Trabajo de fin de período formativo
Reglas de diseño para control predictivo usando Control Dinámico Matricial sin restricciones

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Capítulo 1

Preámbulo

El objeto del presente trabajo es obtener una serie de reglas de diseño para los parámetros del Control Dinámico Matricial (DMC). Para conseguir este objetivo se trabajará con un sistema LTI (Linear Time Invariant) equivalente al algoritmo del DMC y se estudiará el efecto que tienen los parámetros del DMC sobre los polos del sistema (y, por lo tanto, sobre su respuesta).Para simplificar el estudio se trabajará con el equivalente de primer orden del bechmark a estudiar. Una vez se conoce el efecto de cada parámetro sobre la respuesta temporal, se podrán establecer unas reglas de sintonía para dichos parámetros. Para asegurar la validez de dichas reglas se pondrán a prueba mediante simulación en benchmarks extraídos de la bibliografía y en una maqueta térmica real.

Este trabajo segirá el siguiente esquema:

- En primer lugar se hará una introducción al control predictivo. Dicha introducción constará de una breve historia del control predictivo seguida de una explicación de los algoritmos de control más populares (prestando especial atención al DMC) y de un resumen de las aplicaciones de dichos algoritmos.
- A continuación se enumerarán los parámetros del DMC y se hará un pequeño resumen del estado del arte sobre su ajuste.
- Una vez se hayan enumerado los parámetros del DMC, se resumirán los dos artículos en los que ha desarrollado el trabajo y que se adjuntan como anexos.
- Por último, se extrarán conclusiones del trabajo presentado.

Capítulo 2

Introducción al control predictivo

En este capítulo se va a explicar brevemente la historia del control predictivo y sus algoritmos más populares. Se prestará especial atención al DMC, del que se hará su desarrollo matemático completo. También se expondrá una breve comparación de las aplicaciones de cada algoritmo.

El control predicitivo se originó en la segunda mitad de los años setenta. El término control predictivo no se refiera a un único algoritmo sino a una familia de métodos que siguen la misma filosofía:

- Se usa un modelo para predecir las la respuesta de un proceso en instantes futuros
- Se calcula para cada instante una secuencia de incrementos de control que minimizan una función objetivo
- Solo se usa el primer incremento de control de la secuencia calculada. Entonces se desplaza el horizonte hacia el futuro y se vuelve a aplicar el algoritmo.

La diferencia entre los distintos métodos consiste en el modelo usado para representar el proceso y la función de coste a minimizar.

El éxito de esta familia de algortimos se debe a los siguientes motivos:

- Puede usarse con una enorme variedad de procesos, incluyendo aquellos con tiempo, muerto, fase no mínima o inestables.

- Puede implementarse con facilidad en casos multivariables.
- Introduce realimentación de forma natural para compensar perturbaciones medibles.
- la ley de control es fácil de implementar.
- El tratamiento de la restricciones es conceptualmente simple y puede ser sistemáticamente incluido durante el diseño del proceso.
- Es muy útil cuando se conocen las referencias futuras.

Sin embargo, como es de esperar, también tiene inconvenientes:

- Requiere mayor coste computacional que otras soluciones (como el PID).
- El resultado final depende de la exactitud del modelo del proceso.

2.0.1. Historia del control predictivo

A finales de los años 70 aparecieron varios artículos que comenzaban adefinir el control predictivo:

- Richalet et al [19], [20] presentaban el Control algorítmico por modelo (MAC)
- Cutler y Ramaker [12] presentaban el Control Dinámico Matricial

Ambos métodos usan un modelo dinámico del proceso para predecir el efecto de las futuras acciones de control en la salida.

El control predicitivo se volvió popular en la industria química rápidamente. Los principales motivos fueron la sencillez del algoritmo y el uso de modelos de respuesta a escalón o impulso que eran más intuitivos y requerían menos información *a priori* de la planta que los modelos matemáticos. Sin embargo a pesar de su éxito estos aalgoritmos adolecían de teorías formales que proporcionaran estabilidad o robustez. De hecho, el caso del horizonte finito era demasiodo complicado de resolver, salvo en casos especiales.

Aparte de los trabajos anteriores, también se desarrolaron varios métodos desde una línea de trabajo diferente: El control adaptativo. Son de destacar varios trabajos:

- Control auto ajustable basado en predictor [21]: este método minimiza para, para los valores predichos más recientes, el valor esperado de un criterio cuadrático en un horizonte de control.
- Control adaptativo de horizonte extendido [23]: Este método trata de mantener la salida futura (calculada mediante la ecuación diofantina) cercana a la referencia en un período de tiempo posterior al retraso.
- Control auto adaptativo de predicción extendida [22]: Este método propone una señal de control constante empezando desde el momento presente mientras se usa un predictor sub-óptimo en lugar de resolver la ecuación Diofantina.

Sin embargo, entre estos métodos desarrollados en torno a la idea de control adaptativo, el más popular es el control predictivo generalizado (que se describirá más adelante), que se distingue del control predicitvo basado en modelos dinámicos en el uso de la ecuación Diofantina para realizar predicciones

Los controladores predicitvos pueden representarse como diagramas de bloques típicos en la teoría de control. Esto permite que sean tratados mediante la teoría de control y permite abordar problemas complejos como procesos no lienales, con integradores o con ruido. También permite predecir la salida basándose en el análisis de los polos del sistema.

El uso de la teoría de control permitió avanzar en la consecución de robustez y estabilidad. Sin embargo, la falta de estabilidad original para controladores de horizonte finito seguía siendo un problema. Esto propició la apararición de métodos que garantizaban estabilidad en los años 90: CRHPC [24] y SIORHC [25]. Kouvaritakis et al [26] presentaron una formulación estable del GPC que estabiliza el proceso antes de minimizar la función objetivo.

La estabilidad de los problemas con restricciones fue un problema serio, ya que al principio el hecho de que el algoritmo resolviera un problema de optimización no garantizaba que el resultado fuera estable. El uso de restricciones, funciones de Lyapunov o conjuntos de invariantes ha permitido la creación de técnicas que garantizan la estabilidad del sistema.

También se han obtenido resultados prometedores en el campo de la robustez. La idea clave es tener en cuenta ciertas incertidumbres del proceso de manera explícita y diseñar el control predictivo para que optimice la función objetivo para la peor situación de incertidumbres.

El control predicitvo basado en modelos se considera una técnica madura para ser usado con procesos lineales y lentos, típicos en la industria de procesos. Durante muchos años se ha considerado que los procesos más complejos (rápidos, híbridos o no lineales) estaban fuera de su campo de aplicación, aunque esta concepción está empezando a cambiar.

2.0.2. Métodos de control predictivo

Tal y como se ha mencionado anteriormente, la diferencia entre los distintos métodos de control predictivo son principalmente el modelo usado para predecir el comportamiento del proceso y la función de coste a optimizar. Los algoritmos más conocidos se presentan acontinuación.

Control Dinámico Matricial

El Control dinámico matricial usa como modelo la respuesta a escalón unitario del proceso. El proceso debe ser estable y sin integradores.

$$y(t) = \sum_{i=1}^{\infty} g_i \Delta u(t-i) \tag{2.1}$$

Donde g_i son los coeficientes de respuesta a escalón, Δu es el incremento de control, y es la respuesta del sistema y n(t) son las perturbaciones. Los valores predichos serán(comenzando las predicciones en el instante t):

$$\hat{y}(t+k) = \sum_{i=1}^{\infty} g_i \Delta u(t+k) + \hat{n}(t+k) = \sum_{i=1}^{k} g_i \Delta u(t+k-i) + \sum_{i=k+1}^{\infty} g_i \Delta u(t+k-i) + \hat{n}(t+k) \tag{2.2}$$

Considerando las perturbaciones constantes (siendo $y_m(t)$ la respuesta medida):

$$\hat{n}(t+k) = \hat{n}(t) = y_m(t) - \hat{y}(t) = y_m(t) - \sum_{i=1}^{\infty} g_i \Delta u(t-i) \tag{2.3}$$

Entonces la ecuación 2.3 puede escribirse:

$$\hat{y}(t+k) = \sum_{i=1}^{k} g_i \Delta u(t+k-i) + \sum_{i=k+1}^{\infty} g_i \Delta u(t+k-i) + y_m(t) - \sum_{i=1}^{\infty} g_i \Delta u(t-i) = \sum_{i=1}^{k} g_i \Delta u(t+k-i) + f(t+k) \tag{2.4}$$

Siendo $f(t+k)$ la respuesta libre, la parte de la respuesta que no depende de las futuras acciones de control, descrita como sigue:

$$f(t+k) = y_m(t) + \sum_{i=1}^{\infty}(g_{k+i} - g_i)\Delta u(t-i) \tag{2.5}$$

Si el proceso es asintóticamente estable, los coeficientes de la respuesta a, g_i tenderán a un valor constante tras N períodos de muestreo, luego:

$$(g_{k+i} - g_i) \rightarrow 0, i > N \tag{2.6}$$

Y la Ecuación 2.5 puede simplificarse:

$$f(t+k) = y_m(t) + \sum_{i=1}^{N}(g_{k+i} - g_i)\Delta u(t-i) \tag{2.7}$$

Aplicando las ecuaciones anteriores para un horizonte de predicción igual a Pr y un horizonte de control igual a M:

$$\hat{y}(t+1/t) = g_1\Delta u(t) + f(t+1) \tag{2.8}$$

$$\hat{y}(t+2/t) = g_2\Delta u(t) + g_1\Delta u(t+1) + f(t+1) \tag{2.9}$$

$$\vdots$$

$$\hat{y}(t+Pr/t) = \sum_{i=Pr-M+1}^{Pr} g_i\Delta u(t+Pr-i) + f(t+Pr) \tag{2.10}$$

Definiendo la matriz dinámica del sistema como:

$$\mathbf{G} = \begin{pmatrix} g_1 & 0 & ... & 0 \\ g_2 & g_1 & ... & 0 \\ g_M & g_{M-1} & ... & g_1 \\ : & : & ... & : \\ g_{Pr} & g_{Pr-1} & ... & g_{Pr-M+1} \end{pmatrix} \tag{2.11}$$

Usando formulación matricial, se puede escribir:

$$\hat{\mathbf{y}} = \mathbf{G\Delta u} + \mathbf{f} \tag{2.12}$$

Siendo $\hat{\mathbf{y}}$ un vector Pr-dimensional que contiene las predicciones futuras en el horizonte de predicción, $\mathbf{\Delta u}$ un vector M-dimensional vector que contiene los incremenos de control y $\mathbf{f}$ el vector de respuesta libre. Esta expresión relaciona las respuestas futuras con los incrementos de control.

El objetivo del DMC es encontrar un incremento de control que minimice una determinada función de coste que incluye errores y esfuerzos de control.

$$J = \sum_{j=1}^{p} (\hat{y}(t+j|t) - w(t+j))^2 + \sum_{j=1}^{m} \lambda(\Delta u(t+j-1))^2 \tag{2.13}$$

$$\mathbf{J} = \mathbf{e}\mathbf{e}^{\mathbf{T}} + \lambda \mathbf{\Delta u} \mathbf{\Delta u}^{\mathbf{T}} \tag{2.14}$$

Donde $\mathbf{e}$ es el vector de errores y $\mathbf{\Delta u}$ el vector de esfuerzos de control. En un problema sin restricciones, el vector de esfuerzos de control optimizados se obtiene resolviendo la Ecuación 2.15:

$$\frac{\mathbf{dJ}}{\mathbf{d\Delta u}} = 0 \tag{2.15}$$

Para resolver la ecuación 2.15 la dividiremos en dos términos: El término de errores (J_1) y el término de incrementos de control (J_2).

$$J_1 = \sum_{j=1}^{p} (\hat{y}(t+j|t) - w(t+j))^2 \tag{2.16}$$

$$J_2 = \sum_{j=1}^{m} \lambda(\Delta u(t+j-1))^2 \tag{2.17}$$

Teniendo en cuenta que $\hat{y}(t+j|t) = \sum_{i=1}^{j} g_i \Delta u(t+j-i) + f(t+j)$, se puede expresar ($J_1$) de la siguiente manera:

$$J_1 = \sum_{j=1}^{p} (\hat{y}(t+j|t) - w(t+j))^2 = \sum_{j=1}^{p} (\sum_{i=1}^{j} g_i \Delta u(t+j-i) + f(t+j) - w(t+j))^2 =$$
$$\sum_{j=1}^{p} ((\sum_{i=1}^{j} g_i \Delta u(t+j-i))^2 + 2f(t+j) \sum_{i=1}^{j} g_i \Delta u(t+j-i) - 2w(t+j) \sum_{i=1}^{j} g_i \Delta u(t+j-i) -$$
$$2f(t+j)w(t+j) + f(t+j)^2 + w(t+j)^2) \tag{2.18}$$

Para simplificar el análisis, J_1 puede expresarse a su vez como la suma de otros términos que habrá que tratar por separado.

$$J_{11} = \sum_{j=1}^{p}(\frac{1}{2}\sum_{i=1}^{j} g_i \Delta u(t+j-i))^2 = \frac{1}{2}(g_1\Delta u(t))^2 + \frac{1}{2}(g_1\Delta u(t+1) + g_2\Delta u(t))^2 + ...+ \frac{1}{2}(g_1\Delta u(t+p-1) + .. + g_p\Delta u(t))^2 \quad (2.19)$$

$$\begin{aligned} J_{12} = \sum_{j=1}^{p}(f(t+j)\sum_{i=1}^{j} g_i \Delta u(t+j-i)) = (f(t+1)g_1\Delta u(t)) + \\ + (f(t+2)(g_1\Delta u(t+1) + g_2\Delta u(t))) + ...+ \\ + (f(t+p)(g_1\Delta u(t+p-1) + ... + g_p\Delta u(t))) \end{aligned} \quad (2.20)$$

$$J_{13} = \sum_{j=1}^{p}(w(t+j)\sum_{i=1}^{j} g_i \Delta u(t+j-i)) = (w(t+1)g_1\Delta u(t)) + (w(t+2)(g_1\Delta u(t+1) + g_2\Delta u(t))) + ...+ + (w(t+p)(g_1\Delta u(t+p-1) + ... + g_p\Delta u(t))) \quad (2.21)$$

$$J_2 = \lambda\sum_{j=1}^{p}(\Delta u(t+j-1))^2 = \lambda\Delta u(t)^2 + \lambda\Delta u(t+1)^2 + ... + \lambda\Delta u(t+m-1)^2 \quad (2.22)$$

Teniendo en cuenta todo lo anterior, se pueden calcular las derivadas parciales de J respecto a los incrementos de control.

$$J = \sum_{j=1}^{p}(J_{11} + J_{12} - J_{13} - 2f(t+j)w(t+j) + f(t+j)^2 + w(t+j)^2) + J_2 \quad (2.23)$$

$$\frac{\delta J}{\delta \Delta u(t)} = 2(\frac{\delta J_{11}}{\delta \Delta u(t)} + \frac{\delta J_{12}}{\delta \Delta u(t)} - \frac{\delta J_{13}}{\delta \Delta u(t)}) + 2\Delta u(t) = 0$$

$$\frac{\delta J}{\delta \Delta u(t+1)} = 2(\frac{\delta J_{11}}{\delta \Delta u(t+1)} + \frac{\delta J_{12}}{\delta \Delta u(t+1)} - \frac{\delta J_{13}}{\delta \Delta u(t+1)}) + 2\Delta u(t+1) = 0$$

.
.
.

$$\frac{\delta J}{\delta \Delta u(t+m-1)} = 2(\frac{\delta J_{11}}{\delta \Delta u(t+m-1)} + \frac{\delta J_{12}}{\delta \Delta u(t+m-1)} - \frac{\delta J_{13}}{\delta \Delta u(t+m-1)}) + 2\Delta u(t+m-1) = 0 \tag{2.24}$$

Formándose un sistema de m ecuaciones con m incógnitas, los incrementos de control. Si se tienen en cuenta las siguientes igualdades:

$$\begin{pmatrix} \frac{\delta J_{11}}{\delta \Delta u(t)} \\ \frac{\delta J_{11}}{\delta \Delta u(t+1)} \\ . \\ . \\ . \\ \frac{\delta J_{11}}{\delta \Delta u(t+m-1)} \end{pmatrix} = \mathbf{G^T G} \begin{pmatrix} \Delta u(t) \\ \Delta u(t+1) \\ . \\ . \\ . \\ \Delta u(t+m-1) \end{pmatrix} = 0 \tag{2.25}$$

$$\begin{pmatrix} \frac{\delta J_{12}}{\delta \Delta u(t)} \\ \frac{\delta J_{12}}{\delta \Delta u(t+1)} \\ . \\ . \\ . \\ \frac{\delta J_{12}}{\delta \Delta u(t+m-1)} \end{pmatrix} = \mathbf{G^T} \begin{pmatrix} f(t+1) \\ f(t+2) \\ . \\ . \\ . \\ f(t+p) \end{pmatrix} = 0 \tag{2.26}$$

$$\begin{pmatrix} \frac{\delta J_{13}}{\delta \Delta u(t)} \\ \frac{\delta J_{13}}{\delta \Delta u(t+1)} \\ . \\ . \\ . \\ \frac{\delta J_{13}}{\delta \Delta u(t+m-1)} \end{pmatrix} = \mathbf{G^T} \begin{pmatrix} w(t+1) \\ w(t+2) \\ . \\ . \\ . \\ w(t+p) \end{pmatrix} = 0 \tag{2.27}$$

El sistema de ecuaciones puede expresarse de la siguiente manera:

$$(\mathbf{G^T G} + \lambda \mathbf{I})\mathbf{\Delta u} + \mathbf{G^T}(\mathbf{f} - \mathbf{w}) = 0 \tag{2.28}$$

$$(\mathbf{G^T G} + \lambda \mathbf{I})\mathbf{\Delta u} = \mathbf{G^T}(\mathbf{w} - \mathbf{f}) \tag{2.29}$$

Siendo el resultado:

$$\mathbf{\Delta u} = (\mathbf{G^T G} + \lambda \mathbf{I})^{\mathbf{-1}} \mathbf{G^T} (\mathbf{w} - \mathbf{f}) \tag{2.30}$$

Un ejemplo de producto industrial que usa DMC es «AspenTech». Existen unas 1833 aplicaciones conocidas de «AspenTech», mayoría de las cuales pertenecen al campo de la refinería.

Control Algorítmico por Modelo

Este método es muy similar al DMC salvo por las siguientes diferencias:

- Usa un modelo de respuesta a impulso que solo es válido para procesos estables. Calcula el valor de u(t) en lugar de Δu(t).
- No utiliza el concepto de horizonte de control, por lo que se calculan tantas señales de control como futuras salidas.
- Introduce una trayectoria de referencia que evoluciona desde la salida real hasta la referencia deseada de acuerdo a una constante de tiempo.
- La función de coste a minimizar es la varianza del error entre la trayectoria de referencia y el error.

Control Predictivo funcional

Este método fue desarrollado para procesos rápidos. Utiliza un modelo de «estado en espacio» (state space model) del proceso y puede trabajar con modelos internos inestables lineales y no lineales. La dinámica no lineal puede introducirse en la forma de un modelo de estado en espacio no lineal.

El PFC tiene dos características distintivas:

- Puntos de coincidencia: Este concepto se usa para simplificar el cálculo considerando solo un subconjunto de puntos en el horizonte de predicción. Las salidas futuras predichas y deseadas deben coincidir en esos puntos, no en todo el horizonte de predicción.
- El controlador parametriza la señal de control usando un conjunto de funciones de base polinómicas. Esto permite especificar perfiles de entrada relativamente complejos a lo largo de un gran horizonte usando un número pequeño de parámetros. Escoger la familia de estas funciones de base establece muchas de las propiedades del perfil de entrada

computado. Se pueden seleccionar estas funciones para que sigan una referencia polinómica sin retraso.

La función de coste a minimizar es:

$$J = \sum_{j=1}^{n_H} [\hat{y}(t + h_j) - w(t + h_j)]^2 \tag{2.31}$$

Un ejemplo de producto industrial que usa PFC es «Adersa».

Control Predictivo Generalizado

Las predicciones del GPC se basan en un modelo CARIMA:

$$A(z^{-1})y(t) = B(z^{-1})z^{-d}u(t-1) + C(z^{-1})\frac{e(t)}{\Delta} \tag{2.32}$$

Donde las perturbaciones no medibles vienen dadas por un ruido blanco coloreado por $C(z^{-1})$. Como su verdadero valor es difícil de conocer, este polinomio puede usarse para un óptimo rechazo de perturbaciones, aunque su papel en la mejora de la robustez es más convincente.
La derivación de la predicción óptimase hace resolviendo una ecuación Diofantina cuya solución puede obtenerse mediante un algoritmo recursivo eficiente.
La función de coste a minimizar es la siguiente:

$$J(N_1, N_2, N_u) = \sum_{j=N_1}^{N_2} \delta(j)[\hat{y}(t+j|t)-w(t+j)]^2 + \sum_{j=N_1}^{N_2} \lambda(j)[\Delta u(t+j-1)]^2 \tag{2.33}$$

Las secuenciasde peso $\delta(j)$ y Δu se suelen seleccionar constantes o exponencialmente crecientes y la trayectoria de referencia $w(t + j)$ puede generarse por una simple recursión que empieza en la salida actual y tiene exponencialmente a la referencia deseada.

2.0.3. Tecnologías Industriales

Las siguientes empresas (y sus productos) se pueden considerar como representativas del estado del arte del control predictivo en la industria:

- Aspentech: DMC
- Adersa: Identification and Command (IDCOM), Hierarchical Constraint Control (HIECON) y PFC

- Honeywell Profomatics: Robust Model Predictive Technology (RMPCT) y Predictive Control Technology (PCT)
- Setpoint Inc: Setpoint Multivariable Control Architecture (SMCA) e IDCOM-M (multivariable)
- Treiber Controls: Optimum Predictive Control

El Cuadro 2.1 [27] muestra el número de aplicaciones de de cada empresa. Se puede apreciar que el DMC (Aspentech) es, con diferencia, el algoritmo más utilizado.

Cuadro 2.1: Aplicaciones industriales del control predictivo

Área	Aspentech	Honeywell	Adersa	Setpoint Inc	Treiber
Refinería	1200	480	280	320	250
Petroquímica	450	80	0	40	0
Química	100	20	3	20	150
Pulpa y papel	18	50	0	0	5
Gas	0	10	0	0	0
Utilidades	0	10	10	0	0
Separación de Aire	0	0	0	0	5
Minería y metalúrgica	8	6	7	2	6
Procesado de alimentos	0	0	41	0	0
Polímeros	17	0	0	0	0
Hornos	0	0	42	0	0
Aeroespacio/Defensa	0	0	13	0	0
Automoción	0	0	0	0	0
Otros	40	40	1045	20	0
Total	1833	696	1438	402	438

Capítulo 3

Ajuste de parámetros del DMC

En este capítulo se desarrollarán las reglas de sintonía de los parámetros del DMC. Se empezarán describiendo los parámetros que gobiernan el DMC. A continuación se explicará el desarrollo de las reglas de sintonía dividiéndolo en los trabajos publicados o en proceso de publicación.

3.1. Resumen de parámetros

Del desarrollo matemático del capítulo anterior, se extraen los siguientes parámetros que gobiernan el DMC:

- Horizonte de predicción (Pr): Indica el horizonte de timpo durante el que la salida debe seguir a la referencia.también puede expresarse como el período comprendido entre N_1 and N_2.
- Horizonte de control (M): Indica los períodos e muestreo futuros para los que se calculan incrementos de control.
- Tiempo de muestreo (T)
- Horizonte de modelo ($\mathrm{n_g}$): Indica el intervalo de la respuesta a escalón que se usa como modelo.
- Factor de peso (λ): Este coeficiente penaliza la señal de control para evitar que sea demasiado agresiva y se genere una respuesta oscilante. También tiene el efecto de dar robustez matemática al proceso minimizando el efecto de tener una matriz ($\mathbf{G^TG} + \lambda\mathbf{I}$) no posible de invertir.

3.2. Estado del arte

A pesar de ser uno de los métodos más populares en la industria, aún se sigue investigando un método para el ajuste de los parámetros del DMC

Existen diversos métodos matemáticos como el presentado por Shrindar y Cooper [1] que introduce un método para calcular el factor de peso minimizando el número de condición de la matriz del sistema. Permite calcular los parámetros mediante unas sencillas reglas:

1. Aproximar el proceso mediante un sistema de primer orden con retraso (FOPDT)
$$\frac{y(s)}{u(s)} = \frac{K_p e^{\Theta_p s}}{\tau_p s + 1} \tag{3.1}$$
2. Seleccionar, si es posible, un tiempo de muestreo T que cumpla:
$$T \leq 0,1\tau_p T \leq 0,5\Theta_p \tag{3.2}$$
3. Calcular el retraso discreto.
$$k = \frac{\Theta_p}{T} + 1 \tag{3.3}$$
4. Calcular el horizonte de predicción y de modelo.
$$Pr = n_g = 5\frac{\tau_p}{T} + k \tag{3.4}$$
5. Seleccionar el horizonte de control y calcular el factor de peso.
$$f = 0; M = 1f = \frac{M}{500}(\frac{3,5\tau_p}{T} + 2 - \frac{M-1}{2}); M \geq 2\lambda = fK_p^2 \tag{3.5}$$

Este es uno de los métodos más exitosos, pero no el único:

- Trierweiller y Farina [2] proponen un método que usa un número de funcionamiento de robustez (Robustness Performance Number, RPN) que indica lo difícil que es para un sistema alcanzar el punto de funcionamiento deseado con robustez. Este método modifica la función normal de coste cuando factoriza la matriz del sistema.
- Han, Zhao y Qian [3] proponen un algoritmo de maximización-minimización sobre un índice de prestaciones.

- Garriga and Soroush [11] proponen un estudio de los parámetros mediante emplazamiento de autovalores del jacobiano de la planta. Al conocer el efecto de los parámetros en el jacobiano de la planta, se puede predecir su comportamiento.

Los ejemplos anteeriores son métodos analíticos, obtienen sus resultados aplicando el análisis matemático a las ecuaciones que definen el control predictivo y la dinamíca de sistemas. Existe otra familia de métodos menos analíticos que, aparte de en el cálculo, se sustentan en la heurística y en la experiencia. Algunos de estos métodos son:

- Iglesias, Sanjuan y Smith [4] proponen una formula obtenida mediante métodos estadísticos para calcular el factor de peso.

$$\lambda = 1,631\mathrm{K_p}(\frac{\Theta}{\tau})^{0,4094} \tag{3.6}$$

  Siendo $\mathrm{K_p}$ la ganancia del proceso, τ y $\mathrm{t_0}$ la constante de tiempo y el retraso del sistema de primer orden equivalente al proceso.

- En la misma línea del método anterior Bagheri y Khaki-Sedigh [17] proponen el uso de la varianza y obtienen la siguiente fórmula (inspirada en [1]):

$$\lambda = \mathrm{fK_p^2f} = 0,84(\frac{\Theta}{\tau} + 0,94)^{0,15}\Gamma^{0,94} \tag{3.7}$$

  usando lógica difusa para Γ (parámetro exclusivo de este método) se obtiene una expresión más sencilla:

$$\begin{aligned} \lambda = 0,11\mathrm{K_p^2}(\frac{\Theta}{\tau} + 0,94)\ \Gamma = 0,1 \text{ Importancia del error de salida} \\ \lambda = 0,84\mathrm{K_p^2}(\frac{\Theta}{\tau} + 0,94)\ \Gamma = 1 \text{ Intermedio} \\ \lambda = 6,67\mathrm{K_p^2}(\frac{\Theta}{\tau} + 0,94)\ \Gamma = 10 \text{ Importancia del esfuerzo de control} \end{aligned} \tag{3.8}$$

- Wojsznis et al [18] presentan el uso de técnicas heurísticas obtenidas mediante experimentación y simulación.

Los métodos descritos anteriormente coinciden en el coinciden en la importancia del horizonte de predicción y el factor de peso, pero no parece existir un consenso claron sobre cual es el factor clave en el que hay que centrarse. El método de Shrindar y Cooper [1] se orienta claramente al factor de peso, pero otros autores ([5]) hacen notar que el efecto de este parámetro está fuertemente influencido por el horizonte de predicción. Como puede verse en la expresión de la función de coste del DMC (Ecuación 2.13), esta esta compuesta por dos términos: El término de errores y el término de incrementos de control. El factor de peso solo esta presente en el término de incrementos de control y la importancia de este término depende del horizonte de control, mientras que la importancia del término de errores depende del horizonte de predicción. Esto puede ocasionar que, dependiendo de los valores de ambos horizontes, el término de incrementos de control sea mucho menos que el de errores y, por lo tanto, solo valores muy grandes del factor de peso tendrían algún efecto.

3.3. Reglas de diseño

En este capítulo se expondrán las reglas de diseño para los parámetros del DMC. Estas reglas se han desarrollado en dos publicaciones:

- "Design rules for model predictive control based on unconstrained Dynamic Matrix Control"(Anexo I) presentado en el Seminario Anual de Automática y Electrónica Industrial en 2011

- "Tuning rules for a quick start up in Dynamic Matrix Control"(Anexo II, en proceso de revisión en la publicación ISA Transactions)

3.3.1. Primera aproximación al problema

Para hallar unas nuevas reglas de diseño, se ha porcedido a estudiar el efecto que tienen los parámetros del DMC en los polos del sistema. Este estudio se presenta en el trabajo "Design rules for model predictive control based on unconstrained Dynamic Matrix Control"(Anexo I) presentado en el Seminario Anual de Automática y Electrónica Industrial en 2011

Para realizar este estudio se expresó el agoritmo del DMC como un sistema LTI (Linear Time invariant), como se puede ver en la Figura 3.1.

Donde:

$$\mathrm{R_P} = 1 + \sum_{\mathrm{i}=1}^{\mathrm{Pr}} \mathrm{k_i S_n^i} \tag{3.9}$$

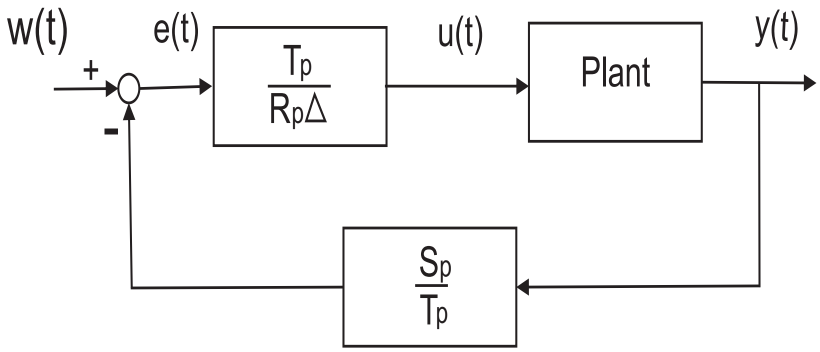


Figura 3.1: DMC como sistema LTI

$$T_P = \sum_{i=1}^{Pr} k_i q^i \quad (3.10)$$

$$S_P = \sum_{i=1}^{Pr} k_i \quad (3.11)$$

Se realizaron varias simulaciones con un benchmark (Ecuación 3.12) y se estudión el efecto que ejercían los parámetros del DMC en los polos en lazo cerrado. Este estudio permitió obtener las siguientes conclusiones:

- El horizonte de control hace que los polos reales positivos aumenten su valor y acaben volviéndose dominantes. Este hace que la respuesta del sistema se vuelva más lenta, pero sin oscilaciones. Se valor máximo deberá igualar la constante de tiempo del sistema de primer orden equivalente

- El factor de peso funciona de forma opuesta al anterior. Al aumentar su valor aumenta el valor de los reales disminuye y aumenta el de los polos complejos. Sin embargo, la parte imaginaria del polo disminuye mientras que la parte real aumenta, por lo que las oscilaciones disminuyen.

- Son aconsejables valores pequeños para el horizonte de control.

- El horizonte de modelo deberá tomar un valor igual al tiempo de establecimiento másel horizonte de predicción.

- los horizontes de control y de predicción tienen un valor máximo útil.

$$G(s) = \frac{e^{-16s}}{(150s+1)(24s+1)} \tag{3.12}$$

3.3.2. Mejora del primer método

El método expuesto en el apartado anterior presentaba una serie de reglas sencillas que acotaban el valor de los parámetros para obtener un control estable, aunque dependiendo del proceso, distaba de ser óptimo por los siguientes motivos:

- No se daba un criterio claro para calcular el factor de peso. Solo se aconsejaban valores bajos.
- No se estudiaba la influencia del horizonte de control en el horizonte de predicción.
- No se validaba el método con más de un bechnmark.
- No se validaba el método con una maqueta real.

En un segundo trabajo ”Tuning rules for a quick start up in Dynamic Matrix Control”(Anexo II, en proceso de revisión en la publicación ISA Transactions) se intentan corregir las deficiencias anteriores

Para poder dar un criterio que permita calcular el factor de peso se tiene en cuenta que para calcular el primer incremento de control, la respuesta libre es cero. Esto nos permite dibujar una curva que muestre el primer incremento de control frente al factor de peso (Figura 3.2). Como era lógico esperar, el valor del primer incremento de control disminuye al aumentar el factor de peso. La curva muestra una zona inicial en la que el primer incremento de control varía muy rápidamente con el factor de peso. Esa es la zona de utilidad del factor de peso. Un valor más allá de esta zona apenas supondría una diferencia en el resultado. Además esta curva ayuda a predecir cunado un factor de peso demasiado bajo hará que un proceso se vualva inestable.

También se ha explorado más la dependencia del horizonte de predicción del horizonte de control y se ha demostrado como al aumentar ligeramente el horizonte de control, se compensa el efecto de un horizonte de control demasiado alto, que tiende a un comportamiento en lazo abierto. Puede verse un ejemplo en la Figura

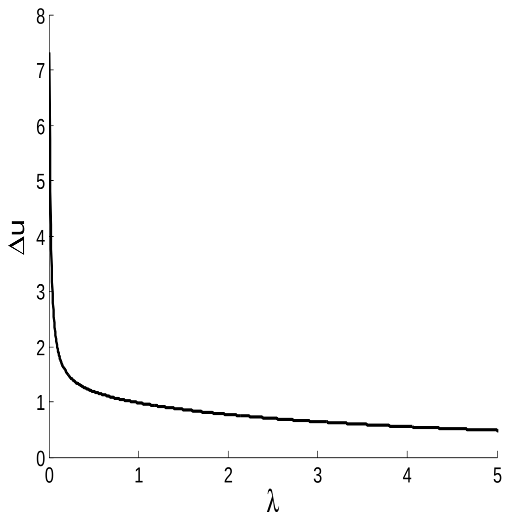
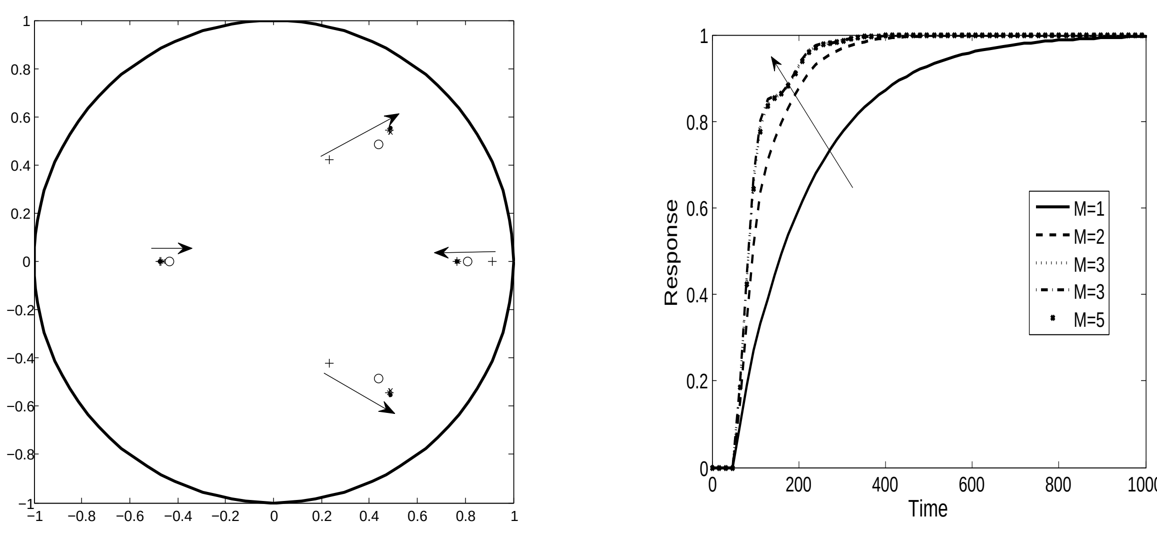


Figura 3.2: Curva Factor de peso-Primer incremento de control



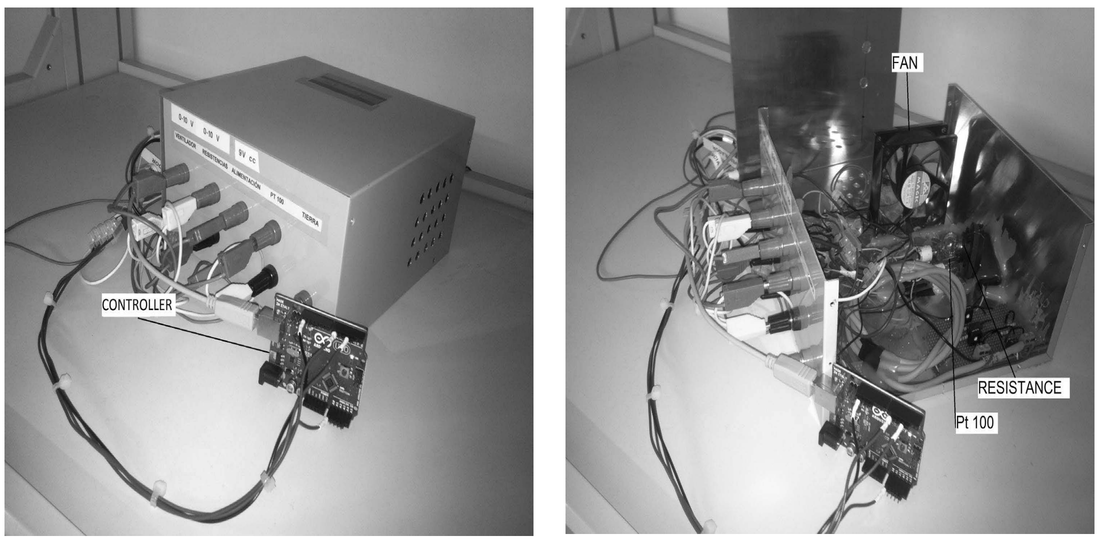
Efecto de M en los polos del sistema y Pr =120 Respuesta para diferentes valores de M y Pr =120

Figura 3.3: Localización de polos y respuesta temporal Pr and M

Con todo lo anterior se han podido refinar algunas de las reglas de diseño dadas anteriormente:

- El valor del factor de peso debe estar en la zona de curva en la que este muestra influencia.
- El horizonte de control no debe ser demasiado elevado, pero debe evitarse un valor de 1. De esta forma se compensan horizontes de predicción altos.

Aparte de refinar las reglas de diseño, se han realizado simulaciones con más benchmarks para validar el método y se han realizado pruebas con éxito en una maqueta térmica real (Figura 3.4)



Maqueta térmica   Interior maqueta térmica

Figura 3.4: Maqueta térmica

Capítulo 4

Conclusiones

Se han presentado una serie de reglas de diseño basadas en un estudio sobre la influencia de los parámetros del DMC en los polos de su sistema LTI equivalente. Este estudio ha permitido conocer el efecto de los parámetros en la respuesta temporal del sistema y ha permitido la obtención de una serie de reglas de diseño para estos parámetros.
Aunque estas reglas no permiten obtener el conjunto óptimo de parámetros para cada caso, si que prporcionan un conjunto de parámetros que garantizan estabilidad al sistema y que pueden ser usados como punto de partida para realizar ajustes. Las simulaciones y pruebas sobre la maqueta térmica demuestran la validez del método.

Capítulo 5

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Bibliografía

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Capítulo 6

Anexos

Anexo I

Design rules for model predictive control using unconstrained Dynamic Matrix Control

Clemente Manzanera Reverter, Julio José Ibarrola Lacalle, José Manuel Cano Izquierdo.

Abstract

This paper pretends to offer design rules for the parameters of the Dynamic Matrix Control (DMC) to allow an easier starting up. The effect on the time response of each parameter that can be tuned by the user is studied in an unconstrained system. To do so the position of the equivalent system's closed loop poles are calculated. To simplify the study and limit the number of poles a First Order Plus Death Time approximation of the real plant will be used. This will allow to obtain more direct conclusions.

1 Introduction

Dynamic Matrix Control (DMC) has become a popular Model Predictive Control (MPC) method since it was first introduced by Cutler and Ramaker [12] in the last seventies. It is one of the most used algorithms in industry, but a method for setting its parameters is still being investigated. There are some mathematical algorithms to tune these parameters. A well known algorithm is the one presented by Shrindar and Cooper [1] who introduced a method to calculate the weighting factor minimizing the condition number of the system matrix. For its calculation the system is approximated by a First Order Plus Dead Time (FOPDT) system. Another example is the algorithm presented by Trierweiller y Farina [2] that uses a Robustness Performance Number (RPN) which indicates how difficult is for a system to reach the required performances with robustness. This method gives directives to calculate the prediction horizon, the control horizon and the sample time. It calculates the system's weighting matrix based on the RPN. This method modifies the normal cost function when it factorizes the system matrix. Han, Zhao y Qian [3] propose a minimization-maximization algorithm over a performance index.

Some works face a more practical approach using “thumb rules” given by the experience obtained from simulations and real controllers. This a usual approach in industry. The work from Iglesias, Sanjuán y Smith [4] is an example of this. They present a formula obtained by correlation with data from several simulations.

Previous works agree on the effect of control horizon and sample time but it is not found a consensus about what parameter, prediction horizon and weighting factor, should be taken as key parameter. Some authors (as Shrindar y Cooper [1]) state that the weighting factor is the key parameter to DMC tuning. But others (as Rossiter [5]) doubt of this parameter and defend that the prediction horizon is the factor DMC users should focus in.

This paper pretends to obtain some “design rules” analysing the effect of changes of DMC parameters on the system closed loop poles (a similar approach to the one used in reference [11]). Time response simulations will be done to evidence the obtained results. These rules will allow users to easily obtain a set of suitable parameters and help them to predict the effect of a parameter's change in the their systems performance. To compute the system poles of a DMC controlled system, it will be expressed as a Linear Time Invariant (LTI) (this development can be seen in [6]).

2 Study of DMC parameters in system response

In the DMC formulation [6] the user must at least select the following parameters: prediction horizon (Pr), sample time (T), weighting factor (λ), control horizon (M) and model Horizon (n_g).
To study their effect on the output response, a LTI formulation [6] is built to make closed poles analysis.
The first process used in [1], but with a time delay of 16 seconds, has been chosen to compute the poles location movements with parameters changes:

$$\frac{e^{-16s}}{(150s+1)(25s+1)} \quad (1)$$

This process has been approximated by a FOPDT reducing the number of poles and facilitating conclusions extrancting:

$$\frac{e^{-32s}}{(157s+1)} \quad (2)$$

The analysis will be done at $T=8$ and $T=16$ seconds . These time steps have been selected to match with the time delay of 32s of the FOPDT and as close as possible to $0.05\tau_p$ and $0.1\tau_p$ as performed by Shrindar and Cooper [1] (τ_p is the time constant of the corresponding FOPDT).

FOPDT discrete (T=8):

$$\frac{0.04968z^{-5}}{(1-0.9503z^{-1})} \quad (3)$$

FOPDT discrete (T=16):

$$\frac{0.09516z^{-3}}{(1-0.9048z^{-1})} \quad (4)$$

The response to analyse shall be the time response to a unit step.

2.1 Effect of prediction horizon and sample time.

As stated before, this analysis will be done at $T=8$ and $T=16$. Tables 1 and 2 show the obtained poles for these sample times. Figures 1 and three allow to observe the evolution of system poles when T and Pr change.

Pr =4 (+, line)	Pr =8 (o, dashed line)	Pr =12 (., dot line)	Pr =20 (x, dash dot line)
-0,5	-0,6	-0,63	-0,63
-0,0728+0,5242i	-0,1402+0,6233i	-0,1747+0,6303i	-0,1887+0,6054i
-0,0728-0,5242i	-0,1402-0,6233i	-0,1747-0,6303i	-0,1887-0,6054i
0,7814+0,3091i	0,6406+0,4251i	0,5787+0,4197i	0,5275+0,3929i
0,7814-0,3091i	0,6406-0,4251i	0,5787-0,4197i	0,5275-0,3929i
0,76	0,85	0,88	0,91

Table 1: Poles for T=8

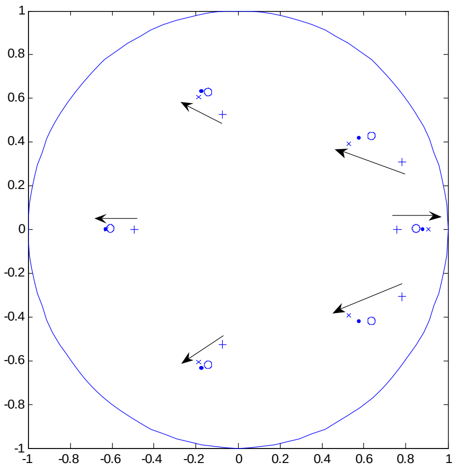

Figure 1: Closed loop Poles for T=8

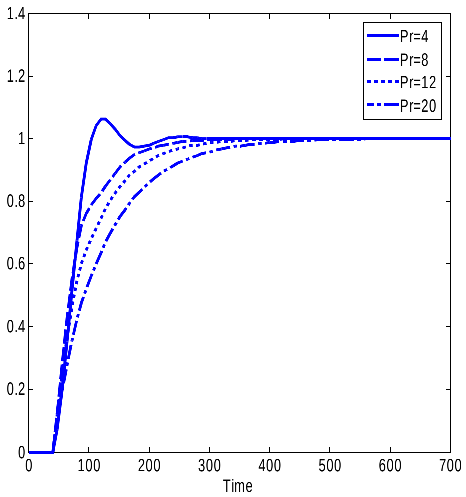

Figure 2: System response for T=8 and different values of Pr

Pr =2 (+)	Pr =4 (o)	Pr =6 (.)	Pr =10 (x)
-0,29	-0,45	-0,52	-0,55
0,7418+0,2668i	0,4938+0,5175i	0,394+0,5588i	0,3145+0,5327i
0,7418-0,2668i	0,4938-0,5175i	0,394-0,5588i	0,3145-0,5327i
0,49	0,77	0,81	0,85

Table 2: Poles for T=16

The prediction horizon has been selected to make prediction time ($Pr\,T$) nearly the same for both sample times and have comparable results.

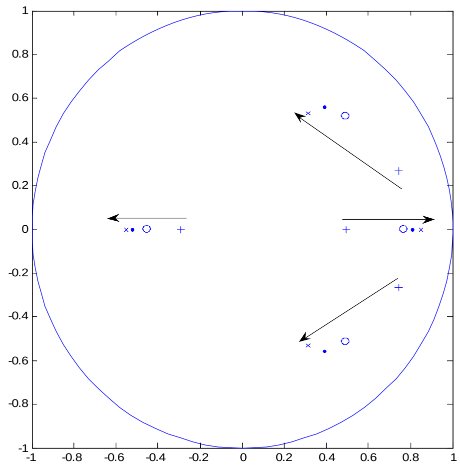

Figure 3: Closed loop Poles for T=16

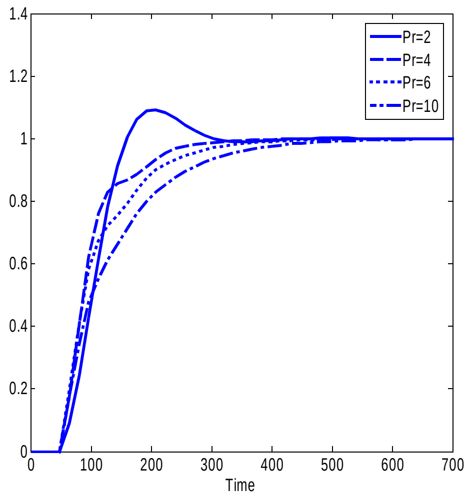

Figure 4: System response for T=16 and different values of Pr

The first conclusion it can be reached is that increasing prediction horizon increases the value of real poles and decreases the module of complex poles. This makes the real positive poles become the dominant ones and dictates the behaviour of the system. The real positive poles make a system respond without oscillations. So if, a time response free of oscillations is required, increasing Pr seems to be the correct choice.

It also can be concluded that as the prediction horizon grows, its effect becomes weaker and the effect on the poles becomes weaker. Pr has a maximum useful value and increasing it beyond it will not vary system's response.

Figure 2 and 4 show that as Pr is increased the system's response is slower (it approaches to an open loop system). As the dominant pole approaches the unit circle, the response time of the system grows. In order to have an oscillations free and not too slow response Pr must be carefully chosen. The key is using a value that makes the real positive pole clearly dominant. For

$T=16$ this value would be $Pr=6$ as the complex poles have a module of 0.68 versus 0.81 of the dominant pole. For $T=8$, it would be $Pr=8$ (module of the dominant pole is 0.85 versus 0.618 of the complex pole).

Tables 1 and 2 show that the lower the sample time is, the closer the poles to the unit circle are. Another effect is that the poles are much closer among them. This makes that for small prediction horizons complex poles are not so dominant, so the response improves with respect the same prediction time but higher sample time. But as the prediction horizon is increased and the real positive poles become dominant, the difference between the poles calculated for each sample times is difficult to appreciate.

2.2 Effect of control horizon

Works by previous researchers (for example Shrindar and Cooper [ref.1]) show that this parameter has a small influence in the process. Various simulations are done varying the control horizon while the other parameters (λ=0.25, $Pr=4$, $T=16$) are kept constant. Obtained poles for process of eq. (3) are shown in table 3.

M=1 (+,)	*M*=2 (o)	*M*=3 (*)	*M*=4 (.)	*M*=5 (x)
0.7680	0.7536	0.7482	0.7475	0.7475
0.4938 +0.5175i	0.5293 + 0.4624i	0.5385 + 0.4505i	0.5396 + 0.4494i	0.5396 + 0.4494i
0.4938 - 0.5175i	0.5293 - 0.4624i	0.5385 - 0.4505i	0.5396 - 0.4494i	0.5396 - 0.4494i
-0.4480	-0.4049	-0.3966	-0.3958	-0.3958

Table 3: Poles for different M values.

As it can be seen, poles (and consequently, response) vary very slightly when M changes.

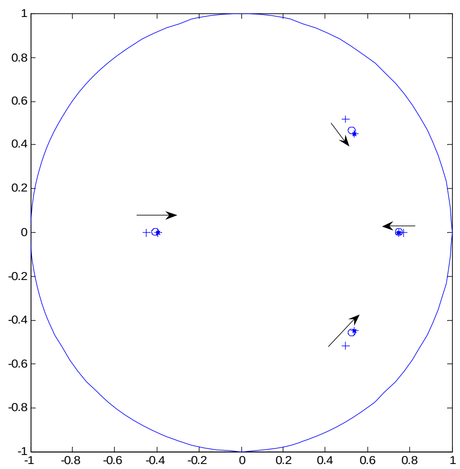

Figure 5: Effect of M in the system poles

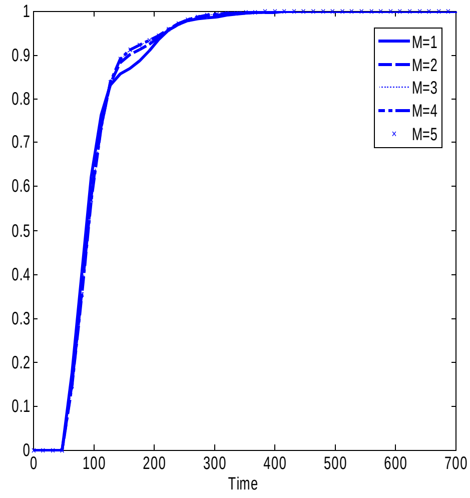

Figure 6: system response for different M values

2.3 Effect of weighting factor

The weighting factor has the effect of softening the system response, but it does in an opposite way than the prediction horizon. As it can be seen in table 4 as λ increases the real positive pole and the imaginary part of the complex poles decrease while the complex pole real part grows. This makes the dominant pole a complex one and the system response presents peaks and oscillations.

The effect of softening the response is explained by the fact that as the real part of the pole is increased and the imaginary part decreased the pole approaches to a real pole and its corresponding response. As the module of the pole is increased, so it is the setting time.

This parameter's effect is conditioned by the prediction horizon. Its effect becomes very difficult to appreciate as Pr and λ grows . Its effects arise when it becomes higher than zero (see table 4) and weakens as λ becomes higher.

λ=0 (+, line)			
Pr=2	Pr=4	Pr=6	Pr=10
-0,97	-0,78	-0,7	-0,61
0,2983+0,8792i	0,3089+0,7199i	0,3001+0,6382i	0,2815+0,5552i
0,2983-0,8792i	0,3089-0,7199i	0,3001-0,6382i	0,2815-0,5552i
0,75	0,79	0,81	0,85
λ=0,25 (o, dashed line)			
Pr=2	Pr=4	Pr=6	Pr=10
-0,29	-0,45	-0,52	-0,55
0,7418+0,2668i	0,4938+0,5175i	0,394+0,5588i	0,3145+0,5327i
0,7418-0,2668i	0,4938-0,5175i	0,394-0,5588i	0,3145-0,5327i
0,49	0,77	0,81	0,85
0	0	0	0
λ=0,50 (*, dotted line)			
Pr=2	Pr=4	Pr=6	Pr=10
-0,22	-0,35	-0,43	-0,5

0,854+0,1877i	0,5646+0,3613i	0,4494+0,4828i	0,342+0,5091i
0,854-0,1877i	0,5646-0,3613i	0,4494-0,4828i	0,342-0,5091i
0,29	0,73	0,8	0,85
0	0	0	0
λ=0,75 (., dash dot line)			
Pr=2	Pr=4	Pr=6	Pr=10
-0,18	-0,3	-0,38	-0,46
0,8876+0,1544i	0,6435+0,2185i	0,4859+0,4139i	0,3651+0,4851i
0,8876-0,1544i	0,6435-0,2185i	0,4859-0,4139i	0,3651-0,4851i
0,23	0,62	0,8	0,85
0	0	0	0
λ=1 (x)			
Pr=2	Pr=4	Pr=6	Pr=10
-0,16	-0,26	-0,34	-0,43
0,904+0,1334i	0,7524+0,1818i	0,5129+0,3498i	0,3847+0,4614i
0,904-0,1334i	0,7524-0,1818i	0,5129-0,3498i	0,3847-0,4614i
0,19	0,43	0,79	0,84
0	0	0	0

Table 4: Poles evolution with λ

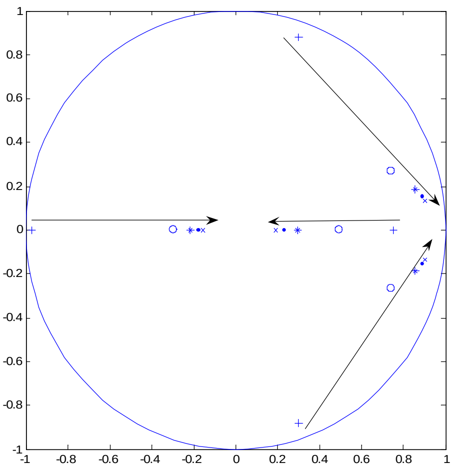

Figure 7. Poles evolution with λ (Pr=2)

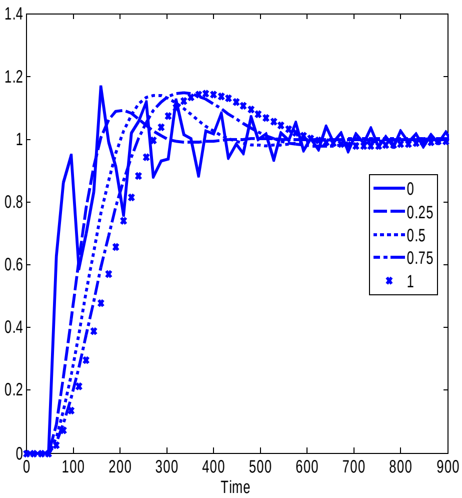

Figure 8: System response according figure 10 poles

2.4 Effect of n_g in the process.

This parameter gives the step response time used as model, n_g is the number of step response coefficients (g) taken . The response will improve increasing n_g up to a point when it will not be affected by it. Model horizon is involved in the calculation of free response in the DMC algorithm.

$$f(t+k) = y_m(t) + \sum_{i=1}^{\infty} (g_{k+i} - g_i) \Delta u(t-i) \quad (5)$$

When all $g(i+k)$ coefficients become equal to g(i) this term will become zero .All additional g coefficients that increase n_g beyond this limit do not have any effect.
DMC is applied to stable processes, where there is a value of n from which $g_k - g_{k-1} = 0$. If this value is the settling time (n), then $n_g = n + Pr$. So n_g depends on the prediction horizon and the sample time. For the example process settling time is 950 seconds approximately. For sample time 16 seconds this means $n=60$. For a prediction horizon of 4, the ideal n_g would be 84. Figure 9 shows the evolution of the system response from $n_g=15$ to $n_g=65$ for the DMC.

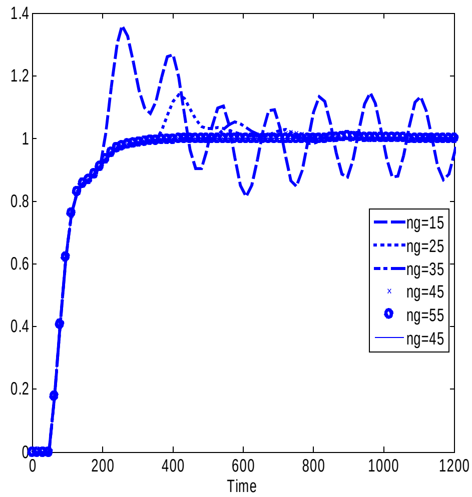

Figure 9: System response varying n_g

Figure 9 shows that only small values of n_g give a poor performance. From $n_g > 1/2\,n$ the output prediction is accurate enough.

3 Design advices.

The objective of this section is studying the conclusions from previous section and elaborating some “design principles” useful to select the right values for DMC parameters.

From previous sections it can be deduced that:

- Small values for M are a good choice.
- System response achieves lower oscillations when T decreases and the prediction time is long enough.
- System response varies with Pr only up to a certain value.
- n_g does not have influence once a certain value has been reached.
- λ works in the opposite way that Pr as it tunes the DMC making a complex pole the dominant one but mitigating the complex part to eliminate oscillations.

For M and Pr horizon limit values can be known from the beginning:

- $n_g = Pr + n$, where n is the required number of g values required to reach the unit step.
- $Pr = \tau_P$, being τ_P the time constant of the FOPDT equivalent (time to reach 63% of step value).

M has little influence on the process but according to Rawlings and Muske [10] this value should be greater or equal than the number of unstable modes in the system to guarantee stability.
To calculate the sample time it will be taken into account the considerations of Rahul Shridhar y Douglas J. Cooper [1] who propose to take $0.1\tau_p$. This has been proven to give an accurate result. This value will be considered as a maximum.
λ contributes to soften response oscillations. But it has been observed that this effect disappears when prediction horizon becomes high enough. The reason is in the cost function minimized to obtain the DMC result (see equation 7).This function has two terms. The first one (depending on the prediction horizon) is the mean quadratic error and the second one (depending on the control horizon) are the control increments. λ influence is limited to the second term which depends on the control horizon that usually has a smaller value than the prediction horizon.

The solution of the DMC problem is [6]:

$$\Delta u(t) = [G^t G + \lambda I]^{-1} G^t E_0(t) \quad (6)$$

G is the step response values matrix and E_0 the free response.

Figure 10 shows the evolution of $G^t G$ determinant. As Pr and M grow this value approach to zero making impossible to invert this matrix. If the weighting factor is not zero the matrix to invert becomes $[G^t G + \lambda I]$ and there is no risk to have a not invertible matrix. The conclusion drawn from this is that the weighting factor must never be zero. A small value (e.g 0.25) is enough to give mathematical robustness to the DMC algorithm. This is another task of this parameter.

From the above rules it can be concluded that the best option to achieve a fast and reasonable free of oscillations response is a not too high prediction horizon corrected with a suitable weighting factor (around a value of one as maximum). This advice should be used as a starting point prior to make any adjustment.

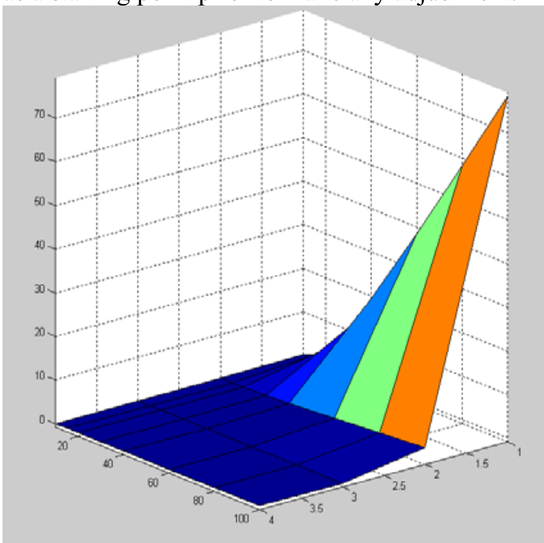

Figure 10: Evolution of G*G' determinant with n_g and Pr.

$$J = \sum_{j=1}^{P} (\hat{y}(t+j/t) - w(t+j))^2 + \sum_{j=1}^{m} \lambda (\Delta u(t+j-1))^2 \quad (7)$$

4 Implementation of design rules

To validate the previous results a benchmark is chosen from [1]. Now the process will be simulated without any simplification. The design advices from section 3 will be used:

$$\frac{(e^{-50s})}{((150s+1)(25s+1))} \quad (8)$$

Figure 11 shows the time constant and the settling can be obtained: 160s and 1000s. This means:

$\tau = 0.1\,x\,160 = 16\,seconds$

$n_g = (1000+160)/16 = 73$

According to the above conclusions this process should be tuned using a prediction horizon lower than 10 (taking into account that sampling time is $0.1\tau_P$) and a weighting factor of 0.25
If a completely oscillation free is required, no matter how slow it is, Pr should be made equal or higher to the above explained limit ($Pr=10$) and λ take a small value to give mathematical robustness to the system ($\lambda=0.25$). Figure 12 Shows this response.
If a faster response is required, then Pr should take values below the limit ($Pr=2$ or $Pr=4$) and the weighting factor higher values than before to stabilize the system. Figure 13 shows the results for different values of the weighting factor.

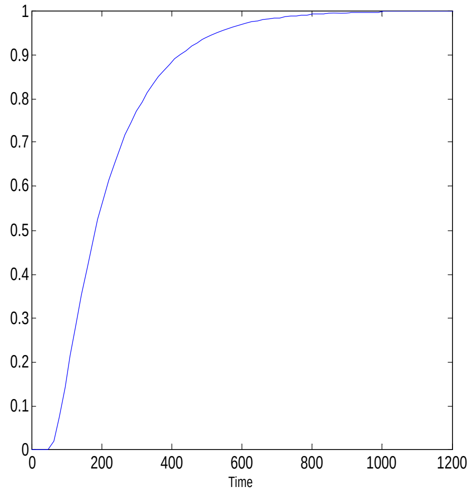

Figure 11: Open loop step response of eq. 8

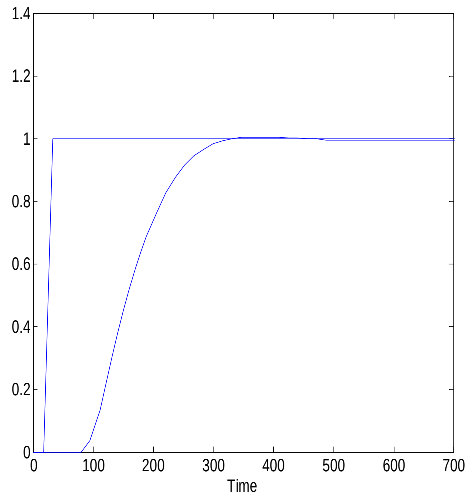

Figure 12: Oscillation free response of eq 8 (M=1).

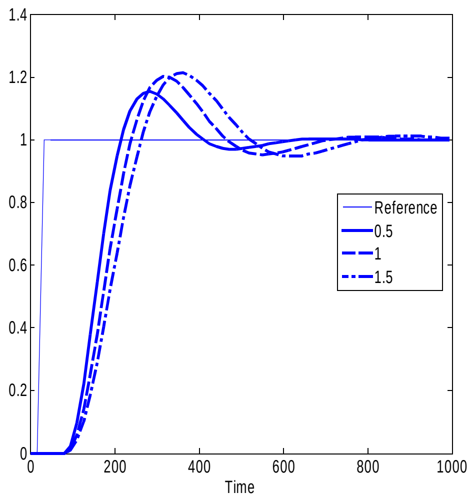

Figure 13: Fast response of eq 8 (Pr=4)

5 Conclusions

In this paper a study of the effect of the main DMC parameters on the time response has been made. A FOPDT approximation has been used to reduce the number of closed loop poles and make the analysis easier.

Once the influence of the parameters on the time response has been studied, some design rules are obtained. These rules can be used as a starting point prior to make adjustments.

As these rules have been obtained using a FOPDT approximation they can be applied to any system that can be simplified by a FOPDT.

They also cover a range of problems not fully solved by other methods (specially when M=1).

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Anexo II

Tuning rules for a quick start up in Dynamic Matrix Control

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1. Introduction

Dynamic Matrix Control (DMC) has become a popular Model Predictive Control (MPC) method since it was first introduced by Cutler and Ramaker [12] in the last seventies. It is one of the most used algorithms in industry, but a method for setting its parameters is still being investigated.

There are some mathematical techniques to tune these parameters. A well known algorithm is the one presented by Shrindar and Cooper [1] who introduced a method to calculate the weighting factor minimizing the condition number of the system matrix. For its calculation the system is approximated by a First Order Plus Dead Time (FOPDT) system.This method is one of the most extended and has been studied by several researchers ([16], [14]). Another example is the algorithm presented by Trierweiller and Farina [2] that uses a Robustness Performance Number (RPN) which indicates how difficult is for a system to reach the required performances with robustness. This method gives directives to calculate the prediction horizon, the control horizon and the sample time. It calculates the system's weighting matrix based on the RPN. This method modifies the normal cost function when it factorizes the system matrix. Han, Zhao and Qian [3] propose a minimization-maximization algorithm over a performance index. Garriga and Soroush propose tuning via eigen value placement [11].

Some works face a more practical approach using thumb rules given by the experience obtained from simulations and real controllers. This a usual approach in industry. The work from Iglesias, Sanjuan and Smith [4] is an example of this. They present a formula obtained by correlation with data from several simulations. Bagheri and Khaki-Sedigh ([17]) propose an analysis of variance. Wojsznis et al present the use of heuristic methods [18]. In this category auto-tuning methods could be included ([8])

Previous works agree on the effect of control horizon and weighting factor but it is not found a consensus about what parameter, prediction horizon and weighting factor, should be taken as key parameter. Some authors (as Shrindar y Cooper [1]) state that the weighting factor is the key parameter to DMC tuning. But others (as Rossiter [5]) doubt of this parameter and defend that the prediction horizon is the factor DMC users should focus in.

Following this goal and trying to make easier the tuning task, this paper pretends to obtain some design rules analysing the effect of changes of DMC parameters on the system closed loop poles (a similar approach to the one used in [11]). Time response simulations will be done to evidence the obtained results. These rules will allow users to easily obtain a first set of suitable parameters and help them to predict the effect of a parameter's change in the systems performance. To compute the poles of a DMC controlled system, it will be expressed as a Linear Time Invariant (LTI) (this development can be seen in [6]).

This paper is structured as follows: The first section will be an introduction to the DMC formulation and the DMC expressed as an LTI system. This will allow a better understanding of the follwing section, an analysis of the effect of DMC parameters in closed loop poles and time response from which useful tuning rules will be obtained. The last section will show a validation of the previously mentioned tuning rules by simulation on a benchmark and test on a real system.

2. DMC algorithm

As the starting point of this paper is transforming the DMC algorithm in a LTI system, it is mandatory to explain this process. The following paragraphs explain the basis of DMC and how it can be expressed as a LTI system.

2.1. DMC formulation

DMC algorithm uses a plant's step response model:

$$y(t) = \sum_{i=1}^{\infty} g_i \Delta u(t-i) \tag{1}$$

Where g_i are the coefficients of the unit step response, Δu is the control increment, y is the system response and $n(t)$ are the disturbances. So predicted values will be (starting predictions from instant t):

$$\hat{y}(t+k) = \sum_{i=1}^{\infty} g_i \Delta u(t+k) + \hat{n}(t+k) = \sum_{i=1}^{k} g_i \Delta u(t+k-i) + \sum_{i=k+1}^{\infty} g_i \Delta u(t+k-i) + \hat{n}(t+k) \tag{2}$$

Considering constant disturbances (being $y_m(t)$ the measured output):

$$\hat{n}(t+k) = \hat{n}(t) = y_m(t) - \hat{y}(t) = y_m(t) - \sum_{i=1}^{\infty} g_i \Delta u(t-i) \tag{3}$$

Then Equation 3 can be written:

$$\hat{y}(t+k) = \sum_{i=1}^{k} g_i \Delta u(t+k-i) + \sum_{i=k+1}^{\infty} g_i \Delta u(t+k-i) + y_m(t) - \sum_{i=1}^{\infty} g_i \Delta u(t-i) = \sum_{i=1}^{k} g_i \Delta u(t+k-i) + f(t+k) \tag{4}$$

Being $f(t+k)$ the free response, the part of the response no depending on future control actions described as follows:

$$f(t+k) = y_m(t) + \sum_{i=1}^{\infty} (g_{k+i} - g_i) \Delta u(t-i) \tag{5}$$

If the process is asymptotically stable, coefficients of step response, g_i will tend to a constant value after N sample periods, so:

$$(g_{k+i} - g_i) \to 0, i > N \tag{6}$$

And Equation 5 can be simplified:

$$f(t+k) = y_m(t) + \sum_{i=1}^{N} (g_{k+i} - g_i)\Delta u(t-i) \tag{7}$$

Applying the previous equations for a prediction horizon equal to Pr and a control horizon equal to M:

$$\hat{y}(t+1/t) = g_1 \Delta u(t) + f(t+1) \tag{8}$$

$$\hat{y}(t+2/t) = g_2 \Delta u(t) + g_1 \Delta u(t+1) + f(t+1) \tag{9}$$

. . .

$$\hat{y}(t+Pr/t) = \sum_{i=Pr-M+1}^{Pr} g_i \Delta u(t+Pr-i) + f(t+Pr) \tag{10}$$

Defining the system dynamic matrix as:

$$\mathbf{G} = \begin{pmatrix} g_1 & 0 & ... & 0 \\ g_2 & g_1 & ... & 0 \\ g_M & g_{M-1} & ... & g_1 \\ : & : & ... & : \\ g_{Pr} & g_{Pr-1} & ... & g_{Pr-M+1} \end{pmatrix} \tag{11}$$

Using matricial formulation, it can be written that:

$$\mathbf{\hat{y}} = \mathbf{G\Delta u} + \mathbf{f} \tag{12}$$

Being $\mathbf{\hat{y}}$ a Pr-dimensional vector that contains the future system predictions in the prediction horizon, $\mathbf{\Delta u}$ a M-dimensional vector that contains the control increments and $\mathbf{f}$ the free response vector. This expression relates the future outputs with the control increments and is used to calculate the necessary action to reach a specific behaviour.

DMC's objective is finding a control increment that minimizes a determined cost function that includes errors and control efforts.

$$J = \sum_{j=1}^{p} (\hat{y}(t+j|t) - w(t+j))^2 + \sum_{j=1}^{m} \lambda(\Delta u(t+j-1))^2 \tag{13}$$

$$\mathbf{J} = \mathbf{ee^T} + \lambda \mathbf{\Delta u \Delta u^T} \tag{14}$$

Where $\mathbf{e}$ is the errors vector and $\mathbf{\Delta u}$ the control efforts vector. In a problem without constraints, the optimized control efforts vector is obtained solving Equation 15:

$$\frac{\mathbf{dJ}}{\mathbf{d\Delta u}} = 0 \tag{15}$$

Being the result:

$$\mathbf{\Delta u} = (\mathbf{G^T G} + \lambda \mathbf{I})^{\mathbf{-1}} \mathbf{G^T}(\mathbf{w} - \mathbf{f}) \tag{16}$$

2.2. Parameters summary

The following parameters can be deduced from previous section.

- Prediction horizon (Pr): Indicates the time horizon during which the output must follow the setpoint. It also may be expressed as the period comprised between N_1 and N_2.
- Control horizon (M): Indicates the future time steps for which control increments are calculated.
- Sample time (T)
- Model horizon (n_g): Indicates the time interval form the step response that is used as model.
- Weighting factor (λ): This coefficient penalizes the control signal to avoid it to be too aggressive and the response oscillating. It also has the effect of giving mathematical robustness to the method minimizing the effect of having a $(\mathbf{G^TG} + \lambda\mathbf{I})$ matrix not possible to invert.

2.3. DMC expressed as LTI model

In order to apply system analysis methods the DMC, it can be expressed as a LTI model. The output prediction will be:

$$\hat{y}(t+k) = G\Delta u(t+k) + f(t+k) \tag{17}$$

The first component of the optimal controls vector will be:

$$\Delta u_{opt} = \mathbf{Ke} \tag{18}$$

Where $\mathbf{e}$ is errors vector, measurable disturbances have not been taken into account and K is:

$$\mathbf{K} = [k_1 k_2 ... k_{Pr}] = [1, 0, ..., 0](\mathbf{G^TG} + \lambda\mathbf{I})^{\mathbf{-1}}\mathbf{G^T} \tag{19}$$

Developing the value of the optimal control increment and taking into account an alternative formulation of the free response:

$$f(t+k) = y_m(t) + \sum_{i=1}^{N} s_n^k \Delta u(t-i) \tag{20}$$

$$S_n^k(q^{-1}) = s_1^k q^{-1} + s_2^k q^{-2} + ... + s_n^k q^{-n} \tag{21}$$

$$s_n^k = g_{n+k} - g_n \tag{22}$$

$$\Delta u_{opt}(t) = \sum_{t=1}^{Pr} k_i w(t+i) - \sum_{t=1}^{Pr} k_i y(t) - \sum_{t=1}^{Pr} k_i S_n^i \Delta u(t) \tag{23}$$

Comparing this expression with a two degree of freedom controller (see Figure 1):

$$R_p(q^{-1})\Delta u(t) = T_p(q^{-1})w(t) - S_p(q^{-1})y(t) \tag{24}$$

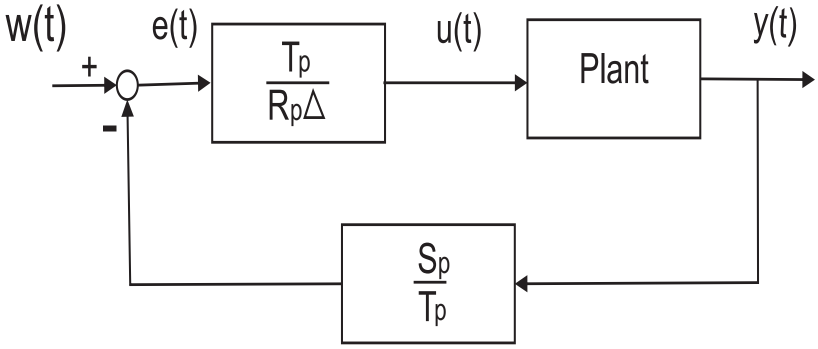


Figure 1: DMC as an LTI system

The DMC can be expressed as a blocks diagram as shown in Figure 1.

Where:

$$R_P = 1 + \sum_{i=1}^{Pr} k_i S_n^i \tag{25}$$

$$T_P = \sum_{i=1}^{Pr} k_i q^i \tag{26}$$

$$S_P = \sum_{i=1}^{Pr} k_i \tag{27}$$

3. Effect of DMC parameters in system response

To study the effect of prediction horizon (Pr), sample time (T), weighting factor (λ), control horizon (M) and model Horizon (n_g) in the time response, an analysis of the closed loop poles will be made. As already explained in section 2.3 a benchmark controlled with a DMC algorithm can be expressed as an LTI system whose poles can be calculated. Those poles will depend on the values of the DMC parameters and the step response coefficients. If the behaviour of the poles when the DMC parameters change can be deduced, we will be able to undestand their influence in the response to a unit step. If the influence in the time response is known, then some useful heuristic desing rules (with the same philosofy than the Ziegler-Nichols method for PID controllers, [13]) can be developed. The first process used in [1], taking a time delay of 16 seconds, has been chosen to compute the poles location movements with parameters changes:

$$G(s) = \frac{e^{-16s}}{(150s+1)(24s+1)} \tag{28}$$

The conclusions taken from the study of the benchmark for Equation 28 must be applicable to many systems. The study will not be made on the benchmark "as it", but to its First Order Plus Dead Time (FOPDT) equivalent (Equation 29). This ensures that the results of the study will be applicable to other first order systems and, consequently, to any system that can be simplified with a FOPDT.

$$G(s) = \frac{e^{-32s}}{157s + 1} \tag{29}$$

The analysis will be done at T=8 seconds and T=16 seconds . These sample times have been selected to match with the time delay of 32 seconds of the FOPDT and as close as possible to 0.05τ and 0.1τ as performed by Shridhar and Cooper [1] (τ is the time constant of the corresponding FOPDT). Equations 30 and 31 are the discrete form of Equation 29 for a sample time of 8 and 16 seconds respectively.

$$G_8(z) = \frac{0.04968z^{-5}}{1 - 0.9503z^{-1}} \tag{30}$$

$$G_{16}(z) = \frac{0.09516z^{-3}}{1 - 0.9048z^{-1}} \tag{31}$$

3.1. Effect of prediction horizon and sample time in the process.

As stated before, this analysis will be done at T=8 seconds and T=16 seconds .Tables 1 and 2 show the obtained poles for these sample times. Figure 2 allows to observe the evolution of system poles when T and Pr change. In Figure 2 each pole has a symbol (indicated in tables 1 and 2) depending of its Pr

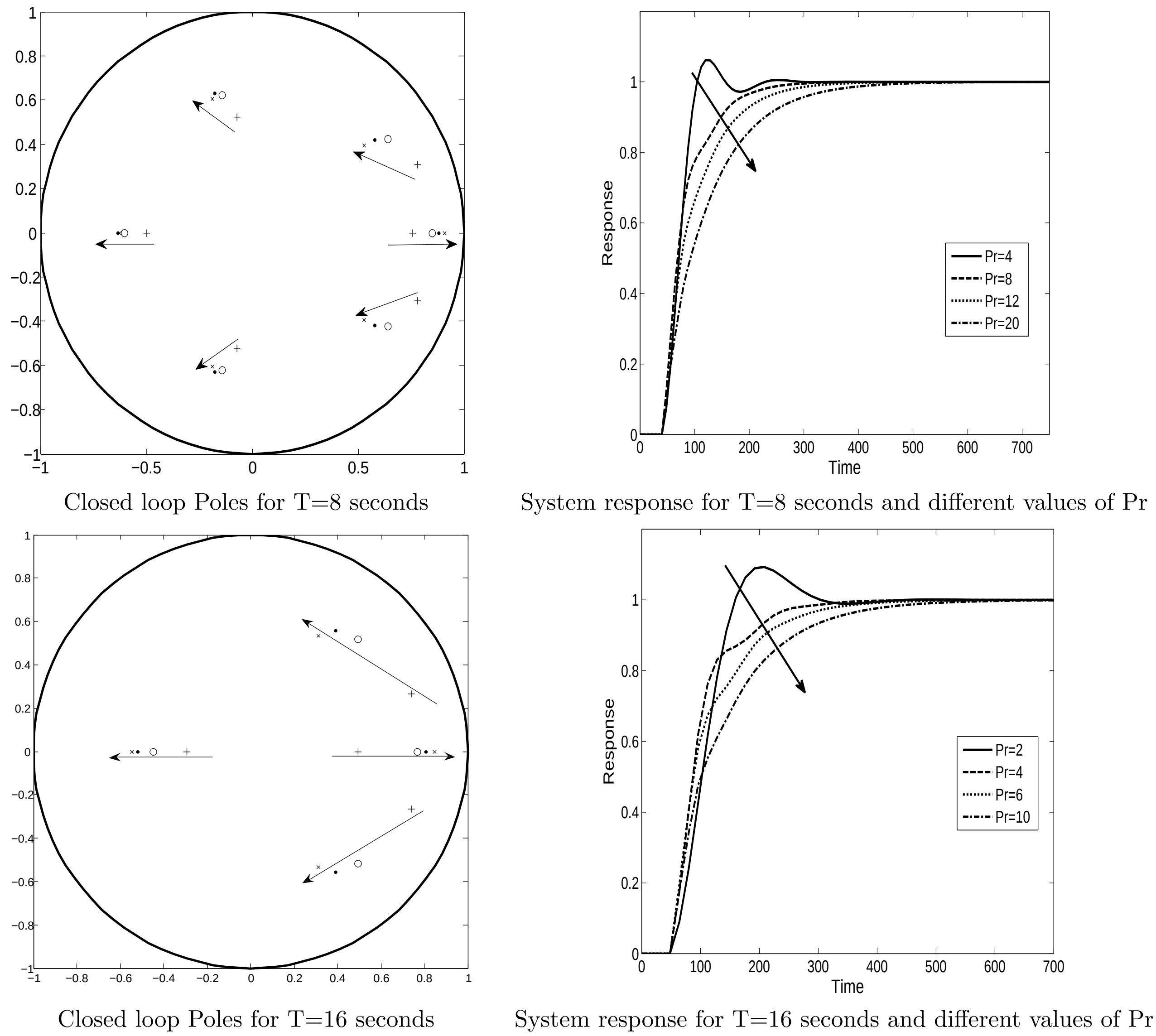
Table 1: Poles for T=8 seconds

Pr=4 (symbol +)	Pr=8 (symbol o)	Pr=12 (symbol .)	Pr=20 (symbol x)
-0,5	-0,6	-0,63	-0,63
-0,0728+0,5242i	-0,1402+0,6233i	-0,1747+0,6303i	-0,1887+0,6054i
-0,0728-0,5242i	-0,1402-0,6233i	-0,1747-0,6303i	-0,1887-0,6054i
0,7814+0,3091i	0,6406+0,4251i	0,5787+0,4197i	0,5275+0,3929i
0,7814-0,3091i	0,6406-0,4251i	0,5787-0,4197i	0,5275-0,3929i
0,76	0,85	0,88	0,91

Table 2: Poles for T=16 seconds

Pr=2 (symbol +)	Pr=4 (symbol o)	Pr=6 (symbol .)	Pr=10 (symbol x)
-0,29	-0,45	-0,52	-0,55
0,7418+0,2668i	0,4938+0,5175i	0,394+0,5588i	0,3145+0,5327i
0,7418-0,2668i	0,4938-0,5175i	0,394-0,5588i	0,3145-0,5327i
0,49	0,77	0,81	0,85

The prediction horizon has been selected to make prediction time ($Pr.T$) nearly the same for both sample times and have comparable results. The first conclusion it can be reached form Tables 1 and 2 and Figure 2 is that increasing prediction horizon increases the value of real poles and decreases the module of complex poles. This makes the real positive poles become the dominant ones and dictates the behaviour of the system. The real positive poles produce an oscillations free time response. So if, a time response free



Closed loop Poles for T=8 seconds

System response for T=8 seconds and different values of Pr

Closed loop Poles for T=16 seconds

System response for T=16 seconds and different values of Pr

Figure 2: Poles location and time response for T =8 seconds and T =16 seconds

of oscillations is required, increasing Pr seems to be the correct choice. It also can be concluded that as the prediction horizon grows, its effect on the poles becomes weaker and the system response approaches to an open loop system. Pr has a maximum useful value and increasing it beyond this value will not vary system response, it will make the system slower.

Figure 2 shows that as Pr is increased the system's response is slower (it approaches to an open loop system). As the dominant pole approaches the unit circle, the response time of the system grows. In order to have an oscillations free and not too slow response Pr must be carefully chosen. The key is using a value that makes the real positive pole clearly dominant. For T=16 seconds this value would be Pr=6 as the complex poles have a module of 0.68 versus 0.81 of the dominant pole. For T=8 seconds , it would be Pr=8 (module of the dominant pole is 0.85 versus 0.618 of the complex pole).

Tables 1 and 2 show that the lower the sample time is, the closer the poles to the unit circle are. Another effect is that the poles are much closer among them. This makes that for small prediction horizons complex poles are not so dominant, so the response improves with respect the same prediction time but higher sample time. But as the prediction horizon is increased and the real positive poles become dominant, the difference between the poles calculated for each sample times is difficult to appreciate.

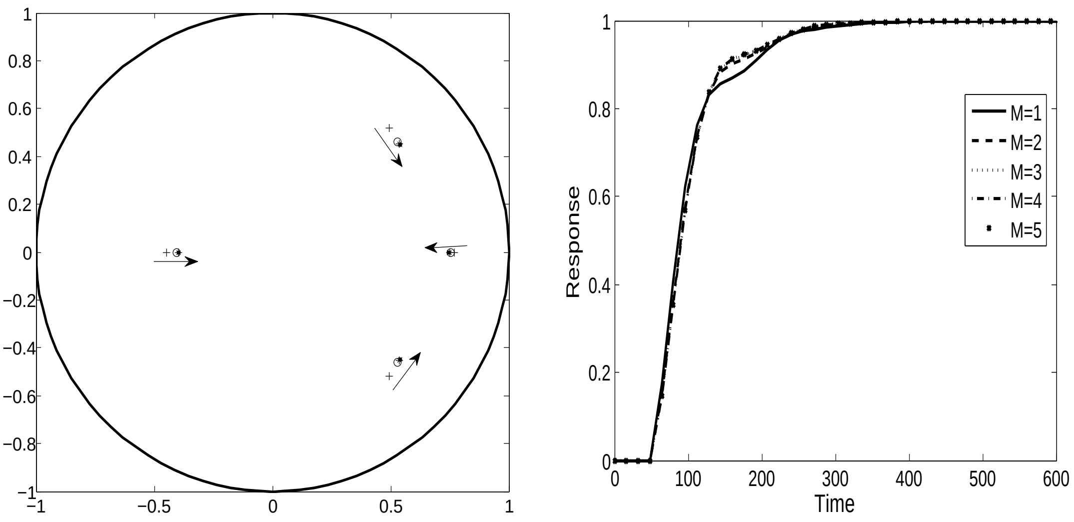
3.2. Effect of control horizon in the process.

Works by previous researchers (for example Shrindar and Cooper [1]) show that this parameter has a small influence in the process. Various simulations are done varying the control horizon while the other parameters (λ=0.25) are kept constant. Obtained poles for process of Equation 31 are shown in Table 3. This table shows too the symbol used for each value of λ in Figure 3.

Table 3: Poles for different M values

M=1 (symbol +)	M=2 (symbolo)	M=3 (symbol *)	M=4 (symbol .)	M=5 (symbol x)
0.7680	0.7536	0.7482	0.7475	0.7475
0.4938 + 0.5175i	0.5293 + 0.4624i	0.5385 + 0.4505i	0.5396 + 0.4494i	0.5396 + 0.4494i
0.4938 - 0.5175i	0.5293 - 0.4624i	0.5385 - 0.4505i	0.5396 - 0.4494i	0.5396 - 0.4494i
-0.4480	-0.4049	-0.3966	-0.3958	-0.3958

As it can be seen in Figure 3, poles (and consequently, response) vary very slightly when M changes.It may seem that M has a small influence in the process as pole's location shows a minimum change when this parameter changes. This is because the influence of M depends on Pr and λ. Let's see what happens when Pr is increased from Pr=4 up to different values (Tables 4, 5 and 6 and Figure 4). As it can be seen with Pr higher than the FOPDT time constant, the effect of M becomes more important reducing notably the dominant pole's value and, consequently, the settling time. However it must be paid attention to the fact that complex poles imaginary and real components are being increased too and this may deteriorate the response if those poles become dominant. When a certain value of M is reached the effect on poles' location gets weaker and only little changes are observed when this parameter increases.



Effect of M in the system poles — System response for different M values

Figure 3: Poles location and time response for different values of M

Table 4: Poles for different M values and Pr=10

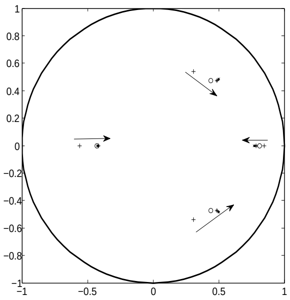
M=1 (symbol +)	M=2 (symbol o)	M=3 (symbol *)	M=4 (symbol .)	M=5 (symbol x)
0.8478	0.8111	0.7830	0.7723	0.7698
0.3084+0.5373i	0.4401+0.4739i	0.4856+0.4722i	0.4985+0.4812i	0.5009+0.4856i
0.3084-0.5373i	0.4401-0.4739i	0.4856-0.4722i	0.4985-0.4812i	0.5009-0.4856i
-0.5578	-0.4279	-0.4177	-0.4217	-0.4242

Table 5: Poles for different M values and Pr=60

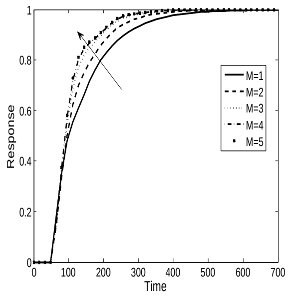
M=1 (symbol +)	M=2 (symbol o)	M=3 (symbol *)	M=4 (symbol .)	M=5 (symbol x)
0.9075	0.8105	0.7687	0.7650	0.7683
0.2368+0.4291i	0.4388+0.4788i	0.4859+0.5369i	0.4886+0.5474i	0.4882+0.5323
0.2368-0.4291i	0.4388-0.4788i	0.4859-0.5369i	0.4886-0.5474i	0.4882-0.5323
-0.4822	-0.4321	-0.4642	-0.4712	-0.4601

Table 6: Poles for different M values and Pr=120

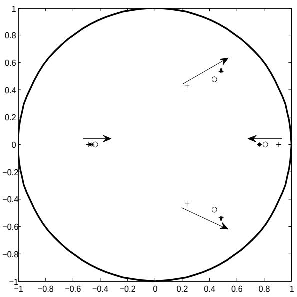
M=1 (symbol +)	M=2 (symbol o)	M=3 (symbol *)	M=4 (symbol .)	M=5 (symbol x)
0.9119	0.8084	0.7673	0.7646	0.7684
0.2320+0.4202i	0.4399+0.4842i	0.4851+0.5456i	0.4869+0.5536i	0.4868+0.5356i
0.2320-0.4202i	0.4399-0.4842i	0.4851-0.5456i	0.4869-0.5536i	0.4868-0.5356i
-0.4737	-0.4362	-0.4709	-0.4763	-0.4630



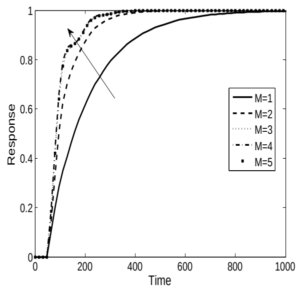
Effect of M in the system poles and Pr =10



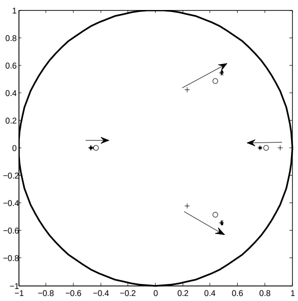
System response for different M values and Pr =10



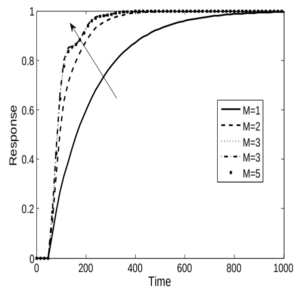
Effect of M in the system poles and Pr =60



Response for different M values and Pr =60



Effect of M in the system poles and Pr =120



Response for different M values and Pr =120

Figure 4: Poles location and time response varying Pr and M

3.3. Effect of weighting factor

The weighting factor has the effect of softening the system response, but it does in an opposite way than the prediction horizon. Table 7 and Figure 6 (the arrow points on the direction of increasing λ) show that as λ increases the real positive pole and the imaginary part of the complex poles decrease while the complex pole real part grows. This makes the dominant pole a complex one and the system response presents peaks and oscillations, as it can be seen in Figure 6. The effect of softening the response is explained by the fact that as the real part of the pole is increased and the imaginary part decreased the pole approaches to a real pole and its corresponding response. As the module of the pole is increased, so it is the setting time. This parameter's effect is conditioned by the prediction horizon. Its effect becomes very difficult to appreciate as Pr and λ grows. Its effects arise when it becomes higher than zero (see Table 7) and weakens as becomes higher.

The effects of λ become clear, but we still need to find the correct value for it. As it has been seen in Table 7 and Figure 6, the effect of λ becomes negligible when a certain value has been reached. It must be pointed that the DMC solution (Equation 16) could be summarized as the product of a matrix, $\mathbf{K}$, independent of time and the vector of errors,$\mathbf{e}$, which changes at every time step (Equation 32). In the first time step the value of the control increment can be calculated without the need of running a simulation or process. It just must be taken into account that the free response, $\mathbf{f}$, in this first time step is zero. This would make the errors vector equal to the setpoint, which is a known datum.

$$\mathbf{\Delta u} = (\mathbf{G^T G} + \lambda \mathbf{I})^{\mathbf{-1}} \mathbf{G^T} (\mathbf{w} - \mathbf{f}) = \mathbf{K}(\mathbf{w} - \mathbf{f}) = \mathbf{Ke} \tag{32}$$

This value of first control increment can be calculated for several values of λ and a curve can be builded. For very small values of λ the first control increment rises to very high values, which points out to an unstable response. For big values of λ, the value of the control increment decreases very slowly (this is in accordance with previous results). There is a range of values of λ in which the control increment changes significantly with λ. Weighting factor should be within that range.

In Figure 5 the relationship between the weighting factor and the first control increment for two fixed values of prediction and control horizon is shown. It can be seen that, as expected, the value of the first control increment decreases when λ becomes higher. It too arrives to a point when further increase of λ has a little impact on the control increment.

3.4. Effect of model horizon

This parameter gives the step response time used as model, n_g is the number of step response coefficients (g) taken . The response will improve increasing n_g up to a point when it will not be affected by it. Model horizon is involved in the calculation of free response in the DMC algorithm, Equation (5). When all g_{i+k} coefficients become equal to g_i this term will become zero. All additional g coefficients that increase n_g beyond this limit do not have any effect. DMC is applied to stable processes, where there is a value of n_g from which $g_k - g_{k-1} = 0$. If this value is the settling time (n), then $n_g = n + Pr$. So n_g depends on the prediction horizon and the sample time. For the example process settling time is 950 seconds approximately. For sample time 16 seconds

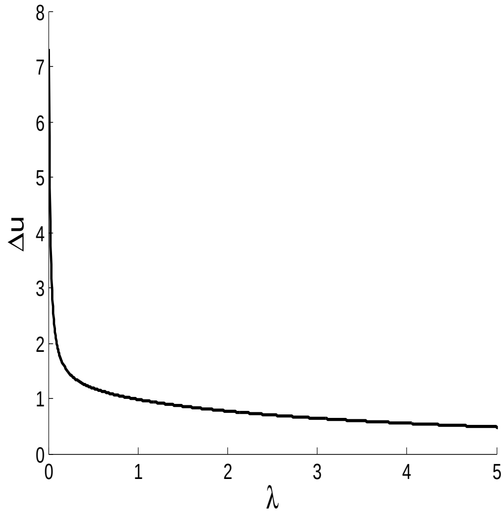
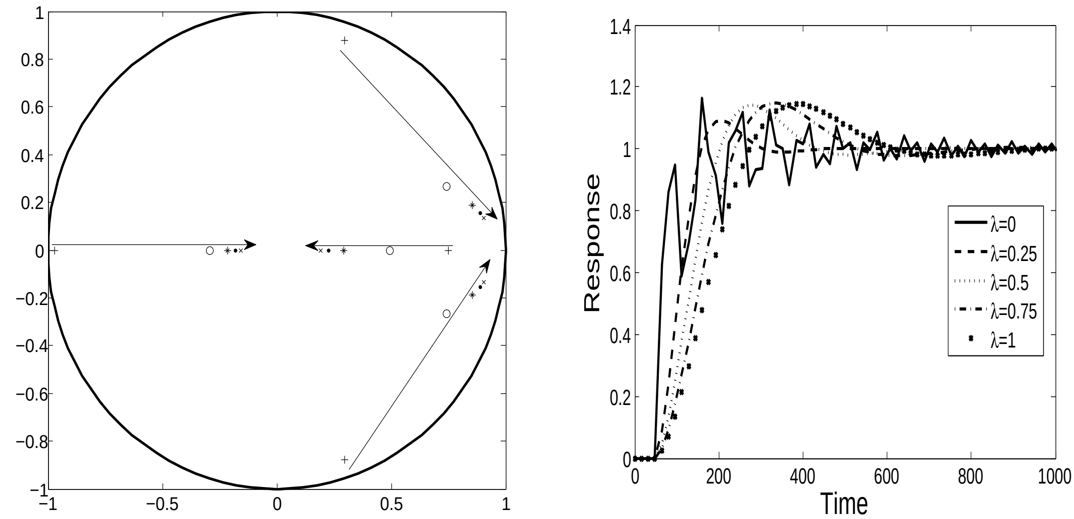


Figure 5: Control increment-Weighting factor curve



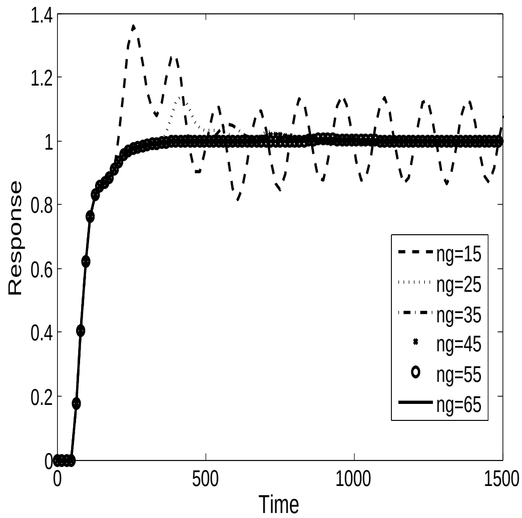
Poles evolution with λ (Pr=2) System response for different λ values

Figure 6: Poles location and time response for different values of λ

Table 7: Poles evolution with λ

λ=0 (symbol in Figure 6 +)			
Pr=2	Pr=4	Pr=6	Pr=10
-0,97	-0,78	-0,7	-0,61
0,2983+0,8792i	0,3089+0,7199i	0,3001+0,6382i	0,2815+0,5552i
0,2983-0,8792i	0,3089-0,7199i	0,3001-0,6382i	0,2815-0,5552i
0,75	0,79	0,81	0,85
λ=0.25(symbol in Figure 6 o)			
Pr=2	Pr=4	Pr=6	Pr=10
-0,29	-0,45	-0,52	-0,55
0,7418+0,2668i	0,4938+0,5175i	0,394+0,5588i	0,3145+0,5327i
0,7418-0,2668i	0,4938-0,5175i	0,394-0,5588i	0,3145-0,5327i
0,49	0,77	0,81	0,85
0	0	0	0
λ=0.50(symbol in Figure 6 *)			
Pr=2	Pr=4	Pr=6	Pr=10
-0,22	-0,35	-0,43	-0,5
0,854+0,1877i	0,5646+0,3613i	0,4494+0,4828i	0,342+0,5091i
0,854-0,1877i	0,5646-0,3613i	0,4494-0,4828i	0,342-0,5091i
0,29	0,73	0,8	0,85
0	0	0	0
λ=0.75(symbol in Figure 6 .)			
Pr=2	Pr=4	Pr=6	Pr=10
-0,18	-0,3	-0,38	-0,46
0,8876+0,1544i	0,6435+0,2185i	0,4859+0,4139i	0,3651+0,4851i
0,8876-0,1544i	0,6435-0,2185i	0,4859-0,4139i	0,3651-0,4851i
0,23	0,62	0,8	0,85
0	0	0	0
λ=1(symbol in Figure 6 x)			
Pr=2	Pr=4	Pr=6	Pr=10
-0,16	-0,26	-0,34	-0,43
0,904+0,1334i	0,7524+0,1818i	0,5129+0,3498i	0,3847+0,4614i
0,904-0,1334i	0,7524-0,1818i	0,5129-0,3498i	0,3847-0,4614i
0,19	0,43	0,79	0,84
0	0	0	0

this means $n = 60$. For Pr=4, the ideal n_g would be 84. Figure 7 shows the evolution of the system response from $n_g = 15$ to $n_g = 65$ for the DMC. As it can be seen only small values of n_g give a poor performance. From $n_g > \frac{1}{2n}$ the output prediction is accurate enough.



System response varying n_g

Figure 7: Time response for different values of n_g

4. Preliminary Design advices

Taking into account the previous sections some useful conclusions can be obtained:

- System response achieves lower oscillations when T decreases and the prediction time is long enough.
- System response varies with Pr only up to a certain value, when the system reaches open loop behaviour. However, this effect can be corrected by an appropriate value of M.
- n_g does not have influence once a certain value has been reached.
- λ works in the opposite way that Pr as it tunes the DMC making a complex pole the dominant one but mitigating the complex part to eliminate oscillations.

So, the following tuning advices can be deduced:

1. $n_g = Pr + n$, where n is the required number of g values required to reach the unit step.
2. $Pr = \tau_p$, being the time constant of the FOPDT equivalent (time to reach 63% of step value).

3. M has little influence on the process for small values of Pr (lower than τ_p) but corrects the open loop behaviour when Pr is too high (greater than τ_p).Also, according to Rawlings and Muske [10], this value should be greater or equal than the number of unstable modes in the system to guarantee stability.
4. To calculate the sample time it will be taken into account the considerations of [1] who propose to take $0,1\tau_p$. This has been proven to give an accurate result. This value will be considered as a maximum.
5. λ contributes to soften response oscillations. But it has been observed that this effect disappears when prediction horizon becomes high enough. The reason is in the cost function minimized to obtain the DMC result (Equation 13).This function has two terms. The first one (depending on the prediction horizon) is the mean quadratic error and the second one (depending on the control horizon) are the control increments. λ influence is limited to the second term which depends on the control horizon that usually has a smaller value than the prediction horizon.The solution of the DMC is given in 16. Figure 8 shows the evolution of $\mathbf{G^tG}$ determinant. As M and Pr grow this value approaches to zero making impossible to invert this matrix. If the weighting factor is not zero the matrix to invert becomes $(\mathbf{GG^t} + \lambda\mathbf{I})$ and there is no risk to have a not invertible matrix. The conclusion drawn from this is that the weighting factor must never be zero. A small value is enough to give mathematical robustness to the DMC algorithm. This is another task of this parameter.

5. Validation of method

To validate the results from previous paragraphs the conclusions from section 4 will be followed to tune the following benchmarks (extracted from [1]), but this time without any simplification as done before:

5.1. Bechmarks

This section presents the bechmarks that will be used to validate the tuning rules presented in section 4. Bechnmark 1 to 4 are extracted from [1].

Bechnmark 1

$$G(s) = \frac{e^{-50s}}{(150s+1)(25s+1)} \tag{33}$$

Bechnmark 2

$$G(s) = \frac{e^{(1-50)e^{-10s}}}{(100s+1)^2} \tag{34}$$

Bechnmark 3

$$G(s) = \frac{e^{(1+50)e^{-10s}}}{(100s+1)^2} \tag{35}$$

Bechnmark 4

$$G(s) = \frac{e^{-10s}}{(50s+1)^4} \tag{36}$$

Bechnmark 5, this process is extracted from [9], section 4.

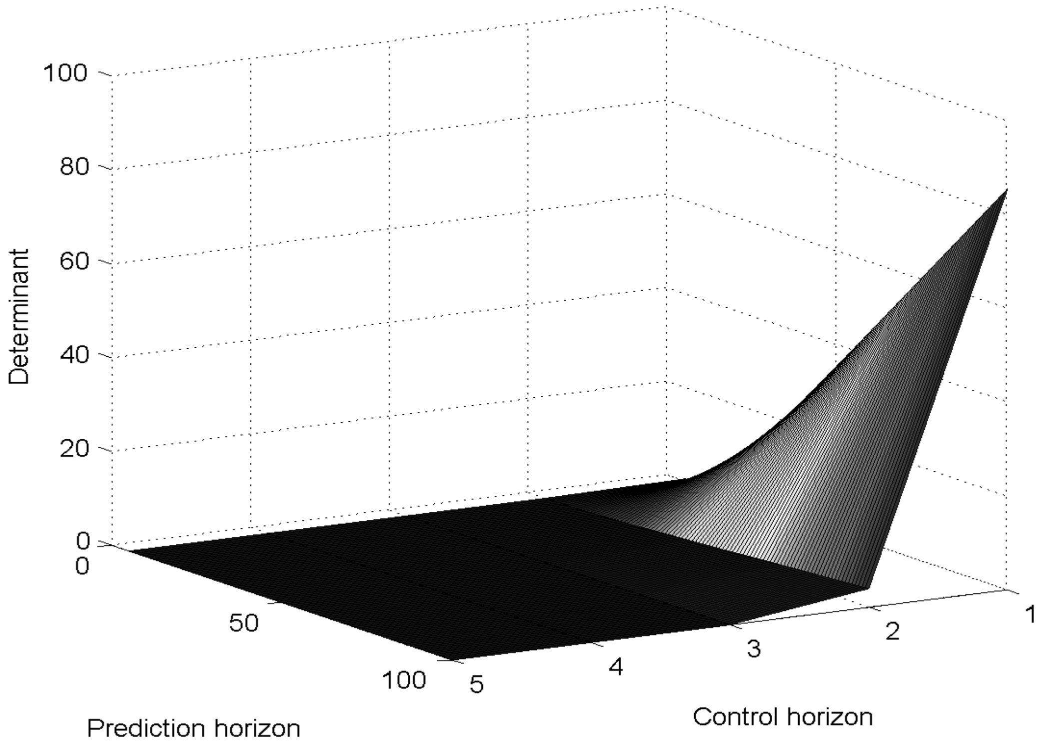


Figure 8: Evolution of GG^t determinant

$$G(s) = \frac{0.0039s + 0.0038}{s^2 + 0.075s + 0.0003} \tag{37}$$

Apart from its transfer function, each benchmark is characterized by its unit step response (required to obtain the time constant and settling time) and a graph showing the relationship between λ and the first Δu, that allows to obtain the correct range of values for λ. Table 9 matches each benchmark with its corresponding curves.

5.2. Results

From the data presented in the previous section and the tuning rules for section 4, the DMC parameters can be calculated for each benchmark. Those results are presented in Table 8

According to the above conclusions benchmarks should be tuned using a prediction horizon lower than 10 (taking into account that sampling time is $0.1\tau_p$). To select a suitable weighting factor it will be used the curve with the first control increment. As it can be seen in Figure 9 (using benchmark 1 as an example), the value of the first control increment changes very slowly once a value of λ =0.25 has been reached. This is the maximum useful value, higher values will only slow down the process. For λ=0 the first control increment reaches values up to 6.5. Taking into account that the system

Benchmark	Time constant	Settling time	τ	Pr	M	n_g
1	160	1000	16	10	2	73
2	180	1100	18	10	2	73
3	180	800	18	10	2	100
4	220	500	22	10	2	33
5	250	1100	25	10	2	54

Table 8: Tuning results for benchmark

gain and the setpoint are 1, this would mean an unstable process.The first part of the curve goes from 0 to the inflexion point of the curve. In this part of the curve, λ has a strong influence on the time response. Once the inflexion point has been surpased, the weighting factor will have only a small influence. This inflexion point becomes then the maximum useful value.
It can also seen how the first four λ curves are different form the last one. The reason is that the first four benchmark have the same gain, 1, while benchmark 5 has a gain of nearly 10. This means that the main factor that has influence on the shape of this curve is the system gain. The system gain is the factor to take into account when λ is calculated. This is in accordance with the method of Shridhar and Cooper that calculates the weighting factor from the system gain [1].

As it is logical to think, Figure 9 and 10 also show that the higher the system gain is, the higher λ must be.

6. Testing on a thermal model

The final exam for the design rules obtained in this paper will be a test on a real system. This system is thermal plant with two inputs and one output. The plant is basically composed of (see Figure 11):

- A fan to decrease temperature
- A set of resistances (lightbulbs) that will increase temperature
- A Pt100 to measure temperature

In this system (Figure 12) the temperature inside the model (output) is controlled by the voltage of the fan and resistances (outputs).
In Figure 13 it can be seen the step response of the system. The time constants for the fan and resistances are:

- 76 seconds for the resistance.
- 64 seconds for the fan.

Taking into account that the step time is 4 seconds (set by the system) this means that the prediction horizon is 20. To make easier the study we will use the same prediction horizon for both inputs. Model horizon will be 100, according to section 3.4. Control horizon will be set to a small value, 2. To select the weighting factor two curves comparing

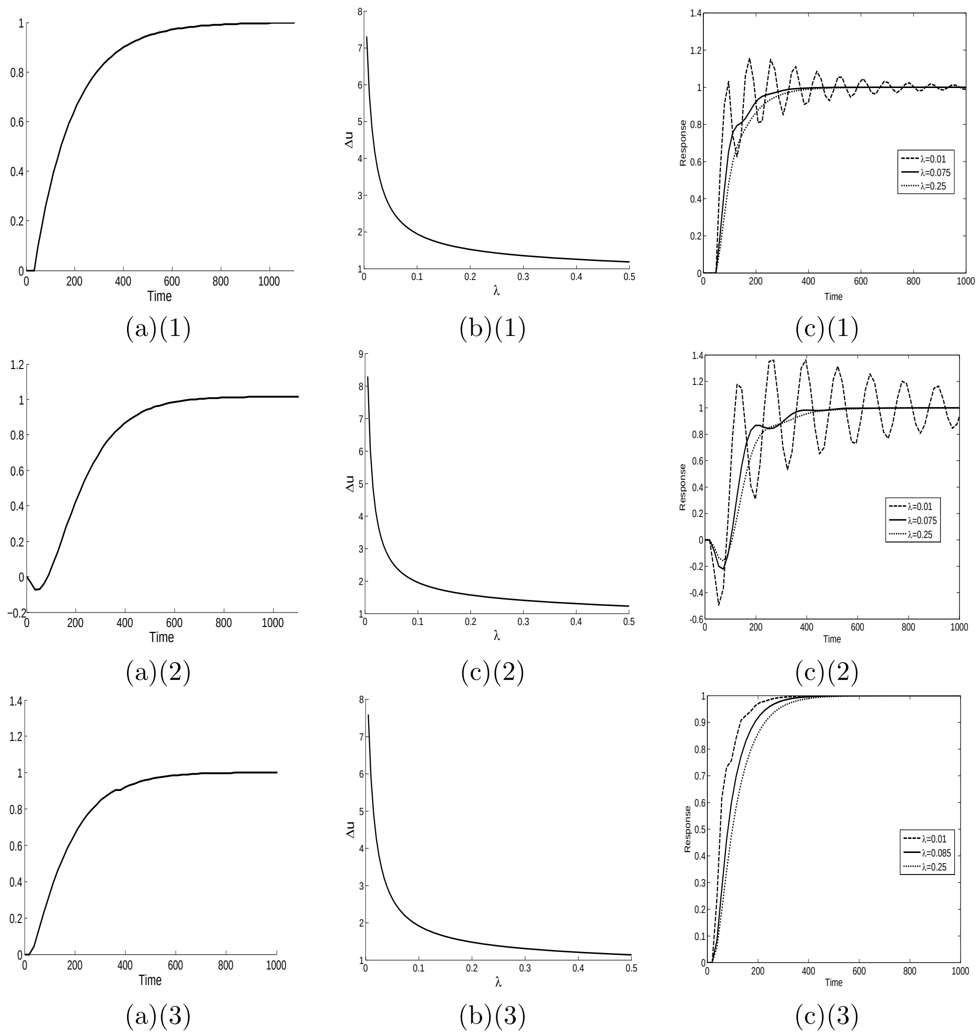
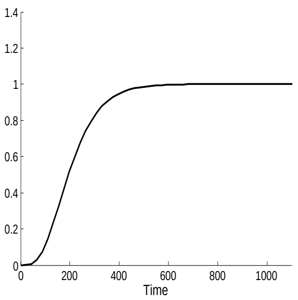
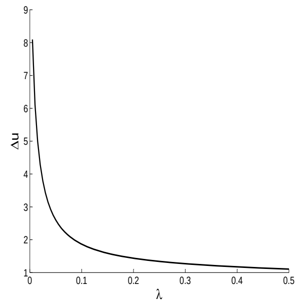


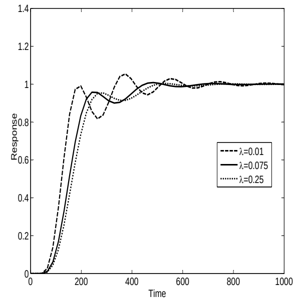
Figure 9: (a) Step response of example, (b) Weighting factor curve (c) Response of example



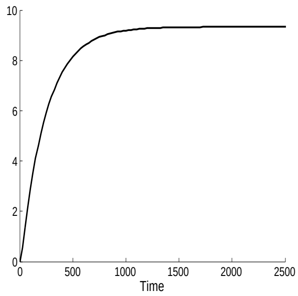
(a)(4)



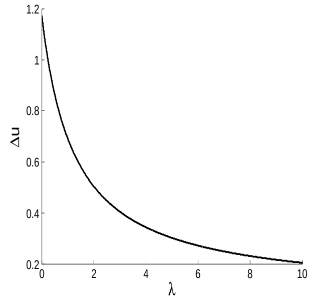
(b)(4)



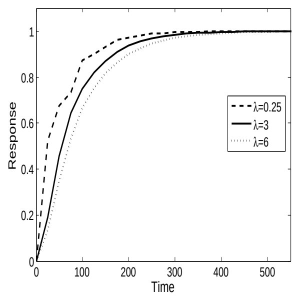
(c)(4)



(a)(5)

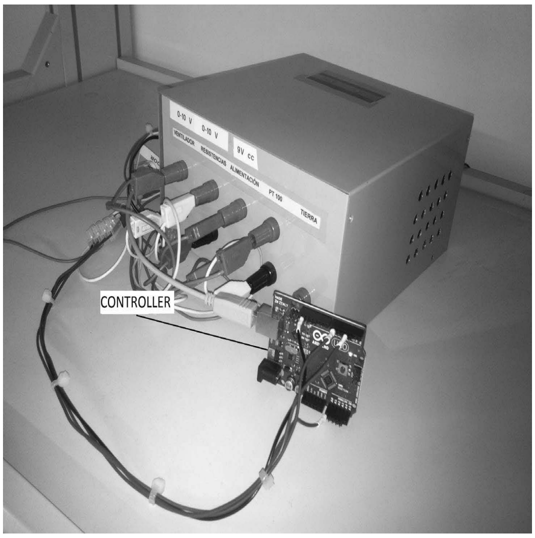


(b)(5)

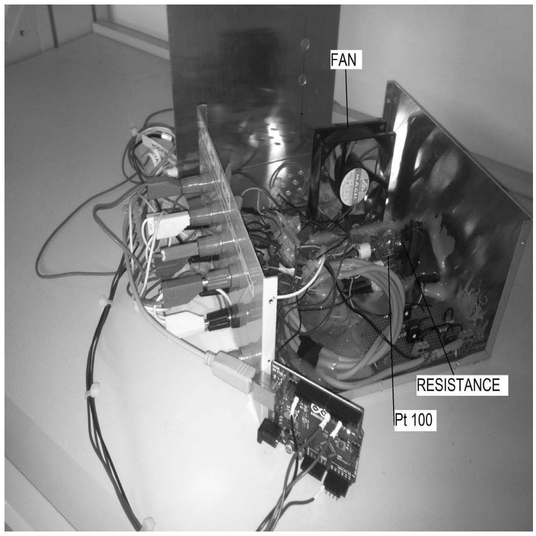


(c)(5)

Figure 10: (a) Step response of example, (b) Weighting factor curve (c) Response of example



Thermal plant



Thermal plant inside

Figure 11: Thermal Plant

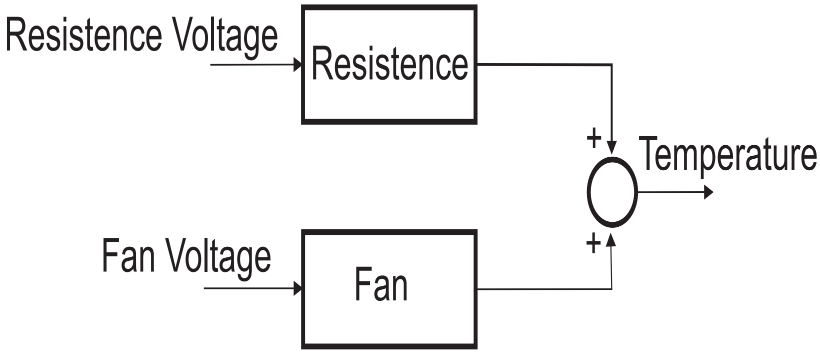


Figure 12: Thermal model block diagram

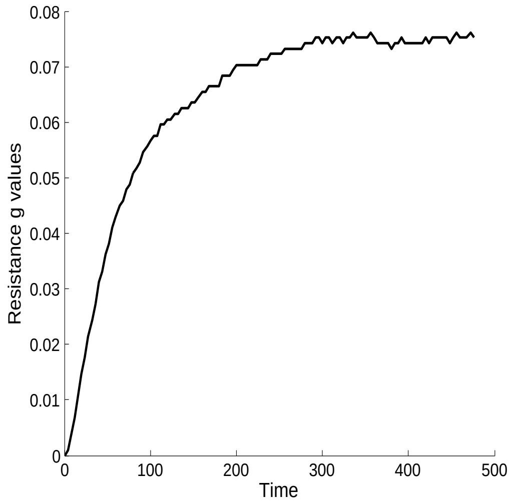
the first control increment with λ are drawn (Figure 13). Both curves show that in this model λ has only useful values up to 0.2. Bigger values will provide very small control increments and, consequently, a slow process. Moreover, too high values for λ will make that the system cannot handle the thermal inertia.

Case	λ	n_g	Pr	M
1	0.2	101	20	2
2	0.1	101	20	2
3	0.01	101	20	2
4	0.1	101	20	1
5	0.1	101	20	5
6	0.1	101	10	2

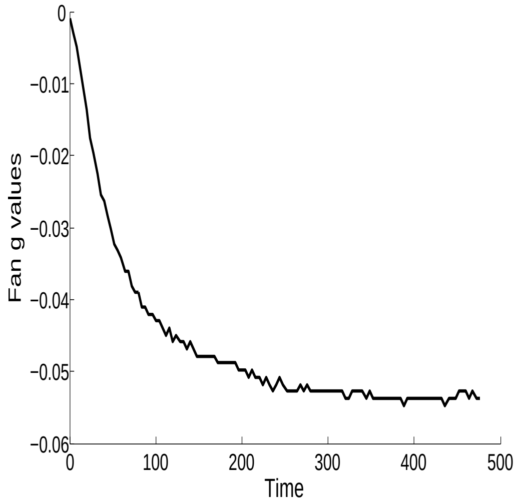
Table 9: Thermal model results

The explanation of cases is as follows (see Table 9 and figure 14.

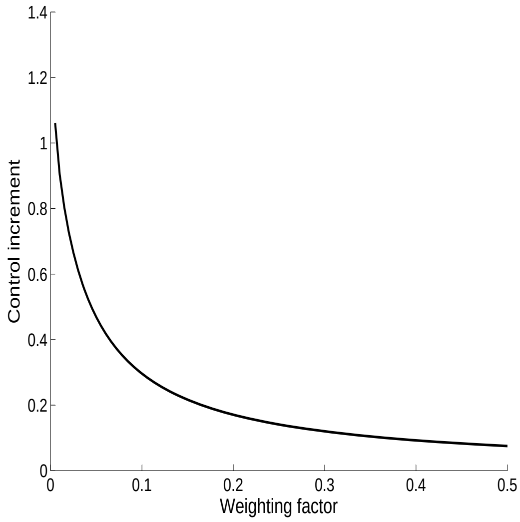
- Case 1: This is the consequence of a too high value of λ. As explained in section 3.3 this value has made dominant the complex poles, resulting in a oscillating system. This inadequate value slows down the fan and resistance actions and makes for then more difficult to defeat the thermal inertia.
- Case 2: It can be seen a better behaviour with only reducing λ. This gives more capability to the fan and resistance to deal with the thermal inertia by allowing bigger control increments. From the point of view of section 3.3, a smaller value of λ weakens the effect of complex poles.
- Case 3: This response shows the effect of a minimum λ. In this process it only means a small overpeak.
- Case 4: The difference between this result and case 2 is hard to differentiate. The only difference between them is the value of M and this parameter only shows a strong influence if Pr reaches high values.



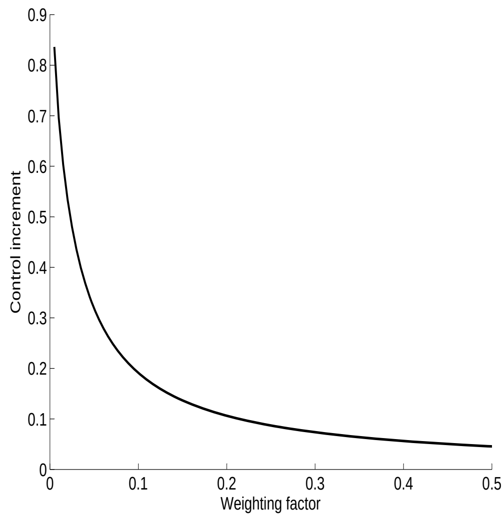
Resistance step response



Fan step response



Resistance weighting factor curve



Fan weighting factor curve

Figure 13: Thermal model definition graphs

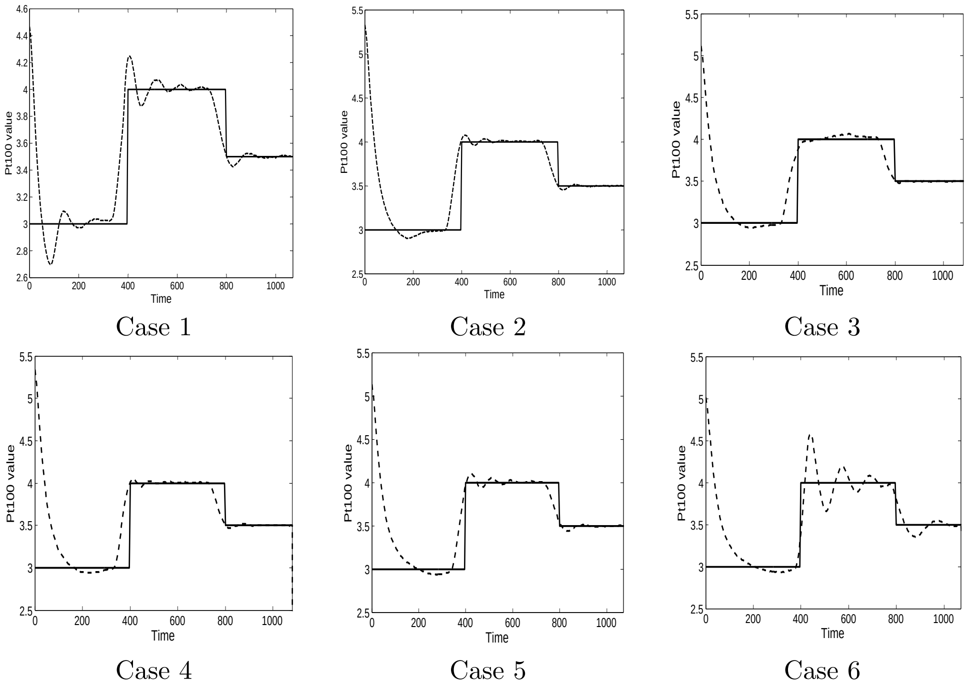


Figure 14: Results of themal model. The continous line shows the setpoint to be followed and the dashed line the system response

- Case 5: It can be seen the effect of a too high value of M. In section 3.2 it was explained that the influence of complex poles grows with M as well as with λ.
- Case 6: This response shows the effect of a too small Pr. The DMC algorithm has not enough data to perform accurate predictions. In section 3.2 it was explained that a too small Pr makes the complex poles become dominant.

7. Conclusions

In this paper some design rules for the DMC tuning are obtained. To this aim, a study of the effect of the main DMC parameters on the time response has been made. The controlled system is transformed to a Linear Time Invariant one in order to compute the closed loop poles that determine the system dynamics. A First Order Plus Dead Time approximation has been used to reduce the number of the existing poles and make the analysis easier. The closed loop poles movement study is traduced to prediction horizon, control horizon, weighting factor and sample time parameters. This yields to some rules to be followed by practitioners to a quick start procedure.
The application of this guide will not provide the optimal solution, but ensure an acceptable control and the parameters set obtained can be used as a good starting point prior to make adjustments. The results of the method over a set of simulation benchmarks and a real plant have been presented to prove the performance reached.

8. Bibliography

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