in this thesis we focus on collision models for a chain of vehicles, particularly those based on physical parameters to assess the collision process itself [63, 82, 114].

However in an attempt for searching related work we find that few work has been done specifically regarding to the parallelization of these VANET mathematical models, strictly speaking. Moreover, to the best of our knowledge, only the vehicle routing problem has been approached using parallelization techniques [34, 43, 62].

2.3 Collision model

We consider a platoon (or chain) of $N+1$ vehicles driving in convoy (see Fig. 2.1), where each vehicle C_i, $i=0,\ldots,N$, moves at constant velocity V_i. The leading vehicle, C_0, faces an emergency situation at time $t_0=0$, and immediately brakes at a high deceleration rate and sends a warning message to the following vehicles. The remaining vehicles start to brake at constant deceleration[1] a_i when they are aware of the risk of collision, that is, after a time lapse δ_i. Let us note that $\delta_i = T_{r,i} + T_{m,i}$ includes both a reaction time and a message reception time respectively, and so it allows to evaluate both contributions separately. Let us remark here that in the presence of communications the reaction of the driver is independent of the movement state of the preceding vehicle. That is, a warned driver will decelerate even if the preceding car has not started to decelerate. In a classical car-following approach, on the contrary, the deceleration would be a consequence of a change in the inter-vehicle spacing or the speed of the preceding vehicle.

For the sake of simplicity, we assume that every vehicle has the same length L and its position is given by the x coordinate of its front bumper. The leading vehicle stops at coordinate $x_0=0$ and the initial inter-vehicle spacing is $s_i = x_i - (x_{i-1}+L)$. To test the worst case situation, vehicles cannot change lane or perform evasive maneuvers. This is a worst-case assumption, commonly used in the literature [36, 42], that leads to upper bounds in the results.

[1] To simplify the notation, in the remaining of the thesis we consider a_i a *deceleration*, and so assign it a positive sign.

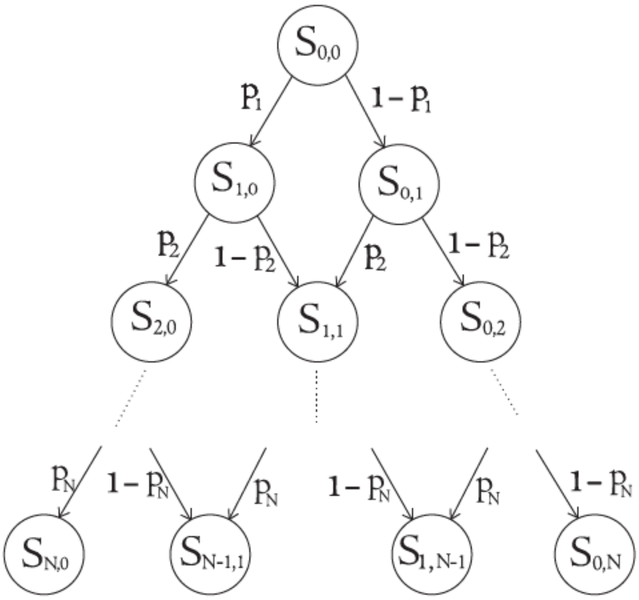
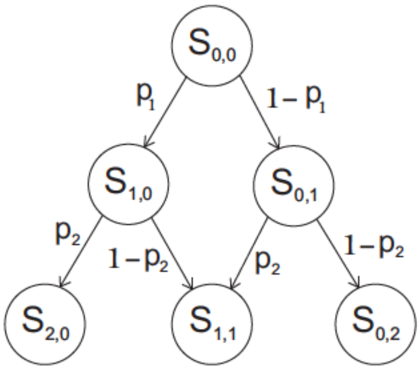


Figure 2.2: Probability tree diagram that defines the model. $S_{i,j}$ represents the state with i collided vehicles and j successfully stopped vehicles.

The model needs five inputs: the number of vehicles, the distribution of the inter-vehicle spacing and vectors for the speeds, delays and decelerations. The first three ones define the initial state whereas the last two are usually controllable. That is, we can imagine that at the time instant of the emergency event we take a snapshot of the system. From this snapshot we extract the speed of each vehicle and the distance between two consecutive vehicles. Therefore, the initial state of the system is defined by the speeds $\{V_i\}_{i=0,\dots,N}$ and inter-vehicle spaces $\{s_i\}_{i=1,\dots,N}$, which will be called *state variables*. On the other hand, the delays before braking $\{\delta_i\}_{i=0,\dots,N}$ and the deceleration rates $\{a_i\}_{i=0,\dots,N}$ will depend on the decisions made by the drivers after the time instant of the emergency event, which may be influenced by a CCA application and will be called *control variables*. We assume that at least inter-vehicle distance is a random variable, but the remaining variables can be considered random or assigned constant values, as it is discussed in the following sections.

With this model the final outcome of a vehicle depends on the outcome of the preceding vehicles. Therefore, the collision model is based on the construction of the probability tree depicted in Fig. 2.2. We consider an initial state in which no vehicle has collided. Once the danger of collision has been detected, the first vehicle in the chain C_1 (immediately after the leading one) may collide or stop successfully. From both of these states two possible



$$
\mathrm{P} = \begin{pmatrix} 0 & p_1 & 1-p_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_2 & 1\ p_2 & 0 \\ 0 & 0 & 0 & 0 & p_2 & 1\ p_2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}
$$

Figure 2.3: Probability tree and transition matrix for a chain with $N = 2$ vehicles.

cases spring as well, that is either the following vehicle in the chain C_2 may collide or stop successfully. And so on until the last vehicle in the chain, denoted by C_N. At the last level of the probability tree there are $N + 1$ possible outcomes (final outcomes) which represent the number of collided vehicles, that is, from 0 to N possible collisions. Observe that $S_{i,j}$ represents the state with i collided vehicles and j successfully stopped vehicles.

The transition probability between the nodes of the tree is the probability of collision of the corresponding vehicle in the chain p_i (or its complementary). These probabilities will be calculated recursively, as described in the following sections, being this computation the main contribution of our model. Let us note how every path in the tree from the root to the leaves leads to a possible outcome involving every vehicle in the chain. The probability of a particular path results from the product of the transition probabilities that belong to the path. Since there are multiple paths that may lead to the same final outcome (a particular leaf node in the tree), the probability of that outcome will be the sum of the resulting probabilities of every possible path reaching it.

In order to compute the probabilities of the final outcomes, we can construct a Markov chain whose state diagram is based on the previously discussed probability tree. It is a homogeneous Markov chain with states:

$$(S_{0,0}, S_{1,0}, S_{0,1}, \ldots, S_{N,0}, S_{N-1,1}, \ldots, S_{1,N-1}, S_{0,N}). \tag{2.1}$$

The transition matrix $\mathbf{P}$ of the resulting Markov chain is a square matrix of dimension $(N + 1)(N + 2)/2$, which is a sparse matrix, since from each state it is only possible to move to two of the other states. For the sake of clarity, a brief example with 2 vehicles is illustrated in Fig. 2.3.

Then, we need to compute the probabilities of going from the initial state to each of the $N+1$ final states in N steps, which are given by $\mathbf{P}^N$. Therefore, the final outcome probabilities are the last $N+1$ entries of the first row of the matrix $\mathbf{P}^N$. Let Π_i be the probability of reaching the final outcome with i collided vehicles, that is, state $S_{i,N-i}$. Therefore,

$$\Pi_i = \mathbf{P}^N \left(1, \frac{(N+1)(N+2)}{2} - i\right). \tag{2.2}$$

Finally, we obtain the average of the total number of accidents in the chain using the weighted sum:

$$N_{acc} = \sum_{i=0}^{N} i \cdot \Pi_i. \tag{2.3}$$

2.4 Preliminary model

In this section we describe the preliminary approximation for the computation of the collision probabilities, which does not describe realistically enough the collision process. However, the method to compute the probabilities of the path outcomes is independent of the correctness or accuracy of the transition probabilities used, and the goal of this section is to evaluate the benefits of parallelization for this technique to compute the average number of accidents. An improved model for the transition probabilities will be explained in Section 2.5.

2.4.1 Computation of the vehicle collision probabilities

In this first approach we consider the inter-vehicle spacing is normally distributed and vehicles react to the first collision of the leading vehicle according to two possible schemes: starting to brake because of a previously received warning message transmitted by a collided vehicle (if the vehicle is equipped with CCA technology) or starting to decelerate after noticing a reduction in the speed of the vehicle immediately ahead (if the vehicle under consideration is not equipped with CCA technology).

As we said in the previous section, the transition probability between the nodes of the tree is the probability of collision of the corresponding vehicle in the chain p_i (or its complementary). These probabilities are calculated recursively, as a function of different kinematic parameters, such as the average velocity of the vehicles in the chain (used to compute the distance to stop), the average inter-vehicle distance and the driver's reaction time, among others.

We start calculating the collision probability of the nearest to the incidence vehicle, C_1. The position of C_i when it starts to decelerate is normally distributed with mean $\mu_i = i \cdot s$ and standard deviation $\sigma = s/2$, where s is the average inter-vehicle distance. Vehicle C_1 will collide if and only if the distance to C_0 is less than the distance that it needs to stop, $d_{s,1}$, so its collision probability is given by:

$$p_1 = 1 - \int_{L+d_{s,1}}^{+\infty} f(x;\mu_1,\sigma)\,dx, \tag{2.4}$$

where L is the average vehicle length and $f(x;\mu,\sigma)$ is the probability density function of the normal distribution with mean μ and standard deviation σ.

Considering constant deceleration a_i and velocity V_i, and a delay before braking δ_i, the distance needed by vehicle C_i to completely stop without colliding is given by

$$d_{s,i} = \frac{V_i^2}{2a_i} + V_i\delta_i. \tag{2.5}$$

To compute the collision probability of the second vehicle we will use the average position of the first vehicle when it has stopped (either by collision or successfully stop). This average position is determined by:

$$\overline{X_1} = \int_{L}^{+\infty} x \cdot f(x;\mu_1 + d_{s,1},\sigma)\,dx \;+\; L \cdot \int_{-\infty}^{L} f(x;\mu_1 + d_{s,1},\sigma)\,dx. \tag{2.6}$$

The second term of the sum means that the vehicle cannot cross the position L when it collides, since we are assuming that when a vehicle collides it stops instantly at the point of collision.

Once we have obtained $\overline{X_1}$ we can compute p_2, and recursively we can obtain all the collision probabilities:

$$p_i = 1 - \int_{\overline{X_{i-1}}+L+d_{s,i}}^{+\infty} f(x;\mu_i,\sigma)\,dx, \quad i = 2,\ldots,N, \tag{2.7}$$

where

$$\overline{X_i} = \int_{\overline{X_{i-1}}+L}^{+\infty} x \cdot f(x;\mu_i + d_{s,i},\sigma)\,dx \;+\; (\overline{X_{i-1}} + L) \cdot \int_{-\infty}^{\overline{X_{i-1}}+L} f(x;\mu_i + d_{s,i},\sigma)\,dx, \quad i = 2,\ldots,N. \tag{2.8}$$

Table 2.1: Number of combinations of $N = \{10, 20, 30\}$ vehicles with and without CCA technology.

CCA%	10 veh.	20 veh.	30 veh.
0%	1	1	1
10%	10	190	4060
20%	45	4845	593775
30%	120	38760	14307150
40%	210	125970	86493225
50%	252	184756	155117520

2.4.2 Parallelization of the preliminary model

Our purpose is to evaluate the functionality of the CCA system depending on the actual penetration rate of this technology. So that, we have to solve the model assuming different technology penetration ratios. This assumption implies that we have to calculate the number of collisions once for each of the possible combinations in the chain of vehicles equipped with and without CCA technology, that is,

$$\binom{N}{m} = \frac{N!}{(N-m)!\, m!}, \tag{2.9}$$

where N is the total number of vehicles in the chain and m is the number of vehicles equipped with the CCA technology. It is worth noting that the number of combinations for m vehicles set with CCA technology and $N-m$ without it is the same that for $N-m$ vehicles with CCA and m without it. Therefore, in order to analyze the computation time, we solve the model varying the CCA penetration rate between 0% and 50%, since the rest of cases are computationally (but not numerically) identical. As we can see in Table 2.1, the number of combinations grows quickly by an increase on the CCA penetration rate as well as by an increase on the number of vehicles.

In addition to that, we also aim at evaluating the impact on the number of accidents of the inter-vehicle distance s, varying this parameter in a wide range.

Here we present the algorithm for the model implementation (Algorithm 1) and then, we explain the method we have used to parallelize it.

Examining the algorithm we can make the following observations:

1. The iterations of the *for* loop that covers the number of *Combinations* resulting from the CCA technology penetration rate are independent for each other, so they can be executed in parallel by different threads.

Algorithm 1 Computation of the number of collisions in a chain of vehicles

for all $comb$ in $Combinations$ **do**
  for all s in $RangeOfDistances$ **do**
    for $i = 1$ **to** N **do**
      $p_i = f(comb, \overline{X_{i-1}}, s, V_i, a_i, \delta_i)$
    end for
    for $j = 0$ **to** N **do**
      $\Pi_j = \mathbf{P}^N(1, \frac{(N+1)(N+2)}{2} - j)$
    end for
    $N_{acc} = \sum_{j=0}^{N} j \cdot \Pi_j$
  end for
end for

2. The same occurs with the *for* loop that covers the *RangeOfDistances* (for the inter-vehicle spacing) to be evaluated.

3. Since the collision probabilities of the vehicles in the platoon is computed recursively, each iteration of the *for* loop that considers each vehicle in the chain needs the results of the preceding iteration, so this loop should be executed sequentially.

4. To obtain the first row of matrix $\mathbf{P}^N$ we have to multiply N times a vector of dimension $(N+1)(N+2)/2$ by a matrix of dimension $(N+1)(N+2)/2 \times (N+1)(N+2)/2$. The vector-matrix multiplication can be also parallelized so that each thread executes the multiplication of the vector by some of the matrix columns. However, the N multiplications should be done one after the other, that is, sequentially.

For the sake of clarity, we will parallelize the following tasks:

- A: Vector-Matrix multiplication.
- B: Average inter-vehicle distance variation.
- C: Technology penetration rate variation.

Next, we will combine the different parallelized tasks (see Table 2.2) and execute the resulting programs in order to assess the actual improvement obtained from each one. In Appendix A we describe the supporting tools for this parallelization process, namely, the OpenMP environment and the Ben-Arabi Supercomputer arquitecture.

Table 2.2: Resulting programs with different parallelized tasks. X means that the corresponding parallelization takes place.

Program	A	B	C
Program 1			
Program 2	$\times$		
Program 3		$\times$	
Program 4			$\times$
Program 5	$\times$	$\times$	
Program 6	$\times$		$\times$
Program 7		$\times$	$\times$
Program 8	$\times$	$\times$	$\times$

2.4.3 Results

In this subsection we summarize the results obtained by executing the programs shown in Table 2.2 in a node of the Arabi cluster. We have used 2, 4 and 8 processors in order to assess the improvement on the execution time achieved by each one.

The parameters used to execute the model are as follows:

- CCA penetration rate: 0% - 50%, in 10% steps.
- Average inter-vehicle distance: 6 - 70 m, in 1 meter steps.
- Number of vehicles: 20 vehicles.
- Velocity: 33 m/s.
- Deceleration: 8 m/s^2.
- Driver's reaction time: 1 s.

The computation times resulting from the execution of the eight programs with the selected penetration rates of CCA technology using 2, 4 and 8 processors are illustrated in Figures 2.4, 2.5 and 2.6, respectively.

Now we focus on the results associated to the 50% CCA penetration rate, since for this value we obtain the highest number of combinations, specifically for a chain of 20 vehicles we obtain a total of 184756 combinations. Therefore, it is for this particular penetration rate when we obtain a higher execution time and it can be considered as the critical case in terms of the solving time.

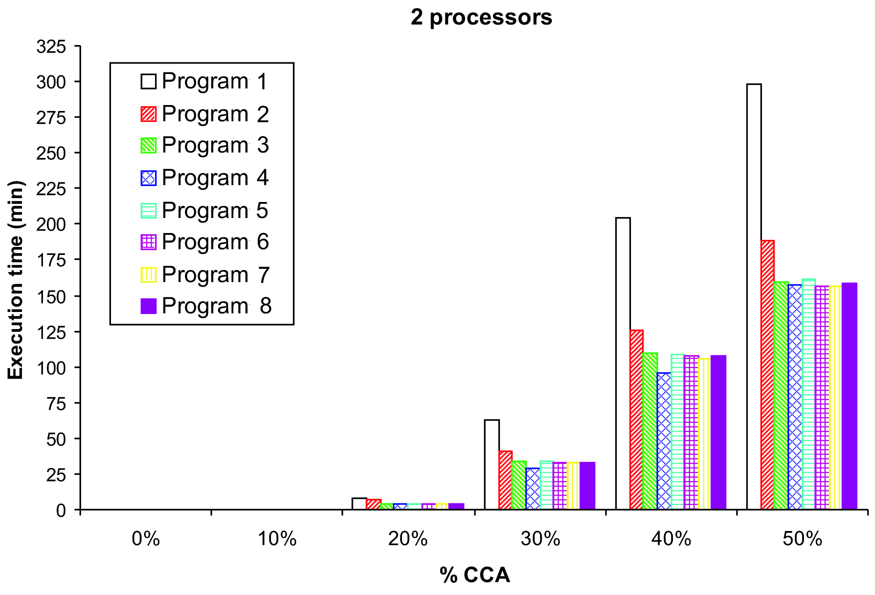


Figure 2.4: Execution times in minutes for each program using 2 processors.

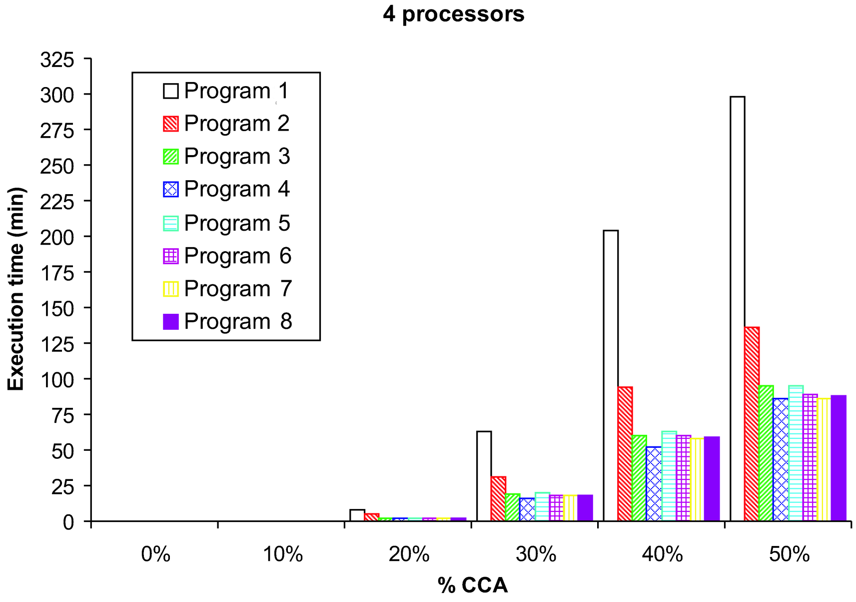


Figure 2.5: Execution times in minutes for each program using 4 processors.

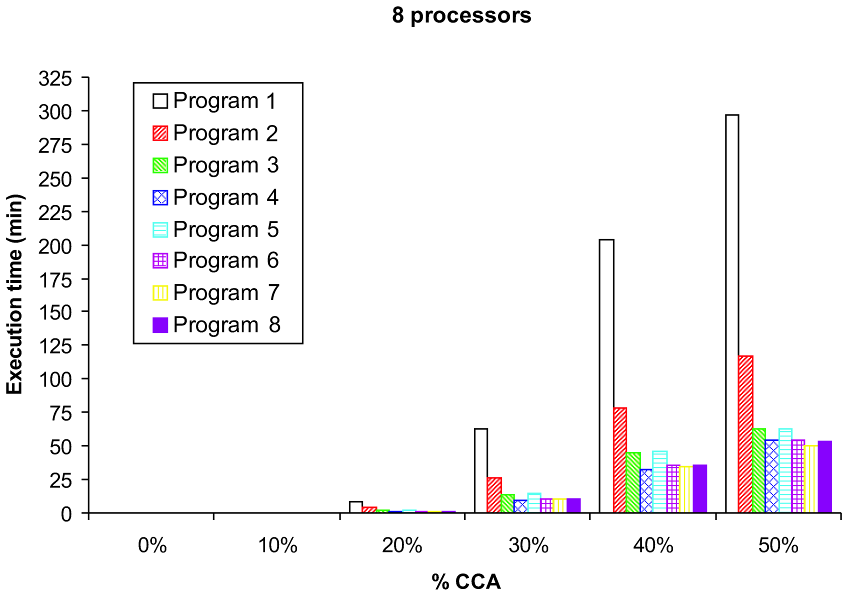


Figure 2.6: Execution times in minutes for each program using 8 processors.

The sequential program (Program 1) lasts for a total of 297.975 minutes, that is approximately 5 hours of computation. If we make a comparison among the parallelized programs we can draw the following conclusions:

- With **2 processors**, the best result is given by the Program 7, with a computation time of 156.433 minutes, what implies around 2.6 hours of calculation time. The achieved speedup[1] is 1.9, which implies an improvement of around 47.5% referred to the execution time.

- With **4 processors**, the best result is given again by the Program 7, with a calculation time of 85.988 minutes (around 1.43 hours). In this case the achieved speedup is 3.46, which implies an improvement of around 71.1% referred to the execution time.

- With **8 processors**, once more, we obtain for the parallelized Program 7 the least computation time, 50.402 minutes with a 50% CCA penetration rate. So if we compare this

[1]In parallel computing, speedup refers to how much a parallel algorithm is faster than a corresponding sequential algorithm. It is computed dividing the execution time of the sequential algorithm by the time of the parallel one.

outcome with the execution time of the sequential program we obtain an improvement of the 83%, that is, a speedup factor of 5.89.

In conclusion, on the one hand, we have achieved an improvement of 83% in the computation time of the most complex case, what can be considered as a pretty much outstanding improvement. On the other hand, if we compare the best execution times between the two technical extremes under study, that is the use of 2 or 8 processors belonging to the shared nodes architecture in the Arabi cluster, we reach to an improvement of 67.78%, which implies an upwards trend with increasing the number of processors, as expected. Moreover, we can observe that those programs including the parallelization of task C, which implies an acceleration on the loop varying the CCA technology penetration rate, are the fastest ones. Nevertheless, the results obtained from Program 2 show that the improvement achieved parallelizing only the vector-matrix multiplication (task A) is already significant, reaching 60.4% using 8 processors.

Analyzing the speedup for programs 7 and 8 it surprises that P7, with two parallelized tasks, wins P8 including one more parallelized task. But this is a common fact in parallel computing due to load balancing and synchronization overhead [10]. This explains also that all programs including parallelized task C have similar execution times, since this is the heaviest computational task and outshines the improvement derived from the A and B tasks' parallelization.

Let us compare now the obtained results for the Program 7, the one with the best execution times, centering on the 50% CCA penetration rate, since as we already mentioned, this is the heaviest option in terms of computational load. We find out an inverse relationship between computation time and the number of processors in use, since when we duplicate the number of processors the execution time of Program 7 is reduced almost to a half. Specifically, the speedup achieved passing from 2 to 4 processors is 1.82, and from 4 to 8 processors, 1.7. However, this speedup is limited according to Amdahl's law [26]. We have calculated for each program the theoretical speedup obtained from this law, as depicted in Figure 2.7.

Amdahl's law states that if α is the proportion of a program that can be made parallel then the maximum speedup, SU, that can be achieved by using n processors is:

$$SU = \frac{1}{(1-\alpha) + \frac{\alpha}{n}}. \tag{2.10}$$

We can estimate α by using the measured speedup SU on a specific number of processors sn as follows:

$$\alpha_{estimated} = \frac{\frac{1}{SU} - 1}{\frac{1}{sn} - 1}. \tag{2.11}$$

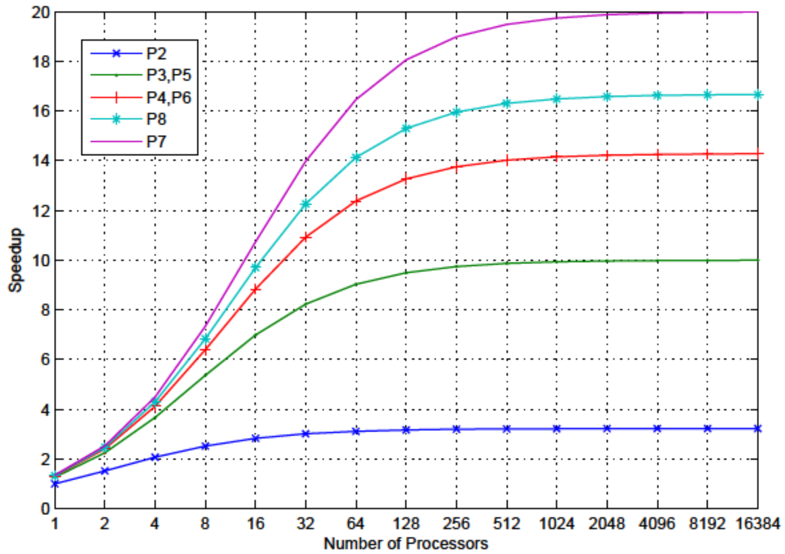


Figure 2.7: Theoretical speedup limits calculated from Amdahl's law.

The results show that for P2 the speedup obtained with 8 processors is almost the limit for it, but the speedup for P7 can still grow up to 20, which implies reducing the execution time to less than 15 minutes.

Unfortunately, we have not been able to check how the results of Amdahl's law approach to reality. We tried to execute the Program 7 in the Superdome Ben, but executing it using 32 cores the time consumed was much higher than using 2 cores in a node of the cluster. It is owing to the computing speed (819 Gflops in the Superdome and 9.72 Tflops in the cluster).

As an alternative, we tried using MPI (Message Passing Interface Standard) [7] in order to execute our programs using different nodes of the cluster simultaneously. However, we encountered the problem of an excessive memory requirement, due to the need to replicate data across processes, and consequently we failed in the execution of the programs by this way too.

2.5 Complete model

In this section we explain the extended model, in which the collision process is described in more detail. Here we assume that all the vehicles in the platoon are equipped with vehicular communications, since this assumption removes the dependence of motion equations on the

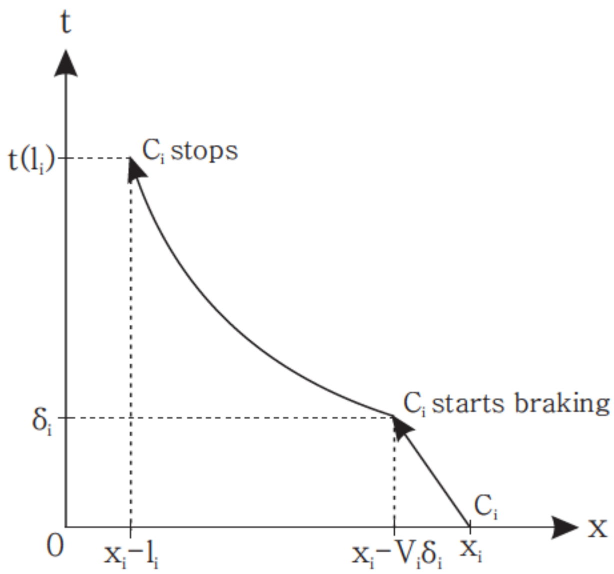


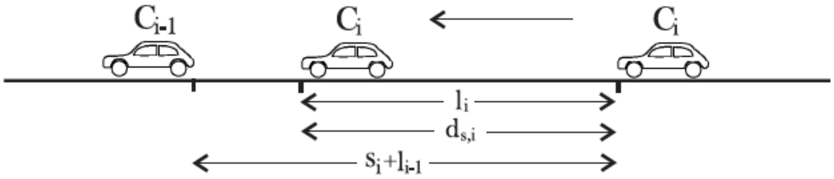
Figure 2.8: Parameters of the kinematic model used to compute the vehicle collision probabilities.

preceding vehicles and facilitates the development of a stochastic model.

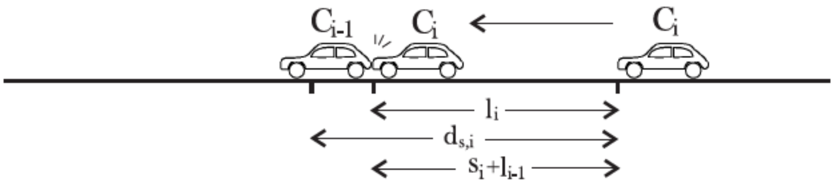
We start from a deterministic kinematic model and compute the collision probabilities when different parameters of this model are considered variables. The results are validated by Monte-Carlo simulations. Hence, we start from a basic kinematic collision model provided by [107], that can be summarized as follows.

Let l_i represent the *total distance traveled* by vehicle C_i since the emergency event occurs at time instant $t_0 = 0$ until the vehicle completely stops or collides with C_{i-1}. Let δ_i be the time lapse that goes between the detection of the emergency event until vehicle C_i actually begins to slow down. We call δ_i the *notification delay* which models the delay between the time instant $t_0 = 0$ and the instant the driver of vehicle C_i is aware of it and starts to brake. These parameters are depicted in Fig. 2.8. The notification delay plays an important role if we consider a communication system in operation between the vehicles. In this case, we can assume that the driver starts to brake when it receives a warning message, so if the emergency event occurs at $t_0 = 0$ the warning message is received at $t = \delta_i$ by the vehicle C_i. However, we assume a more realistic case in which there is also a reaction time before the driver actually starts to brake. Therefore $\delta_i = T_{m,i} + T_{r,i}$, where $T_{m,i}$ is the message reception delay and $T_{r,i}$ is the driver reaction time.

If vehicle C_i does not collide, the distance it needs to completely stop is given by equation (2.5). However, when a collision occurs, the actual distance traveled by the car, $d_{c,i}$, is not given



(a) Vehicle C_i is able to stop successfully, then $l_i = d_{s,i}$.



(b) Vehicle C_i collides with C_{i-1}. In this case, the actual distance covered by C_i up to the collision is shorter than $d_{s,i}$ as given by (2.5). Now it is $l_i = s_i + l_{i-1}$ and depends on the distance covered by C_{i-1}.

Figure 2.9: The distance l_i traveled by a vehicle when there is a collision (b) is shorter than the distance needed by it to stop successfully (a), $d_{s,i}$.

by (2.5) anymore, but one has to consider the way the collision has occurred. For example, if a vehicle crashes, its actual distance to stop is obviously shorter than $d_{s,i}$, as illustrated in Fig. 2.9, and also different when both vehicles are still in motion when the crash occurs.

Let us remark at this point that (2.5) implies that a communication system is in place and all vehicles start to brake when they receive the message, *independently of the behavior of the preceding vehicles*. Otherwise, drivers would start to brake only when they sensed the braking of its nearest forward neighbor as in a car-following approach [37, 115], so (2.5) would become a function of the parameters of the preceding vehicle, that is, $d_{s,i} = f(V_i, V_{i-1}, a_i, a_{i-1}, \delta_i, \delta_{i-1})$ and the problem would become more complex.

In all the cases the probability of collision of vehicle C_i depends on the relationship between its distance to stop $d_{s,i}$, the total distance traveled by the preceding vehicle, l_{i-1}, and the initial inter-vehicle space s_i. That is, when $d_{s,i} < l_{i-1} + s_i$ the vehicle is able to stop without colliding.

At this point we also assume another simplification: if two vehicles collide we consider that they instantly stop at the point of collision. This way we keep on assuming a worst case evaluation. There are more realistic approaches, for instance, to take into account the conservation of the linear moments to compute the displacement due to the crash [65].

As can be seen from the previous equation, the number of collisions depends on the vector of velocities V_i, decelerations a_i, notification delays δ_i, and inter-vehicle distances s_i, which we refer to as *kinematic parameters*. When all the parameters are given, the model is completely deterministic. However, we are interested in a more realistic case involving random variability of the parameters. To study the influence of the different parameters on collisions we introduce variability on different model parameters as follows: for all the cases we consider that s_i is an exponentially distributed random variable with parameter λ. This parameter represents the density of vehicles on the road, defined as the average number of vehicles per meter. Let us remark that s_i can adopt a different distribution and the following model is still valid. The reason for this is that since s_i is the inter-vehicle spacing *when* the emergency event occurs, we can consider it *independent of the rest of parameters of the model*, which means that the following equations would be essentially the same, but substituting the exponential probability density function by the corresponding new one. We have selected an exponential distribution because it simplifies the computations and it has been shown that describes well inter-vehicle spacing when traffic densities are small [122], whereas high traffic densities show log-normal distributions [122].

Once we have described our collision model, we next derive a basic model for the vehicle collision probabilities in which all the parameters are constant except for the inter-vehicle distance. Then, we extend the model by considering variable the rest of the kinematic parameters. This way we can evaluate the effects of the different parameters on the vehicle collision model.

2.5.1 Constant kinematic parameters

Our first step is to evaluate the basic model, considering all the parameters constant, except for s_i, which is assumed exponentially distributed. If a vehicle is able to stop without colliding and the kinematic parameters are constant it always travels the same distance d_s. But if there is a collision, a vehicle only travels the initial inter-vehicle distance plus the distance traveled by the preceding vehicle until it collides. Therefore, we have to compute the collision probability conditioned on the distance traveled by the previous vehicle. In the following subsections we first compute this probability exactly and then we provide an approximation that allows us to simplify the computations when additional variable parameters are considered in the model.

2.5.1.1 Exact computation of collision probabilities

In this case we compute the collision probability exactly. For the sake of clarity, our assumptions are summarized as follows:

- All vehicles move at the same constant velocity V.
- All vehicles begin to slow down at the same constant deceleration a.
- The delay δ is the same for all drivers. It implies that all the drivers receive the warning message at the same instant.

Since V_i, δ_i and a_i are constants, from (2.5) we obtain:

$$d_s = \frac{V^2}{2a} + V\delta. \tag{2.12}$$

For $1 \leq i \leq N$, the collision probability will be computed as follows:

$$\begin{aligned} p_i = P(d_s \geq l_{i-1} + s_i) = P(l_{i-1} + s_i \leq d_s \mid l_{i-1} \leq d_s)P(l_{i-1} \leq d_s) + \\ P(l_{i-1} + s_i \leq d_s \mid l_{i-1} > d_s)P(l_{i-1} > d_s), \end{aligned} \tag{2.13}$$

where l_{i-1} is a random variable that represents the distance traveled by the preceding vehicle (assuming that $l_0 = 0$, since vehicle C_0 stops instantly at $x_0 = 0$), and F is the cumulative distribution function of the exponential distribution, $\exp(\lambda)$, with λ the vehicle density (in veh/m).

In this simple case, if vehicle C_{i-1} does not collide then neither does vehicle C_i, because the velocity, the deceleration and the reaction time are the same for both of them. Moreover, if vehicle C_{i-1} collides, it means that all of the preceding vehicles have collided. From these observations we can conclude that $l_{i-1} = s_1 + s_2 + \ldots + s_{i-1} \sim$ Erlang$(i-1, \lambda)$, and $P(l_{i-1} + s_i \leq d_s \mid l_{i-1} > d_s) = 0$.

Now, we need to compute $p_i = P(l_{i-1} + s_i \leq d_s \mid l_{i-1} \leq d_s)P(l_{i-1} \leq d_s)$. The joint probability density function of $X = l_{i-1} + s_i$ and $Y = l_{i-1}$ is:

$$g(x,y) = \frac{\lambda^2(\lambda y)^{i-2}e^{-\lambda x}}{(i-2)!}, \qquad \text{for} \quad 0 \leq y \leq x. \tag{2.14}$$

So, the joint cumulative distribution function is:

$$\begin{aligned} G(x,y) = \int_0^y \int_0^t \frac{\lambda^2(\lambda s)^{i-2}e^{-\lambda t}}{(i-2)!}\, ds\, dt + \int_y^x \int_0^y \frac{\lambda^2(\lambda s)^{i-2}e^{-\lambda t}}{(i-2)!}\, ds\, dt = \\ = \frac{\gamma(i,\lambda y)}{(i-1)!} + \frac{(\lambda y)^{i-1}}{(i-1)!}(e^{-\lambda y} - e^{-\lambda x}), \qquad \text{for } 0 \leq y \leq x. \end{aligned} \tag{2.15}$$

where γ is the incomplete gamma function, defined as $\gamma(a,x) = \int_0^x t^{a-1}e^{-t}dt$.

Finally, for $1 < i \leq N$ it holds:

$$\begin{aligned} p_i &= P(l_{i-1} + s_i \leq d_s \mid l_{i-1} \leq d_s)P(l_{i-1} \leq d_s) = \frac{G(d_s,d_s)}{F_y(d_s)} \cdot F_y(d_s) = \\ &= G(d_s,d_s) = \frac{\gamma(i,\lambda d_s)}{(i-1)!} + \frac{(\lambda d_s)^{i-1}}{(i-1)!}(e^{-\lambda d_s} - e^{-\lambda d_s}) = \frac{\gamma(i,\lambda d_s)}{(i-1)!}. \end{aligned} \tag{2.16}$$

At this point, if the metric of interest is the average number of accidents, the procedure to obtain it is: once we have computed the collision probability for each vehicle, we have to construct the matrix $\mathbf{P}$ described on Section 2.3. The next step is to calculate the final outcome probabilities, Π_i, and finally the average number of accidents can be obtained through equation (2.3).

As can be seen, in this case it is relatively easy to compute the collision probability conditioned on the distance traveled by the preceding vehicle, l_{i-1}. However, in the following cases it becomes increasingly difficult. Besides, it can be seen that the collision probability basically depends on the difference $d_{s,i} - l_{i-1}$ of any two cars being greater than the initial inter-vehicle distance s_i. From this observation, and in order to simplify the following computations, in the next section we compute the collision probability using the *average* distance traveled by the preceding vehicle and compare it with the results of this subsection.

2.5.1.2 **Approximate computation of collision probabilities**

As discussed previously, in this subsection we compute an approximation to the collision probability for the basic model, where we use the average distance traveled by the preceding vehicle, and compare it with the exact computation. For the sake of clarity, our assumptions are summarized as follows:

- All vehicles move at the same constant velocity V.
- All vehicles begin to slow down at the same constant deceleration a at the same time (the delay δ is the same for all drivers).
- We use the average distance traveled by the preceding vehicle to calculate the collision probabilities.

As in the previous case, the distance traveled by a vehicle until it completely stops if it does not collide is given by (2.12).

For $1 \leq i \leq N$, vehicle C_i will collide with C_{i-1} if and only if the distance needed by C_i to stop is greater than the distance between them plus the average distance traveled by C_{i-1}, $\overline{l_{i-1}}$, so the collision probability of C_i is:

$$p_i = P(d_s \geq \overline{l_{i-1}} + s_i) = F(d_s - \overline{l_{i-1}}). \tag{2.17}$$

The average distance traveled by a vehicle, $\overline{l_i}$, must be computed recursively, starting from $\overline{l_0} = 0$. For $1 \leq i \leq N$, the average distance traveled by vehicle C_i is $\overline{l_i} = d_s(1 - p_i) + d_{c,i}p_i$, where $d_{c,i}$ is the average distance traveled by the vehicle in case of collision:

$$d_{c,i} = \frac{1}{p_i}\int_0^{d_s - \overline{l_{i-1}}} (\overline{l_{i-1}} + x)\lambda e^{-\lambda x} dx = \frac{1}{p_i}\left(\overline{l_{i-1}} + \frac{1}{\lambda} - (d_s + \frac{1}{\lambda})e^{-\lambda(d_s - \overline{l_{i-1}})}\right). \tag{2.18}$$

Then, the equation for $\overline{l_i}$ is:

$$\overline{l_i} = \begin{cases} d_s(1 - p_i) + d_{c,i}p_i, & p_i > 0 \\ d_s, & p_i = 0. \end{cases} \tag{2.19}$$

Now, like in the previous case, we have to construct the matrix $\mathbf{P}$ and calculate the average number of accidents through (2.3).

2.5.1.3 Validation and discussion

Fig. 2.10 shows the results of computing the basic model described in the previous sections. The number of vehicles in the chain is $N = 20$, and the rest of the parameters have been fixed at $a = 8\ m/s^2$, which is the maximum deceleration of what is consider as a normal vehicle [32], $V = 33\ m/s$ and $\delta = T_{m,i} + T_{r,i} = 0.1 + 0.9\ s$. In this case, $T_{m,i} = 0.1\ s$ is the maximum delay for warning messages that vehicular communication standards specify [15], whereas $T_{r,i} = 0.9\ s$ is an average driver reaction time [76]. Fig. 2.10 illustrates the curves for the exact and the approximate basic models. In addition, a Monte-Carlo simulation of the system has been also conducted in order to validate our model. The Monte-Carlo simulations have been performed with 10 replications per simulation point and results are shown with 99.5% confidence intervals. As can be seen, the use of the average distance traveled by the preceding vehicle, $\overline{l_{i-1}}$, provides an excellent approximation to the exact collision probability, since the mean square error between the results of both cases is less than 0.5%. Moreover, simulation

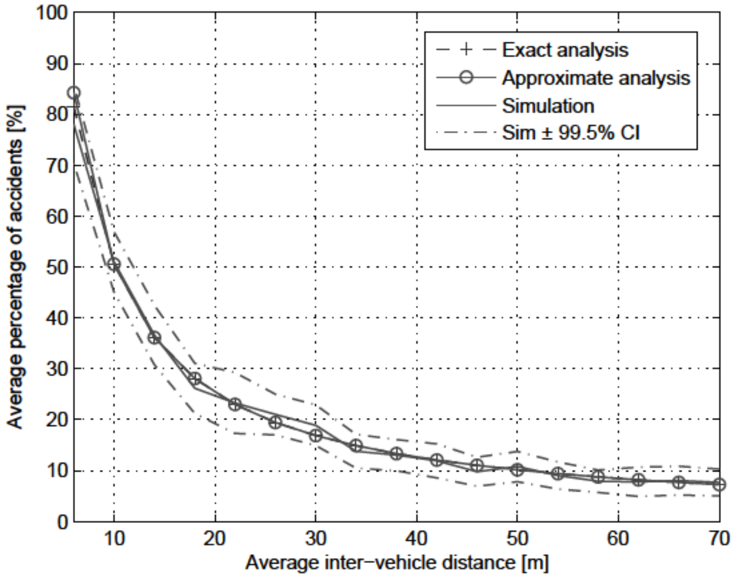


Figure 2.10: Average percentage of accidents versus average inter-vehicle distance $\overline{s} = \frac{1}{\lambda} - L\ m$ for basic model, with exact solution, approximate solution and Monte-Carlo simulation with a 99.5% confidence intervals.

results confirm that the model is correct enough, since the mean square error between the results of the approximate case and the Monte-Carlo simulation does not exceed 2%.

2.5.2 Variable kinematic parameters

In this section the basic model is extended by considering notification delays δ_i, velocities V_i and decelerations a_i as variables. In most of the cases, they should be considered random variables with their appropriate probability density functions to model some particular effect. At this point, we do not assume any particular probability distribution for them. A discussion on this matter is provided later in Section 2.5.2.1.

As in the basic case, the vehicle collision probabilities are calculated recursively. For each vehicle C_i, starting from the leading one, we compute its probability of colliding with the preceding vehicle $p_{c,i}$, which is based on the average distance traveled by the preceding vehicle $\overline{l_{i-1}}$:

$$p_{c,i} = P(d_{s,i} \geq \overline{l_{i-1}} + s_i) = F(d_{s,i} - \overline{l_{i-1}}),\ \ i = 1, \ldots, N, \tag{2.20}$$

where F is the cumulative distribution function (cdf) of the inter-vehicle spacing and $d_{s,i}$ is the distance needed by vehicle C_i to completely stop (defined by eq. (2.5)).

The second step is to compute the average distance traveled by the current vehicle, which is then used in the computation of the next vehicle collision probability. Again, this average distance must be computed recursively, starting from $\overline{l_0} = 0$. However, in this case vehicle collisions may occur in four different ways: (1) vehicles have not started to brake; (2) only one of them is braking; (3) both of them are braking; or (4) the front vehicle has stopped. Each one of these possibilities results in a different distance to stop, $d_{c_j,i}$, that must be weighted by its probability of occurrence, $q_{c_j,i}$, and added to get the average distance traveled $\overline{l_i}$ as:

$$\overline{l_i} = d_{s,i}(1 - p_{c,i}) + \sum_{j=1}^{4} d_{c_j,i} q_{c_j,i}, \tag{2.21}$$

where

$$d_{c_j,i} = \frac{1}{q_{c_j,i}} \int_{inf_{c_j,i}}^{sup_{c_j,i}} D_{c_j,i}(x) f(x)\, dx, \tag{2.22}$$

$$q_{c_j,i} = P(inf_{c_j,i} \leq s_i \leq sup_{c_j,i}) = F(sup_{c_j,i}) - F(inf_{c_j,i}), \tag{2.23}$$

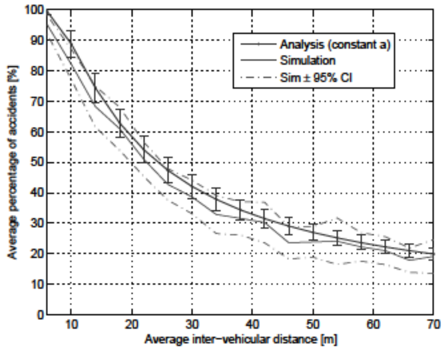
for $i = 1, \ldots, N,\ j = 1, \ldots, 4$, where f represents the probability density function (pdf) of the inter-vehicle spacing distribution and $D_{c_j,i}(x)$ represents the distance traveled by C_i when the inter-vehicle spacing is x and it collides in the way j. The derivation of these distances, as well as the proper values for the integration limits $inf_{c_j,i}$ and $sup_{c_j,i}$, are provided in Appendix B.

Actually, the probability of C_i being crashed is the probability of C_i colliding with C_{i-1} or C_{i+1} colliding with C_i, so the collision probability is computed as follows:

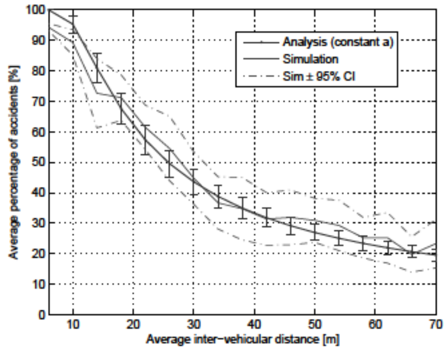
$$\begin{aligned} p_i &= 1 - P\left(C_{i+1} \text{ not coll. } C_i \mid C_i \text{ not coll. } C_{i-1}\right) \cdot P\left(C_i \text{ not coll. } C_{i-1}\right) = \\ &= 1 - F(d_{s,i+1} - l_i)(1 - p_{c,i}). \end{aligned} \tag{2.24}$$

2.5.2.1 **Validation and discussion**

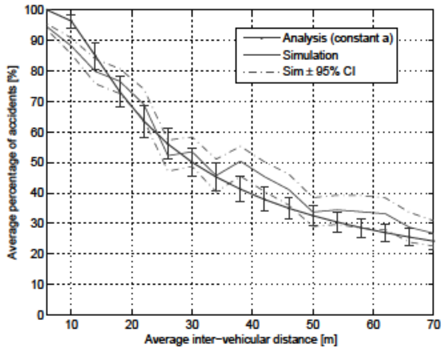
The next stage would be to assign the kinematic parameters and notification delays appropriate values that model realistic scenarios. As an example, in order to take into account an underlying communication model, the notification delay should be assumed to be a random variable with an appropriate probability density function. In this way, information packet collisions in a heavily loaded shared communications channel can be modeled with an appropriate random variable for the access delay and characterized also by $T_{m,i}$ [129]. Furthermore, since vehicles move at different speeds, the velocity should be assumed to be a random variable too. Let



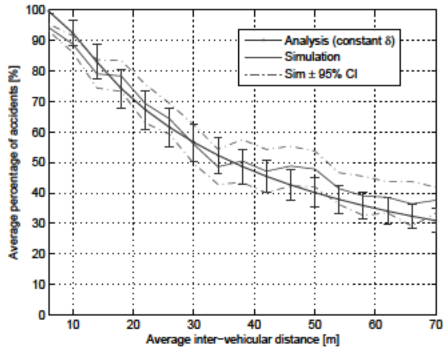
(a) $\delta_i \sim U(0.5, 1.5)\ s$, $V_i = 33\ m/s$ and $a_i = 8\ m/s^2$.



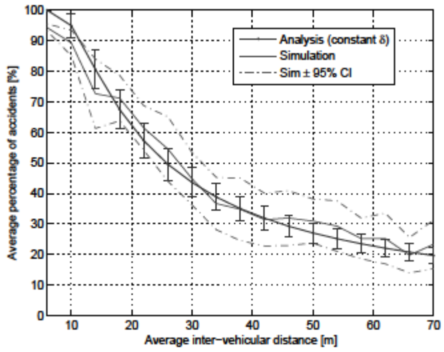
(b) $\delta_i = 1\ s$, $V_i \sim U(30, 36)\ m/s$ and $a_i = 8\ m/s^2$.



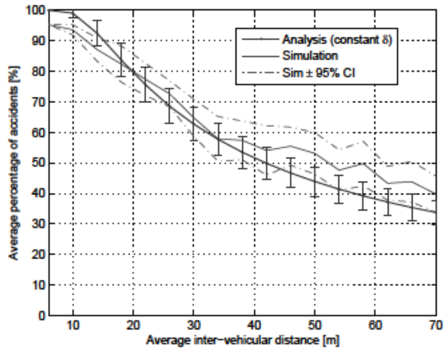
(c) $\delta_i \sim U(0.5, 1.5)\ s$, $V_i \sim U(30, 36)\ m/s$ and $a_i = 8\ m/s^2$.



(d) $\delta_i = 1\ s$, $V_i = 33\ m/s$ and $a_i \sim U(4, 8)\ m/s^2$.



(e) $\delta_i = 1\ s$, $V_i \sim U(30, 36)\ m/s$ and $a_i = 8\ m/s^2$.



(f) $\delta_i = 1\ s$, $V_i \sim U(30, 36)\ m/s$ and $a_i \sim U(4, 8)\ m/s^2$.

Figure 2.11: Validation of the model through the evaluation of six different scenarios.

us note that, in most of the practical cases, inter-vehicle distances and velocities represent the state of the system when the incident occurs, and so they should be considered random variables, though determining their distributions and ranges require a proper characterization of the scenario of interest. Accelerations and delays can be controlled by different means after the incident, and so depending on the application evaluated they can be considered constant or assigned particular values.

Later, in Chapter 3, we provide an evaluation of different types of CCA applications in two scenarios, a freeway and an urban scenario. Those scenarios have been carefully characterized by extracting appropriate parameters' distributions from open literature. However in this section we only intend to validate our model, therefore the parameters are supposed to be uniform random variables and eq. (2.3) has been computed 100 times and averaged. In all the cases we assume a chain of $N = 20$ vehicles.

A solution for the model has been computed for six different scenarios:

a) In the first one, δ_i is assumed to be a uniform random variable ranging between 0.5 and 1.5 s, whereas the velocity and the deceleration have been fixed at $V = 33$ m/s and $a = 8$ m/s^2, respectively.

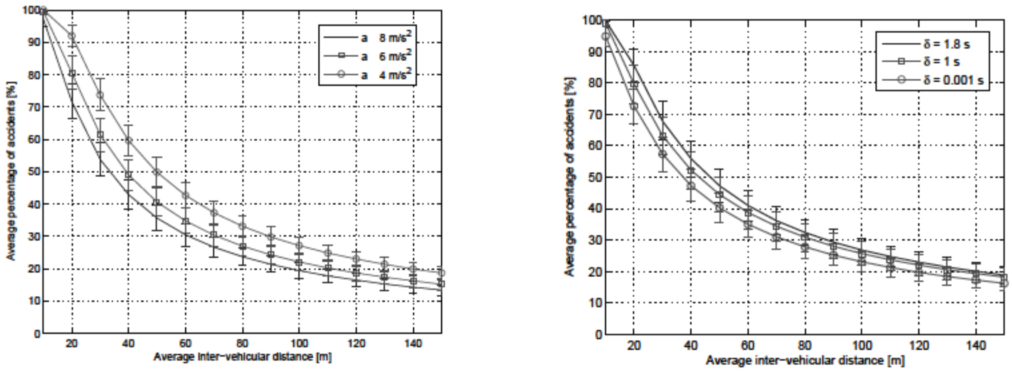
b) In the second scenario, V_i is assumed to be a uniform random variable between 30 and 36 m/s, whereas the notification delay and the deceleration have been fixed at $\delta = 1$ s and $a = 8$ m/s^2, respectively.

c) In this scenario, both the velocity and the notification delay are assumed to be uniform random variables ranging between 0.5 and 1.5 s and between 30 and 36 m/s, respectively, while the deceleration is kept constant at 8 m/s^2.

d) Here the deceleration a_i is assumed to be a uniform random variable between 4 and 8 m/s^2, whereas the velocity and the notification delay have been fixed at $V = 33$ m/s and $\delta = 1$ s, respectively.

e) In this scenario, V_i is assumed to be a uniform random variable between 30 and 36 m/s, whereas the deceleration and the notification delay have been fixed at $a = 8$ m/s^2 and $\delta = 1$ s, respectively.

f) In the last scenario, both the deceleration and the velocity are assumed to be uniform random variables between 4 and 8 m/s^2 and between 30 and 36 m/s, respectively, while the notification delay is kept constant at $\delta = 1$ s.



(a) Average percentage of accidents for different decelerations when $V_i \sim U(30, 36)\ m/s$ and $\delta_i \sim U(0.5, 1.8)\ s$.

(b) Average percentage of accidents for different notification delays when $V_i \sim U(30, 36)\ m/s$ and $a_i \sim U(4, 8)\ m/s^2$.

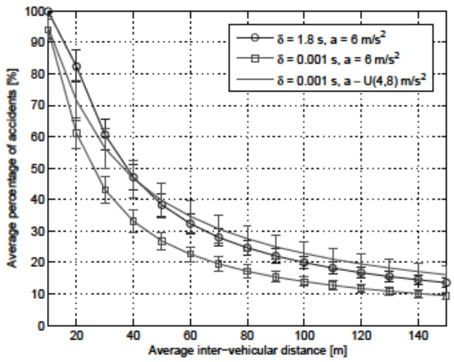
Figure 2.12: Performance of the model with different constant decelerations (a) and message reception delays (b).

Finally, in order to validate the results for our solutions, the corresponding Monte-Carlo simulations have been conducted as well. The results of this evaluation are shown in Figure 2.11. Let us remark that these pictures are provided to validate that our model describes correctly the dynamics of the system. A discussion on the influence of the parameters on the collision process is deferred to the next chapter.

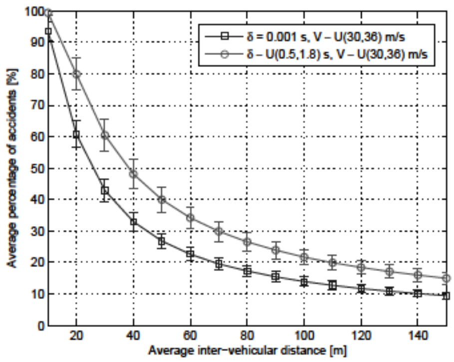
The average number of accidents computed with our model for each of the six cases is compared with the aforementioned Monte-Carlo simulations. The standard deviation has been computed and shown as errorbars. Dashed lines show the 95% confidence interval of the corresponding simulation. In all the cases, the results reasonably confirm the validity of our model, even using $\overline{l_{i-1}}$ as approximation, since the mean square error between the results of the analysis and the simulation remains between 3.5% and 6% for all the cases.

2.5.3 Applications and discussion of the model

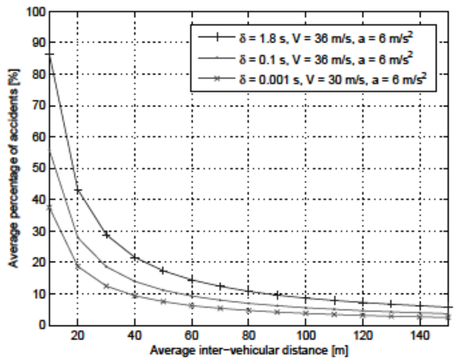
Once our model has been validated, we can use it to evaluate the influence of the different parameters on the vehicle collision process. A systematic evaluation of different scenarios, as well as the development of different metrics, is presented in Chapter 3. Here we present a short discussion of the qualitative and quantitative aspects of the parameters' influence on the average percentage of accidents.



(a) Variability of deceleration with different notification delays.



(b) Variability of velocity and notification delay.



(c) Performance of the model with fixed parameters.

Figure 2.13: Evaluation of the impact of the parameters' variability on the number of vehicle collisions.

As for the qualitative evaluation, we first provide a set of figures that show the influence of the different parameters. Fig. 2.12 shows a family of curves for the model when the deceleration or the notification delay are kept constant, while the rest of the parameters are random. As can be seen in Fig. 2.12(a), the number of accidents is clearly sensitive to the deceleration capabilities of the vehicles, which agrees with the results obtained in [63]. However, there does not appear to be a statistical difference for different notification delays when the deceleration and velocities are variable (Fig. 2.12(b)). This result is also in accordance with [63], where it is shown that moderate changes in the notification delay cause small variations in accident severity. Later in this section we discuss when the delay actually has an important influence on the number of accidents.

Fig. 2.13(a) shows the results when the velocities are randomly distributed. In this case if either deceleration or notification delay are kept constant it causes a reduction of the number of accidents. In fact, in this case it is noticeable the positive effect of a communication system able to deliver warning messages with short maximum delays and automatic vehicle response. Fig. 2.13(b) shows similar results when deceleration is kept constant at $a = 6\ m/s^2$. The results however reveal that in general the variability of the kinetic parameters has a negative impact on the number of accidents. If the system is able to keep constant some of the parameters during the emergency event, an improvement can be achieved. The benefits of a warning collision system are even clearer in Fig. 2.13(c). When all the parameters remain constant, a shorter notification delay always results in fewer vehicle accidents.

Overall, these results suggest that a cooperative warning collision notification system combined with a vehicle control system able to smooth out the variations of speed and deceleration of the platoon of vehicles may improve the driver and passengers safety. In fact, more detailed conclusions can be extracted to provide general guidelines about the design and operation of a CCA application, as we will see in Chapter 3. The usual approach is to consider that the emergency messages must be sent as fast as possible [107, 113], but according to the obtained results a higher delay could be traded off for other features such as reliability of warning message reception. For instance, adding a Request-To-Send/Clear-To-Send mechanism to avoid packet collisions due to hidden nodes [67]. Or more importantly, the CCA application should provide an acceleration control mechanism, so the margin in delay can be used to collect all the necessary information from neighboring vehicles to perform such control properly. This kind of insights on delay requirements is also important for designing CCA applications based on predicting trajectory conflicts, in order to determine the time horizon for trajectory estimation [108].

However, if we consider a low speed and high density scenario, the delay has a remarkable influence. Fig. 2.14 shows the average percentage of accidents when velocities are uniformly distributed within 10 and 16 m/s. This scenario would model an urban road, where speed is relatively low but the vehicle density is high[1]. And in this case, specially at short inter-vehicle distances corresponding to urban roads, the influence of delay is more noticeable, higher than that of deceleration. Therefore we can conclude that the delivery of a warning message might not be sufficient to ensure safety and a special emphasis should be placed on providing automatic deceleration control. Moreover, in this scenario it is specially difficult for a commu-

[1] Just for the sake of example, but let us remark that a log-normal distribution for inter-vehicle distances describes more accurately high density scenarios.

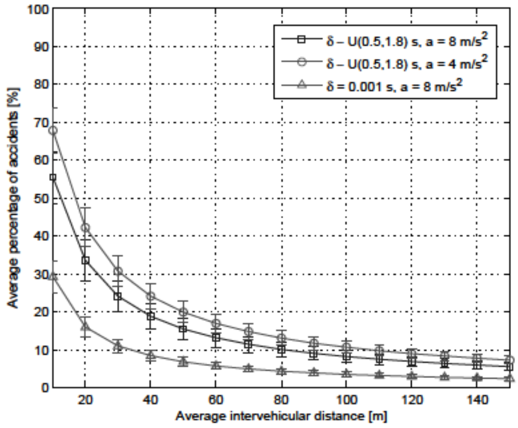


Figure 2.14: Average percentage of accidents in a low speed scenario with $V_i \sim U(10, 16)\ m/s$.

nication system based on contention channel access (CSMA) to provide low delays, since the number of neighbors in range is high, unless additional congestion control mechanisms such as transmit power control are applied.

In fact, some of these conclusions can be drawn by directly examining equations (2.5) and (2.20), that is, for high speeds it is more important to have good deceleration capabilities rather than to press the brake quickly, and conversely for low speeds.

Finally, as for the quantitative aspects of the results, the percentage of accidents might seem higher than expected, above 10% in many cases, as well as the slow decay of it for high inter-vehicular distances. This is first a consequence of the extreme case we are evaluating here, that is, the leading vehicle stops completely and immediately. It makes the collision of the first car of the platoon almost unavoidable in most of the cases. As a worst case approach, better outcomes are expected in reality. But also these results have to be interpreted with care, since using average inter-vehicle distances may lead to misleading conclusions. As an example, with the parameters used in Fig. 2.13(c), $V = 36\ m/s$, $a = 6\ m/s^2$ and $\delta = 0.1$, the distance needed to stop is $111.6\ m$. For an exponentially distributed inter-vehicle distance with mean $\overline{s} = 60\ m$, the probability of s_i being less than $100\ m$ is 0.81, and even with a mean $\overline{s} = 150\ m$, this probability is still 0.48. So the probability of collision is higher than one may intuitively think, specially for the first vehicles in the chain. Therefore, even at relatively high inter-vehicular distances, the collisions are mainly suffered by the first and second vehicle, which accounts for the 10% of accidents in our example with $N = 20$ vehicles.

2.6 Final remarks

In this chapter we have proposed a stochastic model for the probability of collisions in a chain of vehicles where a warning collision system is in operation. The fact that a warning notification system is used allows us to overcome the difficulties for obtaining stochastic models for such vehicular scenarios, since we can assume that all the drivers/vehicles react to the warning message independently, and therefore the motion equations can be simplified. We also propose a good matching approximation to the exact model to further reduce the required computations to calculate the vehicle collision probabilities. In both cases, its validity has been confirmed by Monte-Carlo simulations.

The model is independent of the particular communication system employed as long as its operation can be abstracted and characterized by an appropriate message notification delay including communication latency and driver reaction times. Therefore, it also enables the performance evaluation of different technologies. Similarly, different probability distributions for the inter-vehicular spacing can be incorporated seamlessly into the model, due to the fact that the distribution of the initial inter-vehicle spacing is independent of the actions that drivers make after receiving the warning messages. Here we have used an exponential distribution, which is considered appropriate for low vehicle traffic densities, but using log-normal distributions is better justified under conditions of high vehicular density. Finally, we compute the probability that collisions occur in different forms (both vehicles in motion, one stopped and one in motion, etc.), which constitutes a promising way to define detailed accident severity functions, that is, by assigning different degrees of severity to each collision possibility.

Although we have shown some examples of the application of the model, a quantitative evaluation requires a careful definition of the scenarios of interest. Therefore, in the next chapter we describe a systematic characterization and evaluation of different scenarios to complement the results obtained so far.

3 Using the stochastic model for designing and evaluating Chain Collision Avoidance applications

3.1 Introduction

Simulation is an essential tool to design and evaluate Cooperative/Chain Collision Avoidance (CCA) applications [97, 101]. However, there is a number of issues related to this approach. First, it usually requires the integration of networking and traffic simulation tools, which is not mature yet and requires further work [66]. But, more importantly, available simulation tools are not directly suitable to model accidents and so cannot be effortlessly used for the design of cooperative applications. The reason is that current traffic simulation tools are designed for normal traffic conditions and are based on mobility models that are specifically developed to avoid vehicle crashes, for example, a common metric for the quality of a car-following model is that it is intrinsically *collision-free*. Therefore, those models have to be modified to account for collisions which is either not a straightforward task and may lead to unexpected results or it is difficult to set up controlled experiments. Some of these limitations are pointed out and discussed in the following sections.

In the previous chapter we derived a stochastic model for the number of accidents in a platoon of vehicles equipped with a CCA system. The model enables the computation of the average number of collisions that occur in the platoon, the probabilities of the different ways in which the collisions may take place, as well as other statistics of interest. In this chapter, we discuss its potential as a numerical evaluation tool and as an alternative to simulation, specially at early stages of development. Our goal is to illustrate its use by providing and thoroughly discussing application examples. First, the different metrics the model can provide are described, and its limitations are also discussed. Next, we show how it can be used as a performance evaluation tool and check the validity of the results it provides by comparing them with available independent results. Finally, the model can be used at an early stage to shed relevant guidelines for the design of CCA applications, by disclosing the influence of kinematic parameters on the collision process. To exemplify it, we provide an evaluation of different types of CCA applications in two scenarios, a freeway and an urban scenario. Those scenarios have been carefully characterized by extracting appropriate parameters' distributions from open literature. The results suggest that enabling a coordinated braking policy that removes the variability in deceleration and driver reaction time should be the main concern of a CCA application. It should be noted that particular numerical results have to be considered as upper bounds on the expected number of accidents, since the model is based on the strong assumption that vehicles cannot change lane to avoid the crash (worst case). However, even for the generic scenarios and simplified systems used as examples, it is able to provide a reasonable qualitative insight about the relative benefits of different CCA approaches.

The remainder of this chapter is organized as follows. In Section 3.2 relevant related work is reviewed. Next, in Section 3.3, the limitations of current simulators are discussed and some of the performance metrics that the model can provide as output are described, as well as its current limitations. In Section 3.4 the suitability of the model for evaluating the performance of CCA applications is shown by comparing it with previous results. Section 3.5 provides, as an illustrative example, the evaluation of different CCA systems under two scenarios. Finally, conclusions and future work are remarked in Section 3.6.

3.2 Related Work

The concept of Automated Highway Systems (AHS) goes back several decades and its safety benefits have been studied in the past years [36]. The motion of a platoon of vehicles is usually described as an interconnected (automated or not) system, where one or more leading vehicles influence the driving behavior of the follower. Platoon safety comes as a result of proper stability of the platoon in the presence of perturbations, called string stability. The basis of string stability and safety performance guarantees can be found in [106].

In the absence of safety guarantees collisions may occur and they have been studied mainly by modeling its frequency [25], severity [63] or physical process [42, 65, 82]. In the latter case, very detailed models of vehicle motion and collision dynamics can be found [65, 82], but the equations are completely deterministic, whereas in reality, randomness is always present as an effect of human behavior or noisy operation introduced by sensors or other reasons. To account for it, the usual methodology is to evaluate deterministic models by applying a Monte Carlo or stochastic analysis over an extensive range of their parameters [63, 82, 114].

On the contrary, in this chapter we use the stochastic model proposed in Chapter 2, which assumes that kinematic variables are random. A similar stochastic approach can be found in [41, 42], where the authors assume the “effective” braking is a random variable and analytically compute the probability and expected number of primary collisions and the relative speed at impact in a platoon of vehicles. The inter-vehicle distance and speed of all vehicles are identical, unlike our model, where we consider both parameters as random variables with a known probability distribution function. Their analysis is also based on some strong assumptions, e.g., the expected number of total collisions in the string is assumed to be proportional to the expected number of primary collisions. Therefore, they provide a lower bound on the expected number of collisions. Secondly, they assume that a collision will definitely occur if the deceleration of a following vehicle is less than that of its immediate predecessor. On the contrary, our model considers all the ways in which a collision may occur and provides the average probability of each type of collision, as well as an upper bound for the total number of collisions. In Section 3.4 we validate our model by comparing with theirs.

Carbaugh *et al.* [36] have modeled rear-end crashes and related them to the capacity of AHS. Our work follows a similar approach in several aspects, though it has relevant differences as well. Unlike our work, they restrict themselves to only primary collisions involving only two vehicles. They consider random variables for speed, braking capabilities and reaction times, as we do. However, we introduce them in the analytical model and compute the performance metrics, whereas they discretize the distributions and evaluate all the parameter combinations with Monte Carlo simulations. In both works, parameters have been carefully extracted from open literature. Finally, in both cases, different types of cooperative vehicle systems have been evaluated. The main differences come from the modeling of the cooperative systems: in [36] they are essentially distinguished by the reaction time whereas the speed and inter-vehicle spacing are fixed and constant. In our case, different cooperative systems are not only assigned different reaction times, but also braking and speed behavior, which do not need to be constant. Another major difference is that they explicitly relate safety metrics and road capacity, whereas we use capacity as an independent parameter and do not explicitly mention it. That is, capacity is implicitly given by the random variables used to model the states variables (inter-vehicle spacing and speed) in the scenarios used in Section 3.5 and Table 3.2. For instance, the free-flow highway traffic corresponds approximately to a capacity of 2347 vehicles per hour per lane, by substituting the average speed and inter-vehicle spacing in equation (2) in [36].

With the recent assignment of bandwidth and standardization of communications for vehicular networks, research interest on cooperative vehicular applications has grown again. Kato *et al.* [78] show the feasibility and potential of the technologies for the cooperative driving. In fact, improvement of traffic safety by cooperative vehicular applications is one of the most promising benefits of vehicular ad hoc networks. In a recent work [71] the authors propose an inter-vehicle communication framework for the cooperative active safety system whose operation is based on the dissemination of each vehicle's state information through a wireless network.

As a particular case of cooperative driving, Cooperative Collision Avoidance (CCA) techniques have received special attention in recent years. With CCA systems a fast dissemination of warning messages to the vehicles in the platoon enables them to promptly react in emergency situations. In this way the number of car accidents and the associated damage can be significantly reduced. In [128] the authors identify the application requirements for vehicular cooperative collision warning and achieve congestion control for emergency warning messages based on the application requirements. The authors in [91] develop three cooperative collision warning safety applications: a forward collision warning assistant, an intersection assistant and a blind-spot and lane change "situational awareness" assistant. In [47] we can find a performance evaluation study of cooperative collision warning applications using the Dedicated Short-Range Communications (DSRC) wireless standard. All these studies focus on the communication or implementation aspects of the application. They use simulations that do not involve crashes and do not provide any results related to safety and so the problems with accurate simulation of crashes that we discuss in the next section do not arise. Biswas *et al.* [32] present an overview of highway cooperative collision avoidance and its implementation requirements in the context of a vehicle-to-vehicle wireless network, primarily at the Medium Access Control (MAC) and the routing layers. More recently, Taleb *et al.* [107] proposed an effective collision avoidance strategy for vehicular networks which forms clusters of vehicles that belong to the same group. They also design a risk-aware medium access control (MAC) protocol to increase the responsiveness of the proposed CCA scheme. These studies do evaluate safety aspects of the systems, but as a complement to the communication evaluation. So they develop simple ad hoc mobility models instead of traffic simulators or car-following models, and again the problems of accurate simulation of accidents do not show clearly. Several car-following mobility models have been proposed and analyzed [116, 121], also in the context of VANET simulation [66]. A thorough analysis of car-following models for accident simulation can be found in [126], where the authors propose a car-following model that includes by design accidents behavior as well. Its integration with current simulators still requires to solve additional issues and at the moment has not been incorporated to available tools.

3.3 Simulation and stochastic modeling of accidents

Evaluating CCA applications for vehicular networks requires, as a previous step, appropriate modeling of accidents and driver behavior in such situations. That is, CCA designers need to understand the processes that lead to crashes and the influence of different system variables under such circumstances in order to cooperatively take preventive measures. However, as we discuss next, available simulation tools are not directly suitable to reproduce such processes and so cannot be effortlessly used for designing cooperative CCA applications.

Whereas modeling of vehicle structural deformation and occupant injuries has been widely studied under different contexts [105] and modeling tools are available, current traffic simulation tools are focused on normal traffic conditions and are based on mobility models that are specifically developed to avoid vehicle crashes, for example, a common metric for the quality of a car-following model is that it is essentially *collision-free*. Therefore, those models have to be modified to account for collisions which is either not a straightforward task and may lead to unexpected results or it is difficult to set up controlled experiments. In Section 3.3.1 we further discuss some concerns which arise when using popular simulators and mobility models to simulate crashes. Later, in Section 3.3.2 we discuss the limitations of our stochastic model and define the performance metrics we will use.

3.3.1 Simulation of accidents with current tools

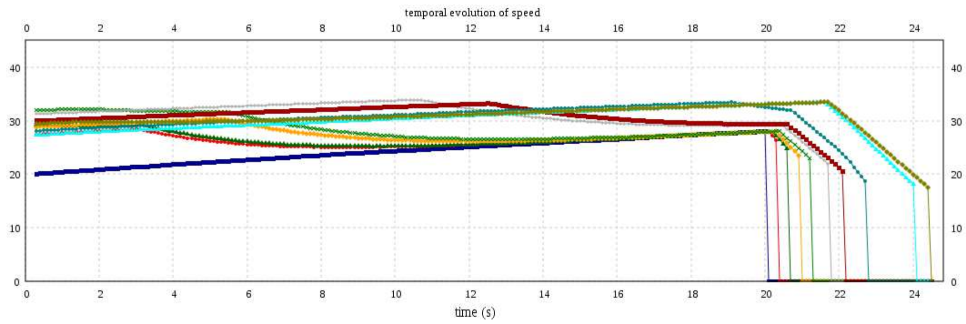
Simulators based on macroscopic magnitudes are not appropriate to simulate accidents, so it is necessary to resort to micro-simulation. Most popular micro-simulation tools [27, 30] are based on car-following models [121, 126]. In particular, the Gipps model is used by AIMSUM [27] and a modified Krauss model is used by SUMO [30]. Both models are called *safety distance models* because it is assumed that drivers try to keep a safety distance with the preceding vehicle to avoid accidents. Both of them use a reaction time τ parameter and an estimation of the preceding car comfortable deceleration in order to compute the next step speed, which is instantaneously updated, together with a maximum comfortable acceleration and deceleration. The Gipps model is collision free as long as the comfortable deceleration of the preceding vehicle is not underestimated [126]. The Krauss model adds a stochastic perturbation to the acceleration but it is collision free because the speed is bounded by a *safety speed* at each updating speed. Moreover, in its implementation in SUMO, every vehicle (or driver) *knows exactly* the deceleration rate and reaction time of the preceding vehicle. Finally, the Intelligent Driver Model (IDM), and its imperfect driver variant, the Human Driver Model (HDM) [116] are becoming popular for simulation in the last years. The former one is collision free by design unless a maximum deceleration is used.

Therefore, in order to simulate accidents, and more particularly chain collisions, one has to modify the models. For the Gipps model, a first obvious approach is to limit the maximum deceleration and remove the safety speed constraint from the model. To obtain a more realistic behavior, the reaction time of each driver can be randomized as well as the estimation of the preceding vehicle deceleration. These changes allow to simulate accidents to some extent, but setting up controlled accident experiments is still hard. The main reason behind is that car-following models lead to an equilibrium state where either all the platoon accelerations are zero or strong instabilities with oscillations occur [121]. Then, if a simulation is started with initial conditions different from those of the equilibrium state it results in an initial transient where all the cars immediately adapt their speed to that prescribed by the model, which tend to avoid collisions. In other words, the platoon behaves as if there is a cooperative safety application in place which automatically and instantaneously dictates the needed speed. For example, if one is interested in the influence of small inter-vehicle gap on the accidents, the model parameters and initial conditions have to be carefully adjusted to overcome the automatic reconfiguration of the platoon. And in many cases it is likely that the model itself has to be tuned as we discuss next. The opposite situation is also common, that is, initial transient leads to strong instabilities, which propagate backwards and result in unexpected crashes even before the programmed emergency event. The IDM/HDM model is particularly sensitive to initial deviations from equilibrium state.

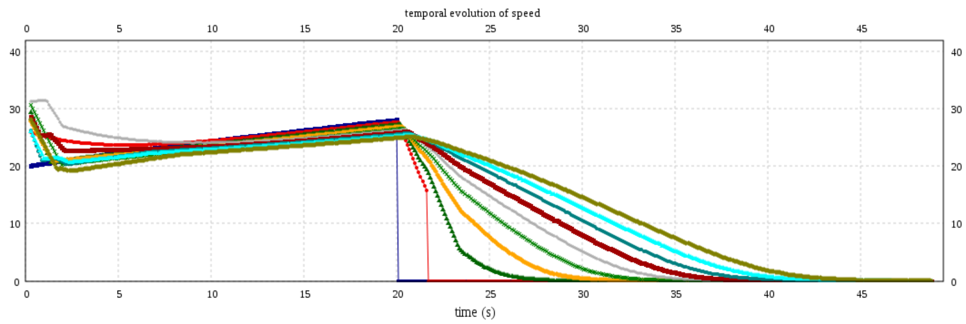
In addition, the influence of model and simulation parameters on the results is not always clear and in some cases their interpretation is different. For instance, Gipss and Krauss models describe the parameter τ as a reaction time. HDM also introduces a reaction time parameter. One would expect high reaction times to increase the risk of accidents. That is, let us consider a simple scenario with two cars where the leading one suddenly decelerates at a high rate. Reaction time is commonly assumed to be the time elapsed since the leading vehicle starts to brake and the following one perceives the change and starts to brake itself, that is, higher reaction times would lead to late brake and more dangerous situations. This is actually the interpretation and behavior of the HDM model. However, in the Krauss model the leader reaction time[1] determines what is considered the safety speed by the follower: a higher reaction time makes the follower to choose a lower safety speed, which makes the model collision free in normal situations. A first objection is that knowledge of the leader reaction time does not seem realistic. But more importantly, that difference makes the results quite the opposite of expected, since in practice it determines the *aggressiveness* of the driver style. This is a consequence of the way the model is constructed. In equilibrium cars follow each other with a time headway that is equal to the reaction time and the safety speed is computed assuming normal conditions comfortable deceleration. Thus, low reaction times lead to short time headways. When an accident occurs, decelerations much higher than the comfortable one can be expected. In that case, short time headways result in more accidents. Fig. 3.1 illustrates this behavior in an extreme scenario where a platoon of 9 vehicles follow a leader which at $t = 20\ s$ stops instantaneously. As can be seen in Fig. 3.1(b), drivers are more conservative. At the beginning all the vehicles reduce their speed to comply with the safety speed and keep higher time headways, which in the end result in few collisions. It also exemplifies the initial automatic adjustment discussed in the previous paragraph. On the contrary, in Fig. 3.1(a) it is shown how vehicles at the beginning accelerate to reduce its headway and start to decelerate later, leading to multiple collisions. In this sense, IDM/HDM provides more flexibility since it separates desired gap and reaction time in a more realistic way.

In any case, more details on the analysis of car-following models in the context of accidents can be found in [126]. Our goal here is to simply point out some of the subtleties involved in accident simulation with current simulation tools. To the best of our knowledge, the only available car-following model that includes by design accident behavior is described in [126], but integration with current simulators still require to solve additional issues and at the moment have not been incorporated to available tools. In summary, simulation of accidents with current simulation tools is neither straightforward nor obvious and need careful design and adjustments, in addition to the usual drawbacks of simulation.

[1] Actually, reaction time is the same for all vehicles in the Krauss model.



(a) Krauss model with $\tau = 0.2$ s.



(b) Krauss model with $\tau = 1.2$ s.

Figure 3.1: Temporal evolution of vehicle speeds with SUMO and a Krauss model with different reaction time τ. If there is a collision, speeds drop instantaneously to zero.

3.3.2 Limitations of the stochastic model and performance metrics

In the previous subsection some of the difficulties of accident simulation with current tools have been discussed. In this subsection we propose, as an alternative to simulation, to numerically evaluate the influence of different system parameters as reaction time, speed or deceleration capabilities on the number of accidents and other metrics. The numerical evaluation is based on the stochastic model for chain collisions derived in the previous chapter.

The model assumes that all vehicles are equipped with a CCA that sends a warning message when an accident occurs. In that case, it can be assumed that drivers react as soon as they receive the warning message, and they start braking just after the time they need to be aware of the danger (reaction time). Hence, the total delay is the sum of the warning message reception and reaction delays. This reaction is independent of the preceding vehicle behavior. The main practical utility of this model lays on its ability to quickly evaluate numerically the influence of different parameters on the collision process without the need to resort to complex simulations at a first stage. Such an evaluation provides relevant guidelines for the design of vehicular communication systems as well as chain collision avoidance (CCA) applications. The limitations of the model as well as the performance metrics it can provide are discussed next.

Our model was derived under the assumption that at least the inter-vehicle spacing is a random variable. The other parameters (velocity, delay and deceleration) can be assigned deterministic or random values before executing the numerical evaluation. However, introducing too much randomness causes unrealistic results. In fact, it results in a pessimistic estimation of the metrics.

Inter-vehicle distances and velocities represent the state of the system when the incident occurs, and so they should be considered random variables in most cases, though determining their distributions and ranges require a proper characterization of the scenario of interest. Accelerations and delays can be controlled by different means after the incident, and so depending on the application evaluated they can be considered constant or assigned particular values. Indeed, different CCA systems are mainly characterized by how they are modeled. As an example, let us consider two different CCA systems. The first one is simply characterized by a warning message delivered to the platoon that makes drivers start braking. In this case, both delay and deceleration should be considered random variables modeling driver reaction time and human-operated braking respectively. The second CCA system is a fully automated braking system that takes over the driver operation and applies a constant deceleration. In that case, both delays and decelerations could be considered deterministic.

Randomness allows the occurrence of situations which rarely occur in reality for both state and control variables. For the state variables, independence may result in samples where a vehicle is traveling very near to its front vehicle and much faster than it, which is unlikely to happen in reality[1]. Actually, there is a correlation between the distance from a vehicle to the preceding one and the relative velocity between them [36]. Unfortunately, this correlation cannot be represented in the model in its current form, so the number of accidents computed by it is an overestimation of the actual number of accidents that may occur in a real situation. For the control variables, the model does not take into consideration that the driver usually reacts to variations in the driving conditions of the preceding vehicle after the incident, as in a car-following approach. That is, if we assign pure random decelerations it may result in a driver which decelerates much softer than its preceding vehicles, leading to a crash. In a real situation, the follower would apply a stronger deceleration to avoid the crash. Fortunately,

[1] Actually its likeliness is arguable since this case does occur when a vehicle is preparing to overtake its front one.

these cases can be corrected since the control variables can be freely assigned, as we describe later in Section 3.5. Finally, as we said previously, the assumption that vehicles cannot change lane or perform evasive maneuvers, often found in literature [36, 42], results again in an over-estimation. Anyway, in the next sections we show that in spite of those limitations the model is suitable for evaluating the performance of CCA-based applications in different scenarios and the influence of the main kinematic parameters on the number of vehicle collisions.

A variety of performance metrics can be provided by the model, either direct metrics, typically used in literature [32, 107], as well as more specialized ones, indirectly derived from the former ones, as follows:

- **Percentage of accidents**. This is a direct global metric that computes the average percentage of collided vehicles in the chain.

- **Relative distance.** It provides for each vehicle in the chain the average distance to the preceding car after stop (in case of collision the relative distance is 0). This metric may be considered as a measure about the margin of safety available to the vehicles.

- **Relative speed.** It provides for each vehicle in the chain the average relative speed with respect to the preceding vehicle at the time of collision (in the absence of collision the relative speed is 0). This metric may be considered as a measure about the severity of collisions.

- **Types of collisions.** As previously mentioned, collisions can occur into four different ways in a single-lane situation. The average probability of each type of collision can be provided, which can be used by itself or as a weight factor for other derived metrics.

- **Accident severity functions.** These are specialized metrics derived by weighting accident severity indexes with probabilities of types of collisions and other metrics. As an example, a collision between two vehicles in movement can be assigned a higher severity than a rear collision with a stopped one. The average severity index results from weighting them by the probability of either type of collision. In addition, the average relative speed at the collision can be used to weight again the severity index.

A number of other metrics can be defined, depending on the application under evaluation, though in next sections we only use the first three ones.

3.4 Evaluation of CCA applications

As said previously, the evaluation of this kind of applications is usually conducted by time-consuming simulations and needs a considerable prior development effort. Using our model, one can quickly evaluate numerically the performance of a CCA application under different situations. In this section, we show that our model can provide similar results, and so it is a good alternative to simulation, by comparing its output with the results reported by previous performance evaluation of CCA mechanisms. The first one is a basic CCA system proposed by Biswas *et al.* [32] and the second one is a more sophisticated mechanism, the Cluster-based Risk-Aware CCA (C-RACCA) scheme proposed by Taleb *et al.* [107], which are briefly described next.

- **CCA:** Upon occurrence of the emergency event, the leading vehicle rapidly decelerates (emergency deceleration, see Table 3.1) and starts sending emergency warning messages to all vehicles behind it. These messages are forwarded in a multi-hop manner in order to ensure a complete coverage within the platoon. Upon reception of a warning message, a driver reacts by decelerating (with a regular deceleration rate, see Table 3.1), even if the brake light on the car ahead is not already lit.

- **C-RACCA:** This mechanism dynamically forms clusters of vehicles in the platoon. The first vehicle of a cluster is the Cluster Head (CH), which is in charge of relaying packets (e.g., emergency warning messages) from a CH in front to the rest of vehicles within the same cluster. This way, the number of redundant retransmissions is reduced.

Although the underlying message exchange mechanism is different, both procedures just make vehicles decelerate at a constant rate when they receive a warning message. Let us remark that our model is mainly concerned with the effects of the kinematic parameters and delays on accidents, unlike those proposals, whose goal is to control the communications broadcast storm. Since this control effectively reduces the warning message delay, it results in fewer accidents and so we can compare with our results.

In fact, our model is intended to be used from a different approach to the design of CCA applications: to quickly decide in which scenarios some parameters may have more influence on the collisions and so design the CCA based on it. As an example, in those proposals the main design goal of the communication system is to quickly deliver emergency messages, but

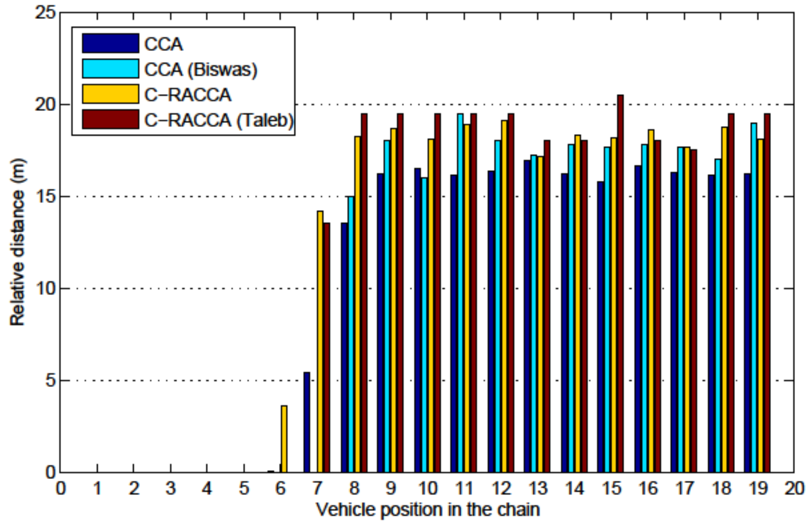
Table 3.1: Parameters for the CCA evaluation.

Number of vehicles	20
Vehicle speed	32 m/s
Inter-vehicle distance	15 m
Emergency deceleration	8 m/s^2
Regular deceleration	4.9 m/s^2
Driver's reaction time	$U(0.75, 1.5)$ s
Average relative delivery latency	CCA: 54 ms C-RACCA: 6.7 ms

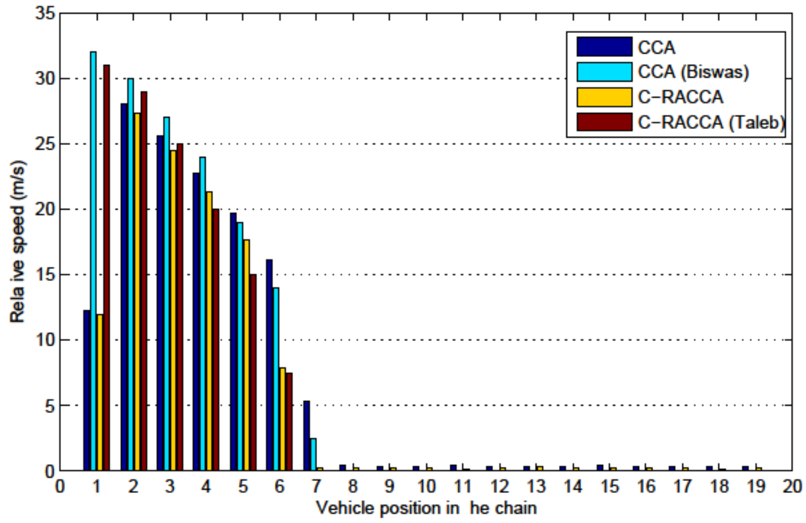
according to our results in some scenarios a low delay is not relevant for the outcome, so we can design a CCA system that trades it off for additional reliability mechanisms.

Regarding the use of the model, the key step is to adequately define and model the input parameters. Kinematic parameters for different scenarios can be extracted from the literature, as is discussed in Section 3.5. Warning message delivery delay (latency) is one of the most difficult to model, since it depends on the transmission range of the nodes, the packet forwarding method used, the additional data traffic present in the channel, etc. In this case, we have modeled it by using the average latency between the reception of the message by two consecutive vehicles in the chain, since it was measured in [32] for different packet error rates. Nevertheless, it can be characterized in other ways, as for example by using the same average latency for all the vehicles in the chain, but this implies that all of them receive the message at the same time.

The kinematic parameters have been set equal to those used in [107] for both CCA methods, which are listed in Table 3.1. Once we have selected the parameters, we run the model 1000 times with different samples of the random parameters and extract the performance metrics of interest. Our model has been implemented with Matlab and it took only 8.65 seconds to run all the samples and extract results on a commodity PC with a quad-core processor at 2.2 GHz and 4 GB of memory.



(a) Relative distance between two consecutive vehicles after stop.



(b) Relative speed between two consecutive vehicles at the time of collision.

Figure 3.2: Performance metrics for the evaluation of CCA and C-RACCA mechanisms.

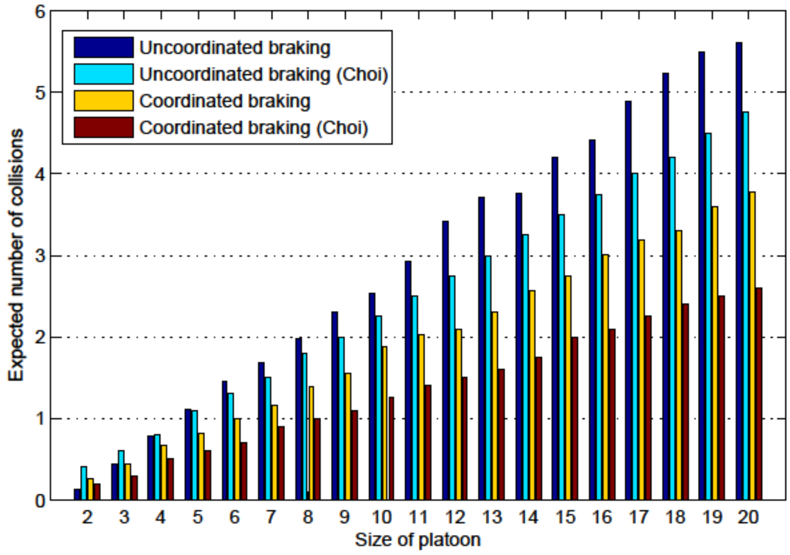
3.4.1 Results

The results obtained by our model for the performance metrics studied are presented in Fig. 3.2, together with the results provided by the authors of the two CCA mechanisms under consideration (tagged as (Biswas) and (Taleb)). From the results of [107] we can only extract data for an inter-vehicle space of 15 m, so we compute the average percentage of accidents only for this distance. For the basic CCA system we obtain 46.5% of accidents while the percentage in [107] is about 50%. On the other hand, for the C-RACCA scheme we obtain 40.2% of accidents, which is very close to the 40% showed in [107]. Both, the results of the model and the simulation show that C-RACCA outperforms the basic scheme due to its reduced delay.

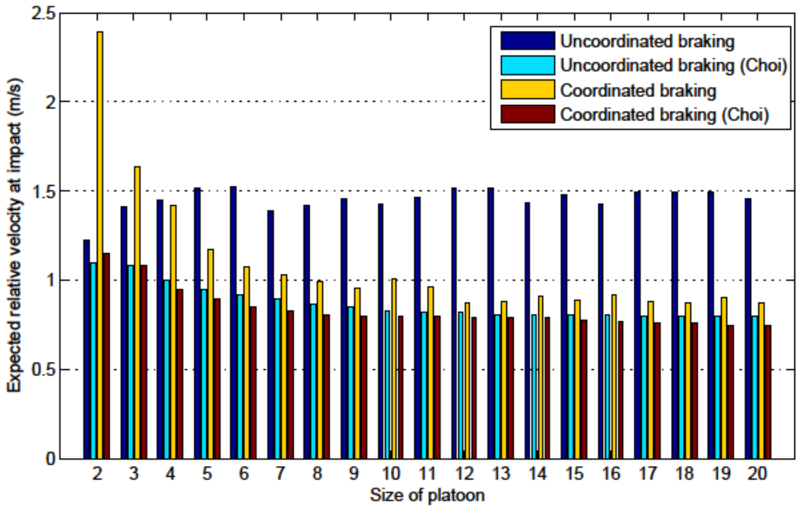
In addition, we compare with other metrics reported in [107]. For each vehicle in the chain, Figs. 3.2(a) and 3.2(b) show the average distance to the front car after stopping and the average relative speed with respect to the front vehicle at the time of collision, respectively. Again, both the results of the model and the simulation coincide. The singularity of the first relative speed in the results comes from the assumption that vehicles stop instantly in case of collision. The first collision usually occurs when either the lead vehicle is braking but the follower is not, or both of them are braking. In both cases, the relative speed at the moment of the crash is not specially high. However, when they crash the speed drops immediately to zero. Thus, if there is a subsequent crash with the next vehicle, the relative speed is exactly the speed of the latter, which is usually high because it started braking recently. Looking at Fig. 5 in [32], this same trend can be seen. In Fig. 3.2(b) we show a comparison with the results reported by Fig. 12 in [107], which do not show this particularity. It seems that the lead vehicle also stops instantly but without further details in the original paper on how their simulation has been done we cannot discuss this discrepancy.

Despite slight differences, the results obtained by our model are in accordance with the results presented by the authors of the mechanisms, since our numerical evaluation matches the simulation and shows that the C-RACCA approach reduces both the number of collisions and the impact of collisions when they inevitably occur. In this way, we have validated that our model is suitable for evaluating this kind of mechanisms.

In order to further validate our model, we compare it with a similar numerical tool previously proposed based on a stochastic model [41]. In Fig. 3.3 we provide a comparison of results provided by both of them. We have tried to faithfully reproduce their experiment by using constant parameters except for decelerations, but our models differ in several aspects, discussed in Section 3.2. In spite of these differences both models show coincident results. Our model is more pessimistic, as expected since it provides an upper bound, whereas theirs provides a lower bound, but the trend is similar. Our model also shows the apparent anomaly of platoons of size 2, discussed in [41].



(a) Average number of collisions



(b) Relative speed between two consecutive vehicles at the time of collision.

Figure 3.3: Comparison with numerical results provided in [41].

3.5 Design of CCA applications

In this section we exemplify the use of our model for CCA design by evaluating two different scenarios under different traffic conditions. It is important to remark that our results provide broad directions for design at an early stage which would typically be refined in later stages.

Our goal is to derive mainly qualitative conclusions about the importance of each kinematic parameter in the development of a CCA application. The first step is to define the scenarios and to model adequately the different input parameters. This step is key in the quality of the results and it requires a research effort from open literature to properly characterize the model parameters. Next, some input parameters are set according to hypothetical CCA systems and performance metrics are computed for the different scenarios and compared. For instance, we can test if a hypothetical CCA system able to just remove reaction times reduces the number of accidents. Finally, we evaluate CCA mechanisms which could control several parameters simultaneously in order to improve the traffic safety.

3.5.1 Scenario and parameter characterization

Let us recall that our model needs that the five input parameters are characterized, number of vehicles by a fixed value, inter-vehicle spacing by a random distribution and speeds, delays and decelerations by deterministic or random values. The first scenario we consider is the freeway studied in [122], where three different time periods with different traffic flows were observed and characterized:

- **Night traffic**: very low traffic density and high speed;
- **Free-flow traffic**: moderate traffic density and high speed;
- **Rush-hour traffic**: very high traffic density and low speed.

Wisitpongphan *et al.* [122] showed that during the night period the inter-vehicle spacing can be modeled by an exponential distribution, while during the other time periods the log-normal distribution[1] provides a better fit. Moreover, they showed that regardless of the time of day, the speed of vehicles follows a normal distribution. The probability distributions used for these parameters are given in Table 3.2.

The second scenario we consider is the urban scenario studied in [132], where two different time periods were considered:

- **Peak hours**: the traffic is in congestion status;

[1]The probability density function of a log-normal distribution is:

$$f(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}, \;\; x > 0.$$

Table 3.2: Probability distributions for the vehicle speed and inter-vehicle distance. *N*, *EXP*, *LN* and *LL* represent the normal, exponential, log-normal and log-logistic distributions respectively.

Scenario		Vehicle speed	Inter-vehicle distance
Freeway	**Night**	$N(30.93, 1.2)\ m/s$	$EXP(256.41)\ m$
	Free-flow	$N(29.15, 1.5)\ m/s$	$LN(3.4, 0.75)\ m$
	Rush-hour	$N(10.73, 2)\ m/s$	$LN(2.5, 0.5)\ m$
Urban	**Peak hours**	$N(6.083, 1.2)\ m/s$	$LL(1.096, 0.314)\ m$
	Non-peak hours	$N(12.86, 1.5)\ m/s$	$LN(0.685, 0.618)\ m$

- **Non-peak hours**: the traffic is in free-flow status.

Yin *et al.* [132] showed that the headway data (and so the inter-vehicle spacing) can be modeled by the log-normal distribution for peak hours and by the log-logistic distribution[1] for non-peak hours data. Table 3.2 shows the parameters used for these probability distributions.

At this point we have the input data for the inter-vehicle distance for all the scenarios and time periods, as summarized in Table 3.2. We use other references to extract the rest of the parameters needed by our model. Let us recall also that delay $\delta_i = T_{r,i} + T_{m,i}$ is actually the sum of message transmission time (latency) and driver reaction time, so we need to characterize both of them.

- **Driver's reaction time.** Taoka [109] estimated the distribution of the driver reaction time by fitting a log-normal distribution to the data collected by Michael Sivak, used also by [36]. We use then a log-normal distribution with mean 1.21 s and standard deviation 0.63 s for the driver reaction time, $T_{r,i}$.

- **Warning message delivery latency.** According to Fracchia and Meo [51], the average time needed by the warning message (using a probabilistic broadcasting scheme) to reach the farthest node in an area of 2 Km is always under 0.1 s. So, we will use this value for the message latency, $T_{m,i}$, as an upper bound.

[1] The probability density function of a log-logistic distribution is:

$$f(x;\mu,\sigma) = \frac{e^{\frac{\ln x - \mu}{\sigma}}}{\sigma x \left(1 + e^{\frac{\ln x - \mu}{\sigma}}\right)^2}, \quad x > 0.$$

- **Deceleration.** We assume that the first vehicle decelerates at $16\ m/s^2$ (which models a vehicle colliding with a fixed obstacle and stopping within a very short distance). The rest of vehicles will decelerate at their maximum braking capabilities. In [36] the authors show that this braking capability (considering light passenger vehicles and dry pavement) fits a normal distribution with mean $7.01\ m/s^2$ and standard deviation $1.01\ m/s^2$.

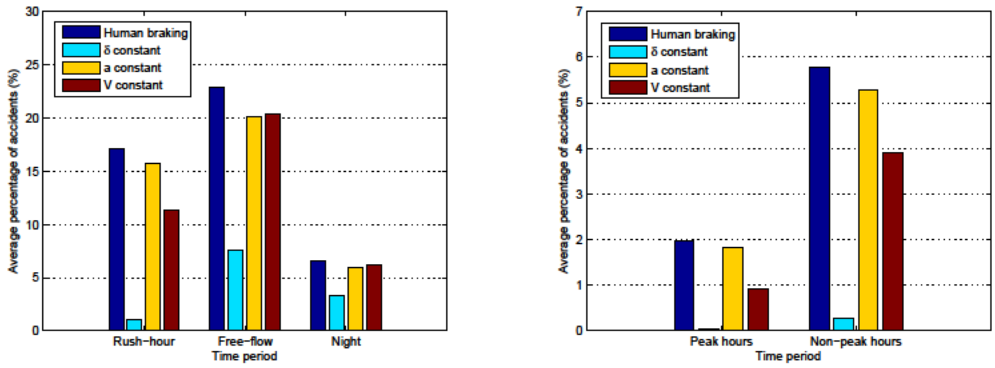
Therefore, during the model initialization we fill the input vectors of speeds, decelerations and delays with a random sample of the appropriate distribution. As we discussed in Section 3.3.2, setting pure random values to all the parameters results in unrealistic values. Fortunately, it can be corrected by adjusting the values during initialization. Therefore, we truncate all the normal distributions within a sufficiently wide range, for instance deceleration is kept between $5.5\ m/s^2$ and $8.5\ m/s^2$. In addition, when setting the value of consecutive vehicles in the input vector of speeds and decelerations, we do not allow the relative speeds and the relative decelerations to exceed a certain threshold. This way we keep the parameters essentially random but avoid unrealistic situations in both the initial state, like a vehicle driving much faster than the preceding one, and the braking process, like starting to decelerate too late and soft.

3.5.2 Influence of isolated controlled parameters

In this subsection we evaluate the influence of each one of the parameters independently of the average number of accidents. The goal is to evaluate the effects of hypothetical CCA systems on the number of accidents. A basic CCA warning message system is always assumed to be in place, since it is the main assumption of the model, so vehicles are notified when the incident occurs. Additionally, a reactive CCA application may control delays, with the communication system, and decelerations, with some automated control response to the warning message. Assuming those systems are used allows to set arbitrary values to them, instead of random variables. In order to assign deterministic values to speeds it would require to assume the use of a proactive CCA system, that is, a system that keeps the relative speeds of vehicles within a certain range at any time.

To test these possibilities, in Fig. 3.4 we compare the average percentage of accidents that occurs in a chain of 20 vehicles driving in the described scenarios with the following conditions:

- **Human braking:** the drivers are only informed about the emergency situation and they react on their own. Reaction times, speeds and decelerations are random to some extent,



(a) Freeway scenario

(b) Urban scenario

Figure 3.4: Influence of the principal kinematic parameters on the average percentage of accidents for the scenarios under evaluation.

since they are human-controlled, which means that all the parameters are assigned the random distributions specified in the previous subsection.

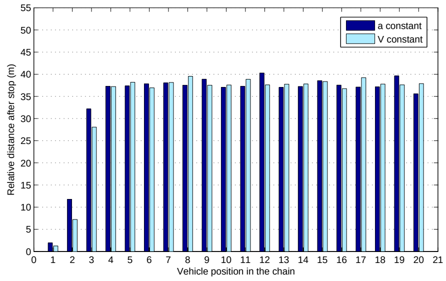
- **δ constant:** a CCA system is able to automatically start braking, so delay is fixed and equal to $0.1\ s$ for all the vehicles in the chain. The rest of the parameters follow the distributions specified in the previous subsection. It means that decelerations are still subject to random variation.

- **a constant:** a CCA system makes all decelerations be equal to $8\ m/s^2$ for all the vehicles in the chain. The rest of the parameters follow the distributions specified in the previous subsection. In this case, random driver reaction time is present.

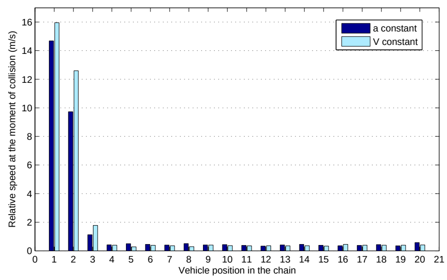
- **V constant:** a proactive CCA system makes the speed constant and equal to the average speed of the scenario for all the vehicles in the chain. The rest of the parameters follow the distributions specified in the previous subsection.

Let us note that fixing delays and decelerations independently is somehow unrealistic. A delay system able to automatically start braking, thus removing the driver reaction time, should brake in a controlled way. However, we are not concerned at this point with the feasibility of these systems, but just evaluate the effect of controlling each one of the principal kinematic parameters.

In all the cases the most significant reduction in the percentage of accidents is obtained by controlling the delays. Therefore, the main goal of a CCA application should be to remove



(a) Relative distance after stop for each vehicle in the chain.



(b) Relative speed at the moment of collision for each vehicle in the chain.

Figure 3.5: Influence of the deceleration rate and the vehicle speed on the relative distance and the relative speed for Free-flow traffic in the Freeway scenario.

the variability of the drivers' reaction time and make cars start braking simultaneously. Hence, although a warning message may help reduce the number of accidents compared to no CCA application at all, to be really effective it needs additional measures involving taking over driver control. In fact, the message delay is not actually relevant to the outcome since it is even in the worst cases, one order of magnitude lower than the driver reaction time. Additionally, for low and medium speeds it seems to be more necessary to control the speed than the deceleration rate.

On the other hand, in high speed scenarios, controlling the deceleration rate and the speed results in a similar reduction in the percentage of accidents. Besides, in Fig. 3.5 we can observe that for the Free-flow traffic in the Freeway scenario (the case of Night traffic results in a similar performance) controlling the deceleration rate, the metrics of relative distance and relative speed show a slightly superior performance than controlling the speed, but there is not a significant improvement.

3.5.3 Influence of combined controlled parameters

In the previous section we evaluated the influence of controlling some parameters independently of others. In this subsection, we evaluate "more realistic" CCA systems based on the results obtained previously, where the system would be able to control several parameters simultaneously as follows:

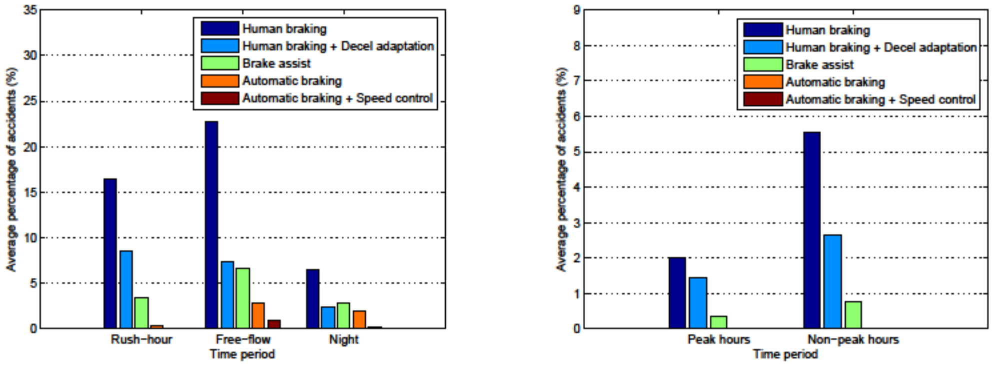
- **Automatic braking**. A reactive CCA mechanism that allows the automatic braking of the vehicles in the chain, that is, when the vehicle receives the warning message it immediately starts to brake, although the driver is not yet aware of the risk. In this case we can assume that the delay of braking and the deceleration rate are controlled (fixed) by the application, whereas the rest of the parameters follow the distributions specified previously.

- **Automatic braking + Speed control**. A proactive CCA mechanism that controls the speed of vehicles before the emergency situation, in addition to allowing the automatic braking of the vehicles in the chain. In this case delays and decelerations are constant and equal to $\delta = 0.1\ s$, $a = 8\ m/s^2$, whereas the inter-vehicle spacing follows the distribution specified in the scenario description and speed is constant and equal to the mean of the distribution.

- **Human braking + Deceleration adaptation**. In the previous subsection we described the “Human braking” behavior. Now we assume a more realistic situation in which the driver adapts its deceleration rate to the velocity and deceleration of the preceding vehicle. In Section 3.5.1 we mentioned that to avoid unrealistic results, the random samples are forced to be within certain ranges. Here we additionally assume a rational deceleration, that is, when the initial deceleration vectors are filled it is checked that the random sample is within the range, but also that the relative values are reasonable. For example, if the preceding vehicle is braking at a high rate, it is not realistic that the follower brakes softer and let itself collide, so its deceleration is set to a higher value, but not above the maximum deceleration. Or if relative speeds seem to allow a safety stop, it is set to a comfortable deceleration.

- **Brake assist**. A mixture of human braking and automatic braking. If the driver has not reacted and started to brake (with rational deceleration) after a given time threshold, the system automatically starts braking at a constant maximum deceleration. The threshold has been set to $0.84\ s$ which is the mode of the log-normal distribution provided in [109]. This case models the behavior of current brake assistance systems, and possibly the most likely CCA to be deployed in the near future.

Fig. 3.6 shows the average percentage of accidents that occur in a chain of 20 vehicles driving in the described scenarios when these types of CCA mechanisms are in operation as well as both types of human braking. Since our model provides upper bounds on the number of accidents, instead of focusing on the particular absolute values it is possibly more useful



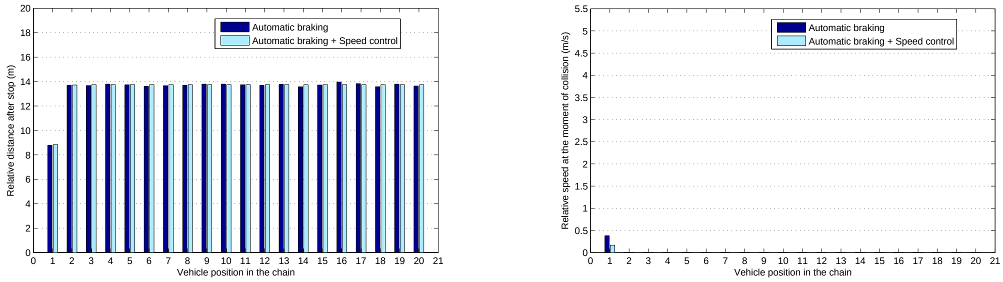
(a) Freeway scenario (b) Urban scenario

Figure 3.6: Average percentage of accidents for different CCA proposals in the scenarios under evaluation.

to compare qualitatively different systems. Looking at the figure, it can be noticed that the percentage of accidents in the case of rational deceleration adaptation is much lower, more than 50%, than when it is not considered. But even considering this human braking behavior, more sophisticated CCA applications still reduce the percentage of accidents. In this sense, our results agree with previous studies [36, 41], as expected.

Brake assist, as we said, models approximately current systems and is the most likely CCA to be deployed in the near future. Its performance is between rational human and automatic (coordinated) braking as should also be expected, but still much closer to human braking, showing that there are still potential gains from enabling systems with more cooperation, as automatic braking. In fact, these gains are dependent on the reaction threshold, which should be carefully selected.

In high speed scenarios (Free-flow and Night period in the Freeway scenario), automatic braking, i.e. coordinated deceleration, effectively reduces the number of accidents with respect to human braking variants, even up to 50% of a Brake assist system in Free-flow. There would be a remarkable reduction in the percentage of accidents if we could employ speed control in addition to automatic braking, but it would require a much more coordinated type of AHS, which would be close to the Platooned Vehicles concept discussed in [36]. Development of such a concept is still technically challenging, whereas implementation of coordinated braking policies seems to be more likely in the near future. Speed control benefits are particularly clear in the Night scenario, where large inter-vehicle spacing makes performance of all the variations



(a) Relative distance after stop for each vehicle in the chain.

(b) Relative speed at the moment of collision for each vehicle in the chain.

Figure 3.7: Comparison of the relative distance and the relative speed between the two CCA proposals for Rush-hour traffic in the Freeway scenario.

of deceleration control practically equal, whereas control of speed would actually reduce the accidents.

However, in low and medium speed scenarios (Urban scenario and Rush-hour in the Freeway scenario), there are no significant differences between the more coordinated CCA policies (automatic and speed control). Besides, if we observe Fig. 3.7 it can be seen that for the case of Rush-hour in the Freeway scenario (the cases in Urban scenario result in a similar performance), the severity of accidents and the margin of safety are very similar as well. In fact, in these scenarios Brake assist is able to remove most of the collisions.

According to these results, it would seem sound to focus on the design of a cooperative CCA system which is able to notify about incidents and, in case of emergency, apply a coordinated braking policy according to the known status of the surrounding vehicles. A less ambitious system based mainly on control of delays, as the Brake assist model can bring relevant improvements in most of the scenarios considered. On the contrary, a cooperative system to control speeds poses much more technical challenges and its cost-benefit ratio is not so evident at a first approach. As a summary of this section, we have exemplified how to use our model to draw qualitative and quantitative results about the influence of the different parameters for the design of CCA applications and have described the key steps.

3.6 Final remarks

In this chapter we have shown how CCA mechanisms can be evaluated numerically by using our stochastic model as an effective alternative to simulation. We have discussed the limitations of current traffic simulators for accident simulation. We also discuss our model's limitations, validate its results against available previous results and provide and discuss abundant application examples. The main limitation of the model in its current form is that independence between state input variables, relative velocity and inter-vehicle spacing, is assumed, which is not realistic in many cases and introduces too much randomness leading to pessimistic results. It can partially be corrected by adjusting the input variables and their relative values, as we have discussed.

We have illustrated its capabilities as an assessment tool for CCA application design while describing the working methodology. To this purpose, we have evaluated different types of CCA applications in two scenarios, a freeway and an urban scenario. The results suggest that the variability due to the drivers reaction time is the main cause of accidents and so removing it should be the main focus of a CCA application. This could be possible by automatic braking, that is, when the vehicle receives the warning message it takes over control and immediately starts to apply a coordinated braking policy, even though the driver is not yet aware of the risk. This is one of the different CCA systems discussed. Results reveal that the benefits of implementing this CCA are relevant. On the contrary, results show that the benefits of implementing a much more challenging cooperative system, able to coordinate speeds, are marginal in most of the cases. In any case, our main goal has been to show the model potential as an aiding design tool rather than proposing a particular CCA system.

As future work, we intend to enhance the model in order to deal with the mentioned limitations. So, the first step is to introduce bivariate distributions in the model, to capture state variable correlations, increasing the model accuracy. As a second step it would be necessary to find appropriate joint distributions for speed and inter-vehicle spacing. There is actually a lack of empirical models that jointly describe inter-vehicle spacing and speed. Similarly, we have shown how to characterize the input variable distributions by using statistical models proposed in the open literature, but additional efforts in the empirical characterization of deceleration, reaction times and communication delays are clearly necessary.

Part III

Efficient geo-routing in VANETs

4 MAC contention distributions for efficient geo-routing in vehicular networks

4.1 Introduction

Communications for Vehicular Ad-Hoc Networks (VANET) have been developed and standardized in the last years. At the moment, a Dedicated Short Range Communication (DSRC) bandwidth has been allocated to vehicular communications at 5.9 GHz, and both American and European standards [22] have adopted IEEE 802.11p as physical and Medium Access Control (MAC) layers, based on Carrier-Sense Multiple Access with collision avoidance (CSMA/CA) [16].

At the network layer, European standards [14, 17, 18, 19, 20, 21, 23] specify the GeoNetworking protocol as the default network layer protocol for vehicle-to-vehicle (V2V) communications. It is a geo-routing protocol, that is, packets forwarding is based on the geographical positions of the nodes. GeoNetworking supports the communication among individual ITS stations, as well as the distribution of packets in geographical areas. If the source node does not belong to the destination geo-area, then the packet should be forwarded until reaching a node which belongs to this area, which takes care on delivering the packet to its destination. As basic forwarding algorithms, the standard proposes *Greedy Forwarding and Contention-Based Forwarding* (CBF) [54]. With the former, the source selects the most distant known neighbor as the next forwarder. With the latter, the packet is broadcast and each receiver station decides whether it becomes the next hop (forwarder router) according to its position. Upon receiving a packet, all routers start a timer whose timeout depends on the specific position of the receiver, usually inversely proportional to the distance to the source. The major advantage of CBF is that it provides an implicit reliability mechanism in case the most suitable forwarder does not receive the packet, which in highly dynamic environments, such as those vehicular networks, is quite likely.

As defined in the standard, CBF is completely implemented at the network layer. However, CBF might be also implemented directly at MAC layer. Implementing CBF at MAC layer should result in lower delays than network layer operation, since forwarding and access delays are integrated. Moreover, CSMA/CA mechanisms can be controlled with several parameters, like contention window size and intervals as well as the probability distribution for the slot selection, which results in multiple degrees of freedom to optimize MAC operation according to the most critical functionality offered by the network. For instance, such an optimization should benefit safety and emergency related applications, which rely and are built on top of the functionality of the geo-routing protocol. As drawback, implementation at the MAC layer may be potentially more complex, requiring at least firmware modification.

CBF operation synchronizes medium access of all nodes, that is, all receivers of a packet immediately become potential forwarders and contend for the medium. In this particular situation, in [110] it is shown that there exists an optimal distribution for the contention slots that maximizes the contention success probability. Although the optimal distribution cannot be implemented in practice, geometric distributions approximate the optimal one. With such a distribution the conditional access probability in case of success is uniformly distributed among all the contenders. However, the main goal of CBF is to prioritize the access of the most suitable node according to its position. Therefore, our objective is to find a mechanism that prioritizes access based on position while retaining the good properties of geometric distributions.

The remainder of this chapter is organized as follows. In Section 4.2 we briefly review the related work. A MAC-layer CBF scheme that prioritizes access according to node position is proposed in Section 4.3. In Section 4.4 we analytically evaluate the total and per-vehicle success probabilities and the average delay bounds. Finally, some concluding remarks are presented in Section 4.5.

4.2 Related work

Because of the dynamic nature of the mobile nodes in the network, finding and maintaining routes is very challenging in VANETs. Routing in VANETs (with pure ad hoc architectures) has been studied recently and many different protocols have been proposed. The authors in [84] classify them into five categories as follows: ad hoc, position-based, cluster-based, broadcast and geocast routing.

The objective of a geocast routing [89] is to deliver the packet from a source node to all other nodes within a specified geographical region. Most geocast routing methods are based on directed flooding, which tries to limit the message overhead and network congestion of simple flooding by defining a forwarding area and restricting the flooding inside it. Non-flooding approaches are also proposed [72, 77], aiming to limit the number of concurrent packets within the network.

We are particularly interested in the methods that use the contention scheme at the MAC layer to select the next forwarding node. This is achieved either by adapting the time when to forward the packet or by introducing rules on whether a given vehicle should forward the packet at all as in [54]. If CSMA/CA is used at the MAC layer, not only the contention window size may be selected [49] but also the distribution function used for the selection of the contention slots may be specifically adapted [110]. In this paper, we use the latter approach, adapting the method in [110] not only to reduce the message overhead, but to prioritize the retransmission of the packet by certain nodes (e.g. the farthest node).

4.3 Adaptations of Sift for prioritized access

Sift is the contention technique proposed in [110] for event-driven networks where a set of nodes tries to send a packet *simultaneously*. That is, when there are synchronized channel access attempts among many nodes. The key idea in Sift is to use a non-uniform, geometrically-increasing probability distribution for choosing the slots $(1, \ldots, CW)$ within a fixed-size contention window (CW), rather than varying the window size as in many traditional MAC protocols. The resulting protocol performs well when the number of nodes trying to send data is large in relation to CW, therefore it scales well when the number of contenders grows.

The Sift protocol assigns the probability that a node chooses the slot r as:

$$p_{CW}(r) = \frac{(1-\alpha)\alpha^{CW}}{1-\alpha^{CW}} \cdot \alpha^{-r}, \quad r = 1, \ldots, CW, \tag{4.1}$$

where $0 < \alpha < 1$ is a characteristic coefficient that determines the shape of the probability distribution.

Let us note that using CBF for GeoNetworking implies that all the packets are broadcast. In this mode, there is no reliability mechanisms, such as acknowledgment packets, and every transmission is independent of each other. In addition, all receiving nodes become potential forwarders (contenders) *simultaneously*. When a node wins the contention, it rebroadcasts the packet and the process is exactly the same, and actually keeps on this way until the packet reaches the destination area.

Therefore, we might expect that Sift as contention distribution optimizes the operation of GeoNetworking hop by hop. However, with the Sift distribution, all the vehicles use the same distribution for the slot selection, so all of them have the same probability of success in accessing the channel. On the contrary, for many applications in VANETs it is needed that certain vehicles have a success probability greater than the rest of them. A clear example is the usual GeoNetworking scenario described in Section 4.1, where the node located farthest away should have priority access. In summary, our purpose is to design a protocol that assigns higher success probability to the nearest node to the destination, but without decreasing the total success probability. This is achieved by allowing that each vehicle uses a different probability distribution for the slot selection, based on its own position. Next, we propose variations of Sift that retain its benefits but adapting the operation to the needs of the GeoNetworking protocol.

4.3.1 **Weighted Sift**

The first method we propose is to weight the Sift distribution according to the respective position of vehicles within the transmission range of the source node, giving a higher success probability to the farthest nodes.

Considering the number of contending vehicles equal to N, each one of these vehicles, $i \in \{1, \ldots, N\}$, will choose the slot $r \in \{1, \ldots, CW-1\}$ with probability

$$g_{CW}(i, r) = \gamma_i \cdot p_{CW}(r), \tag{4.2}$$

while the probability of vehicle i choosing slot CW is

$$g_{CW}(i, CW) = 1 - \gamma_i \sum_{r=1}^{CW-1} p_{CW}(r), \tag{4.3}$$

where p_{CW} is the Sift probability distribution over CW slots, as defined in equation (4.1).

The following step is to select properly the coefficients γ_i, with the condition that they should be bigger for the farthest vehicles. Since the $g_{CW}(i, r)$, $r \in 1, \ldots, CW$, constitutes a probability distribution, the sum of the first $CW - 1$ probabilities should be less than 1. From this observation we obtain $\gamma_i \leq 1/\sum_{r=1}^{CW-1} p_{CW}(r)$.

Let us define

$$\gamma_i = w_i \cdot \left(\sum_{r=1}^{CW-1} p_{CW}(r) \right)^{-1}, \tag{4.4}$$

with $w_i \in (0, 1)$ for $i \in \{1, \ldots, N\}$.

In order to assign higher values of w_i to the farthest vehicles we use the following inverted and truncated exponential distribution:

$$w_i = 1 - \frac{G(R - x_i)}{G(R)}, \tag{4.5}$$

where R denotes the transmission range of the source node, x_i the position of vehicle i in R (with respect to the source node) and finally G denotes the cumulative distribution function of an exponential distribution[1]. Let us remark that knowing the exact number of contenders is not required for this procedure.

4.3.2 **Per groups Sift**

The second method we consider is to divide the total number of vehicles into different groups, depending on their priorities. In particular, as we assume the priority is given by the position, we divide the transmission range into C intervals. The group of vehicles placed in each of these intervals selects their contention slots by using the Sift probability distribution with different values for the contention window (lower values for higher priorities).

Therefore, to each group of vehicles G_j, $j \in \{1, \ldots, C\}$, we associate a contention window CW_{G_j}. So, the probability distribution used by all the vehicles in that group is the following:

$$h_{CW_{G_j}}(r) = p_{CW_{G_j}}(r), \;\; r \in \{1, \ldots, CW_{G_j}\}, \tag{4.6}$$

where $p_{CW_{G_j}}$ is the Sift probability distribution over CW_{G_j} slots, as defined by equation (4.1).

4.4 Comparative Evaluation

In this section we present a comparative study to show the performance of the proposed methods as forwarding algorithms.

[1] Here we use an exponential distribution with mean $R/3$, but its choice is quite arbitrary. A study of the most appropriate parameters is needed.

We consider a one-dimensional scenario in which vehicles are uniformly distributed. The transmission range is assumed to be constant and equal to 300 meters for all the vehicles, and each vehicle knows its own position in the road segment. We assume that after a first transmission of the packet from the source node, all the vehicles in the transmission range have received correctly the packet and all of them contend to be the next forwarder.

We are concerned here exclusively with the probability of success of the proposed distributions, rather than the actual reception probability, which also depends on fading and channel error. We only need to know the total number of contenders and their positions for the analytic computations. In fact, a real implementation of the proposed protocols only would require to approximately know the transmission range and that each vehicle would know its own position.

4.4.1 Performance metrics

Assuming that there are N vehicles contending to be the next forwarder, and the size of the contention window is CW, for each protocol we construct a matrix $\mathbf{P}$ of dimension $N \times CW$, where $\mathbf{P}(i,j)$ is the probability of node i selecting backoff value j. Then, using this probability matrix, we compute the following stochastic metrics.

The probability of a successful transmission (of any node) in the slot S_r is calculated as the sum of the probabilities that one node selects slot r and the $N-1$ remaining nodes do not choose slots from the range of $1,\ldots,r$, which is given by the expression:

$$\Pi_{S_r}(\mathbf{P}) = \sum_{i=1}^{N} \mathbf{P}(i,r) \prod_{j=1,j\neq i}^{N} \left(1 - \sum_{s=1}^{r} \mathbf{P}(j,s)\right). \tag{4.7}$$

The probability of a successful transmission for an arbitrary vehicle V_i (in any slot) is calculated as the sum of the probabilities that the node selects one slot and all the other $N-1$ nodes choose later slots, which is given by the expression:

$$\Pi_{V_i}(\mathbf{P}) = \sum_{r=1}^{CW-1} \mathbf{P}(i,r) \prod_{j=1,j\neq i}^{N} \left(1 - \sum_{s=1}^{r} \mathbf{P}(j,s)\right). \tag{4.8}$$

Immediately from equation (4.8) we can compute the probability of a successful transmission for the last group of vehicles and the total probability of a successful transmission as follows:

$$\Pi_{LG}(\mathbf{P}) = \sum_{i \in LG} \Pi_{V_i}(\mathbf{P}), \tag{4.9}$$

$$\Pi_T(\mathbf{P}) = \sum_{i=1}^{N} \Pi_{V_i}(\mathbf{P}), \tag{4.10}$$

where LG is the last group of vehicles, that is, the nearest group of vehicles to the destination.

Additionally, we compute the mean winner vehicle (veh^*) and the mean slot number (sl^*) in which the successful transmission occurs, provided that the transmission attempt is successful:

$$veh^*(\mathbf{P}) = \frac{\sum_{i=1}^{N} i \cdot \Pi_{V_i}(\mathbf{P})}{\Pi_T(\mathbf{P})}, \tag{4.11}$$

$$sl^*(\mathbf{P}) = \frac{\sum_{r=1}^{CW} r \cdot \Pi_{S_r}(\mathbf{P})}{\Pi_T(\mathbf{P})}. \tag{4.12}$$

Finally, we will compute a lower and upper bound on the delay incurred by the packet to reach the destination area in the described scenario. As a first step, we compute these bounds for one successful transmission, as it is done in [110]. Let us call $L(\mathbf{P})$ to this delay, and T_{packet} to the time duration (in slots) for a packet transmission. If there is a collision, then the delay is at least T_{packet}, so

$$L(\mathbf{P}) \geq (1 - \Pi_T(\mathbf{P})) \cdot T_{packet} = LB_1(\mathbf{P}). \tag{4.13}$$

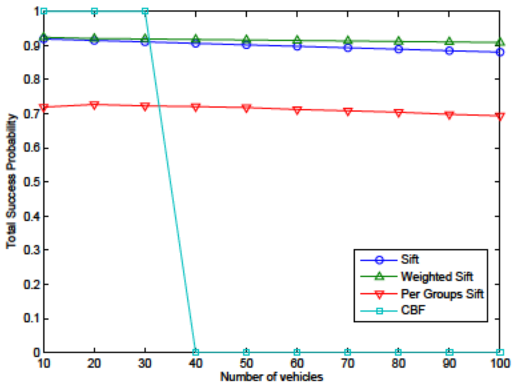
On the other hand, if there is a successful transmission in one round of contention, its latency is $sl^*(\mathbf{P})$. If there is a collision, $L(\mathbf{P})$ is at most $CW + T_{packet} + L(\mathbf{P})$. Hence, $L(\mathbf{P}) < \Pi_T(\mathbf{P}) sl^*(\mathbf{P}) + (1 - \Pi_T(\mathbf{P}))\,(CW + T_{packet} + L(\mathbf{P}))$, and simplifying, we obtain the expression for the upper bound

$$UB_1(\mathbf{P}) = sl^*(\mathbf{P}) + \left(\frac{1}{\Pi_T(\mathbf{P})} - 1\right)(CW + T_{packet}). \tag{4.14}$$

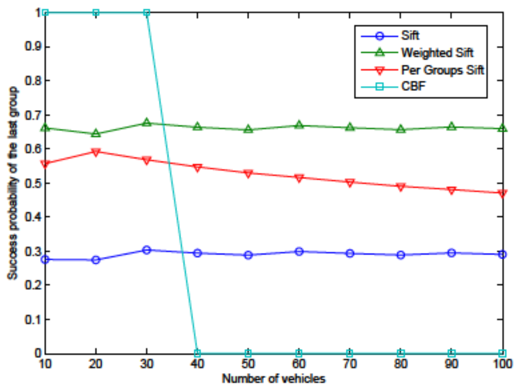
Now, for the total average delay, the lower and upper bounds are computed as follows:

$$LB(\mathbf{P}) = LB_1(\mathbf{P}) \cdot hops(\mathbf{P}), \tag{4.15}$$

$$UB(\mathbf{P}) = UP_1(\mathbf{P}) \cdot hops(\mathbf{P}), \tag{4.16}$$



(a) Total probability of a successful transmission with respect to the number of contenders.



(b) Probability of a successful transmission for the last group of vehicles, with respect to the number of contenders.

Figure 4.1: Total probability of a successful transmission and probability of a successful transmission for the last group of vehicles, with respect to the number of contenders.

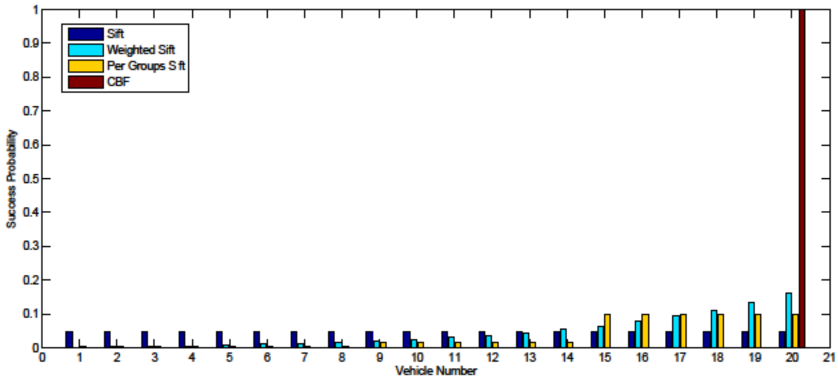
where $hops(\mathbf{P})$ is the average number of hops needed by the packet to reach the destination area when the matrix $\mathbf{P}$ is used for the slot selection. It depends on the distance of the average winner vehicle and the destination area to the source node, denoted as $dist(veh^*(\mathbf{P}))$ and $dist(Dest)$, respectively. It is computed as follows:

$$hops(\mathbf{P}) = \frac{dist(Dest)}{dist(veh^*(\mathbf{P}))}. \tag{4.17}$$

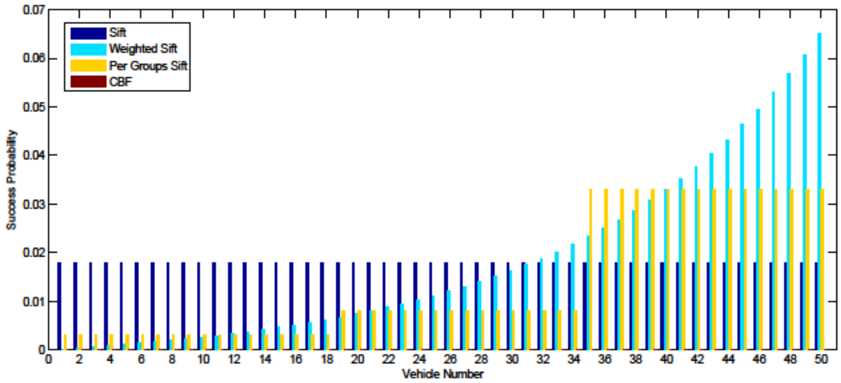
4.4.2 Results

The total probability of a successful transmission and the probability of a successful transmission for the last group of vehicles are shown in Figures 4.1(a) and 4.1(b), respectively, for the two proposed protocols, as well as for the original Sift protocol and for a basic Contention Based Forwarding (CBF) algorithm. The number of vehicles is varied between 10 and 100, while the contention window is always fixed to 32 slots. For the *Per groups Sift* protocol, 3 groups are used and the corresponding contention windows are 8, 16 and 32 slots. For the basic CBF mechanism, we assume that nodes select the slot r as the closest integer to $CW(1 - dist(V_i)/R)$ with probability 1, where $dist(V_i)$ is the distance from the node to the source.

It can be seen in Fig. 4.1(a) that when the number of vehicles is small (up to 30) the success probability for the CBF is 1, clearly outperforming the other protocols. However,



(a) Scenario with 20 contenders.



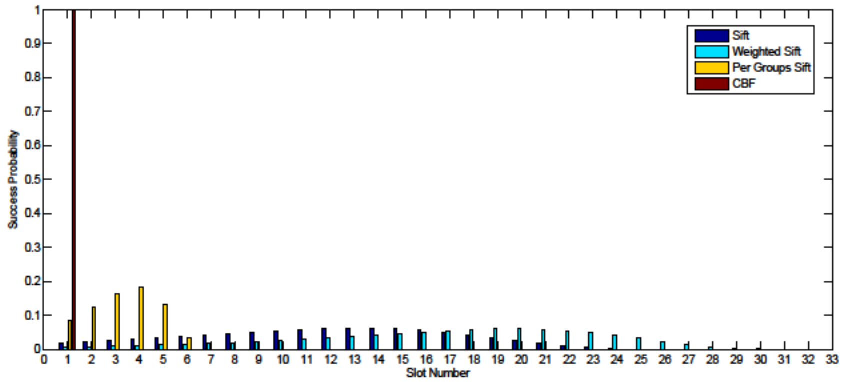
(b) Scenario with 50 contenders.

Figure 4.2: Probability of a successful transmission for each vehicle.

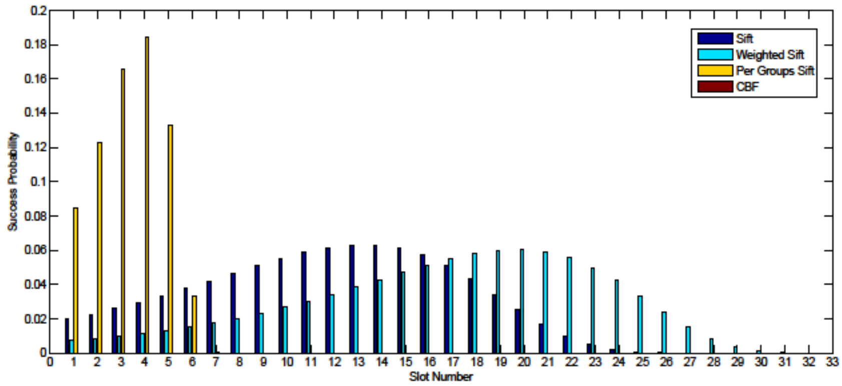
when the number of vehicles increases, the success probability becomes 0, because more than one vehicle is always close enough to select the same slot. On the contrary, the other proposals scale much better, maintaining almost the same success probability. The same holds for the probability of a successful transmission for the last group of vehicles (Fig. 4.1(b)).

Let us recall that our main goal is to give higher priority to the farthest vehicle. Our proposals achieve this goal and outperforms other possibilities. Looking at Fig. 4.1(a) again, the total success probability for the *Weighted Sift* is slightly superior than Sift, whereas for the *Per groups Sift* it is significantly lower than the others. Nevertheless, when we observe the success probability of the last group of vehicles (Fig. 4.1(b)), both the *Weighted Sift* and the *Per groups Sift* clearly outperform the original Sift protocol.

In Fig. 4.2 the probability of a successful transmission for each vehicle is shown when the number of contenders is fixed to 20 (Fig. 4.2(a)) and 50 (Fig. 4.2(b)).



(a) Scenario with 20 contenders.



(b) Scenario with 50 contenders.

Figure 4.3: Probability of a successful transmission in each slot.

As shown in Fig. 4.2(a), the probability of basic CBF is concentrated on the last vehicle, but when the number of vehicles is 50 (Fig. 4.2(b)) the success probability drops to zero for all the contending vehicles. For the two proposed protocols we can see better in Fig. 4.2(b) how the probability grows when approaching the last vehicle/group of vehicles.

In Fig. 4.3 the probability of a successful transmission in each slot is represented when the number of contenders is fixed to 20 (Fig. 4.3(a)) and 50 (Fig. 4.3(b)). For the first case, we can observe that the probability for the CBF is concentrated on the first slot, but when the number of vehicles is 50 the success probability is zero for all the slots. Looking at Fig. 4.3(b) we can observe that the *Per groups Sift* gives a high success probability to the first few slots. On the other hand, the original Sift outperforms the *Weighted Sift* in terms of the slot success probability, since the latter gives more success probability to later slots, which increases the forwarding delay. It seems that Sift would perform slightly better in terms of delay. However,

Table 4.1: Lower and upper bounds on the average delay for one successful transmission.

Protocol	Lower Bound (slots)	Upper Bound (slots)
Sift	3.08	14.31
Weighted Sift	2.57	17.54
Per Groups Sift	8.63	26.09

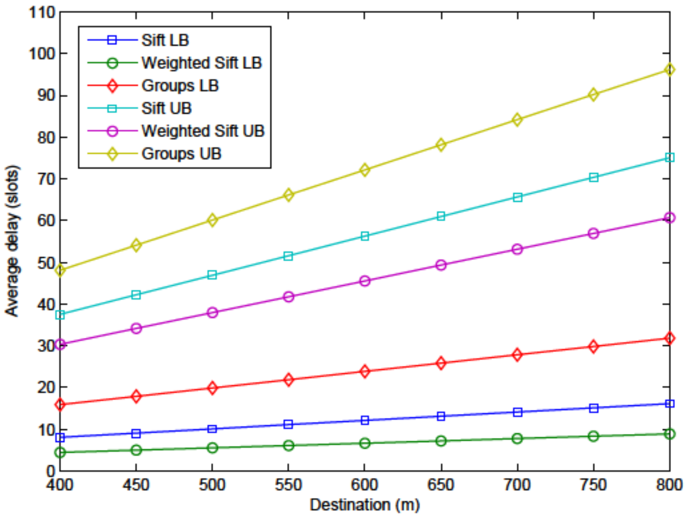


Figure 4.4: Lower and upper bounds on the expected delay incurred by the packet to reach the destination area when it is situated at different distances from the source node and $R = 300\ m$.

if we actually consider the average delay to the destination, that is, the multi-hop or end-to-end delay the results are different, as discussed next.

We consider a multi-hop scenario, where the destination area is situated at different distances (ranging between 400 and 800 m) from the source node, with $R = 300\ m$. We assume the time duration of a packet transmission to be 30 slots, and the number of vehicles in the transmission range equal to 60. For each protocol, the lower and upper bounds on the average delay for one successful transmission, computed through equations (4.13) and (4.14), are shown in Table 4.1, whereas the lower and upper bounds on the total average delay to reach the destination area are shown in Fig. 4.4. Thus, when we take into account the number of hops needed by the packet to reach the destination area, we can see how the *Weighted Sift* outperforms the usual Sift, unlike the *Per Groups Sift*.

4.5 Final remarks

In this chapter we studied CBF mechanisms for GeoNetworking implemented at MAC layer and considered two schemes that use geometrically-distributed contention slots. We have analytically evaluated the total and per-vehicle success probabilities and the average delay bounds and compared them with a basic CBF mechanism and the original Sift protocol. Our results show that a weighted geometric distribution effectively prioritizes the access based on position for a wide range of vehicle densities, while retaining the benefits of geometrical distributions with respect to success probabilities and delay bounds. In particular, while a CBF mechanism with static timers needs to adapt the contention window size to the number of contenders to avoid packet collisions, the proposed mechanisms scale gracefully and do not even need to know the number of contenders. With respect to the end-to-end delay, since the computed lower and upper bounds are weak, a realistic simulation is needed to show the true benefits of the proposed protocols, which is performed in the next chapter. On the other hand, we have arbitrarily fixed several parameters of the distributions, e.g., the window size. Therefore, a more detailed study on how to choose the more appropriate parameters and their influence is needed.

5 A survey and evaluation of MAC contention techniques for efficient geo-routing in vehicular networks

5.1 Introduction

As mentioned in the previous chapter, the standard CBF is completely implemented at the network layer. However, CBF might be also implemented directly at the MAC layer, in order to optimize its operation. For instance, implementing CBF at MAC layer should result in lower latency, since forwarding delay is removed and only access delay counts. This is an example of *cross-layer* design, which is widely used in vehicular networks [74]. Cross-layer design allows information to be exchanged and shared across layer boundaries in order to enable more efficient and robust protocols. Over the last years there have been a number of such cross-layer proposals aimed at optimizing the operation of geo-routing [33, 46, 94] as well as general purpose MAC approaches which could also be used quite effectively in this framework [96, 110].

In this chapter we provide a survey and comparative evaluation of the most relevant MAC-network cross-layer proposals in the context of vehicular networks. We focus on contention-based MAC mechanisms for wireless nodes. The majority of them are based on the CSMA/CA mechanism, whose operation and performance can be controlled by several parameters, namely: contention window size, random and deterministic carrier sense intervals as well as the probability distribution for the contention slots selection. Overall, it results in multiple degrees of freedom to optimize the medium access operation according to the most critical functionality offered by the network. In particular, for vehicular networks, the most important functionality is the delivery of geographically-addressed messages, which as mentioned before is performed by the GeoNetworking protocol. In some cases, this delivery might be critical, for instance, in safety-related applications which rely and are built on top of the functionality of the geo-routing protocol. In that case, optimizing the operation of the MAC layer may be a design requirement. Not only emergency messages benefit from optimized operation of the MAC layer, but also more general-purpose applications, since all of them[1] work on top of GeoNetworking.

In the survey part of this chapter, we discuss both, techniques specifically addressed to vehicular networks as well as general-purpose proposals, which can be adapted to VANETs. For the evaluation, we focus on the most critical functionality, that is, the delivery of emergency messages to a particular location in multi-hop scenarios. Thereby, we intend to define a baseline scenario and a comparison as fair as possible of the performance of different proposals.

The main contributions of this survey are the following:

- We provide a unified formal description of the discussed techniques in terms of the form that takes both the random and/or deterministic delays of the contention mechanism. Unlike previous works [39, 84, 127], we do not only qualitatively describe the operation of the contention mechanism, but also extract a more precise mathematical description of it, which is later used in a common evaluation framework.

- We provide a common framework for the analysis of the performance of the different techniques in the baseline scenario and define several metrics of interest. That is, we perform a simplified stochastic analysis of the proposals. The results are exact for one-hop scenarios and approximated, yet accurate enough, for multi-hop scenarios, as we shall show, which makes it a useful numerical tool for a quick evaluation of new proposals and mechanism variations, as we also illustrate with examples. Moreover, it is specifically aimed at the case in which vehicles select their contention slots in different ways, unlike other available models [73, 100, 118, 124].

- The evaluated proposals have been also simulated to validate our results. We provide a thorough evaluation of the different proposals for both ideal and realistic scenarios and compare them with the basic CBF mechanism specified by the standard. Our results show that there is little difference between them under realistic channel conditions, which should be taken into account in the design of new proposals.

[1] At least most of the Vehicle-to-Vehicle (V2V) applications, since the standard specifies it as "a network layer protocol that provides packet routing in an ad hoc network". Vehicle-to-Infrastructure (V2I) applications might use different network layers.

The remainder of this chapter is structured as follows. Related work is discussed in Section 5.2. Section 5.3 provides a categorization and formal description of the selected mechanisms. In Section 5.4, after the description of the reference scenario, an analytical model for the performance evaluation of the selected protocols is developed, which is then used to provide a comparative evaluation of the different techniques for the single-hop and multi-hop cases. In Section 5.5 we exemplify the use of our model as a quick evaluation tool for different combinations of the techniques. Realistic scenarios are investigated and discussed next in Section 5.6, including the evaluation of the basic CBF protocol proposed by the standard. Concluding remarks are given in Section 5.7.

5.2 Related work

We can find in the literature a number of surveys categorizing into different ways routing protocols for VANETs [39, 84, 127]. Beyond that, in addition to the categorization of routing protocols, in [120] and [103] the authors provide a classification of inter-vehicle communication applications and examine the applicability of different routing protocols to each application class. In general, it is concluded that position-based routing and geo-casting are more effective than other routing protocols for VANETs. Several works [89, 111, 112] survey this specific kind of protocols. In particular, in [111] geo-routing protocols are grouped into sender-based and receiver-based, being this last category in which our evaluated proposals would fall.

On the other hand, a thorough survey and general overview of cross-layer design for VANETs can be found in [74]. In this chapter we focus on the most relevant MAC-network cross-layer proposals in the context of vehicular networks. To the best of our knowledge, there is no work specifically categorizing cross-layer techniques based on the contention stage of the MAC layer for efficient geo-routing in VANETs. And in addition to the protocol classification, we develop a common framework for the performance analysis of the different protocols, which is also validated through simulation. The majority of existing surveys merely describe qualitatively the different protocols and only a few of them perform simulations for comparative purposes. However, unlike our work, formal descriptions and analytical models for quick numerical evaluation of the different proposals are rarely provided. In this latter regard, different analytical models can be found in [73, 100, 118, 124], but they cannot be applied to the specific case in which each vehicle selects its contention time slot in a different way.

5.3 Description of proposals

In this section we categorize and describe MAC proposals which either have been specifically proposed to be used to improve the operation of a geo-routing protocol, such as GeoNetworking, or are general-purpose but can be adapted conveniently for this context. The proposals are briefly described qualitatively but their operation is also formally expressed in terms of random and deterministic delays in a unified way. Previously, in the next subsection we discuss common operational aspects which we assume that hold for all the proposals, establish the main assumptions and the basic notation and compare them to the usual operation of the IEEE 802.11 *Distributed Coordination Function* (DCF) [16].

5.3.1 Common description of the contention mechanisms

From now on we assume that all vehicles use a basic carrier sense multiple access with collision avoidance (CSMA/CA). A station with a new packet to transmit first monitors the channel activity. If the channel is idle for a given *deterministic time interval*, t_D, the station transmits. Otherwise, if the channel is sensed busy (either immediately or during t_D), the station keeps on listening to the channel until it is measured idle for t_D. At this point, the station generates a *random backoff time interval*, t_R, before transmitting. If the channel has been idle for the duration of the random interval, the station transmits the packet. This basic operation coincides with IEEE 802.11 DCF. In that case, t_D equals one of the defined interframe spaces, usually the DCF Interframe Space (DIFS). In our description of the proposals, in order to facilitate a common analysis, we assume that both t_D and t_R times are discretized, that is, that their values are an integer multiple of some arbitrarily small time slot σ.

In DCF, the duration of the slotted backoff time t_R is uniformly chosen in the range $[0, CW]$, called the contention window (CW). The value of CW for unicast packets is doubled every time an unsuccessful transmission occurs. For broadcast packets, there is no acknowledgment or error recovery procedure and so CW remains constant all the time. In fact, vehicular communications are mainly broadcast in nature and all the proposals considered here do not modify the window size as a result of an error. Moreover, unlike DCF, we assume that the slots of t_R can be chosen from arbitrary probability distributions. Additionally, to achieve fairness and avoid channel capture with DCF, the backoff time counter is decremented only when the channel is sensed idle, and “frozen” when a transmission is detected on the channel. In vehicular networks, the main goal of a CBF mechanism is to select the next forwarder and, once a node has retransmitted a packet, the remaining nodes cancel the pending packet. Therefore, we have removed backoff timer suspension from the operation of the MAC. All the previous arguments allow us to assume that a memoryless backoff procedure is in use.

To summarize, in the next subsections we categorize the collected protocols according to the way they modify the standard backoff selection method described above and select the channel sensing delay. Therefore, let us define the total contention delay t_L, which is an integer multiple of a σ time slot and determines the exact time a node has to wait before it is allowed to transmit a frame, that is, the time it founds the medium idle. Then, $t_L = t_D + t_R$ is the sum of two terms: a deterministic term, t_D, and a random one, t_R, any of which can be zero in general.

Throughout the rest of the chapter we also use the following notation: an index n_i is used to identify a vehicle, the function $dist(x, y)$ is the Euclidean distance between two points, R_{tx} is the transmission range of a vehicle and x_s, x_d and x_{n_i} define the position of the source, the destination and the node n_i, respectively.

5.3.2 Deterministic strategies

First, we consider the approaches that use a pure deterministic delay. This approach works well when the delay is selected according to some criterion which makes it unique among the contending nodes. This technique is usually implemented at the network layer, and, in a typical non cross-layer design, it would operate on top of a standard MAC, such as 802.11p, which would add a random delay. Hence, it should be considered the baseline to which compare the advantages of a cross-layer MAC-network design. Thus, t_D takes a deterministic value between t_{min} and a maximum forwarding delay, t_{max}, as a function of selected parameters. For each vehicle n_i within the transmission range this waiting time is given by:

$$t_D(n_i) = t_{max} \cdot (1 - F(n_i)) + t_{min}, \tag{5.1}$$

where F is a function that measures the advantage obtained from a node being the next forwarder. The selection of this function depends on the particular application or the target objective of the broadcast process. $F(n_i)$ is usually a continuous function and therefore t_D takes a continuous value.

In a real implementation, it cannot actually take a continuous value, and so nodes providing a similar advantage (near values of the function F) may select identical backoff value. As a consequence, this method is more efficient the finer the discretization is, that is, in our notation the shorter the time slot is. An advantage of implementing these techniques at the MAC layer is that the delays might be potentially much shorter, but at the cost of a much higher implementation complexity. Though several of these schemes have been proposed in other contexts [99], we describe four representative examples for vehicular networks.

- **CBF** [54] and **Role-Based** [35]. This is one of the mechanisms specified for the GeoNetworking protocol [19]. Basically, in vehicular networks the delay is selected according to the distance from the source, as originally described in [54], and may be refined with other vehicle parameters [35], resulting in a typical advantage function:

$$F(n_i) = \max\left\{0, \frac{dist(x_s, x_{n_i})}{R_{tx}}\right\}. \tag{5.2}$$

  A clear disadvantage of this function is that it depends on the selected value for the transmission range R_{tx}, which is unknown and random in real scenarios, and may result in performance degradation.

- **Link-Lifetime-Based** [28]. For other applications the stability of the links between the sender and the receiver may be more important than the progress towards a final destination. To this purpose the authors in [28] use a combined function:

$$F(n_i) = \alpha \cdot St(n_i) + (1-\alpha) \cdot Pr(n_i), \tag{5.3}$$

  where St is a function that quantifies the stability of the link between the sender and the node n_i and Pr characterizes the progress that the packet achieves in the opposite direction of the movement. The parameter α is used to assign a different priority to any of the criteria.

- **MRSE** [81]. Multi-criteria Receiver Self-Election (MRSE) for vehicular networks is similar to the previous one but with an extended number of criteria. It selects the next forwarder using four criteria: link life-time, $\bar{t}$, optimal distance from sender to receiver, d, optimal transmission range, f, and received power, p. The following four-variable polynomial function of the selected parameters is used:

$$F(n_i) = \frac{\bar{t}_i^{w_1} d_i^{w_2} f_i^{w_3} p_i^{w_4}}{\bar{t}_{max}^{w_1} d_{max}^{w_2} f_{max}^{w_3} p_{max}^{w_4}}, \tag{5.4}$$

where $w_1, \ldots, w_4$ are the weights of each parameter, which are adjusted according to the local vehicle traffic density information.

5.3.3 Random strategies: contention window modification

We now turn to purely MAC approaches. First, we describe approaches that only employ the contention window range to adapt the channel access mechanism to the intended goal. Vehicles dynamically establish a contention range according to some classification criterion. Two main approaches can be found, which differ essentially in whether the selected range can overlap.

5.3.3.1 Overlapped contention windows assignment

In this case, each vehicle n_i computes its individual value $CW(n_i)$ as a function of some given criterion. In fact, most of the proposals are variations of the basic CBF scheme with more refined utility functions, based on multiple parameters. So, for each vehicle n_i the value of t_R is selected from a uniform distribution as:

$$t_R(n_i) \sim U(0, CW(n_i)). \tag{5.5}$$

In all the considered cases, the deterministic term t_D is also used and given a constant value, usually a DIFS, except for the EDCA* proposal, as we discuss later.

- **Fast Broadcast** [94]. This protocol was designed to reduce the time required by a message to propagate from the source to the farthest node in a certain strip-shaped area-of-interest, and it is based on a distributed mechanism for the estimation of the communication range of mobile nodes, $\hat{R}_{tx}$. The contention window for each vehicle is computed based on its position in the estimated communication range using the next formula:

$$CW(n_i) = \left\lfloor CW_{min} + \frac{\hat{R}_{tx} - dist(x_s, x_{n_i})}{\hat{R}_{tx}} (CW_{max} - CW_{min}) \right\rfloor, \tag{5.6}$$

  where CW_{max} and CW_{min} are the maximum and minimum contention window sizes of 802.11p. This is a variation of the basic CBF scheme discussed earlier, but implemented at the MAC layer and refined with the estimation procedure for the transmission range. This latter refinement makes it suitable for a real deployment, but at the cost of increased protocol overhead and complexity.

- **DBA-MAC** [33]. Authors proposed a distributed dynamic clustering algorithm to create a dynamic virtual backbone in a vehicular network. The vehicle members of the backbone are then responsible for efficient broadcast of emergency messages. In the cluster generation, parameter named *Fit Factor* (FF), based on the relative speed, the distance to the backbone node and the transmission range R_{tx}, is computed and then used to dynamically control the contention window, as indicated by the following equation:

$$CW(n_i) = \lfloor CW_{min} + \max\{0, 1 - FF(n_i)\}(CW_{max} - CW_{min}) \rfloor . \tag{5.7}$$

  Again, this mechanism suffers from the basic problem of CBF proposals based on an assumed constant transmission range, which is unrealistic. In addition, the cluster generation algorithm adds extra complexity and overhead to the procedure.

- **Benefit-Based** [46]. It is a general-purpose communication scheme which differentiates data traffic according to the benefit it is likely to provide to potential recipients. This benefit is quantified through a function (MB) that takes into account the message context (e.g. message age, time since last broadcast, etc.), the vehicle context (e.g. driving direction, distance to the last forwarder, vehicle speed, etc.) and the information context (e.g. purpose of traveling, information accuracy, news value, etc.).

  Then, for each vehicle n_i the CW is adapted to the benefit of the currently handled message:

$$CW(n_i) = \lfloor CW_{min} + (1 - MB(n_i))(CW_{max} - CW_{min}) \rfloor . \tag{5.8}$$

  The flexible scheme proposed can be adapted to a variety of applications. For instance, with an adequate definition of the benefit function, the procedure becomes equal to either the Fast Broadcast [94] or DBA-MAC [33] approaches.

- **EDCA*** [16]. Within this category of protocols we also include the EDCA mechanism of the 802.11 standard, by which different classes of frames can be given priority over another in their competition to access the medium. It defines up to four *Access Categories* (ACs) of frames, each of which has its own queue. Each frame arriving at the MAC layer with a priority is mapped into one of the four possible ACs. The priority advantage is the result of modifying two parameters of the protocol. The first one is the contention window size; both the minimum and maximum CW values can be configured per AC.

The second parameter is the delay after the medium goes idle before a contender either begins a transmission or initiates a backoff. So, for each particular AC the delays t_D and t_R are computed as follows:

$$t_D(AC) = SIFS + AIFSN[AC], \tag{5.9}$$

$$t_R(AC) \sim U(0, CW[AC]), \tag{5.10}$$

where SIFS (Short Interframe Space) and AIFSN (Arbitration Interframe Space) are protocol parameters. Obviously, this scheme can be adapted to optimize the operation of the network layer in different ways. Any of the utility functions discussed so far may be used to map the frame to an AC. Let us note that in the following evaluation we focus on broadcast communications and so we assume EDCA works also with no window size increase and backoff timer suspension, that is, as a memoryless backoff, and therefore we call it EDCA*.

5.3.3.2 **Disjoint contention windows assignment**

This method involves dividing the range $[0, CW]$ into m non-overlapping intervals, $I_1, \ldots, I_m$, so that each vehicle, depending on its priority, selects a backoff value randomly from one of the m intervals. Assuming that the length of each interval I_i is W_i, the resulting intervals are:

$$[0,\ W_1 - 1]\,,\ [W_1,\ W_1 + W_2 - 1]\,, \ldots, \left[\sum_{i=1}^{m-1} W_i,\ \sum_{i=1}^{m} W_i - 1\right]. \tag{5.11}$$

We define the function:

$$w_k = \begin{cases} 0, & \text{for } k = 1, \\ w_{k-1} + W_{k-1}, & \text{for } k = 2, \ldots, m. \end{cases} \tag{5.12}$$

Then, the values of t_D and t_R are defined as follows:

$$t_D(n_i) = \text{DIFS} + \sum_{k=1}^{m} w_k \cdot \mu_{I_k}(n_i), \tag{5.13}$$

$$t_R(n_i) \sim U\left(0, \sum_{k=1}^{m} W_k \cdot \mu_{I_k}(n_i) - 1\right), \tag{5.14}$$

where $\mu_{I_k}(n_i)$ is equal to 1 if vehicle n_i is associated with interval I_k and 0 otherwise.

- **Smart Broadcast** [49] and **PBCC** [130]. The authors of [49] defined a distributed position-aware broadcast protocol for highway inter-vehicular networks, which is able to guarantee high reliability, low propagation latency and redundancy reduction, without requiring perfect knowledge of the network topology. Given the transmission range, R_{tx}, it is partitioned into m adjacent and non-overlapping sectors numbered from S_1 to S_m, starting by the farthest sector from the source node. Each sector S_i is associated to a size of the contention window K_i, and the disjoint intervals are constructed as explained before, so that the highest priority corresponds to the farthest nodes from the source.

  In [130] the protocol used is the particular case of Smart Broadcast in which all the contention windows associated to the intervals have the same length. As we shall see, the use of disjoint windows outperforms the overlapped windows approach, but it shares the weakness of using a constant transmission range, which could be further improved with a more realistic estimation procedure.

5.3.4 Random strategies: probability distribution modification

In all the preceding random strategies the backoff counter was selected uniformly within the specified contention window range, which was adjusted in order to prioritize certain contenders. On the contrary, the idea under this methodology is to carefully choose a nonuniform probability distribution that nodes use to randomly select their backoff counters, but keeping the contention window constant. Depending on the shape and the particular characteristics of the probability distribution used, some contenders will have more priority to access the channel than others.

If we choose a discrete probability distribution g_{CW} over the slots of the contention window, then the random term t_R of the waiting time for each vehicle n_i is given by the probability mass function:

$$P\left(t_R(n_i) = j\right) = g_{CW}(j),\ j = 0, \ldots, CW. \tag{5.15}$$

The deterministic term t_D, if present, can be set to a constant value such as DIFS.

- **Sift** [110]. In many situations, the network operation synchronizes the medium access of all nodes, that is, all receivers of a packet immediately become potential forwarders and contend for the medium. In this particular case, in [110] it is shown that there exists an optimal distribution for the contention slots that maximizes the contention success probability. Although the optimal distribution cannot be implemented in practice, geometric distributions approximate the optimal one. So authors proposed an approximation that uses a truncated geometric distribution. The size of the contention window is constant, and the probability $g_{CW}(j)$ of selecting a certain slot j increases with the slot number. The probability of choosing the slot j is given by:

$$g_{CW}(j) = \frac{(1-\alpha) \cdot \alpha^{CW+1}}{1-\alpha^{CW+1}} \cdot \alpha^{-(j+1)},\ j = 0, \ldots, CW, \tag{5.16}$$

where $\alpha = g_{CW}(j)/g_{CW}(j+1)$ is a characteristic coefficient that determines the shape of the probability distribution, and it is adapted to the estimated number of contenders. The geometric distribution assigns low probability to initial slots, and high probability to the last few slots in the contention window, which greatly reduces the probability of packet collision.

- **COMIC** [96]. In this work a scheme for backoff-based collision resolution is proposed. The contention window is fixed for all the contenders, but the uniform contention slot selection distribution over $[0, CW]$ is replaced by a truncated normal distribution:

$$g_{CW}(j) = \frac{f(j)}{\int_0^{CW} f(r)dr},\ j = 0, \ldots, CW, \tag{5.17}$$

where $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ is the normal probability distribution function.

This procedure is designed for non-broadcast communication processes and it is based on the standard Binary Exponential Backoff procedure for contention window expansion and contraction upon collision and success. So, for consecutive backoff stages, the shape of the truncated normal distribution is intelligently tuned, adapting μ and σ, according to the backoff value previously selected, so that the selection likelihood of relatively less collision-probable contention slots is maximized. However, for broadcast communication processes there are no retransmission attempts, and therefore there is no history information available, so μ and σ are taken as $\left\lfloor \frac{CW}{2} \right\rfloor$ and $\sqrt{\frac{CW}{2}}$, respectively.

As we shall show, both procedures perform remarkably well in terms of global transmission success, but unlike the previously discussed ones, all the vehicles have equal success probability. Depending on the application, this might not be desirable, for instance, if we want to maximize the packet progress. In the previous chapter we proposed some modifications to correct it.

5.3.5 Summary

In Table 5.1 we summarize the categorization of the described protocols, together with their particular characteristics. Most of the considered procedures, at least those specifically proposed for vehicular networks, have been designed to optimize the packet advance. And most of them are variations of the basic CBF scheme with refined utility functions involving the physical state of the vehicle and communications, such as distance, position or link quality.

Up to this point we have formally described and qualitatively discussed several alternatives, but attending only to their description it is not obvious which are the real advantages and drawbacks of the different proposals. Moreover, in the literature, they have been evaluated in quite different scenarios and with different assumptions and parameters and the results available are not usually directly comparable. Therefore, in the following sections, we provide a common evaluation stochastic model, a set of performance metrics and a baseline scenario in order to provide a fair comparison of the different proposals.

5.4 Performance evaluation

In this section we present a comparative study to show the performance of the discussed procedures as forwarding algorithms for the GeoNetworking routing [19]. It supports the delivery of packets in geographical areas and is the main service provider for upper transport entities in ad hoc mode. Hence, new proposals for VANET MAC mechanisms may take it into account in their design and it is reasonable to evaluate how its performance is affected by the MAC proposals. CBF is one of the basic forwarding algorithms proposed by GeoNetworking. It is specified at the network layer. Therefore, we additionally consider that any cross-layer MAC-based CBF proposal should be compared with this basic algorithm. Finally, many applications with different requirements can be implemented on top of GeoNetworking, but delivery of emergency messages is usually regarded as the most critical one. In this case, it is normally required that packets advance as much and as quickly as possible, and consequently we define and evaluate related performance metrics in order to compare the proposals.

We provide a common analytical model for the evaluation, based on the definition of an appropriate matrix, each element of it being the probability that a node selects a given number of time slots for channel sensing before transmitting. This way, we incorporate both the deterministic and random terms of the contention delay and can use seamlessly the previous description of the proposals. All the metrics are defined as a function of this matrix, and so

Table 5.1: Summary of broadcasting techniques based on contention

Classification	Kernel Function	Protocol	Particular Characteristics
Deterministic strategies *Timers technique*	$t_D(n_i) = t_{max} \cdot (1 - F(n_i)) + t_{min}$ $t_R(n_i) = 0$ $P(t_L(n_i) = j) = \begin{cases} 1, & j = t_{max} \cdot (1 - F(n_i)) + t_{min} \\ 0, & j \neq t_{max} \cdot (1 - F(n_i)) + t_{min} \end{cases}$	CBF, Role-Based	$F(n_i) = \max \left\{ 0, \frac{dist(x_s, x_d) - dist(x_{n_i}, x_d)}{R_{tx}} \right\}$
		Lifetime-Based	$F(n_i) = \alpha \cdot St(n_i) + (1 - \alpha) \cdot Pr(n_i)$
		MRSE	$F(n_i) = \frac{\bar{t}_i^{w_1} d_i^{w_2} f_i^{w_3} p_i^{w_4}}{\bar{t}_{max}^{w_1} d_{max}^{w_2} f_{max}^{w_3} p_{max}^{w_4}}$
Random strategies CW Modification *Overlapped CW*	$t_D(n_i) = \text{DIFS}$ $t_R(n_i) \sim U(0, CW(n_i))$ $P(t_L(n_i) = j) = \begin{cases} 0, & j = 0, \dots, t_D(n_i) - 1 \\ \frac{1}{CW(n_i)+1}, & j = t_D(n_i), \dots, t_D(n_i) + CW(n_i) \end{cases}$	Fast Broadcast	$CW(n_i) = \left\lfloor \frac{\hat{R}_{tx} - dist(x_s, x_{n_i})}{\hat{R}_{tx}} (CW_{max} - CW_{min}) + CW_{min} \right\rfloor$
		DBA-MAC	$CW(n_i) = \lfloor \max\{0, 1 - FF(n_i)\}(CW_{max} - CW_{min}) + CW_{min} \rfloor$
		Benefit-Based	$CW(n_i) = \lfloor (1 - MB(n_i))(CW_{max} - CW_{min}) + CW_{min} \rfloor$
		EDCA*	$t_D(AC) = SIFS + AIFSN[AC]$ $t_R(AC) \sim U(0, CW[AC])$
Random strategies CW Modification *Disjoint CW*	$t_D(n_i) = \text{DIFS} + \sum_{k=1}^{m} w_k \cdot \mu_{I_k}(n_i)$ $t_R(n_i) \sim U\left(0, \sum_{k=1}^{m} W_k \cdot \mu_{I_k}(n_i) - 1\right)$ $P(t_L(n_i) = j) = \begin{cases} 0, & j = 0, \dots, t_D(n_i) - 1 \\ \frac{1}{\sum_{k=1}^{m} W_k \cdot \mu_{I_k}(n_i)}, & j = t_D(n_i), \dots, t_D(n_i) + \sum_{k=1}^{m} W_k \cdot \mu_{I_k}(n_i) - 1 \end{cases}$	Smart Broadcast, PBCC	$R_{tx} = S_1 \cup S_2 \cup \dots \cup S_m$ S_1 is the farthest sector from the node Nodes in S_i selects their backoff from I_i
Random strategies *Pdf Modification*	$t_D(n_i) = \text{DIFS}$ $t_R(n_i) \sim g_{CW}$ $P\left(t_L(n_i) = j\right) = \begin{cases} 0, & j = 0, \dots, t_D(n_i) - 1 \\ g_{CW}(j), & j = t_D(n_i), \dots, t_D(n_i) + CW \end{cases}$	Sift	$g_{CW}(j) = \frac{(1 - \alpha) \cdot \alpha^{CW+1}}{1 - \alpha^{CW+1}} \cdot \alpha^{-(j+1)}$
		COMIC	$g_{CW}(j) = \frac{f(j)}{\int_0^{CW} f(r)dr}, \quad f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

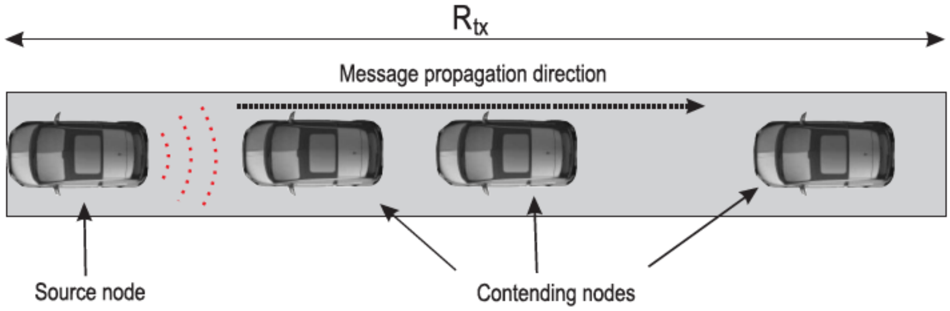


Figure 5.1: Scenario under consideration.

we obtain a unified framework to compare different proposals only by using the corresponding matrices. The results are exact for a single-hop scenario and we provide approximations for multi-hop scenarios.

5.4.1 Scenario description

GeoNetworking [19] supports the communication among individual ITS stations as well as the distribution of packets in geographical areas. If the source node does not belong to the destination geo-area, then the packet should be forwarded until reaching a node which belongs to this area, which takes care on delivering the packet to its destination.

We consider a vehicular ad-hoc network consisting of a strip-shaped area, where vehicles are randomly distributed according to a one-dimensional Poisson process of intensity λ. The parameter λ represents the density of vehicles on the road, which is defined as the average number of vehicles per meter. We assume that each vehicle is equipped with a GPS-like device so that each node knows its own geographical position. We also suppose that all the nodes in the considered area have synchronized time scale.

We assume that a source node (positioned at the beginning of the area) generates a broadcast message that has to be propagated along the strip in the opposite direction of movement, as depicted in Fig. 5.1. Each broadcast message contains a header field that includes the spatial coordinates of the transmitting node, the message propagation direction and information about the destination (a particular node or a geographical area). We also assume that the broadcast message can be correctly received by all vehicles within the transmission range area R_{tx}, that is, we suppose an ideal deterministic radio propagation model with no errors. All nodes try to forward the message, contending to be the next forwarder. Therefore, the number of contending nodes will be a random variable with Poisson distribution of parameter λR_{tx}.

5.4.2 Performance metrics

We start with the stochastic analysis of the protocols, taking into account that in the techniques under evaluation the nodes in the network not necessarily select their waiting time with the same probability distribution nor an equal window size. This stochastic analysis extends the performance metrics presented in 4.4.1, however for completeness and readability we repeat here some of the equations. So, for each protocol we construct a matrix $\mathbf{P}$ where $\mathbf{P}(i,j)$ is the probability of node i selecting j time slots of σ duration for channel sensing before transmitting, that is, $\mathbf{P}(i,j) = P(t_L(n_i) = j)$. Therefore, the dimension of $\mathbf{P}$ is $N \times (\max_i t_L(n_i))$, where N is the number of contenders and $\max_i t_L(n_i)$ is the maximum possible delay that can be chosen by any of the nodes. Let us remark that t_L includes both the deterministic and random number of slots.

For the sake of clarity, we illustrate the construction of the matrix $\mathbf{P}$ with a brief example. Suppose that there are three nodes in the network that select their delay in the following way: the first node has $t_D(1) = 0$ and selects $t_R(1)$ uniformly from $[0,2]$; the second one has $t_D(2) = 1$ and selects $t_R(2)$ uniformly from $[0,2]$; and the last node has $t_D(3) = 1$ and selects $t_R(3)$ uniformly from $[0,1]$. So, the resulting matrix is:

$$\mathbf{P} = \begin{pmatrix} 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 1/2 & 1/2 & 0 \end{pmatrix}. \tag{5.18}$$

Then, by using this probability matrix we compute the probability of a successful transmission by the vehicle i in the slot r, which is the probability of vehicle i selecting slot r multiplied by the probability of all the other vehicles selecting later slots:

$$\Pi_{V_i,S_r}(\mathbf{P}) = \mathbf{P}(i,r) \prod_{j=1,j\neq i}^{N} \left(1 - \sum_{k=0}^{r} \mathbf{P}(j,k)\right). \tag{5.19}$$

Then, by addition of the corresponding probabilities we can obtain the success probability in a specific slot (Π_{S_r}), the probability of a successful transmission by a particular vehicle

(Π_{V_i}) and the total success probability (Π_T):

$$\Pi_{S_r}(\mathbf{P}) = \sum_{i=1}^{N} \Pi_{V_i,S_r}(\mathbf{P}), \tag{5.20}$$

$$\Pi_{V_i}(\mathbf{P}) = \sum_{r=0}^{W} \Pi_{V_i,S_r}(\mathbf{P}), \tag{5.21}$$

$$\Pi_{T}(\mathbf{P}) = \sum_{i=1}^{N}\sum_{r=0}^{W} \Pi_{V_i,S_r}(\mathbf{P}), \tag{5.22}$$

where W is the maximum slot number that can be chosen by any of the nodes, that is, $W = \max_i t_L(n_i)$.

Next, we compute the winner vehicle (veh^*), provided that the transmission attempt is successful:

$$veh^*(\mathbf{P}) = \frac{\sum_{i=1}^{N} i \cdot \Pi_{V_i}(\mathbf{P})}{\Pi_T(\mathbf{P})}. \tag{5.23}$$

From this equation we can extract the average position of the winner vehicle, pos^*, if we know the positions of each vehicle in our experiment.

Similarly, the mean slot number when the successful transmission starts is given by the following expectation:

$$t_s(\mathbf{P}) = \frac{\sum_{r=0}^{W} r \cdot \Pi_{S_r}(\mathbf{P})}{\Pi_T(\mathbf{P})}. \tag{5.24}$$

On the other hand, in the slot r there is no collision if one of the following situations occurs: there is success or collision before slot r; there is success in slot r; or all the nodes choose their slots after slot r. So, the probability of a collision in the slot r is:

$$\Omega_{S_r}(\mathbf{P}) = 1 - \sum_{k=0}^{r-1} (\Pi_{S_k}(\mathbf{P}) + \Omega_{S_k}(\mathbf{P})) - \Pi_{S_r}(\mathbf{P}) - \prod_{j=1}^{N} \sum_{k=r+1}^{W} \mathbf{P}(j,k). \tag{5.25}$$

Therefore, the mean slot number when the collision occurs is given by:

$$t_c(\mathbf{P}) = \frac{\sum_{r=0}^{W} r \cdot \Omega_{S_r}(\mathbf{P})}{1 - \Pi_T(\mathbf{P})}, \tag{5.26}$$

where t_c is defined for $N \geq 2$ since the collision may only happen if more than one node compete for the channel access.

Now, from these stochastic metrics, we compute the critical performance metrics in the considered emergency-message scenario: the *Mean Access Delay* (T_{acc}) and the *End-to-end*

Delay (T_e), which are expressed in seconds.

Let σ and L_{Pkt} be the time duration for a slot and a packet transmission, also expressed in seconds. The Mean Access Delay is defined as the average time from the instant the nodes start trying to send a packet until the beginning of a successful transmission. It is computed as follows:

$$T_{acc}(\mathbf{P}) = (E[A(\mathbf{P})] - 1) \cdot (\sigma \cdot t_c(\mathbf{P}) + L_{Pkt}) + \sigma \cdot t_s(\mathbf{P}), \tag{5.27}$$

where $E[A(\mathbf{P})]$ represents the expected number of attempts until a node wins the contention. Let us note that the probability of succeeding at the i-th attempt equals $(1-\Pi_T(\mathbf{P}))^{i-1}\Pi_T(\mathbf{P})$, so the expected number of attempts is computed as follows:

$$E[A(\mathbf{P})] = \sum_{i=1}^{\infty} i\,(1 - \Pi_T(\mathbf{P}))^{i-1}\,\Pi_T(\mathbf{P}) = \frac{1}{\Pi_T(\mathbf{P})}, \tag{5.28}$$

being the last equality a consequence of the infinity sum of a geometric series. The number of attempts may be restricted by a *hop limit* parameter, in which case, the summation would be truncated to such value.

Finally, the End-to-end Delay is defined as the average delay incurred by the packet to reach the destination area, which in a one-hop scenario is simply:

$$T_e(\mathbf{P}) = T_{acc}(\mathbf{P}) + L_{Pkt}. \tag{5.29}$$

The above metrics are exact for a single-hop network, where all the nodes are in range of each other. They can be used for a fair comparison in a number of ideal situations. However, we are interested in the performance of the different proposals in a more realistic scenario where the emergency message has to advance multiple hops. In that case, we cannot take advantage of a memoryless model, but it depends on the position of the colliding nodes, and the analysis become more complex. In order to keep it simple, we provide the Algorithm 2 to approximate the metrics of interest in a multi-hop scenario.

The rationale for this algorithm is as follows, for an example with two hops. For the first hop, we have to compute the one-hop metrics as above. In case of successful transmission, in the next hop we will be exactly under the same conditions as in the first hop, so we only have to compute the basic metrics for the new contenders, as we do in step 3(a). Now, in case of collision, it should be noticed that a packet always “advances”, in the sense that there are nodes that receive the packet correctly because they are in range of one transmitter but out of

Algorithm 2 Approximation of multi-hop metrics.

Require: $\mathbf{P}$, R_{tx}, $Dest$.

1: $Hops = 1$.
Compute the basic metrics in the first hop, that is, for the interval $(0, R_{tx})$:

$$\Pi_T,\ pos^*,\ t_s,\ t_c,\ T_{acc},\ T_e.$$

2: Initialize the success probability in the previous hop:

$$\Pi_T^0 = \Pi_T.$$

3: While $pos^* < Dest - R_{tx}$:

a) *Assuming that in the current hop there is a successful transmission:*

- Compute the basic metrics in the next hop, that is, for the interval $(pos^*, pos^* + R_{tx})$:

$$\Pi_T^1,\ pos^{*1},\ t_s^1,\ t_c^1,\ T_{acc}^1,\ T_e^1.$$

b) *Assuming that in the current hop a collision occurs:*

- Estimate the position of the two vehicles implicated in the packet collision, $posCol1$ and $posCol2$.

- Compute the basic metrics in the next hop, that is, for the interval $(posCol1 + R_{tx},\ posCol2 + R_{tx})$:

$$\Pi_T^2,\ pos^{*2},\ t_s^2,\ t_c^2,\ T_{acc}^2,\ T_e^2.$$

c) Update the hops number and the global metrics:

$Hops = Hops + 1$;
$\Pi_T = \Pi_T + \Pi_T^1 \Pi_T^0 + \Pi_T^2 (1 - \Pi_T^0)$;
$pos^* = pos^* + pos^{*1} \Pi_T^0 + pos^{*2} (1 - \Pi_T^0)$;
$ts = ts + ts^1 \Pi_T^0 + ts^2 (1 - \Pi_T^0)$;
$tc = tc + tc^1 \Pi_T^0 + tc^2 (1 - \Pi_T^0)$;
$T_{acc} = T_{acc} + T_{acc}^1 \Pi_T^0 + T_{acc}^2 (1 - \Pi_T^0)$;
$T_e = T_e + T_e^1 \Pi_T^0 + T_e^2 (1 - \Pi_T^0)$.

d) Update the success probability in the previous hop:

$$\Pi_T^0 = \Pi_T^1 \Pi_T^0 + \Pi_T^2 (1 - \Pi_T^0).$$

4: Compute the average of the global metrics (except for T_e, which is the accumulation of successive hops delays):

$\Pi_T = \Pi_T / Hops$;
$pos^* = pos^* / Hops$;
$ts = ts / Hops$;
$tc = tc / Hops$;
$T_{acc} = T_{acc} / Hops$.

return $\Pi_T,\ pos^*,\ t_s,\ t_c,\ T_{acc},\ T_e$.

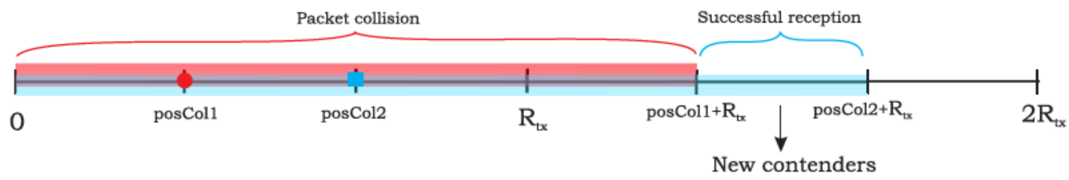


Figure 5.2: Interval where the new contenders are placed after a packet collision.

range of the other one (see Fig. 5.2). Therefore, in this case, the number of new contenders at the next hop depends on the relative distance between the colliding nodes, as can be seen in Fig. 5.2. Hence, in step 3(b) we first estimate the average position of the two[1] colliding nodes, which depends on the particular mechanism (for instance, for the Uniform one they are placed at $R_{tx}/3$ and $2R_{tx}/3$), and afterwards we compute the metrics for the new contenders.

Once we have computed the metrics for the two possible cases, success or collision, we compute in step 3(c) their averages multiplying them by the probability of success or collision in the previous hop. In this way we obtain the average metrics for the new hop, which turns into current hop for the next iteration. Then, the process starts again from step 3(a), and continues until the packet reaches the destination. Finally, in step 4 we compute the average per-hop metrics, which are returned together with the total end-to-end delay.

As we show in Section 5.4.4, our simulation results validate this approximation.

5.4.3 Single-hop scenarios

In this section we verify the correctness of our analytical model and perform a comparative study between some of the selected protocols, as well as an evaluation of the influence of different parameters on the performance metrics.

The protocols considered in the comparative study are shown in Table 5.2, including an standard contention procedure labeled as "Uniform". We exclude from the evaluation those protocols that need too much extra context information, except for Fast Broadcast, for which we have implemented the transmission range estimation. We consider at least one protocol from each category and we use a parameter, K, to homogenize the size of the contention windows for the different protocols, trying to make the comparative study as fair as possible. All the values are shown in Table 5.2 as a function of the parameter K. For the proposals based on position we assume they know their position exactly, and those based on groups use a number

[1]We assume here that only two vehicles transmit at the same time in a packet collision.

Table 5.2: Deterministic delays and contention window sizes for the protocols under evaluation.

Protocol	**Delay and contention window size**
Uniform	$t_D = DIFS, CW = 2K - 1$
Fast Broadcast	$t_D = DIFS$ $CW_{min} = K - 1$ $CW_{max} = 4K - 1$
EDCA*	$AIFSN[0] = 9, CW[0] = 2K - 1$ $AIFSN[1] = 6, CW[1] = 2K - 1$ $AIFSN[2] = 3, CW[2] = K - 1$ $AIFSN[3] = 2, CW[3] = K/2 - 1$
Smart Broadcast (with $m = 4$)	$t_D = DIFS$ $I_1 = [0, K - 1]$ $I_2 = [K, 2K - 1]$ $I_3 = [2K, 3K - 1]$ $I_4 = [3K, 4K - 1]$
Sift	$t_D = DIFS, CW = 2K - 1$
COMIC	$t_D = DIFS, CW = 2K - 1$

of m groups. For EDCA* we have defined a map that replicates that of Smart Broadcast, that is, there are m groups and higher priority access categories are assigned to more distant groups.

To validate our analysis, we have simulated the procedures with the OMNeT++ network simulator and its Inetmanet 2.0 extension [9]. In the simulations, the source sends a new packet every 10 s. All the simulations are run for 5000 s and all the scenarios, for every vehicle density, have been replicated with different seeds. For all the metrics, their 95% confidence intervals have been computed and are shown as error bars in the figures. Let us note that there are slight differences in the simulation with respect to the ideal situation analyzed in previous sections. The simulations are more realistic in the sense that nodes involved in a packet collision are not aware of the collision. Since there are no acknowledgement or error recovery, the involved nodes do not participate in a retransmission.

The evaluation is conducted for the scenario described at the beginning of the present section, varying the vehicle density and the size of the contention windows (with the parameter K). The values of the parameters used for the performance metrics computation are shown

Table 5.3: Parameters used in the evaluation.

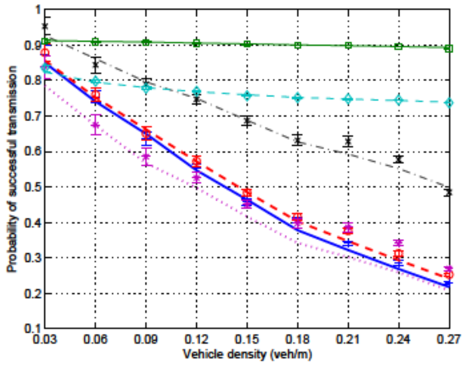
Parameter	Fig. 5.3	Fig. 5.4
λ	$[0.03, 0.27]\ veh/m$	$0.21\ veh/m$
K	16	$[16, 64]$
L_{Pkt}	768 μs	
σ	9 μs	
DIFS	28 μs	
SIFS	10 μs	
R_{tx}	300 m	
$dist(Dest)$	600 m	
Hop Limit (HL)	10	

in Table 5.3. We consider no background data traffic, so that only the broadcast message is propagated over the network. The impact of node mobility is disregarded in this evaluation, since the variation of node positions is negligible for the duration of a packet exchange[1] and it has a minor influence on the performance of message broadcast with high data rates and short safety message lengths [88].

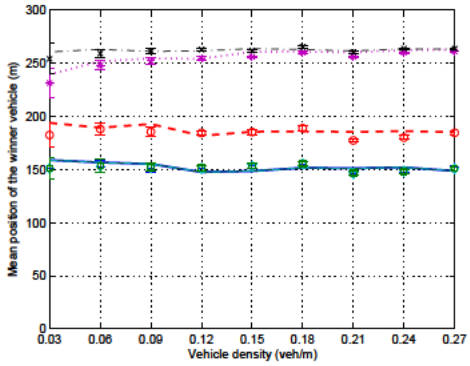
First, we validate our model in a single-hop scenario with a deterministic free-space signal propagation model, where all the nodes are in range of each other. Therefore, all the proposals based on knowledge of the transmission range, R_{tx}, are using the exact value. Fig. 5.3 shows the performance metrics computed for different values of the vehicle density, while keeping the parameter K fixed to 16. The lines represent the results of our analytic model, computed with a Monte Carlo simulation with 50 replications per vehicle density, whereas the marks refer to the results obtained from the OMNeT++ simulation, with 15 replications per density. We only show results for success probability, average position of the winner vehicle and average access delay.

As expected our performance model approximates very well all the performance metrics, in spite of being more pessimistic. As we said, for proposals prone to collision, such as EDCA*, fewer nodes participate in successive retransmissions (as expected in reality), which increases

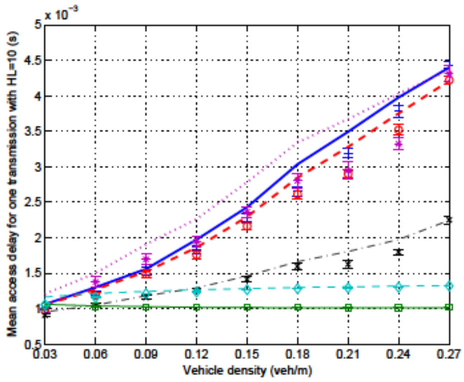
[1] In 10 ms, which is above the maximum end-to-end delay we obtain, a vehicle moving at 32 m/s only advances 0.32 m.



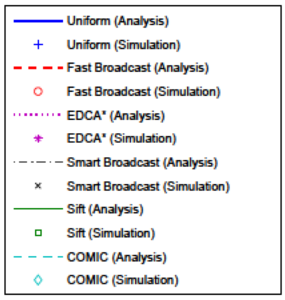
(a) Probability of successful transmission.



(b) Average position of the winner vehicle (m).



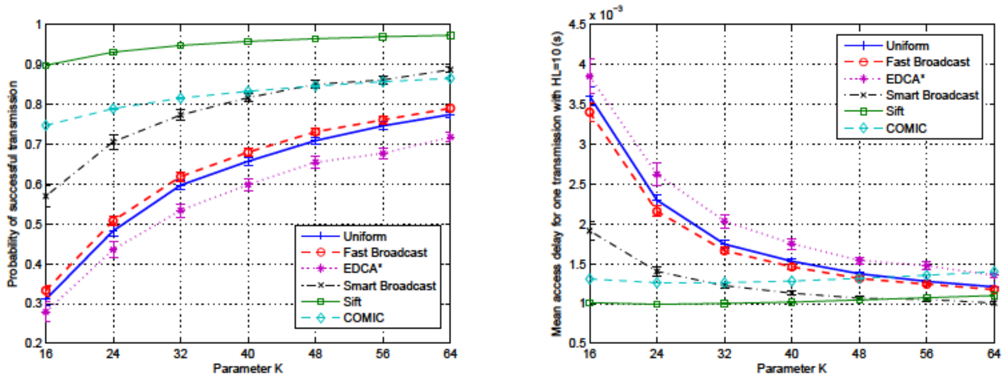
(c) Average delay for one successful transmission (s).



(d) Legend.

Figure 5.3: Performance metrics for vehicle densities varying between 0.03 and 0.27 veh/m and parameter $K = 16$. Single-hop scenario with all vehicles in range.

the success probability and decreases the mean access delay. This is the reason for the differences observed between the analysis and simulation, specially for high vehicle densities. In Fig. 5.3(a), we can see how the Sift protocol outperforms the rest of them with respect to success probability, since it is an approximation of the optimal distribution that can be used in this scenario. EDCA*, on the contrary, shows poor performance due to the use of too small window sizes and overlapped CW ranges, whereas Smart Broadcast benefits from greater window sizes and disjoint windows. In the cases of Uniform, Fast Broadcast and EDCA*, the poor success probability is reflected in high access delays, as shown in Fig. 5.3(c). The good success probability of Sift and COMIC is the reason for their low access delay, and more importantly, in both cases it is independent of the vehicle density, whereas the performance of the other proposals noticeably degrades as the number of vehicles in range increases.



(a) Probability of successful transmission.

(b) Average delay for one successful transmission (s).

Figure 5.4: Performance metrics for parameter K varying between 16 and 64 with vehicle density fixed to 0.21 veh/m. Single-hop scenario with all the vehicles in range.

However, as we said before, Sift and COMIC, being general-purpose proposals, do not take into account the position of the nodes. Therefore, all the vehicles have equal probability of success, and so the average position of the winner vehicle is in the middle of the range. On the contrary, both Smart Broadcast and EDCA*, and to a lesser extent Fast Broadcast, achieve their goal of increasing the average advance of the packet, *provided there is success*. Obviously in a single-hop situation packet advance is irrelevant, so we have to turn to multi-hop scenarios to find if this optimization is a real advantage for geo-routing or it is simply better to achieve a high success probability.

Before discussing multi-hop scenarios we examine the influence of the contention window size on the proposals. Once our model is validated, we can safely evaluate further experiments without requiring simulations. Fig. 5.4 shows the performance metrics computed for different values of the contention window size, while keeping the vehicle density fixed to 0.21 veh/m. Again, the computation is conducted through a Monte Carlo simulation with 50 replications per contention window size and the 95% confidence intervals are shown as error bars.

The success probability increases for all the proposals when increasing the contention window size, and it is more stable for the Sift and COMIC protocols. In fact, except for them, the rest of protocols clearly benefit from higher contention window sizes, reducing significantly the channel access delay. Obviously, the small extra delay due to higher window sizes is amply compensated by avoiding the delay due to collisions. For Sift and COMIC, on the contrary, the delay is slightly higher. This is due to the definition of the distribution for the slot selection, which concentrates the probability on the last slot (Sift) or on the middle one (COMIC), and therefore it grows with the contention window size. As expected, the influence of the contention window size on the position of the forwarder is not significant and it is not shown in the figure.

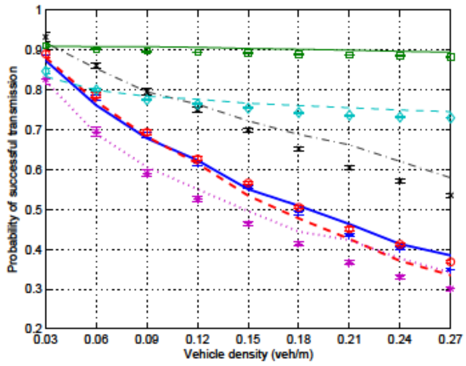
5.4.4 Multi-hop scenarios

In this case vehicles are located on a road segment of 600 m length and we set a deterministic free-space propagation model with transmission range $R_{tx} = 300$ m. These particular values are arbitrary and could have scaled up but at the cost of more simulation time because of the increased number of vehicles. In real deployments one should probably expect higher transmission ranges. We think that two hops is also a reasonable distance as range for emergency messages, though the evaluation framework can be used with more hops. This scenario has also been simulated with OMNeT++ with the same settings: the source sends a message every 10 s and simulations are run for 5000 s. All the simulations have been replicated 40 times to better capture the influence of position on the result. The rest of the parameters are shown in Table 5.3.

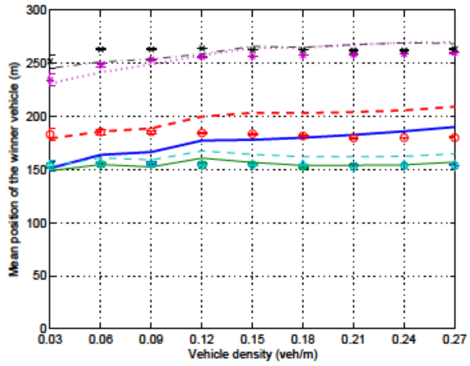
In Fig. 5.5 we show the results again for probability of success, average channel access delay, average position of the winner vehicle, and we have added the relevant metric end-to-end delay. First, as can be observed, again the simulations validate our evaluation model, in particular our approximation for multi-hop scenarios. Therefore, our evaluation framework provides a simple yet accurate tool to test this type of proposals. In the next section we exemplify its utility as a design tool for new proposals made up as combinations of the considered ones.

Regarding the performance of the different mechanisms, there is little variation with respect to the conclusions stated for one hop. The probability has improved for all of them, except for Sift and COMIC, which remains equal. The reason is that, as described in sect. 5.4.2, after a packet collision the number of nodes competing in the next attempt reduces proportionally to the length of the segment between the nodes involved in it. Therefore, the global success probability increases. For the same reason, the probability of success for Sift and COMIC remains equal because it is almost independent of the number of contenders [110]. This is again reflected in the average channel access delay.

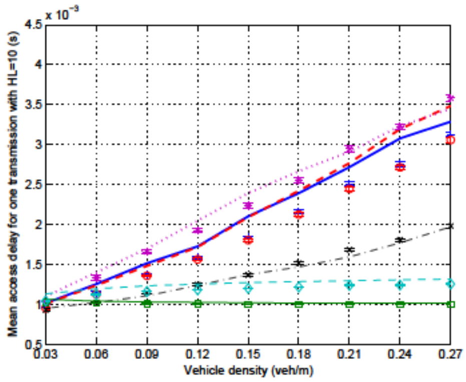
And, in the end, it is also determinant for the most relevant metric, the end-to-end delay. As can be seen in Fig. 5.5(d), the supposed benefits of making the packet advance as much as



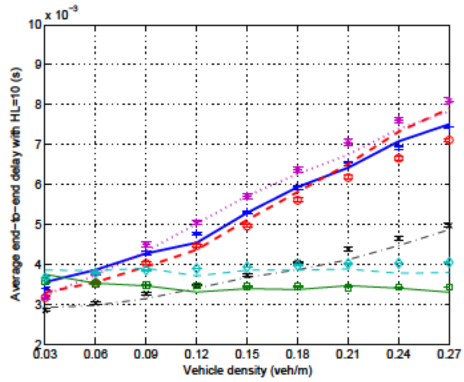
(a) Probability of successful transmission.



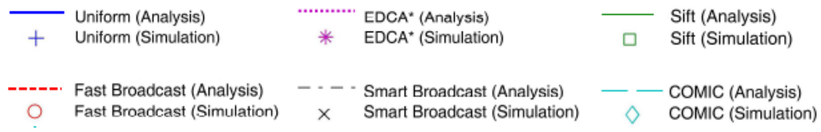
(b) Average position of the winner vehicle (m).



(c) Average channel access delay (s).



(d) Average end-to-end delay (s).



(e) Legend.

Figure 5.5: Performance metrics for vehicle densities between 0.03 and 0.27 veh/m and parameter $K = 16$. Multi-hop scenario with a length of 600 m and $R_{tx} = 300$ m.

possible are only noticeable for Smart Broadcast and low vehicle densities. For EDCA* and Fast Broadcast the delay penalties due to the high number of packet collisions take over any advantage due to making the packet advance as much as possible, except for very low vehicle densities.

The overall conclusion of this section is clear: for this kind of emergency applications it is preferable to achieve better probability of success rather than trying to make the packet advance as much as possible. Moreover, these delay penalties due to collisions depend on the packet size, so for greater sizes one should expect worse results. On the contrary, increasing the contention window size results in a general improvement for high vehicle densities and it tends to equalize the performance of the proposals.

Let us also remark that the real goal for an emergency warning should be to reach as many vehicles in the vicinity as possible, that is, inform all nodes, rather than to quickly reach a distant location. In this sense, achieving lower end-to-end delays may be misleading because in a packet collision the packet may be received by a distant neighbor but lost for most of the closer ones. However, we have checked, though it is not shown in the figures, that under ideal channel conditions, the time to inform all nodes is actually only slightly higher than the end-to-end delay.

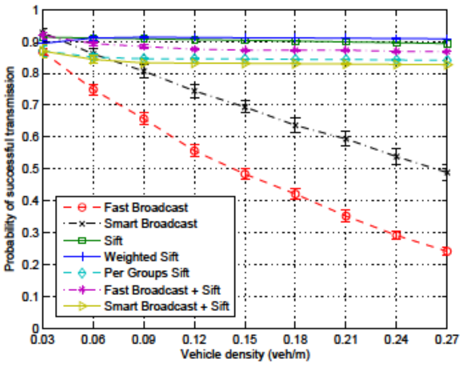
5.5 Evaluation of new proposals

Our performance evaluation framework can be used as a quick design tool for new proposals. In this section we exemplify it and test the use of combined procedures in order to obtain the best of each one. More specifically, we wish to obtain a method with such a high and stable success probability as the Sift distribution, while getting a forwarder positioned as far as possible. To this aim we proposed in Chapter 4 two modifications to the Sift distribution which improve its performance with respect to the position of the forwarder, but not to the mean access delay. Here we additionally test the results of combining the Smart Broadcast and the Fast Broadcast with the Sift distribution, as described next.

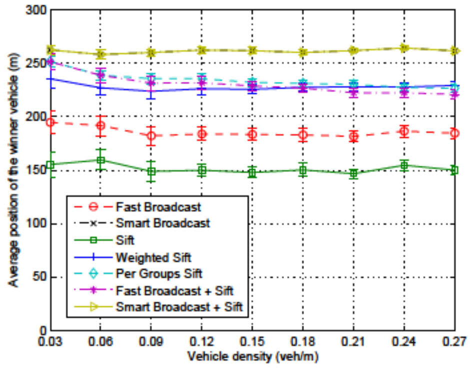
- **Fast Broadcast + Sift.** In this case the contention window for each vehicle is computed according to its position with eq. (5.6), as in the Fast Broadcast protocol. However, in this case, instead of selecting the random delay $t_R(n_i)$ uniformly between 0 and $CW(n_i)$ slots, each vehicle uses the corresponding Sift distribution over $CW(n_i)$ slots.

- **Smart Broadcast + Sift.** Similarly, in this proposal each group of vehicles is assigned a contention window as in the Smart Broadcast protocol, but the random delay t_R is selected using the Sift distribution instead of the Uniform one.

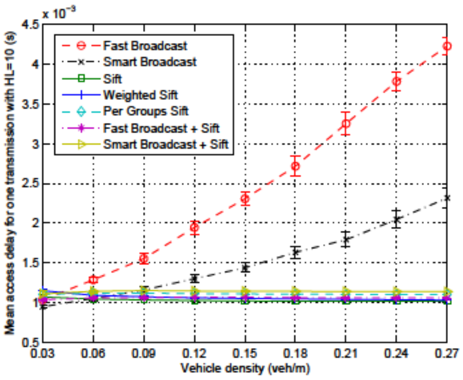
In Fig. 5.6 we show the resulting performance metrics for the described proposals, as well as for the usual Sift, Fast Broadcast and Smart Broadcast protocols, in order to compare them. These performance metrics are computed for different values of the vehicle density parameter and $K = 16$. The results show that all the new proposals achieve a higher success probability



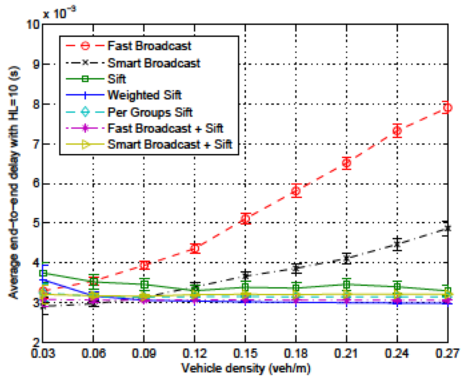
(a) Probability of successful transmission.



(b) Average position of the winner vehicle (m).



(c) Average channel access delay (s).



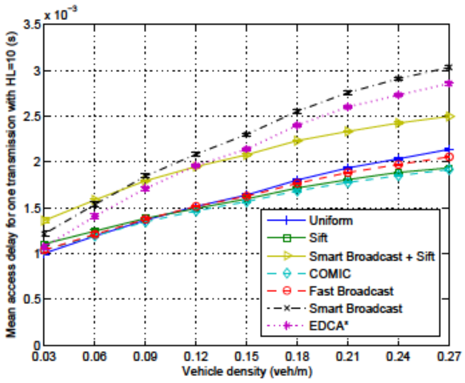
(d) Average end-to-end delay (s).

Figure 5.6: Performance metrics for new proposals for vehicle densities between 0.03 and 0.27 veh/m and $K = 16$ in a multi-hop scenario.

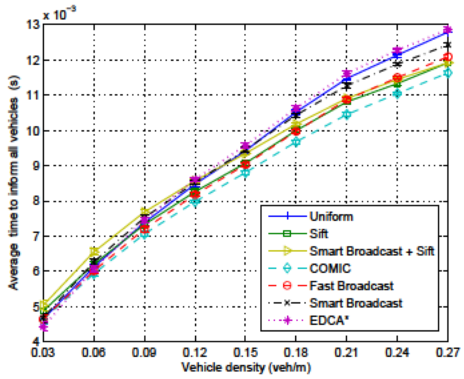
than the original ones for Fast Broadcast and Smart Broadcast, getting closer to the Sift success probability and achieving a similar stability with respect to the number of vehicles.

Regarding the average winner vehicle, or forwarder position, the improvement is more significant. For the Weighted Sift, the Fast Broadcast + Sift and the Per Groups Sift, the forwarder position is farther than for Sift and Fast Broadcast. On the other hand, a similar distance is achieved for the Smart Broadcast and the Smart Broadcast + Sift, but the latter results in much higher success probabilities and lower channel access and end-to-end delays.

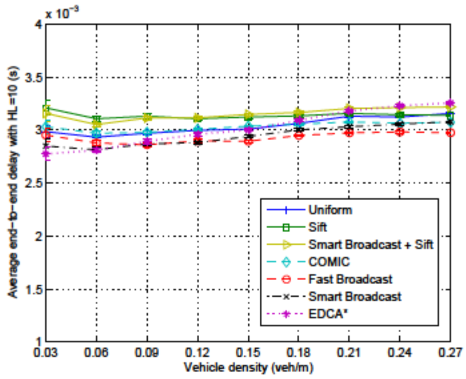
According to these results the intended goals can be achieved, and it seems reasonable to combine Smart Broadcast and Sift, since the implementation may be simpler than for the other ones. In any case, we have shown the utility of our framework as a quick (no need of simulations) evaluation and design tool.



(a) Average channel access delay (s).



(b) Average time to inform all vehicles (s).



(c) Average end-to-end delay (s).

Figure 5.7: Performance metrics for vehicle densities between 0.03 and 0.27 veh/m and parameter $K = 16$. Multi-hop scenario with Nakagami-m fading model with $m = 1$.

5.6 Realistic scenarios

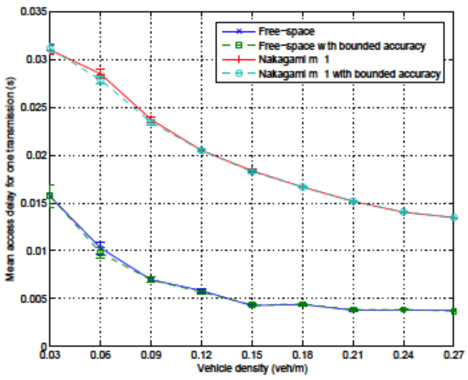
Up to this point, we have compared the contention mechanisms that involve random procedures under ideal assumptions, such as deterministic free-space propagation. In this section we evaluate the standard CBF specification [19] implemented at the network layer and introduce more realistic effects. On the one hand, it has been shown that radio propagation in vehicular networks is subject to strong fading [40]. In that case, the assumptions for our evaluation model do not hold anymore, since there is a chance that nodes in the close vicinity of a contender do not sense the channel busy and defer transmission. Therefore, we have simulated the proposals, as well as the standard CBF, using a more realistic fading model. In particular, we use the Nakagami-m distribution, which can model a wide class of fading channel conditions and fits well the empirical data [40]. The noise level is set to -110 dBm and the sensitivity to -85 dBm.

In addition, the simulator takes into account capture effects, since packets are only discarded when the *Signal to Interference Noise Ratio* (SINR) is below 4 dB. On the other hand, both real timers and location information have a certain accuracy. These effects may have particular influence on the deterministic CBF algorithm and have been simulated as well.

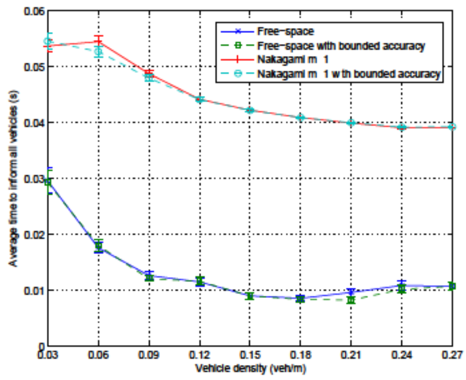
Performance metrics for the previous proposals in a strong fading scenario, modeled by Nakagami-*m* with *m*=1, are shown in Fig. 5.7. In presence of strong fading the basic carrier sense medium access is broken, since neighbor nodes may not sense the channel busy due to transmissions. The probability of reception decreases with the distance to the source, and, with the simulated parameters, on average 12% of nodes in the R_{tx} range do not receive a given transmission [131]. Therefore, specifically designed MAC mechanisms have little influence on most of the evaluated metrics.

Let us recall that, in realistic situations, collisions are locally experienced and so we cannot use the global notion of success probability anymore. Therefore, we use average delays to evaluate these scenarios. As can be seen in Fig. 5.7(b) and 5.7(c), both average time to inform all nodes and average end-to-end delay are almost independent of the MAC mechanism in use. Strong fading actually benefits the end-to-end delay since there is a chance that packets reach directly the destination even if it is out of the ideal deterministic range R_{tx} of the forwarder. In addition, as discussed before, packet always effectively advances, even in the presence of collisions. The overall result is that end-to-end delay is practically independent of the MAC mechanism and the vehicle density. On the contrary, vehicle density has more influence on the time to get all nodes informed. As in the ideal case, it is directly related to the average channel access delay, and it increases with the vehicle density, but there is little difference between proposals.

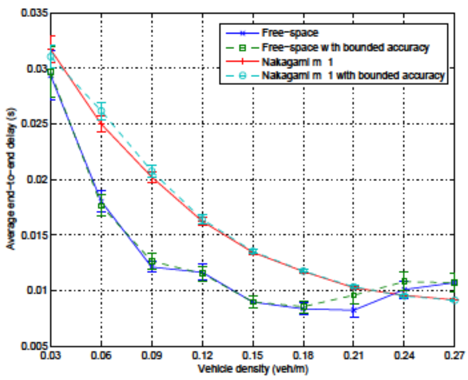
If we look at the average channel access delay in Fig. 5.7(a), proposals with good performance in ideal scenarios, such as Smart Broadcast and Smart Broadcast + Sift, remarkably increase their channel access delay in realistic scenarios. The cause of this performance degradation is actually the fact that they have been designed to maximize the distance of the forwarder to the source. Accordingly, the more distant nodes select earlier slots, win contention and transmit, but then, since reception probability decreases with the distance, nodes closer to the source have a higher probability of not sensing the channel busy and so they also transmit, generating collisions. On the contrary, in the proposals that do not attempt to maximize the packet advance, the forwarder is at the middle of the range on average and so its transmission has a higher probability of being sensed by the surrounding nodes. That is, the MAC



(a) Average channel access delay (s).



(b) Average time to inform all vehicles (s).



(c) Average end-to-end delay (s).

Figure 5.8: Performance metrics of GeoNetworking for vehicle densities between 0.03 and 0.27 veh/m and parameter $K = 16$. Multi-hop scenario with free-space and Nakagami-m fading model with $m = 1$. Clock accuracy set to $1\mu s$ and position accuracy set to 1 m.

mechanism works properly more frequently. As the vehicle density increases, there are simply more packet collisions, though the use of Sift slightly improves the probability of success in the group closer to the source for Smart Broadcast + Sift.

The conclusion from these results is that any new proposal which intends to optimize the operation of GeoNetworking must take into account in its design the effects of realistic radio propagation models. In fact, as we discuss next, the basic CBF protocol specified by the standard works remarkably well under realistic conditions.

GeoNetworking CBF specifies that upon reception of a packet, nodes start a deterministic timer whose value depends on the distance to the source, that is, $to(n_i) = maxTime + (minTime - maxTime)dist(x_s, x_i)/R_{tx}$. The goal is to select the most distant node as next forwarder. The standard specified parameters are $maxTime = 100\ ms$ and $minTime = 1\ ms$, and we set $R_{tx} = 300\ m$ to compare with previous proposals. Nodes use the standard IEEE 802.11p as MAC layer with a contention window of 32 slots. In Fig. 5.8 we show the average channel access delay, average time to inform all vehicles and average end-to-end delay for the standard GeoNetworking CBF. From these results it is clear that CBF GeoNetworking works well in all the cases. Moreover, its performance improves as the vehicle density increases, since there are more nodes available to forward the packets. Limited accuracy in vehicle position and timers have little influence on the performance. Delay values are higher than for MAC implementations, as expected, but they actually depend on the configured values for $maxTime$ and $minTime$, and there is margin for tuning them. Let us just mention that, for the ideal free-space case, there are scenarios where no packet is actually transmitted. The reason is that the particular fixed positions of the vehicles result in continuous collisions. However, this pathological scenarios should be very rare in realistic dynamical situations.

5.7 Final remarks

In this chapter we provide a survey and evaluation of the most relevant MAC-network cross-layer proposals for efficient geo-routing in the context of vehicular networks. They are described and a unified formal description of different contention-based mechanisms is extracted. This formal description is later used in a common framework for their performance analysis in the critical scenario of emergency messages delivery. As a novelty, our performance model allows to analyze the case in which each vehicle selects its contention slot in a different way.

This model has been used to rigorously evaluate the selected proposals in single-hop and multi-hop scenarios under ideal propagation conditions. Additionally, the evaluation has also been done by simulation, whose results further validate our approach. The evaluation shows the strengths and weaknesses of the different mechanisms and allows to conclude that it is preferable to achieve better success probability rather than trying to make the packet advance as much as possible, at least for small contention window sizes. We have also evaluated the proposals under more realistic channel fading conditions. This has been done by simulation, since our model cannot be directly applied to these cases. In this situation, however, there is actually little difference in the performance of the protocols. In fact, those proposals which attempt to maximize the progress of the packet suffer a noticeable degradation in performance.

On the contrary, the basic CBF protocol specified by the GeoNetworking standard performs well in most of the cases.

According to these results, our next step is to modify our analytical model to introduce realistic radio propagation effects. In this case, it is difficult to keep it simple because the reception probabilities depend on the positions of the receivers and there is no global notion of success probability. Even though, we also plan to work on new MAC proposals which take into account those effects in their design.

Part IV

Final remarks

6 General conclusions and future work

6.1 Main summary

The initial motivation of this dissertation was to contribute to the improvement of safety on the road by means of vehicular communications. Cooperative Collision Avoidance (CCA) applications are a new emerging means of reducing the number of accidents by providing cars with collaborative communication capabilities, thus allowing them to react against possible accident risks. However, to design and implement such applications, a deep understanding of the vehicle collision process is needed. The influence of different driving parameters on the collision event must be assessed at an early design stage to develop applications that can timely adapt vehicle dynamics to avoid or at least mitigate the danger. In this context, this thesis has presented and evaluated a novel stochastic model for the computation of the average number of vehicle collisions that occur in a platoon of vehicles where a warning collision system is in operation. The fact that a warning notification system is used allowed us to overcome the difficulties for obtaining stochastic models for such vehicular scenarios, since we could assume that all the drivers/vehicles react to the warning message independently, and therefore the motion equations could be simplified. We also proposed a good matching approximation to the exact model to further reduce the required computations to calculate the vehicle collision probabilities. In both cases, the results were validated by Monte-Carlo simulations.

It should be noted that the establishment of this VANET applications will be deployed gradually, equipping vehicles with the proper hardware and software to be able to communicate in an effective way within the vehicular environment. Therefore, it is highly convenient to study how the system of vehicles in a platoon will behave at different stages of technology deployment until full penetration in the market. When the CCA penetration ratio is taken into account, the growth in the number of operations of the analytical model is such that the sequential computation of a numerical solution is no longer feasible. To deal with this matter, we have shown how parallelization techniques coordinated with supercomputing resources make the simulation process a more suitable and efficient one, allowing a thorough evaluation of the CCA application.

On the other hand, the developed model is independent of the particular communication system employed since its operation has been abstracted and characterized by an appropriate message notification delay, including communication latency and driver reaction times. Therefore, it also enables the performance evaluation of different communication technologies. Similarly, different probability distributions for the parameters (inter-vehicle distance, velocity, driver reaction time, etc.) can be evaluated with the model. In addition to the average number of collisions, the analytic model enables the computation of the probabilities of the different ways in which the collisions may occur: both vehicles in motion, one stopped and one in motion, etc. By assigning different degrees of severity to each collision possibility, detailed accident severity functions can be defined.

Continuing with the model, we have shown in Chapter 3 how it can be used as an effective alternative to simulation for the numerical evaluation of CCA mechanisms, after discussing the limitations of current traffic simulators for accident simulation. We have also discussed our model's limitations, being the main one the independence between the state input variables (relative velocity and inter-vehicle spacing), which is not realistic in many cases and introduces too much randomness, leading to pessimistic results. However, it has been partially alleviated by adjusting the input variables and their relative values. To illustrate the model's capabilities as an assessment tool for CCA application design we have evaluated different types of CCA applications in two scenarios, a freeway and an urban scenario. The results suggest that the variability due to the drivers reaction time is the main cause of accidents and so removing it should be the main focus of a CCA application. This could be possible by automatic braking, that is, when the vehicle receives the warning message it takes over control and immediately starts to apply a coordinated braking policy, even though the driver is not yet aware of the risk. This is one of the different CCA systems discussed. Results suggest that the benefits of implementing this CCA are relevant. On the contrary, results show that the benefits of implementing a much more challenging cooperative system, able to coordinate speeds, are marginal in most of the cases.

In the second part of the thesis, we decided to focus our research on the efficiency and reliability of emergency messages propagation, which should reach all the vehicles within a certain area in a limited time. The delivery of these geographically-addressed messages is performed by the GeoNetworking protocol [19], which uses a forwarding mechanism to route packets through intermediate nodes until reaching the destination. As basic forwarding algorithms, the standard defines Greedy Forwarding (GF) and Contention-Based Forwarding (CBF). As defined by the standard, CBF is completely implemented at the network layer. However, CBF might be also implemented directly at the MAC layer, in order to optimize its operation. For instance, implementing CBF at MAC layer should result in lower latency, since forwarding delay is removed and only access delay counts. We have assessed here how cross-layer techniques, allowing the exchange of information between the different communication layers, can help to improve the operation of GeoNetworking by optimizing the forwarding algorithm in use.

In Chapter 4, two CBF schemes implemented at MAC layer that use geometrically distributed contention slots were proposed. We have analytically evaluated the total and per-vehicle success probabilities and compared them with a basic CBF mechanism. We have shown that a weighted geometric distribution effectively prioritizes the access based on position for a wide range of vehicle densities, while retaining the benefits of geometrical distributions with respect to success probabilities. In particular, while a CBF mechanism with static timers needs to adapt the contention window size to the number of contenders to avoid packet collisions, the proposed mechanisms scale gracefully and do not even need to know the number of contenders.

Then, the necessity to compare these and other existent cross-layer techniques under a common framework emerged. Therefore, in Chapter 5 we provided a survey and comparative evaluation of the most relevant MAC-Network cross-layer proposals for efficient geo-routing in the context of vehicular networks. We have focused on contention-based MAC mechanisms for wireless nodes. The majority of them are based on the CSMA/CA mechanism, whose operation and performance can be controlled by several parameters, namely: contention window size, random and deterministic carrier sense intervals as well as the probability distribution for the contention slots selection. Overall, it results in multiple degrees of freedom to optimize the medium access operation according to the most critical functionality offered by the network. We have discussed both, techniques specifically addressed to vehicular networks as well as general-purpose proposals, which can be adapted to VANETs. They have been described and a unified formal description of different contention-based mechanisms has been derived. This formal description has been later used in a common framework for their performance analysis in the critical scenario of emergency messages delivery. As a novelty, our performance model allows to analyze the case in which each vehicle selects its contention slot in a different way.

This model has been used to rigorously evaluate the selected proposals in single-hop and multi-hop scenarios under ideal propagation conditions. Additionally, the evaluation has also been done by simulation, whose results further validate our approach. With this evaluation, the strengths and weaknesses of the different mechanisms have been shown, allowing to conclude that it is preferable to achieve better success probability rather than trying to make the packet advance as much as possible, at least for small contention window sizes. We have also evaluated the proposals under more realistic channel fading conditions. This has been done by simulation, since our model cannot be directly applied to these cases. In this situation, however, there is actually little difference in the performance of the protocols. In fact, those proposals which attempt to maximize the progress of the packet suffer a noticeable degradation in performance. On the contrary, the basic CBF protocol specified by the GeoNetworking standard performs well in most of the cases. Therefore, it is extremely important to take into account the effects of realistic radio propagation for the design of new MAC protocols.

6.2 Future work

To close this work we briefly outline some open issues and possible research lines that may be of further interest.

Regarding the stochastic model presented in Chapters 2 and 3, it should be enhanced in order to deal with its current limitations. First of all, bivariate distributions should be introduced in the model, to capture state variable correlations, increasing the model accuracy. As a second step, it would be necessary to find appropriate joint distributions for speed and inter-vehicle spacing. There is actually a lack of empirical models that jointly describe inter-vehicle spacing and speed. Similarly, we have shown how to characterize the input variable distributions by using statistical models proposed in the open literature, but additional efforts in the empirical characterization of deceleration, reaction times and communication delays are clearly necessary.

On the other hand, concerning the study of efficient geo-routing in VANETs, realistic radio propagation effects should be introduced in the analytical model developed for the protocols' performance assessment, since they have a great impact on the results. This is not straightforward because the reception probabilities depend on the positions of the receivers and there is no global notion of success probability. Therefore, it would be interesting to analytically define new reliability metrics which take into account the effects of channel fading and hidden nodes. It would also be interesting to work on new MAC proposals which take into account those effects in their design.

A Supporting tools for parallelization

A.1 The OpenMP Technique

OpenMP is a well-known open standard for providing parallelization mechanisms to multi-processors with shared memory [38]. OpenMP API supports shared memory programming, multi-platform techniques for the programming languages like Fortran, C and C++, and for every architecture including Unix and Windows platforms. OpenMP is a scalable and portable model developed for hardware and software distributors which provides shared memory programmers with a simple and flexible interface for developing parallel applications which can run not only in a personal computer but also in a supercomputer.

OpenMP uses the parallel paradigm known as *fork-join* with the generation of multiple threads, where a heavy computational task is divided into k threads (*forks*) with less weight and afterwards it collects their results and combines them at the end of the execution in a single result (*join*). The master thread runs sequentially till it finds an OpenMP guideline and since this moment a bifurcation is generated with the corresponding slave threads. These threads can be distributed and executed in different processors, decreasing in this way the execution time.

A.2 The Ben-Arabi Supercomputer

Our model is executed under the Ben-Arabi supercomputer resources, which is placed in the Scientific Park of Murcia (Spain). The Ben-Arabi system consists of two different architectures; on the one hand the central node HP Integrity Superdome SX2000 with 128 cores of the Intel Itanium-2 dual-core Montvale (1.6 Ghz, 18 MB of cache L3) processor and 1.5 TB of shared memory, called Ben. On the other hand, Arabi is a cluster consisting of 102 nodes, which offers a total of 816 Intel Xeon Quad-Core E5450 (3 GHz y 6 MB of cache L2) processor cores and a total of 1072 GB of shared memory.

We run our mathematical model within a node of the Arabi cluster environment using 2, 4 and 8 processors in order to compare the resulting execution times. Les us remark that we are using a shared memory parallelization technique, so we are not allowed to combine the use of processors from different nodes.

Next we summarize the technical features of the cluster:

- Capacity: 9.72 Tflops.
- Processor: Intel Xeon Quad-Core E5450.
- Nodes number: 102.
- Processors number: 816.
- Processors/Node: 8.
- Memory/Node: 32 nodes of 16 GB and 70 of 8 GB.
- Memory/Core: 3 MB (6 MB shared among 2 cores).
- Clock frequency: 3 Ghz.

B Computation of the distance traveled by a vehicle in case of collision

Let us recall that, when the parameters are not constant, collisions may occur in four different ways: 1) vehicles have not started to brake; 2) only one of them is braking; 3) both of them are braking; or 4) the front vehicle has stopped. Each one of these possibilities results in a different distance to stop, $d_{c_j,i}$, that must be weighted by its probability of occurrence, $q_{c_j,i}$, and added to get the average distance traveled $\overline{l_i}$, as in equation 2.21. Next, we describe the process followed to compute these distances.

B.1 Collision when the vehicles have not started to brake

This event may happen if the difference of initial velocities makes the vehicles crash before receiving the warning message.

For a given initial inter-vehicle spacing s_i, a time instant t should exist so that

$$V_i t = V_{i-1} t + s_i, \tag{B.1}$$

$$0 \leq t \leq \min\{\delta_i, \delta_{i-1}\}. \tag{B.2}$$

Solving equation (B.1) we obtain

$$t_{c_1,i}(s_i) = \frac{s_i}{V_i - V_{i-1}}. \tag{B.3}$$

Therefore, the distance traveled by C_i in this case is

$$D_{c_1,i}(s_i) = V_i t_{c_1,i}(s_i) = \frac{V_i s_i}{V_i - V_{i-1}}, \quad inf_{c_1,i} \leq s_i \leq sup_{c_1,i}, \tag{B.4}$$

where $inf_{c_1,i}$ and $sup_{c_1,i}$ define the range of s_i in which this type of collision can happen, and they are computed as follows:

- If $V_i - V_{i-1} \leq 0$, there is no solution (this type of collision cannot occur). Let us define appropriate limits $inf_{c_1,i} = sup_{c_1,i} = 0$.
- If $V_i - V_{i-1} > 0$, then the condition (B.2) holds if and only if $0 \leq s_i \leq (V_i - V_{i-1}) \cdot \min\{\delta_i, \delta_{i-1}\}$. Let us define

$$inf_{c_1,i} = 0, \tag{B.5}$$

$$sup_{c_1,i} = (V_i - V_{i-1}) \cdot \min\{\delta_i, \delta_{i-1}\}. \tag{B.6}$$

B.2 Collision when only one vehicle is braking

In this case, the collision event depends on the relative reaction times of the drivers. That is, due to a high reaction time, one of the drivers starts to brake too late.

- If $\delta_i = \delta_{i-1}$, then we have to skip to Section B.3, and so let us define $inf_{c_2,i} = sup_{c_2,i} = sup_{c_1,i}$.
- If $\delta_i < \delta_{i-1}$, then vehicle C_i starts to brake before C_{i-1} does.

  For a given initial inter-vehicle spacing s_i, a time instant t should exist so that

$$V_i t - \tfrac{a_i}{2}(t - \delta_i)^2 = V_{i-1} t + s_i, \tag{B.7}$$

$$\delta_i \leq t \leq \delta_{i-1}. \tag{B.8}$$

  Solving (B.7) we obtain the following solutions:

$$t^a_{c_2,i}(s_i) = \frac{V_i - V_{i-1}}{a_i} + \delta_i - \sqrt{\left(\frac{V_i - V_{i-1}}{a_i}\right)^2 + 2\delta_i\left(\frac{V_i - V_{i-1}}{a_i}\right) - \frac{2s_i}{a_i}} \tag{B.9}$$

$$t^b_{c_2,i}(s_i) = \frac{V_i - V_{i-1}}{a_i} + \delta_i + \sqrt{\left(\frac{V_i - V_{i-1}}{a_i}\right)^2 + 2\delta_i\left(\frac{V_i - V_{i-1}}{a_i}\right) - \frac{2s_i}{a_i}} \tag{B.10}$$

  The term in the square root is positive if and only if $s_i \leq \delta_i(V_i - V_{i-1}) + \frac{(V_i - V_{i-1})^2}{2a_i}$.

It can be proved that condition (B.8) does not hold for $t^b_{c_2,i}$, so the only possible solution is $t^a_{c_2,i}$. Therefore, the distance traveled by C_i in this case is

$$D_{c_2,i}(s_i) = V_{i-1} t^a_{c_2,i}(s_i) + s_i, \quad inf_{c_2,i} \leq s_i \leq sup_{c_2,i}. \tag{B.11}$$

Now, it remains to compute the range of s_i in which this type of collision can happen:

- If $V_i - V_{i-1} \leq 0$, then (B.8) does not hold, so we define $inf_{c_2,i} = sup_{c_2,i} = sup_{c_1,i}$.
- If $0 < V_i - V_{i-1} \leq a_i(\delta_{i-1} - \delta_i)$, then (B.8) holds for $t^a_{c_2,i}$ if and only if

$$\delta_i(V_i - V_{i-1}) \leq s_i \leq \delta_i(V_i - V_{i-1}) + \frac{(V_i - V_{i-1})^2}{2a_i}. \tag{B.12}$$

  Let us define

$$\begin{aligned} inf_{c_2,i} &= \delta_i(V_i - V_{i-1}), && \text{(B.13)} \\ sup_{c_2,i} &= \delta_i(V_i - V_{i-1}) + \frac{(V_i - V_{i-1})^2}{2a_i}. && \text{(B.14)} \end{aligned}$$

- If $V_i - V_{i-1} > a(\delta_{i-1} - \delta_i)$, then (B.8) holds for $t^a_{c_2,i}$ if and only if

$$\delta_i(V_i - V_{i-1}) \leq s_i \leq \delta_{i-1}(V_i - V_{i-1}) - \frac{a_i(\delta_i - \delta_{i-1})^2}{2}. \tag{B.15}$$

  Let us define

$$\begin{aligned} inf_{c_2,i} &= \delta_i(V_i - V_{i-1}), && \text{(B.16)} \\ sup_{c_2,i} &= \min\left\{\delta_i(V_i - V_{i-1}) + \frac{(V_i - V_{i-1})^2}{2a_i}, \right. \\ &\qquad \left. \delta_{i-1}(V_i - V_{i-1}) - \frac{a_i(\delta_i - \delta_{i-1})^2}{2}\right\}. && \text{(B.17)} \end{aligned}$$

- If $\delta_i > \delta_{i-1}$, then vehicle C_{i-1} starts to brake before C_i does.

  For a given initial inter-vehicle spacing s_i, a time instant t should exist so that

$$\begin{aligned} V_i t = V_{i-1} t - \tfrac{a_{i-1}}{2}(t - \delta_{i-1})^2 + s_i, && \text{(B.18)} \\ \delta_{i-1} \leq t \leq \delta_i. && \text{(B.19)} \end{aligned}$$

Solving (B.18) we obtain the following solutions:

$$t^a_{c_2,i}(s_i) = \frac{V_{i-1}-V_i}{a_{i-1}} + \delta_{i-1} - \sqrt{\left(\frac{V_{i-1}-V_i}{a_{i-1}}\right)^2 + 2\delta_{i-1}\left(\frac{V_{i-1}-V_i}{a_{i-1}}\right) + \frac{2s_i}{a_{i-1}}} \tag{B.20}$$

$$t^b_{c_2,i}(s_i) = \frac{V_{i-1}-V_i}{a_{i-1}} + \delta_{i-1} + \sqrt{\left(\frac{V_{i-1}-V_i}{a_{i-1}}\right)^2 + 2\delta_{i-1}\left(\frac{V_{i-1}-V_i}{a_{i-1}}\right) + \frac{2s_i}{a_{i-1}}} \tag{B.21}$$

The term in the square root is positive if and only if $s_i \geq \delta_{i-1}(V_i - V_{i-1}) - \frac{(V_{i-1}-V_i)^2}{2a_{i-1}}$. It can be proved that (B.19) does not hold for $t^a_{c_2,i}$, so the only possible solution is $t^b_{c_2,i}(s_i)$. Therefore, the distance traveled by C_i in this case is

$$D_{c_2,i}(s_i) = V_i t^b_{c_2,i}(s_i), \quad inf_{c_2,i} \leq s_i \leq sup_{c_2,i}. \tag{B.22}$$

Now, it remains to compute the limits $inf_{c_2,i}$ and $sup_{c_2,i}$:

– If $V_{i-1} - V_i \leq 0$, then condition (B.19) holds for $t^b_{c_2,i}$ if and only if

$$\delta_{i-1}(V_i - V_{i-1}) \leq s_i \leq \delta_i(V_i - V_{i-1}) + \frac{a_{i-1}}{2}(\delta_i - \delta_{i-1})^2. \tag{B.23}$$

Let us define

$$inf_{c_2,i} = \delta_{i-1}(V_i - V_{i-1}), \tag{B.24}$$

$$sup_{c_2,i} = \delta_i(V_i - V_{i-1}) + \frac{a_{i-1}}{2}(\delta_i - \delta_{i-1})^2. \tag{B.25}$$

– If $0 < V_{i-1} - V_i \leq a_{i-1}(\delta_i - \delta_{i-1})$, then (B.19) holds for $t^b_{c_2,i}$ if and only if

$$0 \leq s_i \leq \frac{a_{i-1}}{2}(\delta_i - \delta_{i-1})^2 - \delta_i(V_{i-1} - V_i). \tag{B.26}$$

Let us define

$$inf_{c_2,i} = sup_{c_1,i}, \tag{B.27}$$

$$sup_{c_2,i} = \delta_i(V_i - V_{i-1}) + \frac{a_{i-1}}{2}(\delta_i - \delta_{i-1})^2. \tag{B.28}$$

– If $V_{i-1} - V_i > a_{i-1}(\delta_i - \delta_{i-1})$, then condition (B.19) does not hold, so we define $inf_{c_2,i} = sup_{c_2,i} = sup_{c_1,i}$.

B.3 Collision when both vehicles are braking

In this case, both vehicles are aware of the danger and have started to brake but they are not able to avoid the collision, due to their initial speeds and reaction times, and they collide in motion.

For a given initial inter-vehicle spacing s_i, a time instant t should exist so that

$$V_i t - \tfrac{a_i}{2}(t-\delta_i)^2 = V_{i-1}t - \tfrac{a_{i-1}}{2}(t-\delta_{i-1})^2 + s_i, \tag{B.29}$$

$$\max\{\delta_i, \delta_{i-1}\} \le t \le \min\left\{\tfrac{V_i}{a_i} + \delta_i,\ T_{i-1}(\overline{l_{i-1}})\right\}, \tag{B.30}$$

where $T_{i-1}(\overline{l_{i-1}})$ is the time needed by vehicle C_{i-1} to travel the distance $\overline{l_{i-1}}$, and it is calculated by the function:

$$T_i(x) = \begin{cases} \frac{x}{V_i}, & \text{if } x \le V_i\delta_i, \\ \frac{V_i}{a} + \delta_i - \sqrt{\frac{2}{a}(d_{s,i} - x)}, & \text{if } x > V_i\delta_i. \end{cases} \tag{B.31}$$

In order to simplify the notation, we call $t_{min} = \max\{\delta_i, \delta_{i-1}\}$ and $t_{max} = \min\left\{\frac{V_i}{a_i} + \delta_i,\ T_{i-1}(\overline{l_{i-1}})\right\}$.

If $a_i - a_{i-1} = 0$, solving (B.29) we obtain

$$t_{c_3,i}(s_i) = \frac{s_i + \frac{a_i}{2}(\delta_i^2 - \delta_{i-1}^2)}{V_i - V_{i-1} + a_i(\delta_i - \delta_{i-1})}. \tag{B.32}$$

Therefore, the distance traveled by C_i in this case is

$$D_{c_3,i}(s_i) = V_i t_{c_3,i}(s_i) - \frac{a_i}{2}(t_{c_3,i}(s_i) - \delta_i)^2, \quad inf_{c_3,i} \le s_i \le sup_{c_3,i}. \tag{B.33}$$

Now, we compute the integration limits in this case:

- If $V_i - V_{i-1} = a_i(\delta_{i-1} - \delta_i)$, then (B.30) does not hold, so let us define $inf_{c_3,i} = sup_{c_3,i} = sup_{c_2,i}$.

- If $V_i - V_{i-1} > a_i(\delta_{i-1} - \delta_i)$, then (B.30) holds for $t_{c3,i}$ if and only if

$$(V_i - V_{i-1} + a_i(\delta_i - \delta_{i-1}))t_{min} - \frac{a_i}{2}(\delta_i^2 - \delta_{i-1}^2) \leq s_i \leq$$
$$\leq (V_i - V_{i-1} + a_i(\delta_i - \delta_{i-1}))t_{max} - \frac{a_i}{2}(\delta_i^2 - \delta_{i-1}^2). \quad \text{(B.34)}$$

Let us define

$$inf_{c3,i} = (V_i - V_{i-1} + a_i(\delta_i - \delta_{i-1}))t_{min} - \frac{a_i}{2}(\delta_i^2 - \delta_{i-1}^2), \quad \text{(B.35)}$$
$$sup_{c3,i} = (V_i - V_{i-1} + a_i(\delta_i - \delta_{i-1}))t_{max} - \frac{a_i}{2}(\delta_i^2 - \delta_{i-1}^2). \quad \text{(B.36)}$$

- If $V_i - V_{i-1} < a_i(\delta_{i-1} - \delta_i)$, then eq. (B.30) holds for $t_{c3,i}$ if and only if

$$(V_i - V_{i-1} + a_i(\delta_i - \delta_{i-1}))t_{max} - \frac{a_i}{2}(\delta_i^2 - \delta_{i-1}^2) \leq s_i \leq$$
$$\leq (V_i - V_{i-1} + a_i(\delta_i - \delta_{i-1}))t_{min} - \frac{a_i}{2}(\delta_i^2 - \delta_{i-1}^2). \quad \text{(B.37)}$$

Let us define

$$inf_{c3,i} = (V_i - V_{i-1} + a_i(\delta_i - \delta_{i-1}))t_{max} - \frac{a_i}{2}(\delta_i^2 - \delta_{i-1}^2), \quad \text{(B.38)}$$
$$sup_{c3,i} = (V_i - V_{i-1} + a_i(\delta_i - \delta_{i-1}))t_{min} - \frac{a_i}{2}(\delta_i^2 - \delta_{i-1}^2). \quad \text{(B.39)}$$

If $a_i - a_{i-1} \neq 0$ solving (B.29), we obtain the following solutions

$$t^a_{c3,i} = \frac{\eta - \sqrt{\eta^2 - \gamma(a_i - a_{i-1}) - 2(a_i - a_{i-1})s_i}}{a_i - a_{i-1}}, \quad \text{(B.40)}$$
$$t^b_{c3,i} = \frac{\eta + \sqrt{\eta^2 - \gamma(a_i - a_{i-1}) - 2(a_i - a_{i-1})s_i}}{a_i - a_{i-1}}, \quad \text{(B.41)}$$

where $\eta = V_i - V_{i-1} + a_i\delta_i - a_{i-1}\delta_{i-1}$ and $\gamma = a_i\delta_i^2 - a_{i-1}\delta_{i-1}^2$.

Therefore, the distance traveled by C_i in this case is

$$D_{c3,i}(s_i) = \begin{cases} V_i t^a_{c3,i} - \frac{a_i}{2}(t^a_{c3,i} - \delta_i)^2, & inf^a_{c3,i} \leq s_i \leq sup^a_{c3,i} \\ V_i t^b_{c3,i} - \frac{a_i}{2}(t^b_{c3,i} - \delta_i)^2, & inf^b_{c3,i} \leq s_i \leq sup^b_{c3,i} \end{cases} \quad \text{(B.42)}$$

The term in the square root is positive if and only if

$$\begin{cases} s_i \leq \frac{\eta^2}{2(a_i - a_{i-1})} - \frac{\gamma}{2}, & \text{for } a_i - a_{i-1} > 0, \\ s_i \geq \frac{\eta^2}{2(a_i - a_{i-1})} - \frac{\gamma}{2}, & \text{for } a_i - a_{i-1} < 0. \end{cases} \tag{B.43}$$

First we compute the limits for $t^a_{c_3,i}$:

- If $a_i - a_{i-1} > 0$, then
  - If $\frac{\eta}{a_i - a_{i-1}} < t_{min}$, then (B.30) does not hold for $t^a_{c_3,i}$, and so let us define $inf^a_{c_3,i} = sup^a_{c_3,i} = sup_{c_2,i}$.
  - If $t_{min} \leq \frac{\eta}{a_i - a_{i-1}} \leq t_{max}$, then (B.30) holds for $t^a_{c_3,i}$ if and only if

$$\eta t_{min} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{min} \leq s_i \leq \frac{\eta^2}{2(a_i - a_{i-1})} - \frac{\gamma}{2}. \tag{B.44}$$

    Let us define

$$inf^a_{c_3,i} = \eta t_{min} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{min}, \tag{B.45}$$

$$sup^a_{c_3,i} = \frac{\eta^2}{2(a_i - a_{i-1})} - \frac{\gamma}{2}. \tag{B.46}$$

  - If $\frac{\eta}{a_i - a_{i-1}} > t_{max}$, then (B.30) holds for $t^a_{c_3,i}$ if and only if

$$\eta t_{min} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{min} \leq s_i \leq \eta t_{max} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{max}. \tag{B.47}$$

    Let us define

$$inf^a_{c_3,i} = \eta t_{min} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{min}, \tag{B.48}$$

$$sup^a_{c_3,i} = \min\left\{ \eta t_{max} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{max},\ \frac{\eta^2}{2(a_i - a_{i-1})} - \frac{\gamma}{2} \right\}. \tag{B.49}$$

- If $a_i - a_{i-1} < 0$, then
  - If $\frac{\eta}{a_i - a_{i-1}} < t_{min}$, then (B.30) holds for $t^a_{c_3,i}$ if and only if

$$\eta t_{min} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{min} \leq s_i \leq \eta t_{max} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{max}. \tag{B.50}$$

Let us define

$$inf^a_{c_3,i} = \max\left\{\eta t_{min} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2}t^2_{min},\ \frac{\eta^2}{2(a_i - a_{i-1})} - \frac{\gamma}{2}\right\}, \tag{B.51}$$

$$sup^a_{c_3,i} = \eta t_{max} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2}t^2_{max}. \tag{B.52}$$

– If $t_{min} \leq \frac{\eta}{a_i - a_{i-1}} \leq t_{max}$, then (B.30) holds for $t^a_{c_3,i}$ if and only if

$$\frac{\eta^2}{2(a_i - a_{i-1})} - \frac{\gamma}{2} \leq s_i \leq \eta t_{max} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2}t^2_{max}. \tag{B.53}$$

Let us define

$$inf^a_{c_3,i} = \frac{\eta^2}{2(a_i - a_{i-1})} - \frac{\gamma}{2}, \tag{B.54}$$

$$sup^a_{c_3,i} = \eta t_{max} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2}t^2_{max}. \tag{B.55}$$

– If $\frac{\eta}{a_i - a_{i-1}} > t_{max}$, then (B.30) does not hold for $t^a_{c_3,i}$, and so let us define $inf^a_{c_3,i} = sup^a_{c_3,i} = sup_{c_2,i}$.

It remains to compute the limits for $t^b_{c_3,i}$:

- If $a_i - a_{i-1} > 0$, then

  – If $\frac{\eta}{a_i - a_{i-1}} < t_{min}$, then (B.30) holds for $t^b_{c_3,i}$ if and only if

$$\eta t_{max} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2}t^2_{max} \leq s_i \leq \eta t_{min} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2}t^2_{min}. \tag{B.56}$$

Let us define

$$inf^b_{c_3,i} = \eta t_{max} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2}t^2_{max}, \tag{B.57}$$

$$sup^b_{c_3,i} = \min\left\{\eta t_{min} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2}t^2_{min},\ \frac{\eta^2}{2(a_i - a_{i-1})} - \frac{\gamma}{2}\right\}. \tag{B.58}$$

  - If $\frac{\eta}{a_i - a_{i-1}} > t_{min}$, then (B.30) does not hold for $t^b_{c_3,i}$, and so let us define $inf^b_{c_3,i} = sup^b_{c_3,i} = sup^a_{c_3,i}$.

- If $a_i - a_{i-1} < 0$, then
  - If $\frac{\eta}{a_i - a_{i-1}} \leq t_{max}$, then (B.30) does not hold for $t^b_{c_3,i}$, and so let us define $inf^b_{c_3,i} = sup^b_{c_3,i} = sup^a_{c_3,i}$.
  - If $\frac{\eta}{a_i - a_{i-1}} > t_{max}$, then (B.30) holds for $t^b_{c_3,i}$ if and only if

$$\eta t_{min} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{min} \leq s_i \leq \eta t_{max} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{max}. \tag{B.59}$$

Let us define

$$inf^b_{c_3,i} = \max\left\{\eta t_{min} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{min},\ \frac{\eta^2}{2(a_i - a_{i-1})} - \frac{\gamma}{2}\right\}, \tag{B.60}$$

$$sup^b_{c_3,i} = \eta t_{max} - \frac{\gamma}{2} - \frac{a_i - a_{i-1}}{2} t^2_{max}. \tag{B.61}$$

B.4 Collision when vehicle C_{i-1} has stopped

The preceding vehicle has been able to stop safely but a rear collision still occurs.

In this case s_i should directly satisfy $s_i \leq d_{s,i} - \overline{l_{i-1}}$. The distance traveled by C_i in this case is

$$D_{c_4,i}(s_i) = \overline{l_{i-1}} + s_i, \quad inf_{c_4,i} \leq s_i \leq sup_{c_4,i}. \tag{B.62}$$

And we set

$$inf_{c_4,i} = sup^b_{c_3,i} \tag{B.63}$$

$$sup_{c_4,i} = d_{s,i} - \overline{l_{i-1}}. \tag{B.64}$$

References

[1] Advanced Safety Vehicle Project in Japan.
http://www.mlit.go.jp/jidosha/asv_english/asv3.htm. 6

[2] Car-to-Car Communication Consortium official webpage.
http://www.car-to-car.org/. 6

[3] Connected Vehicle Research program official webpage.
http://www.its.dot.gov/connected_vehicle/connected_vehicle.htm. 6

[4] European Commission's Directorate General for Mobility and Transport.
http://ec.europa.eu/transport/road_safety/index_en.htm. 3

[5] The European Project CarTALK 2000.
http://www.cartalk2000.net. 6, 16

[6] Fundación Parque Científico de Murcia official webpage.
http://www.parquecientificomurcia.es/. 19, 31

[7] MPI official webpage.
http://www.mcs.anl.gov/research/projects/mpi/. 47

[8] Nationwide Insurance.
http://www.nationwide.com/. 3

[9] OMNeT++ simulation library.
http://www.omnetpp.org. 120

[10] OpenMP official webpage.
http://openmp.org. 46

[11] IEEE 802.11e-2005. "IEEE Standard for Information Technology - Telecommunications and Information Exchange Between Systems - Local and Metropolitan Area Networks - Specific Requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications - Amendment 8: Medium Access Control (MAC) Quality of Service Enhancements", November 2005. 11

[12] GeoNet STREP No. 216269 - D2.2 Final GeoNet Specification, January 2010. 16

[13] IEEE 802.11p-2010. "Standard for Information Technology - Telecommunications and Information Exchange Between Systems - Local and Metropolitan Area Networks - Specific Requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications - Amendment 6: Wireless Access in Vehicular Environments", July 2010. 11, 13

[14] EN 302 636-3. "Intelligent Transport Systems (ITS); Vehicular Communications; GeoNetworking; Part 3: Network architecture", October 2011. 16, 89

[15] IEEE 1609.4-2010. "Standard for Wireless Access in Vehicular Environments (WAVE) - Multi-Channel Operation", 2011. 11, 53

[16] IEEE 802.11-2012. "Standard for Information Technology - Telecommunications and Information Exchange Between Systems - Local and Metropolitan Area Networks - Specific Requirements Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications", March 2012. 11, 12, 89, 104, 108

[17] EN 302 636-1. "Intelligent Transport Systems (ITS); Vehicular Communications; GeoNetworking; Part 1: Requirements", August 2013. 16, 89

[18] EN 302 636-2. "Intelligent Transport Systems (ITS); Vehicular Communications; GeoNetworking; Part 2: Scenarios", November 2013. 16, 89

[19] EN 302 636-4-1. "Intelligent Transport Systems (ITS); Vehicular Communications; GeoNetworking; Part 4: Geographical addressing and forwarding for point-to-point and point-to-multipoint communications; Sub-part 1: Media-Independent Functionality", October 2013. 16, 20, 89, 106, 112, 114, 128, 137

[20] EN 302 636-5-1. "Intelligent Transport Systems (ITS); Vehicular Communications; GeoNetworking; Part 5: Transport Protocols; Sub-part 1: Basic Transport Protocol", October 2013. 16, 89

[21] EN 302 636-6-1. "Intelligent Transport Systems (ITS); Vehicular Communications; GeoNetworking; Part 6: Internet Integration; Sub-part 1: Transmission of IPv6 Packets over GeoNetworking Protocols", October 2013. 16, 89

[22] EN 302 663. "Intelligent Transport Systems (ITS); Access layer specification for Intelligent Transport Systems operating in the 5 GHz frequency band", July 2013. 89

[23] TS 102 636-4-2. "Intelligent Transport Systems (ITS); Vehicular Communications; GeoNetworking; Part 4: Geographical addressing and forwarding for point-to-point and point-to-multipoint communications - Sub-part 2: Media-dependent functionalities for ITS-G5", October 2013. 16, 89

[24] Abboud, K., and Zhuang, W. “Modeling and Analysis for Emergency Messaging Delay in Vehicular Ad Hoc Networks”. In *IEEE Global Telecommunications Conference* (2009), pp. 1–6. 35

[25] Abdel-Aty, M. A., and Radwan, A. E. “Modeling Traffic Accident Occurrence and Involvement”. *Accident Analysis & Prevention 32*, 5 (2000), 633–642. 34, 64

[26] Amdahl, G. M. “Validity of the Single Processor Approach to Achieving Large Scale Computing Capabilities”. In *Proceedings of the ACM Spring Joint Computer Conference* (1967), pp. 483–485. 46

[27] Barceló, J., Codina, E., Casas, J., Ferrer, J., and García, D. “Microscopic Traffic Simulation: A Tool for the Design, Analysis and Evaluation of Intelligent Transport Systems”. *Journal of Intelligent and Robotic Systems 41*, 2-3 (2005), 173–203. 67

[28] Barghi, S., Benslimane, A., and Assi, C. “A Lifetime-Based Routing Protocol for Connecting Vanets to the Internet”. In *IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks & Workshops* (2009), pp. 1–9. 106

[29] Barney, B. “Introduction to Parallel Computing”. *Lawrence Livermore National Laboratory* (2010). 35

[30] Behrisch, M., Bieker, L., Erdmann, J., and Krajzewicz, D. “SUMO - Simulation of Urban MObility - an Overview”. In *The Third International Conference on Advances in System Simulation* (2011), pp. 55–60. 67

[31] Bishop, R. “A Survey of Intelligent Vehicle Applications Worldwide”. In *Proceedings of the IEEE Intelligent Vehicles Symposium* (2000), pp. 25–30. 9

[32] Biswas, S., Tatchikou, R., and Dion, F. “Vehicle-to-Vehicle Wireless Communication Protocols for Enhancing Highway Traffic Safety”. *IEEE Communications Magazine 44*, 1 (2006), 74–82. 14, 15, 53, 66, 72, 73, 74, 76

[33] Bononi, L., and Di Felice, M. “A Cross Layered MAC and Clustering Scheme for Efficient Broadcast in VANETs”. In *IEEE Internatonal Conference on Mobile Adhoc and Sensor Systems* (2007), pp. 1–8. 101, 108

[34] Bouthillier, A. L., and Crainic, T. G. “A cooperative parallel meta-heuristic for the vehicle routing problem with time windows”. *Computers & Operations Research 32*, 7 (2005), 1685–1708. 36

[35] Briesemeister, L., and Hommel, G. “Role-Based Multicast in Highly Mobile but Sparsely Connected Ad Hoc Networks”. In *Proceedings of the 1st ACM International Symposium on Mobile Ad Hoc Networking & Computing* (2000), pp. 45–50. 106

[36] Carbaugh, J., Godbole, D. N., and Sengupta, R. "Safety and capacity analysis of automated and manual highway systems". *Transportation Research Part C: Emerging Technologies 6*, 1-2 (1998), 69–99. 36, 64, 65, 66, 71, 72, 79, 80, 84

[37] Chakravarthy, A., Song, K., and Feron, E. "Preventing Automotive Pileup Crashes in Mixed-Communication Environments". *IEEE Transactions on Intelligent Transportation Systems 10*, 2 (2009), 211–225. 32, 34, 49

[38] Chandra, R. *"Parallel Programming in OpenMP"*. Morgan Kaufmann, 2001. 141

[39] Chen, W., Guha, R. K., Kwon, T. J., Lee, J., and Hsu, Y.-Y. "A survey and challenges in routing and data dissemination in vehicular ad hoc networks". *Wireless Communications & Mobile Computing 11*, 7 (2011), 787–795. 102, 103

[40] Cheng, L., Henty, B. E., Stancil, D. D., Bai, F., and Mudalige, P. "Mobile Vehicle-to-Vehicle Narrow-Band Channel Measurement and Characterization of the 5.9 GHz Dedicated Short Range Communication (DSRC) Frequency Band". *IEEE Journal on Selected Areas in Communications 25*, 8 (2007), 1501–1516. 128

[41] Choi, W., and Swaroop, D. "Assessing the benefits of coordination in automatically controlled vehicles". In *IEEE Intelligent Transportation Systems* (2001), pp. 70–75. xvi, 65, 76, 77, 84

[42] Choi, W., and Swaroop, D. "Assessing the safety benefits due to coordination amongst vehicles during an emergency braking maneuver". In *American Control Conference* (2001), vol. 3, pp. 2099–2104. 36, 64, 65, 72

[43] Cook, W., and Rich, J. L. "A Parallel Cutting-Plane Algorithm for the Vehicle Routing Problem With Time Windows". *Technical Report, Computational and Applied Mathematics, Rice University, Houston, TX* (1999). 36

[44] Djenouri, D., Nekka, E., and Soualhi, W. "Simulation of mobility models in vehicular ad hoc networks". In *Ambi-Sys workshop on Software Organisation and MonIToring of Ambient Systems* (2008), no. 4. 35

[45] Du, S., Liu, F.-q., Xu, S.-z., and Wang, X.-h. "On-Demand Power Control Algorithm Based on Vehicle Ad Hoc Network". *Computer Engineering 16* (2009). 35

[46] Eichler, S., Schroth, C., Kosch, T., and Strassberger, M. "Strategies for Context-Adaptive Message Dissemination in Vehicular Ad Hoc Networks". In *Third Annual International Conference on Mobile and Ubiquitous Systems: Networking & Services* (2006), pp. 1–9. 101, 108

[47] ElBatt, T., Goel, S. K., Holland, G., Krishnan, H., and Parikh, J. "Cooperative collision warning using dedicated short range wireless communications". In *3rd International Workshop on Vehicular Ad Hoc Networks* (2006), pp. 1–9. 66

[48] FARNOUD, F., AND VALAEE, S. “Reliable Broadcast of Safety Messages in Vehicular Ad Hoc Networks”. In *IEEE INFOCOM* (2009), pp. 226–234. 15

[49] FASOLO, E., FURIATO, R., AND ZANELLA, A. “Smart Broadcast algorithm for inter-vehicular communication”. *IWS/WPMC* (2005). 91, 110

[50] FASOLO, E., ZANELLA, A., AND ZORZI, M. “An Effective Broadcast Scheme for Alert Message Propagation in Vehicular Ad Hoc Networks”. In *IEEE International Conference on Communications* (2006), vol. 9, pp. 3960–3965. 35

[51] FRACCHIA, R., AND MEO, M. “Analysis and Design of Warning Delivery Service in Intervehicular Networks”. *IEEE Transactions on Mobile Computing 7*, 7 (2008), 832–845. 79

[52] FRANZ, W., HARTENSTEIN, H., AND MAUVE, M. “Inter-Vehicle Communications Based on Ad Hoc Networking Principles”. *The FleetNet Project, Karlsruhe, Germany: Universtatverlag Karlsruhe* (2005). 6, 16

[53] FUKUYAMA, J. “A delay time analysis for multi-hop V2V communications over a linear VANET”. In *IEEE Vehicular Networking Conference* (2009), pp. 1–7. 35

[54] FÜßLER, H., WIDMER, J., KÄSEMANN, M., MAUVE, M., AND HARTENSTEIN, H. “Contention-Based Forwarding for Mobile Ad Hoc Networks”. *Ad Hoc Networks 1*, 4 (2003), 351–369. 89, 91, 106

[55] **GARCÍA-COSTA, C.**, EGEA-LÓPEZ, E., AND GARCÍA-HARO, J. “Desarrollo de un Modelo Estocástico Para el Estudio de Accidentes en Cadena”. *V Jornadas de Introducción a la Investigación de la UPCT* (2012), 105–107. 23

[56] **GARCÍA-COSTA, C.**, EGEA-LÓPEZ, E., AND GARCÍA-HARO, J. “A stochastic approach for vehicle safety modeling in a platoon of vehicles equipped with vehicular communications”. In *15th International Conference on Transparent Optical Networks* (2013), pp. 1–4. 23

[57] **GARCÍA-COSTA, C.**, EGEA-LÓPEZ, E., AND GARCÍA-HARO, J. “A stochastic model for design and evaluation of chain collision avoidance applications”. *Transportation Research Part C: Emerging Technologies 30* (2013), 126–142. 20, 21, 22

[58] **GARCÍA-COSTA, C.**, EGEA-LÓPEZ, E., AND GARCÍA-HARO, J. “MAC Contention Distributions for Efficient Geo-routing in Vehicular Networks”. In *IEEE International Symposium on Wireless Vehicular Communications* (2013). 23

[59] **GARCÍA-COSTA, C.**, EGEA-LÓPEZ, E., TOMÁS-GABARRÓN, J. B., GARCÍA-HARO, J., AND HAAS, Z. J. “A Stochastic Model for Chain Collisions of Vehicles Equipped With Vehicular Communications”. *IEEE Transactions on Intelligent Transportation Systems 13*, 2 (2012), 503–518. 19, 21, 22

[60] **García-Costa, C.**, Tomás-Gabarrón, J. B., Egea-López, E., and Garcia-Haro, J. "Speeding Up the Evaluation of a Mathematical Model for VANETs Using OpenMP". In *Simulation and Modeling Methodologies, Technologies and Applications. Advances in Intelligent Systems and Computing*, vol. 197. Springer, 2013, pp. 23–37. 21, 23

[61] Geddes, N. B. "Magic motorways". (1940). 5

[62] Ghiani, G., Guerriero, F., Laporte, G., and Musmanno, R. "Real-time vehicle routing: Solution concepts, algorithms and parallel computing strategies". *European Journal of Operational Research 151*, 1 (2003), 1–11. 36

[63] Glimm, J., and Fenton, R. E. "An accident-severity analysis for a uniform-spacing headway policy". *IEEE Transactions on Vehicular Technology 29*, 1 (1980), 96–103. 32, 34, 35, 36, 59, 64, 65

[64] Goumas, G., Kourtis, K., Anastopoulos, N., Karakasis, V., and Koziris, N. "Performance evaluation of the sparse matrix-vector multiplication on modern architectures". *The Journal of Supercomputing 50*, 1 (2009), 36–77. 35

[65] Grime, G., and Jones, I. S. "Car Collisions - the Movement of Cars and Their Occupants in Accidents". *Proceedings of the Institution of Mechanical Engineers: Automobile Division 184*, 1 (1969), 87–136. 32, 49, 64

[66] Harri, J., Filali, F., and Bonnet, C. "Mobility models for vehicular ad hoc networks: a survey and taxonomy". *IEEE Communications Surveys & Tutorials 11*, 4 (2009), 19–41. 19, 35, 63, 67

[67] Hartenstein, H., and Laberteaux, K. *"VANET: Vehicular Applications and Inter-Networking Technologies"*, vol. 1. Chichester: Wiley, 2010. 5, 6, 7, 13, 34, 60

[68] Hartenstein, H., and Laberteaux, K. P. "A tutorial survey on vehicular ad hoc networks". *IEEE Communications Magazine 46*, 6 (2008), 164–171. 9

[69] Hartman, K., and Strasser, J. "Saving Lives Through Advanced Vehicle Safety Technology: Intelligent Vehicle Initiative Final Report". Tech. Rep. FHWA-JPO-05-057, Federal Highway Administration, 2005. 6

[70] Höfs, W., Lappin, J., Schagrin, M., Cronin, B., Resendes, R., Hess, S., Pincus, M., Schade, H.-J., Sill, S., Harding, J., et al. "International Deployment of Cooperative Intelligent Transportation Systems - Bilateral Efforts of the European Commission and United States Department of Transportation". Tech. Rep. FHWA-JPO-12-081, Research and Innovative Technology Administration and Federal Highway Administration, 2012. 7

[71] Huang, C.-L., Fallah, Y. P., Sengupta, R., and Krishnan, H. "Intervehicle Transmission Rate Control for Cooperative Active Safety System". *IEEE Transactions on Intelligent Transportation Systems 12*, 3 (2011), 645–658. 31, 66

[72] IMIELINSKI, T., AND NAVAS, J. C. "GPS-based geographic addressing, routing, and resource discovery". *Communications of the ACM 42*, 4 (1999), 86–92. 91

[73] JANG, I., CHOI, W., AND LIM, H. "An opportunistic forwarding protocol with relay acknowledgment for vehicular ad hoc networks". *Wireless Communications and Mobile Computing 11*, 7 (2011), 939–953. 102, 103

[74] JARUPAN, B., AND EKICI, E. "A survey of cross-layer design for VANETs". *Ad Hoc Networks 9*, 5 (2011), 966–983. 101, 103

[75] JIANG, D., AND DELGROSSI, L. "IEEE 802.11p: Towards an International Standard for Wireless Access in Vehicular Environments". In *IEEE Vehicular Technology Conference* (2008), pp. 2036–2040. 13

[76] JOHANSSON, G., AND RUMAR, K. "Drivers' Brake Reaction Times". *Human Factors: The Journal of the Human Factors and Ergonomics Society 13*, 1 (1971), 23–27. 53

[77] KARP, B., AND KUNG, H. T. "GPSR: Greedy perimeter stateless routing for wireless networks". In *6th Annual International Conference on Mobile Computing and Networking* (2000), pp. 243–254. 91

[78] KATO, S., TSUGAWA, S., TOKUDA, K., MATSUI, T., AND FUJII, H. "Vehicle control algorithms for cooperative driving with automated vehicles and intervehicle communications". *IEEE Transactions on Intelligent Transportation Systems 3*, 3 (2002), 155–161. 66

[79] KAWASHIMA, H. "Japanese perspective of driver information systems". *Transportation 17*, 3 (1990), 263–284. 5

[80] KHABAZIAN, M., AND ALI, M. "A Performance Modeling of Connectivity in Vehicular Ad Hoc Networks". *IEEE Transactions on Vehicular Technology 57*, 4 (2008), 2440–2450. 36

[81] KHOKHAR, R. H., NGADI, M. A., LATIFF, M. S., GHAFOOR, K. Z., AND ALI, S. "Multi-criteria Receiver Self-Election Scheme for Optimal Packet Forwarding in Vehicular Ad Hoc Networks". *International Journal of Computers, Communications & Control 7*, 5 (2012), 865–878. 106

[82] KIM, T., AND JEONG, H.-Y. "Crash Probability and Error Rates for Head-On Collisions Based on Stochastic Analyses". *Intelligent Transportation Systems, IEEE Transactions on 11*, 4 (2010), 896–904. 32, 34, 35, 36, 64, 65

[83] KOTAKEMORI, H., HASEGAWA, H., KAJIYAMA, T., NUKADA, A., SUDA, R., AND NISHIDA, A. "Performance Evaluation of Parallel Sparse Matrix–Vector Products on SGI Altix3700". In *OpenMP Shared Memory Parallel Programming. Lecture Notes in Computer Science*, vol. 4315. Springer, 2008, pp. 153–163. 35

[84] Li, F., and Wang, Y. "Routing in vehicular ad hoc networks: A survey". *IEEE Vehicular Technology Magazine 2*, 2 (2007), 12–22. 90, 102, 103

[85] Li, L.-J., Liu, H.-F., Yang, Z.-Y., Ge, L.-J., and Huang, X.-Y. "Broadcasting methods in vehicular ad hoc networks". *Ruan Jian Xue Bao (Journal of Software) 21*, 7 (2010), 1620–1634. 35

[86] Liu, S., Zhang, Y., Sun, X., and Qiu, R. "Performance Evaluation of Multithreaded Sparse Matrix-Vector Multiplication Using OpenMP". In *11th IEEE International Conference on High Performance Computing and Communications* (2009), pp. 659–665. 35

[87] Luo, J., and Hubaux, J.-P. "A Survey of Research in Inter-Vehicle Communications". In *Embedded Security in Cars*. Springer, 2006, pp. 111–122. 9

[88] Ma, X., and Chen, X. "Delay and Broadcast Reception Rates of Highway Safety Applications in Vehicular Ad Hoc Networks". In *Mobile Networking for Vehicular Environments* (2007), pp. 85–90. 121

[89] Maihofer, C. "A survey of geocast routing protocols". *IEEE Communications Surveys & Tutorials 6*, 2 (2004), 32–42. 91, 103

[90] Miao, L., Djouani, K., van Wyk, B. J., and Hamam, Y. "Evaluation and Enhancement of IEEE 802.11p Standard: A Survey". *Mobile Computing 1*, 1 (2012), 15–30. 13

[91] Misener, J. A., Sengupta, R., and Krishnan, H. "Cooperative Collision Warning: Enabling Crash Avoidance with Wireless Technology". In *12th World Congress on Intelligent Transport Systems* (2005). 66

[92] Murcia-Hernández, R., **García-Costa, C.**, Tomás-Gabarrón, J. B., Egea-López, E., and García-Haro, J. "Parallelization of a Mathematical Model to Evaluate a CCA Application for VANETs". In *1st International Conference on Simulation and Modeling Methodologies, Technologies and Applications* (2011), pp. 5–13. 19, 21, 22

[93] Ning, Z., Jung, Y., Jin, Y., and Kim, K.-C. "Route Optimization for GPSR in VANET". In *IEEE International Advance Computing Conference* (2009), pp. 569–573. 35

[94] Palazzi, C. E., Ferretti, S., Roccetti, M., Pau, G., and Gerla, M. "How Do You Quickly Choreograph Inter-Vehicular Communications? A Fast Vehicle-to-Vehicle MultiHop Broadcast Algorithm, Explained". In *3rd IEEE CCNC International Workshop on Networking Issues in Multimedia Entertainment* (2007). 101, 107, 108

[95] Prasanth, K., Duraiswamy, K., Jayasudha, K., and Chandrasekar, C. "Minimizing end-to-end delay in Vehicular Ad Hoc Network using Edge Node Based Greedy Routing". In *First International Conference on Advanced Computing* (2009), pp. 135–140. 35

[96] PUDASAINI, S., KANG, M., SHIN, S., AND COPELAND, J. A. "COMIC: Intelligent Contention Window Control for Distributed Medium Access". *IEEE Communications Letters 14*, 7 (2010), 656–658. 101, 111

[97] REZAEI, S., SENGUPTA, R., KRISHNAN, H., GUAN, X., AND BHATIA, R. "Tracking the position of neighboring vehicles using wireless communications". *Transportation Research Part C: Emerging Technologies 18*, 3 (2010), 335–350. 63

[98] ROSEN, D. A., MAMMANO, F. J., AND FAVOUT, R. "An electronic route-guidance system for highway vehicles". *IEEE Transactions on Vehicular Technology 19*, 1 (1970), 143–152. 5

[99] RUHRUP, S., KALOSHA, H., NAYAK, A., AND STOJMENOVIC, I. "Message-Efficient Beaconless Georouting With Guaranteed Delivery in Wireless Sensor, Ad Hoc, and Actuator Networks". *IEEE/ACM Transactions on Networking 18*, 1 (2010), 95–108. 106

[100] SAHOO, J., WU, E.-K., SAHU, P. K., AND GERLA, M. "Binary-Partition-Assisted MAC-Layer Broadcast for Emergency Message Dissemination in VANETs". *IEEE Transactions on Intelligent Transportation Systems 12*, 3 (2011), 757–770. 15, 102, 103

[101] SANTA, J., TOLEDO-MOREO, R., ZAMORA-IZQUIERDO, M. A., ÚBEDA, B., AND GÓMEZ-SKARMETA, A. F. "An analysis of communication and navigation issues in collision avoidance support systems". *Transportation Research Part C: Emerging Technologies 18*, 3 (2010), 351–366. 63

[102] SCHOCH, E., KARGL, F., WEBER, M., AND LEINMULLER, T. "Communication patterns in VANETs". *IEEE Communications Magazine 46*, 11 (2008), 119–125. 9

[103] SICHITIU, M. L., AND KIHL, M. "Inter-vehicle communication systems: a survey". *IEEE Communications Surveys & Tutorials 10*, 2 (2008), 88–105. 103

[104] SONG, J., GHOSH, M., MIAOU, S., AND MALLICK, B. "Bayesian multivariate spatial models for roadway traffic crash mapping". *Journal of Multivariate Analysis 97*, 1 (2006), 246–273. 34

[105] SPETHMANN, P., HERSTATT, C., AND THOMKE, S. H. "Crash simulation evolution and its impact on R&D in the automotive applications". *International Journal of Product Development 8*, 3 (2009), 291–305. 67

[106] SWAROOP, D., AND HEDRICK, J. "String stability of interconnected systems". *IEEE Transactions on Automatic Control 41*, 3 (1996), 349–357. 64

[107] TALEB, T., BENSLIMANE, A., AND BEN LETAIEF, K. "Toward an Effective Risk-Conscious and Collaborative Vehicular Collision Avoidance System". *IEEE Transactions on Vehicular Technology 59*, 3 (2010), 1474–1486. 15, 48, 60, 66, 72, 73, 74, 75, 76

[108] TAN, H.-S., AND HUANG, J. "DGPS-Based Vehicle-to-Vehicle Cooperative Collision Warning: Engineering Feasibility Viewpoints". *IEEE Transactions on Intelligent Transportation Systems 7*, 4 (2006), 415–428. 31, 35, 60

[109] TAOKA, G. “Break reaction times of unalerted drivers”. *ITE Journal 59*, 3 (1989), 19–21. 13, 79, 83

[110] TAY, Y., JAMIESON, K., AND BALAKRISHNAN, H. “Collision-minimizing CSMA and its applications to wireless sensor networks”. *IEEE Journal on Selected Areas in Communications 22*, 6 (2004), 1048–1057. 90, 91, 95, 101, 110, 124

[111] TAYSI, Z. C., AND YAVUZ, A. G. “Routing Protocols for GeoNet: A Survey”. *IEEE Transactions on Intelligent Transportation Systems 13*, 2 (2012), 939–954. 103

[112] TEE, C., AND LEE, A. C. “Survey of position based routing for Inter Vehicle Communication system”. In *First International Conference on Distributed Framework and Applications* (2008), pp. 174–182. 16, 18, 103

[113] TORRENT-MORENO, M., MITTAG, J., SANTI, P., AND HARTENSTEIN, H. “Vehicle-to-Vehicle Communication: Fair Transmit Power Control for Safety-Critical Information”. *IEEE Transactions on Vehicular Technology 58*, 7 (2009), 3684–3703. 15, 60

[114] TOURAN, A., BRACKSTONE, M. A., AND MCDONALD, M. “A collision model for safety evaluation of autonomous intelligent cruise control”. *Accident Analysis & Prevention 31*, 5 (1999), 567–578. 32, 34, 36, 65

[115] TREIBER, M., HENNECKE, A., AND HELBING, D. “Congested traffic states in empirical observations and microscopic simulations”. *Physical Review E 62*, 2 (2000), 1805–1824. 32, 49

[116] TREIBER, M., KESTING, A., AND HELBING, D. “Delays, inaccuracies and anticipation in microscopic traffic models”. *Physica A: Statistical Mechanics and its Applications 360*, 1 (2006), 71–88. 67, 68

[117] VAHIDI, A., AND ESKANDARIAN, A. “Research advances in intelligent collision avoidance and adaptive cruise control”. *IEEE Transactions on Intelligent Transportation Systems 4*, 3 (2003), 143–153. 31, 35

[118] WATTEYNE, T., AUGÉ-BLUM, I., DOHLER, M., AND BARTHEL, D. “Reducing Collision Probability in Wireless Sensor Network Backoff-Based Election Mechanisms”. In *IEEE Global Telecommunications Conference* (2007), pp. 673–677. 102, 103

[119] WILLIAMS, S., OLIKER, L., VUDUC, R., SHALF, J., YELICK, K., AND DEMMEL, J. “Optimization of sparse matrix–vector multiplication on emerging multicore platforms”. *Parallel Computing 35*, 3 (2009), 178–194. 35

[120] WILLKE, T. L., TIENTRAKOOL, P., AND MAXEMCHUK, N. F. “A survey of inter-vehicle communication protocols and their applications”. *IEEE Communications Surveys & Tutorials 11*, 2 (2009), 3–20. 10, 103

[121] WILSON, R. E. "An analysis of gipps's car-following model of highway traffic". *IMA Journal of Applied Mathematics 66*, 5 (2001), 509–537. 67, 68

[122] WISITPONGPHAN, N., BAI, F., MUDALIGE, P., SADEKAR, V., AND TONGUZ, O. "Routing in Sparse Vehicular Ad Hoc Wireless Networks". *IEEE Journal on Selected Areas in Communications 25*, 8 (2007), 1538–1556. 34, 50, 78

[123] WISITPONGPHAN, N., BAI, F., MUDALIGE, P., AND TONGUZ, O. K. "On the Routing Problem in Disconnected Vehicular Ad-hoc Networks". In *26th IEEE International Conference on Computer Communications* (2007), pp. 2291–2295. 35

[124] WOLTERINK, W. K., HEIJENK, G., AND VAN DEN BERG, J. "An analytical model for the performance of geographical multi-hop broadcast". In *IEEE Vehicular Networking Conference* (2012), pp. 242–249. 102, 103

[125] XIE BO, XIAO XIAO-QIANG, X. M. L. Z.-Q. "Survivability model for vehicular Ad-Hoc network based on Markov chain". *Journal of Computer Applications 28*, 10 (2008), 2577–2579. 36

[126] XIN, W., HOURDOS, J., MICHALOPOULOS, P., AND DAVIS, G. "The Less-Than-Perfect Driver: A Model of Collision-Inclusive Car-Following Behavior". *Transportation Research Record: Journal of the Transportation Research Board 2088* (2008), 126–137. 67, 68, 69

[127] YAN, G., MITTON, N., AND LI, X. "Reliable Routing in Vehicular Ad Hoc Networks". In *IEEE 30th International Conference on Distributed Computing Systems Workshops* (2010), pp. 263–269. 102, 103

[128] YANG, X., LIU, L., VAIDYA, N. H., AND ZHAO, F. "A vehicle-to-vehicle communication protocol for cooperative collision warning". In *First Annual International Conference on Mobile and Ubiquitous Systems: Networking and Services* (2004), pp. 114–123. 66

[129] YANG, Y., AND YUM, T.-S. "Delay distributions of slotted ALOHA and CSMA". *IEEE Transactions on Communications 51*, 11 (2003), 1846–1857. 34, 55

[130] YE, F., YIM, R., GUO, J., ZHANG, J., AND ROY, S. "Prioritized Broadcast Contention Control in VANET". In *IEEE International Conference on Communications* (2010), pp. 1–5. 110

[131] YE, F., YIM, R., ROY, S., AND ZHANG, J. "Efficiency and Reliability of One-Hop Broadcasting in Vehicular Ad Hoc Networks". *IEEE Journal on Selected Areas in Communications 29*, 1 (2011), 151–160. 129

[132] YIN, S., LI, Z., ZHANG, Y., YAO, D., SU, Y., AND LI, L. "Headway distribution modeling with regard to traffic status". In *IEEE Intelligent Vehicles Symposium* (2009), pp. 1057–1062. 78, 79

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I herewith declare that I have produced this work without the prohibited assistance of third parties and without making use of aids other than those specified; notions taken over directly or indirectly from other sources have been identified as such. This work has not previously been presented in identical or similar form to any other examination board.

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