

# When Should We Work? A theoretical approach

Antonio Garcia-Sanchez

Mar Vazquez

Technical University of Cartagena. Spain

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## ABSTRACT

This paper analyzes flexibility in working hours from a theoretical point of view. The aim of this analysis is not only how much work is performed, but also when it is performed. In this context flexibility is understood as the capacity to match the preferences of workers well to the firm's needs. We draw up a model for determining the schedules in an economy with heterogeneous agents. Types are distinguished by their preferences for leisure time or production time. First we define the economy without flexibility. Then we analyze, under flexibility, what the optimal working schedule would be for each worker type. This is conditioned by the capital used in the plant. The baseline model solves the assignment of different types of workers to different production plants in a competitive equilibrium. The model enables us to determine what capital stock the plants must have and during what period they should be used in order that they adjust to the preferences. The flexible results, that conform to the preferences of firms and workers, it is optimal, efficient and reaches the maximum instantaneous aggregate production of the economy.

As an extension of the model we analyze the possibility of working at home. In this case the capital stock does not condition the work schedule. Thus, not only the time of work, but also the workplace turn into endogenous variables.

Keywords: flexibility, heterogeneity, work at home, production plant, work schedule.

*JEL Codes:* E22, J21, J22.

## 1 Introduction

Flexibility in the labor market is considered as part of a more far-reaching change in working arrangements, designed to increase the capacity of firms to innovate and to adapt to changes in product markets and in the demands brought about by new life styles. In addition, in the recessive phase of the cycle the need for more flexibility on the labor market is often argued as a premise to be able to confront economic recovery in some advanced economies. Nevertheless, the concept of flexibility, particularly on the labor market, is complex and includes several facets and instruments to put it into practice. Our interest in flexibility focuses on the time of work especially, not so much how long a worker is employed but when he/she works. Flexibility, in the sense of when and how long the employees are working, is better for firms and workers than flexibility in the

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<sup>0</sup>Mar Vazquez  
Department of Economics  
Technical University of Cartagena,  
Real 3, 30201 Cartagena. Spain.  
Tel.: +34968325629; fax: +34968325781  
E-mail: mar.vazquez@upct.es

sense of the number of employees. As one will see along this article the time in which we work is important enough to be studied from the standpoint of economic analysis.

In this paper we focus on the flexibility of working hours, a flexibility practice that has already been implemented in many companies<sup>1</sup>, but this has not been sufficiently analyzed from the point of view of economic analysis. We adopt the meaning of the term flexibility used in studies about work-life time balance as in Tausing and Fenwick (2001): flexibility implies that workers have some control or choice over their work schedules. We draw up a tractable general equilibrium model that determines work schedules as an equilibrium outcome between the firm's decision and workers' preferences. In doing so, we assume heterogeneity in worker preferences and in production plants to encourage flexibility. Workers are heterogeneous in relation to their leisure (or non-market time) preferences. Firms organize their production in plants with different working hours. Each individual type is classified by the time of day around which they prefer to organize their workday, and this is consistently the time of day in which leisure is less valued by workers. As regards firms, we also assume that there is a time of maximum production for a given unit of time. This paper analyzes the optimal determination of work schedules in the presence of such heterogeneity. Workers and firms take their decisions about when to work on the basis of the available alternatives. When the worker is employed on his/her optimal workday (lowest disutility) his/her salary will be less than if he/she is employed in a workday with more disutility for him/her. Consequently, on the optimal workday the employee will demand a wage on the basis of his/her lowest reservation wage. On the other hand, higher productivity during a workday enables the firm to pay higher wages to the employees. We look into the circumstances under which flexibility leads to an efficient assignment.

In 2004, about 30% of full-time wage workers had flexible schedules in the United States, which is more than double that in 1985<sup>2</sup>. In the European Union, social policy demands that social partners negotiate the modernization of working time to generate positive flexibility, that is, the synchronization of employer and employee time needs, contributing to sustainable work-life balance policies for employees as well as high performance for workplaces (Morris and Pillinger, 2008). According to the EU Labour Force Survey (ad-hoc module about work organization and working time arrangements) in 2004, 64.6% of employees report a "fixed start and end of a working day". The rest of employees had some form of variable working hours<sup>3</sup>. From the Fourth European Working Conditions Survey (EWCS) (2007), around 40% of the European workers do not have fixed starting and finishing working hours, and around 50% do not work the same number of hours every day. Bettio et al (1998) confirm that work arrangements with flexible working time such as part-time work, telecommuting, compressed workweeks, annualized hours contracts, and time banks have increased substantially since 1990 in advanced industrial economies. Specifically, some forms of flexible schedule are: *Flexitime*: workday start and end times differ from the workgroup's standard, although the same number of hours per day is usually maintained. *Compressed Workweeks*: full-time options that enable employees to work longer days for part of a week or pay period in exchange for shorter days or a day off. Other practices are *part-time*: working fewer than 35 hours per week, and *job sharing*: full-time position shared by two people, each working part-time. Beside know how many workers enjoy flexibility, it is important to know if it is because they wish it or because the company imposes it. In this sense, following the EWCS, in northern European countries, around half of employees declare they can choose to adapt working time to their needs to a large extent. This is in sharp contrast to southern and eastern European countries, where more than 75% of employees have no possibility of adapting their work schedules, as they are set by the company.

As an extension of the model we analyze the possibility of working at home, that we consider the way to achieve the maxima flexibility. It does not treat itself of household production that is nonmarket activity. We introduce the paid work at home that is characterized by a few limited requirements of capital, more the availability and utilization of the technologies of the information and communication, which they allow to work from any place and at any time of day or week. From the EWCS, 12% of European workers report working at least a quarter of the time from home without a PC, and 8% at home with a PC. This suggest that telework or working from home is used by a substantial proportion of people as a complement to their

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<sup>1</sup>Some examples of successful workplace flexibility implementation can be found in Intel, JPMorgan Chase, IBM. See <http://www.worldatwork.org>.

<sup>2</sup>Data from Current Population Survey, Supplement, May 2004.

<sup>3</sup>In the LFS module variable working hours are classified into the following categories: Staggered working hours, banded start and end; working time banking with possibility only of taking hours off; working time banking with possibility of taking full days off (as well as taking hours off); start and end of working day varying by individual agreement; each determines their own work schedule (no formal boundaries).

normal working arrangements. Respect to the relationship between the place of work and working hours, those working at company premises show much less variation in their weekly working hours than all the others. Working from home is by nature much more flexible time-wise.

It is difficult to find evidence about schedules in and out of work. We observe statistics of weekly working hours that focus on how many people work full-time or part-time, or how many people work less than 35 hours a week or more than 50 hours. There is also no evidence about the time of day that workers prefer to be employed, only about the number of hours. From the time use statistics we know what percentage of the population are working every hour of the day. In Figure 1 we show data on three countries confirming that the work activity is concentrated between 8 am and 6 pm.

Fagan (2005) presents evidence about the number of hours that men and women would prefer to work, and their assessments of the degree of compatibility between their work schedules and their family and other commitments, drawn from the *European Working Conditions Survey 2000*. It is noted that preferences adapt when changes in economic conditions, workplace innovations or policy interventions alter the context, and people make decisions based on the alternatives that they consider to be open to them<sup>4</sup>. Working-time preferences are rarely fully carried out in practice because of the constraints imposed by the job, the firm or the market. To analyze the compatibility between work schedules and family life, the survey classifies workers according to their different types of day and asks if their working hours fit in with their family or social commitments outside work very well, fairly well or not at all well<sup>5</sup>. Workers said that daytime, weekday work without long days is the most compatible. Variable start and finish times are less compatible than fixed ones, particularly when the variation is set by the employer, but also when the workers have some influence in varying their hours. In this latter case, the problem is that this autonomy may be associated with jobs that require a commitment of long hours.

Our motivation lies not only on the importance for society to reconciling work with personal life, but also on their effects on employment and output. If the working hours are tailored to the preferences, the activity rate of the labor force will be higher, and this could increase the usage time of equipment and increase output. More specifically, with flexible work schedules, employees experience several benefits, some of which could be: external childcare costs may decrease; reduced burnout; increased feeling of personal control over schedule; avoiding commuting during rush hours; greater flexibility to meet family needs and life responsibilities and allowing people to work when they accomplish most, feel most refreshed and enjoy working (e.g. morning person versus night person). With regard to the firms, employers experience the following benefits: increased employee morale; engagement and commitment to the organization; reduced turnover; extended hours of operation; increased ability to recruit outstanding employees; reduced absenteeism and lateness and creation of an image as an employer with a choice with family-friendly, flexible work schedules. An important aspect to bear in mind also is that flexitime and to have a variety of work schedules are clear advantages for the environment because they reduce congestion and traffic problems.

The main factors that influence the determination of working hours are: technological conditions; capital available; existing legislation and institutional framework; the requirements imposed by demand or by output characteristics and also the preferences of employers and workers. In this paper we focus on the relationship between technological characteristics and workers' preferences to determine the schedule. We assume that all firms produce the same output. In this context, flexibility is understood as the capacity to match the preferences of workers will with the firms' needs.

To illustrate the usefulness of the model, we use it to consider a scenario where the technology used cannot enforce the preferences of workers and companies, which allows us to graphically illustrate the instantaneous aggregate output in different situations compared to the one obtained in a "flexible" economy.

A second finding is that the flexible result, when firms and workers can assert their preferences, is optimal, efficient and achieves the maximum instantaneous aggregate production of the economy. The "non flexible" result, never exceeds the aggregate instantaneous production obtained as a result of a "flexible" economy. So the aggregate production during one workday is greater when firms and workers can adjust preferences about the part of the day that they want to work.

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<sup>4</sup>For example, mothers tend to prefer shorter working hours where public child care services are limited.

<sup>5</sup>The schedule types were, among others: Daytime-weekday; Daytime-some weekends; Daytime-regular weekends; Some evenings/nights-but not rotating shifts; Rotating shifts or permanent nights; Fixed own start and finish times; Start and finish times fixed by employer; Varies own start and finish times; Start and finish time varied by employer; Little working time autonomy; More working time autonomy.

An outline of the paper follows. First, in Section 2 we explain the assumptions of the model on the technology and preferences, and we review briefly the literature of reference. In Section 3 we describe the model economy. Section 4 shows the competitive equilibrium. The analysis of flexibility is developed in Section 5. An extension to consider working at home is in Section 6. Section 7 provides a numerical example and the final remarks are in 8.

## 2 Preferences, technology and related literature

The introduction of a flexible schedule requires technology with a production function where the capital-labor ratio can also change throughout the day. It is also necessary to distinguish between working time and capital utilization time. Therefore, in the model the production is organized in production plants. We define a production plant by means of the capital-labor ratio and by the workday of their employees. Our definition of a production plant is a variation of the plant in Hornstein and Prescott (1993), where both the length of time in which a plant can be operated and the number of workers operating it can be varied. However, in Hornstein and Prescott the workers are homogeneous. Fitzgerald (1998) also extends the plant concept of Hornstein and Prescott in his general equilibrium model of team production. But in each of Fitzgerald's teams there are only two different types of workers that must be coordinated. Hansen and Prescott (2005) construct a real business cycle model where the production takes place at individual plants and the number of plants operated varies over the cycle. In comparison with the plant concept of those models, we introduce the time of starting work and consequently the analysis of the timing of work. As with Hansen and Prescott (2005), the capital is identified with the equipment rather than locations. Furthermore, we assume mobile and exchangeable equipment between workers and over time. In equilibrium, the capital utilization time and the percentage of utilization of the installed capital are determined.

Although in this area the workweek is the usual measurement, to discuss the schedule it is better to limit ourselves to just one day. We define a workday as a combination of starting time and duration. Therefore, plant refers to a way of producing, not to something physical. The plants differ for the  $k/n$  ratio and for the workday. This introduces the possibility of adaptation in the utilization of the capital and in the workday. Afterwards, the definition of plant will be expanded including a time throughout the day in which the production is maximized.

First, there is a predefined workday (starting time and duration) and plants and then an analysis is made as to how workers would provide their work and what workdays are going to be carried out as the adjustment result. We assume nonlinear mapping from hours of work in a given period to labor services provided in that same period. This assumption is also a key feature in Prescott, Rogerson and Wallenius' model (2007), where at low hours of work this mapping is convex, due to such factors as the costs associated with getting set up in a job, communicating with coworkers and so on. On the other hand, at long hours of work this mapping is assumed to be concave due to fatigue, weariness, etc. Therefore, in our model the function that relates time of work and services of the work initially increases and later, from a certain moment, it falls. When we observe all the workers, the function looks like figure 1, which shows when people work, created from the time use survey.

With regard to the preferences, literature on the matter usually considers the disutility of the time of work as increasing and convex. As we use the instantaneous use of time to be worked, the disutility of the time of work depends not only on the quantity of worked hours, but also on the moment of the day in which they occur. Workers prefer to organize their working day around a certain time, so disutility increases as working hours differ from this time. Our analysis focuses on the timing of production activities, so we must also agree that some activities should be conducted at certain parts of the day. Equally, at other times there are activities that cannot be done. The work schedule results from the assignment of different types of workers to different plants. Depending on workers' preferences, availability of capital and technology, working times are scheduled. The use of the instantaneous use of time provides us with relevant information about effort, behavior of workers and firms' production and profitability, which allows us to us come up with relevant results. As noted by Hamermesh (1999, p.37), studying the instantaneous use of time as opposed to time used integrated over days, weeks, years or a working life, can yield insights into questions about behavior that are not obtainable from examining other labor market outcomes. The equilibrium exists and is unique.

Secondly, as in the reference model (Hornstein and Prescott, 1993) the formation of plants is allowed to form part of the set of possibilities of production and not to exist independently of this set (exogenous).

Our analysis thus also enables us to determine what the optimal plants should be like. In this way we can determine the best set of possibilities. The result obtained is an efficient assignment and, after comparing with what we observe in reality, this enables guidelines to be established for the policymakers on how act to improve the efficiency of the productive system.

As in Hornstein (2002), production can increase with the incorporation of one more shift of work. This allows us to determine the time of utilization of the capital that does not necessarily have to coincide with that of work. In this type of model, the capital installed normally is not used to full capacity, that is, the whole capital during all the time. This is so because there is a marginal increasing cost in the utilization of the capital, since it increases the depreciation or/and because it implies using workers during more untimely hours and it is necessary to pay them more. Nevertheless, in our model if the utilization of the capital spreads using workers who prefer working at these hours, the marginal increasing cost of the capital does not exist due to the more wages/hour motive. Therefore, if there are people with preferences for being employed at different schedules the plant will be operative for more hours and with this the added production and the utilization of capital of the firm will increase. The model also allows to explain how the firm must organize the capital among the plants.

As the number of plant types is finite the model lies within the framework of the indivisible labor and employment lotteries of Rogerson (1988) and Hansen (1985), although to get analytical results this environment will change towards divisible labor in a similar way to Hornstein and Prescott (1993) and Fitzgerald (1998).

With regard to the related literature, besides the one already commented previously, in general the timing of work has received little attention in literature concerning economic analysis. Wooden, Warren and Drago (2009) analyze the role played by mismatches between hours actually worked and working time preferences in contributing to reported levels of job and life satisfaction. They show that it is not the number of hours worked per se that matters but whether these hours are in line with workers' preferences.

Some studies analyze the distribution of the time of work throughout the working life. Prescott, Rogerson and Wallenius (2007) present a model of lifetime aggregate labor supply where both the fraction of life spent in employment and the hours of work while employed are characterized as an equilibrium outcome. But the model does not explain the timing of work. Hence it is necessary to include life cycle effects such as age-varying productivity or age-varying disutility of work. As usual in the life cycle labor supply literature, Rogerson and Wallenius (2006) use a function that represents exogenous life cycle variation in individual productivity. This function is assumed to be single peaked, twice continuously differentiable and has zero derivative only at its global maximum. In our model there is considered to be a similar relation between over time and individual productivity, but during the workday.

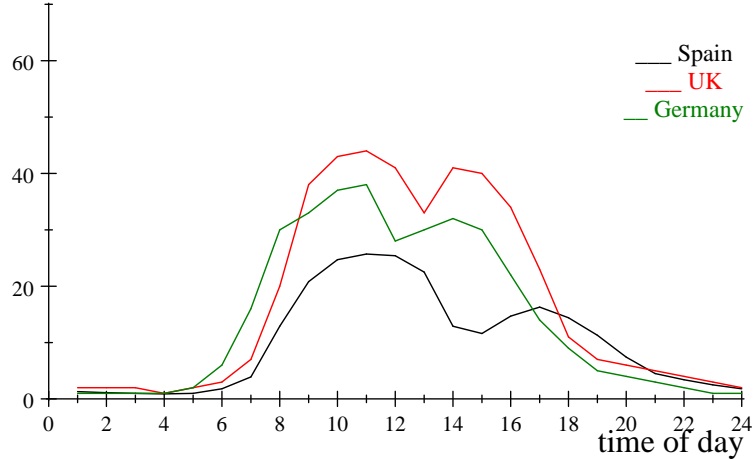
The issue of the effects of deregulation in shop opening hours is the theme in some theoretical and applied papers. Inderst and Irmen (2005) analyze the impact on prices of deregulation in shop opening hours in a model of imperfect duopolistic competition. Burda and Weil (2005) investigate the real effects of deregulation in a general equilibrium model. Both papers stress that the results depend on the value placed on leisure/shopping time for consumers. Note that the paper by Burda and Weil focuses on arguments against flexible schedules such as positive externalities arising from enjoying free and work time collectively.

Among literature dealing with work schedules, Weiss (1996) constructs a lifetime labor supply model to explain the synchronization of work schedules. An analysis is made as to how workers choose the optimal work schedule to maximize their lifetime utility subject to the constraints derived from the exogenous time pattern of the worker's productivity. The length of each working day and the beginning of the first (and subsequent) work intervals are deduced. But, in Weiss' paper there is a partial equilibrium model and the interaction between capital utilization and work schedules is only mentioned. From an empirical point of view, Hamermesh (1999) analyzes the timing of work in the United States and shows indicators on the fraction of workers who work at each hour of the day. He explains the decline in evening and night work by the rise in real earnings leading workers to move away from such work. Dupaigne (2001) is probably the pioneer who incorporates the idea that the value of leisure varies throughout the day. He studies the effects of shift work on workers' welfare, although workers can only choose the starting time, as the length of the work period has already been established.

Liu, Wen and Zhu (2005) draw up a partial equilibrium model to analyze the synchronization of working hours between workers of different skills, but they refer to the synchronization in the number of hours and not in the schedule. In this model, all the people start working at the same time. Lührmann and Weiss

(2010) analyze the relation between longer work hours and higher labor force participation with outsourcing of domestic tasks. As a consequence, the demand for unskilled labor rises and unemployment falls. They develop a general equilibrium model and find quantitative relevant effects of these relations from the empirical evidence. Apart from all these papers, we have not found models that treat the determination of when people work from a theoretical perspective, including microeconomic fundamentals and equilibrium conditions.

Figure 1: Proportion of employees working at the beginning of each hour



Source: United Kingdom and Germany: Comparable Time Use Statistics, 2005, Eurostat

Spain: Time Use Survey, 2009-2010 (the proportion is over all population, but UK and in Germany is over population aged 20 to 74).

### 3 The economy

The main features of our model economy with flexible schedules are: the existence of different types of plants and the possibility for workers to choose between different workdays. Also, unlike models with homogeneous agents, in equilibrium not all work at the same time.

Although it is a question of time, the analysis refers to one period. The model ignores inter-temporal considerations. We are assuming that the issues about the work schedule will be the same henceforth. So, this economy lasts for one period (a day) and the length of the day is normalized to unity.

The economy is populated by a continuum of people with measure 1. There is a finite number of different agent types,  $i \in I = \{1, 2, \dots, N_I\}$ . The measure of agent type  $i$  is  $\lambda^i$  and  $\sum_i \lambda^i = 1$ . People types are distinguished by their different preferences regarding leisure or preferences as regards no market working time (necessary to reconcile working and personal life, for example), especially with regard to when they prefer to enjoy leisure time. Each type can be characterized by a moment  $\tau_i \in \{\tau^-, \dots, \tau^+\} \subset [0, 1]$  that is the preferred moment to work in a day for this individual type or when leisure is less valued. So, individuals  $i$  prefer their work schedule to be organized around that moment. We define  $\tau_0 = \tau^-$ , and  $\tau_i = \tau_{i-1} + \bar{\tau}$ , where  $\bar{\tau}$  is a constant satisfying:  $\bar{\tau} = \frac{(\tau^+ - \tau^-)}{N_I}$ . In this manner, the distribution of types is going to determine the distribution of working times in equilibrium. Individuals are identical in their endowment: a time endowment of 1 can be allocated to either work or leisure, and  $\bar{k} > 0$  units of capital. Individuals are also identical in their preferences about consumption goods.

The distribution of types may capture the idea that consumers value the synchronization of leisure time with other types positively. In this case the respective  $\tau_i$  would be very close. Burda and Weil (2005) stress that positive externalities can arise from resting or enjoying free time collectively<sup>6</sup>. On the contrary, the different types of agents may be very different in their preferences,  $\tau_i$ , probably as a consequence of different needs of making work compatible with family or studies, for example.

Given that our interest focuses on the work schedule, it is necessary to distinguish between duration of work and timing of work. So, we define the working time during a day:

<sup>6</sup>They also consider that negative externalities may result from coordinated leisure or synchronized economic activity. With these considerations Burda and Weil draw up a model to analyze *blue laws* with two types of households: manufacturing families and retail families.

**DEFINITION.** A workday  $s$  is a pair  $(t, h)$  where  $t$  is the moment at which work starts and  $h$  is the length.

Assuming that individuals prefer to organize their working time around a moment of the day, we adopt the function introduced by Dupaigne (2001) that sums up the instantaneous value of leisure between the time of starting and finishing<sup>7</sup>. So, in our model the preferences of each individual type are represented by a function  $v^i(s)$  that sums up the instantaneous value of leisure between  $t$  and  $t + h$ , which is measured by the function  $\vartheta^i(\tau)$  that will be defined around the preferred moment for the  $i$  type<sup>8</sup> such as:

$$v^i(s) = \int_t^{t+h} \vartheta^i(\tau) d\tau \quad (1)$$

and where the integrand sign obeys this, during the period when time passes continuously. The function  $\vartheta^i(\tau)$  is strictly convex, decreasing for  $\tau < \tau_i$  and increasing for  $\tau > \tau_i$ , so  $\vartheta^i(\tau_i) = 0$ . For a given number of hours the disutility increases as the central moment deviates from the preferred time of day.

With respect to production technology, output is produced by a large but finite number of types of production plants to which workers are assigned, and they use capital during one type of workday. So, a production plant is characterized by the ratio of capital per worker  $k$  and by the workday  $s$ . The same stock of capital can be utilized in different plants if the respective workdays do not overlap. That is, the capital is equipment that is mobile between workers. If the working time is organized in shift systems, the operating time of capital utilized is longer than a corresponding workday.

Concerning the output of a plant, we distinguish between the instantaneous production and the total production. The output per instant in a plant with  $n$  workers and with a ratio of capital per worker  $k$ , assuming, as usual, constant returns to scale in the instantaneous production, is:  $f(k)n$ , with  $f' > 0, f'' < 0$ , but the resulting output depends on the workday the plant operates. Both the length and the time of starting matter because we could consider, for example, that the productivity in an 8-hour workday is not the same during the day as it is at night. So, the output per worker in a type  $(k, s)$  plant is:  $f(k)g(s)$ , where  $g(s)$  measures the effective working time of a workday starting at  $t$  and ending at  $t + h$ . If the set of feasible workdays is denoted by  $S$ , then  $g : S \rightarrow R$ , multiplies the instantaneous output of the plant that operates the workday  $(t, h)$ . The function  $g(s)$  sums up the value of an instantaneous index of productivity between  $t$  and  $t + h$ , which is denoted by the function  $\gamma(\tau)$ :

$$g(s) = \int_t^{t+h} \gamma(\tau) d\tau \quad (2)$$

This definition of effective working time allows for the consideration of several hypotheses. So, according to Mulligan (2001) the instantaneous productivity function could incorporate the *fatigue* caused by extended intervals of work. Booth and Ravallion (1993) consider that the number of efficiency hours obtained from a given number of clock hours is a strictly concave function of hours worked because there is a *warm-up* period followed by a *fatigue* period. Corrado and Mattey (1997) use the domain of the production function for the definition of different technologies: continuous production, two discontinuous shifts, etc. All these questions can be included by means of different specifications of  $\gamma(\tau)$ . We assume that  $\gamma(\tau)$  is strictly concave with a maximum at about  $\hat{\tau} \in [0, 1]$ . Productivity is maximized at this time, and for a given number of hours, output is lower the greater the distance from this point. Although workers and jobs are heterogeneous, output is effectively homogeneous. Thus in Walrasian equilibrium only some types of plants will be operated.

We represent this economy in a McKenzie-type general equilibrium language. Let a commodity point  $x$  be an element of the Euclidean space  $L$ . The consumption set  $X^i$  of a type  $i$  agent is a subset of the commodity space  $L$ . Preferences over consumption bundles in  $X^i$  are represented by the utility function

<sup>7</sup>In Dupaigne (2001) the number of hours is given and workers choose the time to start to work. In addition, workers are homogeneous.

<sup>8</sup>Different types of preferences about leisure can be considered depending on the functional form of  $v(\tau)$ . If the workers care only about the number of hours spent at work and the work schedule does not matter,  $v(\tau) = \text{constant}$ , which is the case in Weiss (1996). In literature the most usual form of defining preferences about leisure is assuming that the value of leisure is increasing and convex throughout the day, Fitzgerald (1998), Hornstein and Prescott (1993). In the cited papers, the relevant variable is the length of the leisure time and not its distribution.

$U^i : X^i \rightarrow R$ . Production is described by an aggregate production possibility set  $Y$ , which is a convex cone in  $L$ . An allocation  $[(x^i)_{i \in I}, y]$  is feasible if  $x^i \in X^i$  for all  $i \in I$ ,  $y \in Y$ , and  $\sum_{i \in I} \lambda^i x^i = y$ .

### 3.1 Traded commodities

The model is in continuous time. However, it is simplified by assuming that there is a finite number of feasible workdays,  $S = \{s_1, s_2, \dots, s_{N_s}\}$  which is a subset of a bigger set of workdays,  $T \times H$ . That is, the set of possible workday lengths denoted by  $H$ , where  $H \subset [0, 1]$ ,  $H = \{h_0, h_1, \dots, h_{N_H}\}$  and the set  $T = \{t_0, t_1, \dots, t_{N_T}\}$ ,  $T \subset [0, 1[$ , that contains the possible starting times. Thus, by limiting the number of elements of the set  $S$ , we consider only the standard workdays<sup>9</sup>. As we will see later, we consider flexible working hours if the elements of  $S$  are determined jointly by firm and workers.

Introducing a finite number of different workdays creates an indivisibility, that is, the nonconvexity of the set of consumption and production possibilities. The introduction of employment lotteries, following Rogerson (1988), is a useful device to convexify these sets. So, we assume that people supply a lottery contract that specifies the probability of working different workdays, and they will work only one workday depending on the lottery's outcome.

The commodity space  $L$  is  $R^2 \times M(S)$ , where  $M(S)$  denotes the set of signed measures on the Borel sigma algebra of  $S$ . An element of  $L$  is given by  $(c, k, n)$ , where  $c$  is the consumption good,  $k$  denotes the services of the capital stock, and  $n$  is a measure over labor workdays. One unit of capital produces one unit of capital services. When  $S$  is a finite set,  $n$  is a vector and  $n(s)$  is the measure of type  $s$  workday (with start at  $t$  and length  $h$ ). The agent  $i$  that chooses a point in  $L$  receives  $c^i$  units of the consumption good in exchange for providing  $k^i$  units of capital<sup>10</sup> and some measure  $n^i$  over labor workdays.

### 3.2 Production possibility set

A production plant is characterized by the capital per worker ratio  $k$  and by the workday  $s$ . Let  $K$  and  $S$  be finite sets and let  $J \subset K \times S$  be the set of feasible plants with generic element  $(k, s)$  and cardinality  $N_J$ . We can index plant types by  $j : 1, 2, \dots, N_J$ . The output per worker of a type  $(k, s)$  plant is:  $f(k) g(s)$ , where  $g(s)$  measures the effective working time of a workday starting at  $t$  and ending at  $t + h$ . A production *plan* organizes the distribution of inputs across plants of different types, given that workers are available for certain workdays, while capital is available at the beginning of the period, and each time a shift finishes the capital utilized is available for another shift. In this way, the production plan determines the total time of utilization of capital, the distribution of different ratios throughout the day and the distribution of workers. Let  $m_j$  denotes the measure of type  $j$  plant operated, and also the measure of workers in it. Then the production *plan* is a vector of  $N_J$  numbers,  $\{m_1, m_2, \dots, m_{N_J}\}$ ,  $m \in R_+^{N_J}$ ,  $m_j \geq 0$ , which describes how the inputs are allocated across plants of different types. The production possibility set,  $Y$ , is defined as:

$$\begin{aligned}
Y \equiv \{ \{C, K, N\} : \text{there exists a production plan } m \in R_+^{N_J} \text{ such that} \\
C \leq \sum_j m_j f(k_j) g(s_j) \\
\sum_{\{j: t_j \leq t < t_j + h_j\}} m_j k_j \leq K, \quad \text{for each } t \in T \\
\sum_{\{j: h_j = h, t_j = t\}} m_j \leq N(s), \quad \text{for each } s \in S \}
\end{aligned} \tag{3}$$

The first constraint highlights that the total amount of the consumption good is less than or equal to the total output produced by all plant types. The second constraint states: for each feasible starting time the capital allocated across all the plant types with this starting time, or with the previous starting time but not yet finished is less than or equal to the total capital available. The third constraint states that the amount of

<sup>9</sup> Afterwards, it will be necessary to consider that the set  $S$  is  $S = T \times H$ , and  $T = [0, 1[$  and  $H = [0, 1]$ . Also it would be possible to consider the existence of working time regulation by means of imposing constraints on the sets  $T$  and  $H$ .

<sup>10</sup> The component  $k$  of the commodity space is not the same as the element  $k$  of a plant because the latter is an element of a finite set. Thus they are denoted differently.



type  $s$  workdays allocated across all plant types is less than or equal to the total amount of type  $s$  workdays available. It is immediate that  $Y$  is a convex cone.

### 3.3 Preferences

Turning to individuals' preference ordering and their feasible consumption bundles: the utility of a type  $i \in I$  person choosing the commodity point  $x = (c, k, n)$  is given by:

$$U^i(x) = u(c) - \sum_s n^i(s) v^i(s) \quad (4)$$

where  $v^i : S \rightarrow R_+$  represents the accumulated disutility of working the workday  $s$ , and where  $u : R \rightarrow R$ ;  $v(0, 0) = 0$ , and  $\lim_{c \rightarrow 0} u(c) = \infty$ . The function  $u(c)$  is assumed to be continuously differentiable and strictly concave. Notice that  $\sum_s n^i(s) v^i(s)$  is the expected disutility of working for a type  $i$  person. The consumption possibility set of an agent type  $i$  is:

$$X^i(\bar{k}) = \left\{ (c, k, n) : k \leq \bar{k}, c \geq 0, k \geq 0, \sum_{s \in S} n^i(s) = 1, n^i(s) \geq 0 \right\} \quad (5)$$

which contains the standard nonnegativity constraints and the conditions that capital services are restricted by the capital stock endowment, and  $n$  is a probability measure.

The economy is completely described by  $E = \{(U^i, X^i, \lambda_i)_{i \in I}, Y\}$

## 4 Competitive equilibrium

The commodities traded are given by  $x = (c, k, n)$ . Prices are in terms of the consumption good. The rental price of capital is  $r$ . The wage is a function  $w$  mapping signed measures into  $R$ . With a finite set of possible workdays,  $w$  is a vector of prices, where  $w(s)$  is the price of the type  $s$  workday. That is, if a person works the workday  $s$  with probability 1,  $w(s)$  units of the consumption good are received.

The firm rents capital, employs workers for workdays of different types and decides how to allocate these resources across all the plants. On hiring workers, the firm buys lottery contracts that specify the probability of a person working workdays of different types, possibly including a workday of length 0. All the individuals of the same type will sell the same lottery contract, but people of different types will choose different probabilities. In fact, each workday will be worked by a measure of agents similar to the sum of the probability specified by the contract of working that workday of each type multiplied by the measure of that type.

Given prices  $(r, w)$ , the firm chooses quantities  $(C, K, N)$  to solve:

$$\max C - rK - \sum_s w(s) N(s) \quad (6)$$

$$s.t. : (C, K, N) \in Y \quad (7)$$

where  $N(s)$  is the measure of workdays of type  $s$ .

In this economy, individuals purchase the consumption good and sell capital and labor services to firms. The labor services are supplied in the shape of a lottery contract that specifies the probability of working different workdays. The amount an individual receives for a given lottery contract does not depend on the lottery's outcome, that is, on the type of workday the individual works ex post, but the probabilities of work supplied. A type  $i$  person faces the decision problem:

$$\max u(c) - \sum_s n^i(s) v^i(s) \quad (8)$$

$$s.t. : (c, k, n) \in X^i(\bar{k}) \quad (9)$$

$$c \leq rk + \sum_s w(s) n^i(s) \quad (10)$$

**Definition of equilibrium.** A competitive equilibrium for this economy is an allocation  $[(x^{i*})_{i \in I}, y^*]$  and a price system  $(r, w)$  such that

- i)  $x^{i*}$  maximizes  $U^i(x)$  subject to  $x^i \in X^i(\bar{k})$  and the budget constraint (10),  $i \in I$ .
- ii)  $y^*$  maximizes (6) subject to  $y \in Y$ .
- iii)  $\sum_{i \in I} \lambda^i x^{i*} = y^*$ .

Given that the set of feasible workdays is finite, the commodity space is finite dimensional and according to Stokey and Lucas (1989) the first and second welfare theorems hold. Therefore we can study the properties of the anonymous Pareto optima of this economy to establish properties of competitive equilibrium allocations. In this economy, with heterogeneous agents, individuals of the same type choose the same commodity point, although this does not imply that all of them work the same workday, since the chosen commodity point will involve randomizing over different workdays.

The social planner's problem, assuming equal weight for each type, is:

$$\begin{aligned} & \max \sum_i \lambda^i U^i(x^i) \\ \text{s.t.} \quad & : \quad x^i \in X^i(\bar{k}), \quad y \in Y, \quad \text{and} \quad \sum_i \lambda^i x^i = y \end{aligned}$$

As there is a finite number  $N_J$  of  $(k, s)$  pairs, the measure of workers in each type of plant  $n_j$  is the sum of different types of workers who supply work on the workday of this plant, and it is also the measure  $m_j$  of type  $j$  plant operated. So the Pareto problem can be rewritten as:

$$\max_{c^i, n_j^i} \sum_i \lambda^i \left\{ u(c) - \sum_j n_j^i v^i(s_j) \right\} \quad (11)$$

$$\text{s.t.} : \sum_i \lambda^i c^i \leq \sum_j f(k_j) g(s_j) n_j \quad (12)$$

$$\sum_{j: t_j \leq t < t_j + h_j} n_j k_j \leq \bar{k} \quad \text{all } t \in T \quad (13)$$

$$n_j \leq \sum_i \lambda^i n_j^i \quad \text{all } j \in J \quad (14)$$

$$\sum_j n_j^i \leq 1 \quad \text{all } i \in I \quad (15)$$

That is, the planner assigns workers of different types to plants with different workdays.

We divide the Pareto problem into two subproblems: one that is a linear program and one that is a nonlinear problem. Let the function  $V : R^{N_J \times N_I} \rightarrow R$  be defined as:

$$V(n) = - \sum_i \lambda_i \sum_j n_j^i v^i(s_j) \quad (16)$$

This function gives the disutility of work associated with the working plan  $n$ . Notice that  $V(n)$  is linear in  $n$ .

Let us suppose:

Assumption 1: the slope of the contours of the function  $V(n)$  does not equal the slope of any of the boundary faces of the set defined by the constraints (12)-(15) for any value given by  $\sum_i \lambda^i c^i$ .

Then we establish the following:

**PROPOSITION 1.** *The solution of maximizing (11) subject to (12)-(15) is unique and the number of pairs  $(k, s)$  receiving strictly positive mass is less than or equal to  $N_I + N_T + 1$ .*

*Proof*<sup>11</sup>. Given the division of the Pareto problem, and by substituting the constraint (14) in (12) and

<sup>11</sup>The reasoning followed in this proof is similar to that provided in Fitzgerald (1998).

(13) with equality, it is possible to consider:

$$W(C) = \max_{n \geq 0} V(n) \quad (11.a)$$

$$s.t. : C \leq \sum_j f(k_j) g(s_j) \sum_i \lambda^i n_j^i \quad (12.a)$$

$$\sum_{j: t_j \leq t < t_j + h_j} k_j \sum_i \lambda^i n_j^i \leq \bar{k} \quad \text{all } t \in T \quad (13.a)$$

$$\sum_j n_j^i \leq 1 \quad \text{all } i \in I \quad (14.a)$$

where  $C = \sum \lambda^i c^i$ . Let  $C_{\max}$  be the solution to  $\max_{n \geq 0} C$  subject to (12.a)-(14.a).  $W(C)$  is the smallest sum of the disutilities of working associated with producing  $C$  units of output, and  $C_{\max}$  is the maximum feasible value of  $C$  which can be produced. For  $0 \leq C \leq C_{\max}$  there is a solution which has at most a number of nonzero unknowns equal to the number of constraints, that is to say  $N_T + N_I + 1$ . The original problem can be rewritten as:

$$\begin{aligned} \max_{c^i \geq 0} \quad & \sum u(c^i) + W(C) \\ s.t. : \quad & \sum \lambda^i c^i \leq C_{\max} \end{aligned} \quad (17)$$

where it is straightforward that the solution is  $C_{\max}$  given the continuity and strict concavity of  $u(c)$  and the concavity of  $W$ . Associated with this unique value of  $C$  is the unique  $n$  that solves (11.a)-(14.a), which has at most  $N_T + N_I + 1$  nonzero elements.  $\square$

Proposition 1 states that the Pareto optimal allocation will assign positive values to most  $N_T + N_I + 1$  different  $n_j^i$ . This finite-linear program is solved by the simplex algorithm which searches for optimal basic solutions. The column vector associated to each  $n_j^i$  is  $[-f(k_j)g(s_j)\lambda^i, k_j\lambda^i, \dots, 1]'$ . The set of column vectors defined by the basic solutions must be linearly independent. Thus plants that do work can employ more than one type of worker. A basic solution is degenerated if the number of points with strictly positive mass is less than the number of constraints. The number of different plants that will start to run, the measure and the type of workers allocated to them depend on the coefficients both in the objective function and in the constraints, and on the parameter values in the constraints. That is to say, it is a question of  $k_j, f(k_j), g(s_j), v^i(s_j)$ , and  $\bar{k}$ . We can see this through the dual constraints or first-order conditions with respect to  $n_j^i$  of the linear program:

$$f(k_j)g(s_j)\lambda^i \rho_0 - k_j \left( \sum_{p=t_j}^{p=t_z-1} \rho_p \right) \lambda^i - v^i(s_j)\lambda^i - \mu_i \leq 0 \quad \forall n_j^i \quad (18)$$

where  $\rho_0$  is the Lagrange multiplier associated with the constraint (12); each  $\rho_p$  is the multiplier on the constraint of the capital corresponding to the starting time  $t_p$ , that is, the constraints denoted (13). So, the condition (18) corresponding to a type  $j$  plant, with starting time  $t_j$  and ending time  $t_z = t_j + h_j$ , includes the multipliers associated with the starting times from  $t_j$  to  $t_{z-1}$ , given that, at the moment  $t_z$  the capital allocated to this plant can be used for another plant. The multiplier  $\mu_i$  is associated with the labor supply constraint of the individual type  $i$ . That is,  $\mu_i$  is zero if type  $i$  workers are not all working in equilibrium. Equation (18) must hold with equality if  $n_j^i$  is strictly positive, that is, the output minus the cost of capital and labor must be equal to 0<sup>12</sup>. Just for each  $n_j^i > 0$  we denote:  $F(k_j, s_j) = f(k_j)g(s_j) \rho_0$ ,  $R_j = k_j(\sum_{p=t_j}^{p=t_z-1} \rho_p)$  and  $W^i(s_j) = v^i(s_j) + \mu_i/\lambda^i$ , so the plants and types that work fulfill the condition:

$$F(k_j, s_j) - R_j - W^i(s_j) = 0 \quad (19)$$

<sup>12</sup>From the individual maximization problem, equations (8), (9) and (10), it is deduced the reserve wage for type  $i$  works the workday  $s_j$ , and from the firm's problem (6)-(7) the condition of 0 profit for the plants that are going to operate is obtained. Substituting the reserve wage in this condition gives the condition in (18).

Therefore, the types of workers who are employed obtain a wage  $w^i(s_j)$  greater or at least equal to the reservation wages, that is  $v^i(s_j) + \mu_i/\lambda^i$ . (if there is unemployment in this type of workers then the reservation wage is only:  $v^i(s_j)$  as  $\mu$  equal to zero). Among the employed people, the value of employment less than or equal to one depends on the available amount of capital in relation to the capital of the plant.

The term  $F(k_j, s_j) - W^i(s_j)$  is the net output of the plant.

## 5 Analysis of flexibility in working schedules

### 5.1 The economy without a flexible workday

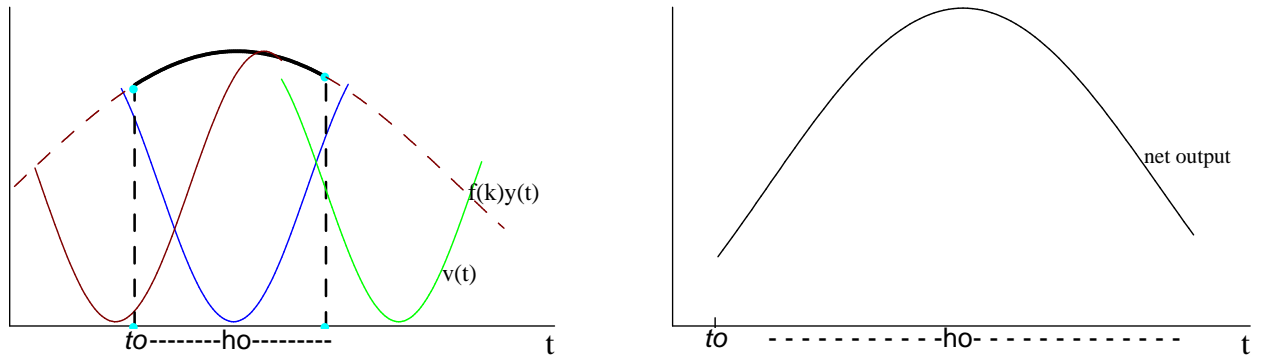
An economy without a flexible workday is characterized by a set of production plants  $J$ , which is finite, and with working schedules determined previously by the firm. As the set  $J$  contains more elements and more variety of workdays, it is easier for workers to find a plant with a workday that suits their preferences.

In equilibrium, worker types will be assigned to plants if they fulfil condition (19).

For illustrative purposes only, in Figure 2 we plot the instantaneous production function:  $f(k)\gamma(\tau)$ , and the function  $\vartheta(\tau)$  for three worker types<sup>13</sup> (type 1 with the red function, type 2 with the blue one). The set  $J$  is formed by one plant:  $(k, s_0)$ , where  $s_0 = (t_0, h_0)$ . The output per worker is the area under  $f(k)\gamma(\tau)$  and between the two dashed lines. The disutility of each worker is the area under his/her respective function  $\vartheta(\tau)$  and the two dashed lines. In equilibrium, type 2 workers will be assigned to the plant  $(k, s_0)$ , because of the cumulative disutility:  $\int_{t_0}^{t_0+h_0} \vartheta^i(\tau) d\tau$ , and therefore the reservation wage is lower for this type. If there is enough capital, then type 1 workers could be employed too (because the value of  $\mu$  would equal the reservation wages). But type 3 would hardly work in this plant.

Figure 2

- a) Instantaneous production function, three types of instantaneous disutility of work and the only workday  $(t_0, h_0)$ .  
 b) Resulting net output throughout the day



As the set  $J$  contains more elements, the workdays would be more appropriate and more people would work.

Figure 2 could represent the case of a standard workday and three worker types that prefer to work at nights, another during mornings and another during evenings.

We analyze the properties of the solutions of the problem when the set  $J$  is finite and contains some standard workdays. This is shown in the Appendix.

<sup>13</sup>The functions utilized in all the graphics have the form:

$$\vartheta^i(\tau) = a + b \cos(c\pi\tau - d)$$

$$\gamma(\tau) = p + q \sin(r\pi\tau - z)$$

and parameters  $a, b, c, d, p, q, r, z$  will be chosen to illustrate the main issues.

## 5.2 The economy with a flexible workday

If we choose progressively finer grids on  $J$  the solutions converge toward the solution when the set  $J$  is a rectangular subset on  $\mathbb{R}^3$ . In this way it is possible to determine which elements set  $J$  should contain to maximize production and employment.

We examine more closely the determination of work schedules that results from the interaction between individuals' preferences and plants' technology. Hence, it is necessary to specify the preferences about working time for each worker type and the shape of the function  $g(s)$ . It is also necessary to consider that the set of feasible workday  $S$  is no longer a finite set, but it is the convex set<sup>14</sup>:

$$\widehat{S} = \{(t, h) \mid t \in [0, 1], h \in [0, 1], t + h \leq 1, \text{ and } h = 0 \Rightarrow t = 0\}$$

Now set  $K$  is  $\mathbb{R}_+$ . Therefore, the set  $\widehat{J}$ , the set of all feasible production plants, is:  $\mathbb{R}_+ \times \widehat{S}$ .

From the necessary conditions given in (18) any plant that is operated in equilibrium and the type of worker in it must maximize the left hand side of (18) with respect to  $k$ ,  $t$ , and  $h$ .

For a given  $k$ , the optimal working hours that each type  $i$  should perform will be obtained from<sup>15</sup>:

$$\max_{(t, h) \in \widehat{S}} f(k)g(s) - v^i(s) \quad (20)$$

From the first order conditions and the definitions of the functions  $v$  and  $g$ , equations (1) and (2), the optimal moment to start,  $t$ , and the optimal moment to finish,  $t + h$ , for a worker  $i$  in a plant with  $k$  are respectively:

$$f(k)\gamma(t) = \vartheta^i(t) \quad (21)$$

$$f(k)\gamma(t + h) = \vartheta^i(t + h) \quad (22)$$

that is, the optimal moment to start is the instant at which the marginal utility of leisure coincides with the marginal productivity at that instant, and the same concerning the optimal moment to finish. The strict concavity of  $\gamma(\tau)$  and the strict convexity of  $\vartheta^i(\tau)$  ensure the fulfillment of the sufficient conditions,  $f(k)\gamma'(t) > \vartheta^{i'}(t)$  and  $f(k)\gamma'(t + h) < \vartheta^{i'}(t + h)$ , provided that  $t < \tau_i < t + h$ .

Thus, the components of the workday, for the types that meet the first and second order conditions, can be denoted as  $t(k, \tau_i)$  and  $h(k, \tau_i)$ . Now,  $g(s(t, h))$  is defined as  $g(k, \tau_i)$ .

It is easy to see that  $\frac{\partial t}{\partial k} < 0$  and  $\frac{\partial h}{\partial k} > 0$ , greater capital per worker ratio implies that the optimal workday starts earlier and finishes later.

**PROPOSITION 2:** *If the economy provides a flexible workday, the net output of each plant is maximized with the workday resulting from the interaction between workers preferences and technology.*

*Proof:* as demonstrated in the problem (20).

We obtain  $\{(k, s_1), (k, s_2), \dots, (k, s_{N_i})\}$ , that is a candidate to form the set  $J$ , and where each element contains the same ratio  $k$  and a workday depending on the preferences of each type. Comparing between types for a given  $k$ , types with greater  $f(k)g(s) - v^i(s)$  will have greater  $k\rho$  to bring the profit to 0, in condition (18), so as the  $\tau_i \rightarrow \widehat{\tau}$  the corresponding  $\rho$  is greater.

While with the inflexible workday work only one type, now from the conditions (21) and (22) could be an optimal workday for everyone and capital will be used for longer.

Then we analyze what the optimal capital-labor ratio is for each plant given the time in which the capital will be operational. It is important to distinguish whether the workdays of different plants overlap or not because this determines both the shape and the number of constraints on capital allocation in the original problem. That is, they have implications for the assignment of the capital between plants.

### A) The distribution of types determines that the workdays do not overlap

<sup>14</sup>In this case the commodity space is infinite dimensional, but the number of constraints remains finite. This is a semi-infinite linear program where an optimal measure assigns positive mass to no more points than there are constraints (Hornstein and Prescott, 1989).

<sup>15</sup>Given that  $\varrho_0$  is the same for all plants and dividing the left hand side of (18) by  $\lambda_i$ , the term to maximize is simplified in this way.

Assuming that the preferences are so different that when one type works then the other types do not, now the left hand side of condition in (18) that we denote as  $\Pi(k)$  is:

$$\Pi(k) = f(k)g(k, \tau_i)\rho_o - k\rho - v^i(k, \tau_i) - \mu_i/\lambda^i \quad (23)$$

The ratio  $k$  which is assigned to the plants that operate in equilibrium is such that it maximizes  $\Pi(k)$ . The first order condition is:

$$f'(k)g(k, \tau_i)\rho_o - \rho = 0 \quad (24)$$

given that from (21) we get:  $f(k)\gamma(t)\frac{\partial t}{\partial k} - \vartheta(t)\frac{\partial t}{\partial k} = 0$  and from (22):  $f(k)\gamma(t+h)\frac{\partial(t+h)}{\partial k} - \vartheta(t+h)\frac{\partial(t+h)}{\partial k} = 0$ .

The  $k$  ratio satisfying the condition is  $k_i^*$ , such that the marginal productivity in this plant is equal to the value imputed to the utilization of capital during this time, divided by  $\rho_o$ .

And the second order condition:

$$f''(k)g(k, \tau_i) + f'(k)\frac{\partial g}{\partial k} < 0 \quad (25)$$

states that for this ratio the decreasing effect on the marginal productivity is greater than the increasing effect on the effective working time.

We state the following proposition:

**PROPOSITION 3.** *If the technology and the worker preferences determine that the workdays do not overlap, the set of pairs  $(k, s)$  receiving strictly positive mass consists of plants whose respective ratios of capital per worker,  $k_i^*$ , are such that  $k_i^* > \bar{k}$ , and the employment within each type depends on  $\lambda^i, \bar{k}$ , and  $k_i^*$ .*

*Proof.* If the workdays do not overlap, the amount of capital utilized in each plant must satisfy only one constraint (13.a) and only for one worker type. That is:  $k\lambda^i n^i \leq \bar{k}$ .

Associated with this constraint there is one multiplier,  $\rho$ , corresponding to the utilization of the capital from the moment  $t$ . The condition in (24) must be satisfied for  $k_i^*$  such that  $k_i^*\lambda^i n^i = \bar{k}$ , because if the constraint (13.a) does not satisfy equity,  $\rho$  would be 0 (by the complementary slackness condition), and for  $k > 0$ , it is not possible that  $f'(k)g(k, \tau_i) = 0$ . Therefore, given that  $\lambda^i < 1$  and  $n^i \leq 1$ , necessarily  $k_i^* > \bar{k}$ .

The distribution of the workers by types ( $\lambda^i < 1$ ) and which are heterogeneous enough (workdays do not overlap) lead to fully capital utilization and full employment is possible for the types who work.

**PROPOSITION 4.** *Comparing between worker types, the ratio  $k_i^*$  depends on the distance  $|\hat{\tau} - \tau_i|$  as follows:*

*Shorter  $|\hat{\tau} - \tau_i|$  leads to a lower or equal ratio  $k_i^*$ . The ratio could be higher only when the decrease in the marginal productivity is compensated with the increase in the effective time as the type of worker is more appropriate.*

*Proof.* The value of  $g(k, \tau_i)$  increases when  $\tau_i \rightarrow \hat{\tau}$ . With a continuum of types the maximum  $g$  with respect to  $\tau_i$  and  $\gamma(\tau)$  implies that  $\gamma'(\tau) = v^i'(\tau) = 0$  which is reached when  $\hat{\tau} = \tau_i$ . From (23) and (24) as  $\tau_i$  approaches  $\hat{\tau}$  both  $g(k, \tau_i)$  and  $\rho$  increase. Thus, lower or equal  $k$  implies higher or equal  $f'(k)$  and the condition (24) is met. Only when comparing between types, for example type 1 and 2, when  $|\hat{\tau} - \tau_1| < |\hat{\tau} - \tau_2|$  and  $g(k, \tau_1) > g(k, \tau_2)$ , and the corresponding  $\rho$  is also higher for type 1 than for type 2, it is certain that with  $k_1^* > k_2^*$ :

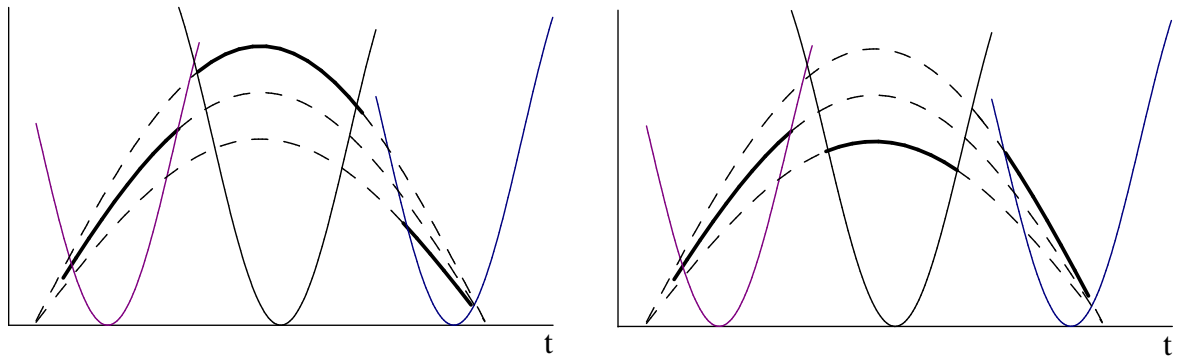
$$f'(k_1)g(k_1, \tau_1) > f'(k_2)g(k_2, \tau_2) \quad (26)$$

more appropriate types would use a higher ratio.  $\square$

Figure 3 shows the two cases: in a) the most remote types handle a lower ratio, and in the case b) the contrary. The instantaneous production appears (in bold) multiplied by  $f(k)$  corresponding to every type of workday. Remember that the distribution of capital also affects the length of the workday, so types of workers who manage more capital work more hours. To meet with the capital constraint, the planner assigns the employment such that  $n^i = \frac{\bar{k}}{\lambda^i k_i^*}$ . Thus, unemployment (which implies  $n^i < 1$ ), could exist in such type

of workers. In case a) types with lower measure and whose moment  $\tau$  is more distant with respect to  $\hat{\tau}$  work in plants whose capital ratio is lower, so the employment within this type of workers will be greater. In case b) the employment could be greater within types closer to  $\hat{\tau}$ .

Figure 3: Instantaneous production functions of three plants depending on  $k$  and disutility of three types of workers.



a)  $k$  decreases with  $|\hat{\tau} - \tau_i|$

b)  $k$  increases with  $|\hat{\tau} - \tau_i|$

### B) The distribution of types means that the workdays do overlap

Let us assume that after solving the conditions in (20) and (21) and the respective second order conditions, the optimal workdays of some types are overlapped. If set  $J$  contains plants whose workdays are overlapped and these plants work in equilibrium, the capital-labor ratio must satisfy the constraint (13.a) for each starting time. Assuming workdays of types  $i = 1, 2, 3$ , and so on, are overlapped such that:  $\dots t_1 < t_2 < (t_1 + h_1) < t_3 < (t_2 + h_2) < (t_3 + h_3) \dots$  now the conditions in (18) are for each type:

$$\begin{aligned} F(k_1, \tau_1) - k_1(\rho_1 + \rho_2) - W^1(k_1, \tau_1) &= 0 \\ F(k_2, \tau_2) - k_2(\rho_2 + \rho_3) - W^2(k_2, \tau_2) &= 0 \\ F(k_3, \tau_3) - k_3(\rho_3 + \rho_4) - W^3(k_3, \tau_3) &= 0 \end{aligned} \quad (27)$$

Therefore, if the capital stock is utilized by different types of workers simultaneously this is because the unit profit derived simultaneously using the capital must be the same:

$$\frac{F(k_1, \tau_1) - W^1(k_1, \tau_1)}{k_1} - \rho_1 = \frac{F(k_2, \tau_2) - W^2(k_2, \tau_2)}{k_2} - \rho_3; \frac{F(k_2, \tau_2) - W^2(k_2, \tau_2)}{k_2} - \rho_2 = \frac{F(k_3, \tau_3) - W^3(k_3, \tau_3)}{k_3} - \rho_4; \dots \quad (28)$$

**PROPOSITION 5.** *If the technology and the worker preferences determine that the workdays do overlap, the set of pairs  $(k, s)$  receiving strictly positive mass consists of plants whose respective ratios of capital per worker,  $k_i^*$ , are such that  $k_i^* \lesssim \bar{k}$  and depend negatively on the distance  $|\hat{\tau} - \tau_i|$ .*

*Proof.* The ratio  $k$  that maximizes  $\Pi$  in plants with overlapped workdays must satisfy the first order condition: :

$$\begin{aligned} f'(k_1)g(\tau_1, k_1) \rho_0 &= \rho_1 + \rho_2 \\ f'(k_2)g(\tau_2, k_2) \rho_0 &= \rho_2 + \rho_3 \\ &\dots \end{aligned} \quad (29)$$

(for plants overlapping two by two). The capital is assigned between different plants so that:

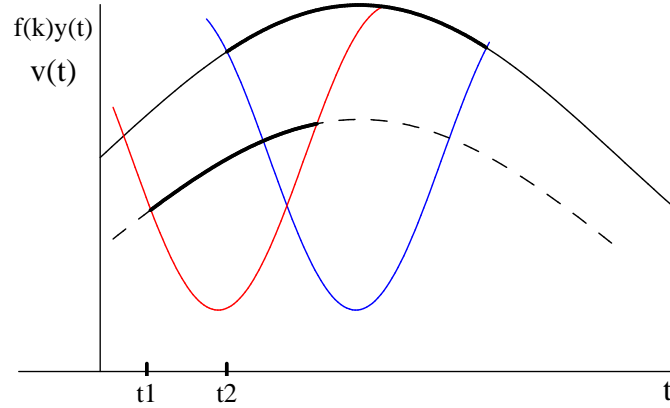
$$f'(k_1)g(\tau_1, k_1) \rho_0 - \rho_1 = f'(k_2)g(\tau_2, k_2) \rho_0 - \rho_3; \quad f'(k_3)g(\tau_3, k_3) \rho_0 - \rho_4 = f'(k_4)g(\tau_4, k_4) \rho_0 - \rho_5, \dots \quad (30)$$

If two types work simultaneously during part of their workday, the type with lesser distance  $|\hat{\tau} - \tau_i|$  reaches greater  $g(\tau_i, k_i)$  and utilizes larger  $k$ , which implies lower  $f'$ . So, both types will have the same imputed value.

As regards to the total available capital, fulfillment with constraint (13.a) leads to  $k_i^* \leq \bar{k}$  because what counts is the sum of the products of capital and employment of each plant.  $\square$

In Figure 4 we plot two types of workers and the instantaneous production function as in Figure 2. The workdays  $(t_1, h_1)$  and  $(t_2, h_2)$  are the optimal ones for each type of workers. The instantaneous production of type 2 is larger than type 1 because they utilize different capital-labor ratios.

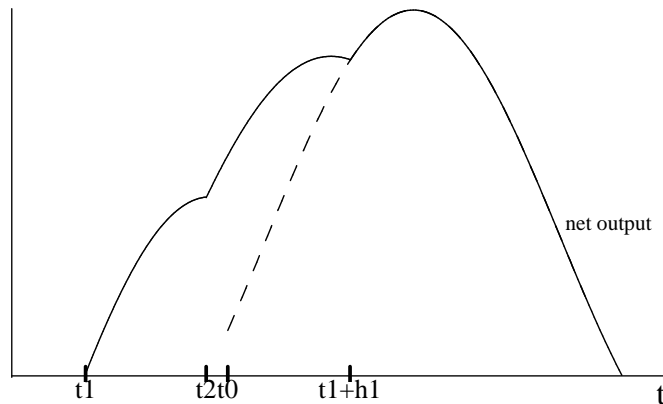
Figure 4: Overlapped workdays



Nevertheless, could also occur that only one plant meets the condition (27) and it is the only operating.

As we see, the proposed model is a framework in which to analyze variables we consider relevant to determine the flexibility of working hours. After analyzing possible findings the flexibility of the working time leads to the net output being maximized through the day. In Figure 5 we plot the instantaneous net output corresponding to the plants of Figure 4, that is the continuous line measuring the output of the first plant starting at  $t_1$ , followed by the sum of two plants when overlap at  $t_2$  and when the plant 1 has stopped at  $t_1 + h_1$ , the output of the second plant. In order to compare with the case of Figure 2 this is showed with a dashed line starting at  $t_0$ .

Figure 5: Net output with and without flexible workday





## 6 An extension: paid work at home versus work at the office

Now let's assume that it is possible to be working both at the home and at the plant of the firm. For example, certain sectors of activity that are characterized by being intensive in work and in which the requirements of the installed capital are limited.

Nevertheless, we suppose that in the plant or office the employees must share the existing capital, whereas at home the necessary capital is minimal and often owned by the worker. Obviously, the flexibility is easier to be applied in productive activities by fewer needs of fixed capital.

In order to distinguish between working at home and working at the plant we consider two sets of feasible workdays:  $S_1 = \{s_{11}, s_{12}, \dots, s_{1N}\}$  that contains workdays to be worked in the plant and  $S_2 = \{s_{21}, s_{22}, \dots, s_{2N}\}$ , with workdays at home. Now the working time in the plant is limited by legal or technical reasons such that the starting time is fixed at  $t^*$ , and workers can choose the number of hours to work in the plant (by assigning a measure to different lengths). Therefore the elements of the set  $S_1$  are  $s_{1j} = (t^*, h_{1j})$ . The set  $S_2$  contains several workdays to be worked at home. As equilibrium result, a worker may work only at the plant, or only at home, or partly at the plant and partly at home.

The commodity space  $L$  is  $R^2 \times M(S_1) \times M(S_2)$ . An element of  $L$  is given by  $(c, k, n_1, n_2)$ , where  $n_1$  is a measure over workdays at the plant and  $n_2$  is a measure over workdays at home.

With respect to the production possibility set this contains the production in plants plus the production at home. Let  $J_1 \subset K \times S_1$  be the set of feasible plants with generic element  $(k, s_1)$  and cardinality  $N_{J_1}$ . The output per worker of a type  $(k, s_1)$  plant is:  $f(k) g(s_1)$ . Also the set of feasible production at home is  $J_2 \subset \{\underline{k}\} \times S_2$  with generic element  $(\underline{k}, s_2)$  and cardinality  $N_{J_2}$ . We normalize  $f(\underline{k}) = 1$  and the output per worker of a type  $(\underline{k}, s_2)$  plant is:  $g'(s_2)$ . Now the function  $g'(s)$  sums up the value of an instantaneous index of productivity of working at home which is denoted by the function  $\gamma'(\tau)$ , different from the productivity at the plant,  $\gamma(\tau)$ . Then the production plan is a pair of measures  $m_1$  and  $m_2$ , which describes how the inputs are allocated across plants of different types. The production possibility set,  $Y$ , is defined as:

$$\begin{aligned}
 Y \equiv \{ \{C, K, N_1, N_2\} : \text{there exists a production plan } m_1 \in R_+^{N_{J_1}} \text{ and } m_2 \in R_+^{N_{J_2}} \text{ such that} \\
 C \leq \sum_{j_1} m_{1j} f(k_j) g(s_{1j}) + \sum_{j_2} m_{2j} g'(s_{2j}) \\
 \sum_{j_1} m_{1j} k_j \leq K, \\
 \sum_{\{j: h_j=h, t_j=t\}} m_{1j} \leq N(s_1), \quad \text{for each } s \in S^1 \\
 \sum_{\{j: h_j=h, t_j=t\}} m_{2j} \leq N(s_2), \quad \text{for each } s \in S^2 \}
 \end{aligned} \tag{31}$$

The possibility to work at home affects the individuals' preference ordering and their feasible consumption bundles. So the utility of a type  $i \in I$  person choosing the commodity point  $x = (c, k, n_1, n_2)$  is given by:

$$U^i(x) = u(c) - \sum_{s_1} n_1^i(s_1) v^i(s_1) - \sum_{s_2} n_2^i(s_2) v^i(s_2) \tag{32}$$

And the consumption possibility set of an agent type  $i$  is:

$$X^i(\bar{k}) = \left\{ (c, k, n) : k \leq \bar{k}, c \geq 0, k \geq 0, \sum_{s \in S_1 \cup S_2} n^i(s) = 1, n^i(s) \geq 0 \right\} \tag{33}$$

The definition of equilibrium is the same as in Section 4 and following the same steps we deduce the first-order conditions with respect to  $n_{1j}^i$  and  $n_{2j}^i$ , that are:

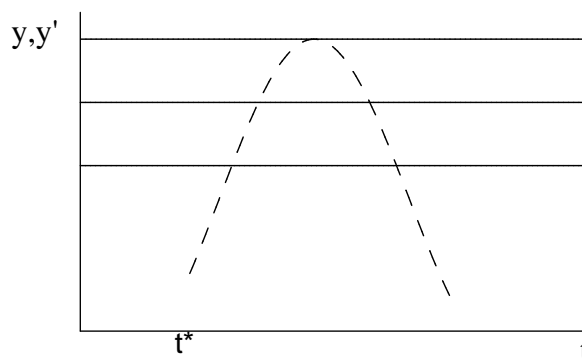
$$f(k_j) g(s_{1j}) \lambda^i \rho_0 - k_j \rho_k \lambda^i - v^i(s_{1j}) \lambda^i - \mu_i \leq 0 \quad \forall n_{1j}^i \tag{34}$$

$$g'(s_{2j}) \lambda^i \rho_0 - v^i(s_{2j}) \lambda^i - \mu_i \leq 0 \quad \forall n_{2j}^i \tag{35}$$

where  $\rho_0$  is the Lagrange multiplier associated with the output constraint.  $\rho_k$  is the multiplier on the constraint of the capital, that is the same for all the plant types  $s_1$  because all of them start at time  $t^*$ . The multiplier  $\mu_i$  is associated with the labor supply constraint of the individual type  $i$ , and he can supply work both at the plant or at home.

Following some empirical contributions, Gariety and Shaffer (2007), Eldridge and Pabilonia (2010), working at home is associated with significant wage differentials, positive overall, for both men and women. A positive productivity effect may stem from either the selective granting of working at home to more productive employees or a productivity-enhancing factor intrinsic to working at home, such as less time spent in unproductive activities or less fatigue associated with commuting. We adopt this hypothesis and consider that working at home does not need warming period, or fatigue as it accumulates at the plant, so the function  $\gamma'(\tau)$  that measures the instant productivity at home is constant. The question is at what level compared with the plant. Figure 6 shows several cases where the productivity at home (continuous line) is higher than in the plant during all the day or at least during some part of the day.

Figure 6



In equilibrium any plant that operates and/or anyone working at home must maximize the left hand side of (34) and (35) with respect to  $k$ ,  $h_1$ ,  $t_2$ , and  $h_2$ .

For a given  $k$ , the optimal working hours ( $t^*$  is given) that each type  $i$  should perform at the plant will satisfy :

$$f(k)\gamma(t^* + h_1) = \vartheta^i(t^* + h_1) \quad (36)$$

and at home:

$$\begin{aligned} \gamma'(t_2) &= \vartheta^i(t_2) \\ \gamma'(t_2 + h_2) &= \vartheta^i(t_2 + h_2) \end{aligned} \quad (37)$$

and the second order conditions, too. The results depend on the preferences of worker types and the productivity index.

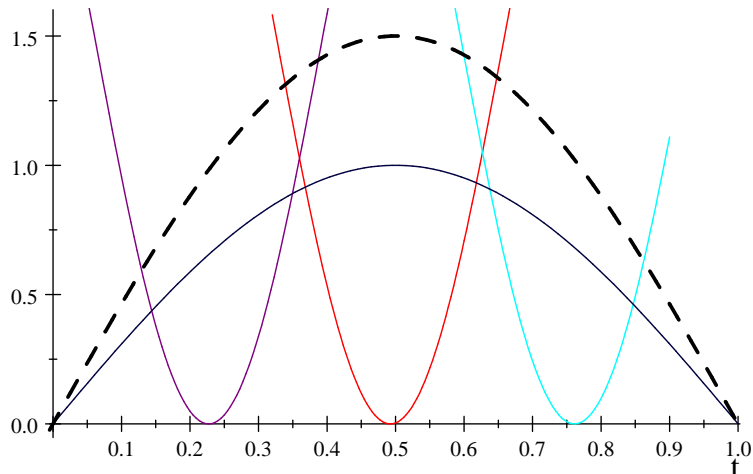
Therefore, the possibility of being employed at home allows to compensate the reduction of the productivity generated by the more or less fixed schedules imposed by the plant or the office.

## 7 Numerical example

Through an easy example we can show the results that the model predicts and the relevance of some key parameters. Assuming three types of workers, whose preferences as regards working time are given by the function:  $\vartheta(\tau) = a + b \cos(c\pi\tau - d)$ , the parameters  $c$  and  $d$  define the value of  $\tau_i$  of each type. So, the functions are:  $\vartheta^1(\tau) = 1.5 + 1.5 \cos(3\pi\tau + 1)$ ,  $\vartheta^2(\tau) = 1.5 + 1.5 \cos(3\pi\tau - 1.5)$ , and  $\vartheta^3(\tau) = 1.5 + 1.5 \cos(3\pi\tau + 2.25)$ , and the respective  $\tau_i$  are:  $\tau_1 = 0.23$ ,  $\tau_2 = 0.5$ , and  $\tau_3 = 0.77$ . That is, translating into 24h., the time of day that type 1 prefers working is 5:31, type 2: 12:00, and type 3: 18:28. With respect to the production plant, we assume the plant can operate throughout the day. This could be the case of continuous production. But

instantaneous production reaches the highest value at  $\hat{\tau} = 0.5$ . The function  $\gamma(\tau)$  can be described by the sine wave, specifically:  $\gamma(\tau) = \sin(\pi\tau)$ . The function  $f(k)$  is  $k^\alpha$ , where  $\alpha = 0.5$ . Therefore, the instantaneous production per worker is  $f(k)\gamma(\tau) = k^\alpha \sin(\pi\tau)$ . In Figure 7 we plot the functions  $\vartheta^i(\tau)$ , and the function:  $k^\alpha \sin(\pi\tau)$  for  $k = 1$  (the solid line), and for  $k = 2.25$  (the dashed line).

Figure 7



From the conditions (21) and (22), given  $k = 1$ , this gives:  $s_1 = (0.14, 0.2)$ ;  $s_2 = (0.36, 0.25)$ ;  $s_3 = (0.63, 0.21)$ , which means the workday for type 1 is from 3:21 to 8:09, for type 2: from 8:38 to 14:38, and for type 3, from: 15:07 to 20:09.

For  $\bar{k} = 1, \lambda_i = 1/3$ , the planner assigns  $k_i^* = 1$ , and  $n_i = 1$ . Total output is  $C = 0.53$ . Assuming  $\rho_0 = 1, \rho = 0.01$ , the respective wages would be:  $w^1 = 0.13, w^2 = 0.232, w^3 = 0.138$ .

Given  $k = 2.25$ , we get  $s_1 = (0.12, 0.26)$ ;  $s_2 = (0.33, 0.31)$ ;  $s_3 = (0.59, 0.27)$ . The workdays are partially overlapped. In this case, for  $\bar{k} = 1, \lambda_i = 1/3$ , the planner assigns  $k_i^* = 2.25$ , and  $n_1 = 0.18, n_2 = 0.21$  and  $n_3 = 0.18$ , and output is  $C = 0.2$ . It is obvious that if there is more available capital employment would be higher. For example with  $\bar{k} = 3$  the result is  $n_1 = 0.2, n_2 = 1$  and  $n_3 = 0.003$ , and output is  $C = 0.5$ .

## 8 Final Remarks

In this paper, we have addressed the analysis of flexible schedules. In this context, flexibility is understood as the capacity to match the firms' needs well but also the preferences of workers. Therefore, the starting point is heterogeneity both among firms and among workers. We develop a tractable general equilibrium model that delivers the work schedules as an equilibrium outcome between the firm's decision and workers' preferences. We propose to analyze heterogeneous workers in relation to their leisure preferences and firms that organize their production in plants or jobs with different working hours. Technological advances and changes in the organization of family and personal life are the encouragement, but not the only one, to promote new forms of arranging working time. Looking deeper into the theoretical foundations of the organization of working time can help achieve more rational and efficient results.

In our model, the work schedule results from the assignment of different types of workers to different plants. Depending on workers' preferences, availability of capital and technology, working times are scheduled. Equilibrium exists and it is unique. The result obtained is an efficient assignment and, after comparing with what we observe in reality, it enables guidelines to be established for the policy makers on how to act to improve the efficiency of the productive system.

It is deduced that if all the plants operate with the same technology, the distribution of worker types together with the amount of capital available determines the types of plants that operate in equilibrium.

Variations in productivity and the assessment of leisure throughout the day are key factors, and incorporating others variables we build the analysis of working hours. We have presented some innovations in this context. That is, the characterization of worker types which are different in terms of their time preferences, and also the definition of a workday, including the time to start work. Hence, the utility function and the production function defined on the workday.

We have presented a model broad enough to address some other related issues, such as working from home. The impact of labor policies: working time regulations, overtime taxation, policies for balancing work and family life, remain for future research. We have made restrictive, simplifying assumptions that should be reconsidered in the future.

## APPENDIX

### A. Characteristics of the solutions when set $J$ is finite

From the complementary-slackness relationship between primal and dual problems in linear programming, if  $n_j^i > 0$  condition (18) is satisfied with equality:  $f(k_j)g(s_j) \rho_0 - k_j(\sum_{p=t_j}^{p=t_j-1} \rho_p) = v^i(s_j) + \mu_i/\lambda^i$ . The term  $v^i(s_j) + \mu_i/\lambda^i$  is the supply reservation wage of the workday  $j$  for the worker  $i$  ( $v^i(s_j)$  measures the disutility of workday  $j$  and  $\mu_i$  is positive when the type  $i$  offered work any workday other than zero). The output of the plant  $j$ ,  $f(k_j)g(s_j)$  is weighted by the imputed value of every plant in the total output,  $\rho_0$  (which is the same for all plants).

Operating plants meet the condition:

$$F(k_j, s_j) - R_j - w^i(s_j) = 0$$

a) Two types of workers in the same plant require that at least, within one type, there is no unemployment. Two types of workers,  $a$  and  $b$ , in the same plant  $j$  imply the same wage, that is:

$$v^a(s_j) + \mu_a/\lambda^a = v^b(s_j) + \mu_b/\lambda^b$$

and given that necessarily  $v^i(s_j) \neq v^{ii}(s_j), \dots$  the worker with lower disutility compensates with greater  $\mu/\lambda$ .

b) One type in two different plants  $c$ , and  $d$  implies that:

$$F(k_c, s_c) - R_c - w^i(s_c) = F(k_d, s_d) - R_d - w^i(s_d)$$

c) Two types of plants,  $c$  and  $d$ , with different workers and whose workdays are partially overlapped:

$$\begin{aligned} f(k_c)g(s_c) \rho_0 - k_c(\rho_c + \rho_d) - w^i(s_c) &= 0 \\ f(k_d)g(s_d) \rho_0 - k_d(\rho_d + \rho_e) - w^{ii}(s_d) &= 0 \end{aligned}$$

that is, if the capital is fully utilized  $\rho_c > 0$ ,  $\rho_d > 0$ , and in the range where they overlap the profit per unit of capital must be equal for both.

$$\frac{f(k_c)g(s_c) \rho_0 - k_c \rho_c - w^i(s_c)}{k_c} = \frac{f(k_d)g(s_d) \rho_0 - k_d \rho_e - w^{ii}(s_d)}{k_d}$$

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