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An Efficient Integral Equation Technique for the Analysis of Arbitrarily Shaped Capacitive Waveguide Circuits

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***Abstract*— In this paper a new and efficient integral equation formulation is presented for the analysis of arbitrarily shaped capacitive waveguide devices. The technique benefits from the symmetry of the structure in order to reduce the dimensions of the problem from three to two dimensions. For the first time, this technique formulates the waveguide capacitive discontinuity problem as a 2D scattering problem with oblique incidence, combined with an efficient calculation of the parallel plate Green's functions. Results for a capacitive impedance transformer are successfully compared with measurements for validation of the proposed theory.**

I. INTRODUCTION

Capacitive rectangular waveguide structures are typically used for the design of low pass filters[1], impedance converters[2], and matching networks. In the past, several numerical techniques have been used for computing the electrical response of this kind of devices, such as the finite elements method employed by widely used commercial software packages like HFSS©, modal analysis techniques [3], or the boundary integral resonant mode expansion (BI-RME) [4]. Although general purpose finite elements codes can deal properly with E-Plane (capacitive) structures, their main drawback is the lack of efficiency, since one has to setup a three dimensional model of the circuits. On the other hand, modal analysis [3] are very efficient for the study of E-plane devices composed of canonical rectangular waveguide sections, but can not handle easily complex geometries such as rounded corners introduced during mechanical manufacturing procedures, or other useful elements such as conducting rounded posts. This limitation is due to the need to compute the modal chart of the basic waveguide sections defining the whole structure. This drawback has been overcome by other techniques allowing the efficient evaluation of modal charts in complex geometries, such as the BI-RME method. Alternatively, the analysis of E-plane arbitrarily shaped microwave components can also be performed by computing the admittance matrix of the structure with the 2D BI-RME technique, as presented in [4]. The technique is based on solving an eigenvalue problem where the whole structure is surrounded by an auxiliary rectangular cavity resonator, and the input and output ports are short circuited. The dynamic variation of the fields inside the structure are expanded in terms of the resonant modes of the surrounding cavity used as a reference. Once the eigenvalue problem is solved, the broadband admittance parameters of the structure are obtained opening the input and output ports.

This paper presents a new technique, based on the integral equation method, for the analysis of E-plane discontinuities in rectangular waveguides. As a difference with respect to modal approaches is that there is no need to compute the modal chart of the different waveguide sections. The proposed integral equation only needs the parallel plate Green's functions of line sources, efficiently computed in [5]. Another advantage of the integral equation formulated in this paper, is that the scattering parameters of the devices are directly computed, and it can be easily extended to homogeneous or in-homogeneous dielectric objects. The main advantage with respect to other numerical techniques such as Finite Elements, is that the formulation exploits the E-plane geometry to reduce the computational effort to a 2D structure, thus being very efficient. This is the first time, to the authors' knowledge, that capacitive discontinuities inside waveguides are reduced to a simple 2D scattering problem of oblique incidence, solved efficiently through integral equation techniques.

This contribution presents the basic formulation and the theory underlying the developed integral equation. The technique is demonstrated in an E-plane impedance transformer, including measurements for the validation. A structure composed of three capacitive rounded posts is also analyzed, to show the ability to treat very complex geometries. The results obtained show the accuracy and efficiency of the new integral equation technique applied to E-plane discontinuities inside rectangular waveguides.

II. THEORY

The formulation of the proposed method is setup considering a rectangular waveguide where all the discontinuities are invariant along the x-axis, as shown in Fig. 1. In this figure, the relevant dimensions (width a and height b) of the waveguide are indicated, together with the orientation of the coordinate axes.

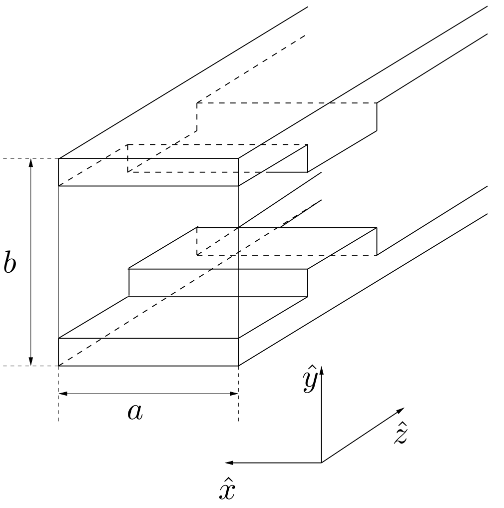

Fig. 1. A capacitive step inside a rectangular waveguide of width a and height b. The coordinate axis used in this paper are also shown.

The original structure shown in Fig. 1 is replaced after the application of the surface equivalent principle [6] by induced electric current densities on the boundaries of the conducting discontinuities. In this way, the rectangular waveguide is reduced to a parallel plate waveguide due to the invariant geometry of the problem along the x-axis. After these considerations, an electric field integral equation (EFIE) [7] is employed for computing the response of the equivalent two-dimensional problem shown in Fig. 2.

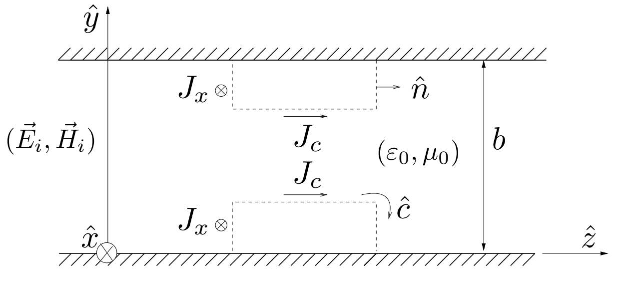

Fig. 2. Equivalent parallel plate waveguide problem considered for the evaluation of the response of the device. The constitutive parameters of the medium inside the waveguide are the same as those corresponding to vacuum (ε_0, μ_0). The electromagnetic fields exciting the structure $(\vec{E}_i, \vec{H}_i)$ correspond to the fundamental mode of the rectangular waveguide TE_{10}^z; $(\hat{c})$ is a unit vector tangent to the contour, whereas $(\hat{n})$ is the normal unit vector.

The excitation of the problem $(\vec{E}_i, \vec{H}_i)$ is the dominant mode of the rectangular waveguide TE_{10}^z. Although the geometry of the capacitive device is invariant along the x-axis, this fundamental mode exhibits a half period trigonometric variation of sine-type along the waveguide width, as can be observed from its basic expression:

$$\vec{E}_i = -\frac{A_{10}}{\varepsilon_0}\frac{\pi}{a}\sin\left(\frac{\pi}{a}x\right)e^{-jk_z z}\hat{y} \tag{1}$$

where k_z is the propagation constant along the waveguide z-axis, and A_{10} is an arbitrary amplitude constant [6]. This variation of the exciting field prevents to formulate the problem as a simple scattering problem of normal incidence contained in the (y, z)-plane, [5], [8], [9], [10].

Fortunately, the previous variation of the field along the x-axis is known and does not change due to the symmetry of the problem. After applying the Euler's formula to Eq. (1), the incident electric field can be considered as that produced by two plane waves with an oblique incident angle with respect to the x-axis ($\vec{k}_1 = -\frac{\pi}{a}\hat{x} + k_z\hat{z}$ and $\vec{k}_2 = \frac{\pi}{a}\hat{x} + k_z\hat{z}$):

$$\vec{E}_i = -\frac{A_{10}}{\varepsilon_0}\frac{\pi}{a}\left(\frac{e^{j\frac{\pi}{a}x} - e^{-j\frac{\pi}{a}x}}{2j}\right)e^{-jk_z z}\hat{y} \tag{2}$$

This decomposition of the exciting field in two plane waves propagating at an oblique angle with respect to the x-axis, allows to formulate the problem as a 2D scattering problem of oblique incidence [7]. Following this theory, the electromagnetic fields scattered by the conducting capacitive posts inside the rectangular waveguide can be computed as the sum of the contributions of the two previous plane waves. A very efficient way for solving this problem is based on writing the EFIE in the spatial frequency domain (spectral variable $k_x = \pm\frac{\pi}{a}$), resulting:

$$\hat{n} \times \left[\tilde{\vec{E}}^i + \tilde{\vec{E}}^s\left(\tilde{\vec{J}}(c')\right)\right] = 0 \tag{3a}$$

$$\begin{aligned}\tilde{\vec{E}}^i(c)\Big|_{\tan} &= j\omega\int_{c'}\bar{\bar{G}}_A(\vec{\rho}, \vec{\rho}\,')\left[\tilde{J}_x(c')\hat{x}' + \tilde{J}_c(c')\hat{c}'\right]dc' + \\ &\left(jk_x\hat{x} + \frac{d}{dc}\hat{c}\right)\int_{c'}\frac{\left[jk_x\tilde{J}_x(c') + \frac{d\tilde{J}_c(c')}{dc'}\right]}{-j\omega}G_V(\vec{\rho}, \vec{\rho}\,')\,dc'\Big|_{\tan}\end{aligned} \tag{3b}$$

$$\vec{\rho} = y\hat{y} + z\hat{z} \quad ; \quad \tilde{\nabla} = \left(jk_x\hat{x} + \frac{d}{dc}\hat{c}\right) \tag{3c}$$

The previous expression (3b) comes from a mixed-potential representation of the electric field. In this equation all the relevant magnitudes are written in a mixed spatial-spectral domain (spatial variables of the contour (c) in the (y, z)-plane and the spectral variable k_x). Another important difference with respect to an inductive problem [5], [8], [10], is that the unknown induced electric current $\vec{J}(c') = \tilde{J}_x(c')\hat{x} + \tilde{J}_c(c')\hat{c}'$ presents two components, one along the longitudinal x-axis, and the other along the contour (c) of the capacitive problem. On the other hand, $\bar{\bar{G}}_A(\vec{\rho}, \vec{\rho}\,')$ is a diagonal dyadic Green's function corresponding to the magnetic vector potential, whereas $G_V(\vec{\rho}, \vec{\rho}\,')$ is the electric scalar potential Green's function. The mathematical forms of these Green's functions for the geometry under consideration are described in the next subsection.

TABLE I

GREEN'S FUNCTIONS COMPONENTS NEEDED FOR SOLVING THE CAPACITIVE EQUIVALENT PROBLEM.

-	ξ	f_n	g_n
G_A^{xx}	μ_0	$\sin(k_y\, y)$	$\sin(k_y\, y')$
G_A^{yy}	μ_0	$\cos(k_y\, y)$	$\cos(k_y\, y')$
G_A^{zz}	μ_0	$\sin(k_y\, y)$	$\sin(k_y\, y')$
G_V	$1/\varepsilon_0$	$\sin(k_y\, y)$	$\sin(k_y\, y')$

Finally, the integral equation (3) has been solved expanding the unknown electric current with triangular subsectional basis functions, and using the same set of functions for testing. Once the unknown current has been found, the scattering parameters can directly be computed by evaluating the ratio between the incident and the scattered fields on the ports of the device, as described in [11]. Although the excitation of the capacitive problem is split into two different plane waves, it is only necessary to solve the previous integral equation for one of them, since the results are mathematically related.

A. Parallel Plate Green's Functions

In this section, the parallel plate Green's functions used for solving the capacitive equivalent problem are summarized. The general expression of these Green's functions in their spectral form is given by:

$$G_{ppw}(z-z',y,y') = \frac{h_n \xi}{b\pi} \sum_{n=0}^{\infty} f_n(k_y y') g_n(k_y y) \frac{e^{-jk_z(z-z')}}{jk_z} \tag{4a}$$

$$k_z = \sqrt{k_0^2 - k_x^2 - k_y^2} \quad ; \quad k_y = \frac{n\pi}{b} \quad ; \quad k_x = \frac{\pi}{a} \tag{4b}$$

where $h_n = \begin{cases} 1 & n = 0 \\ 2 & n \neq 0 \end{cases}$, f_n and g_n are trigonometric functions, and ξ is a constant depending on the constitutive parameters of the rectangular waveguide. It is interesting to observe the definition of the longitudinal wavenumber (k_z), which is modified with respect to the inductive case with the fixed oblique incident angle ($k_x = \pi/a$) used in this formulation. All the relevant definitions for the Green's functions needed in order to solve the capacitive equivalent problem are given in Tab. I.

It is worth stressing the low convergent performance of the kind of series represented in Eq. (4) when the observation point along the propagation z-axis is very close to the source at z'. For these situations, one has to employ summation acceleration techniques like the Kummer's method [5], [11] for an efficient implementation of the integral equation technique.

III. RESULTS

The usefulness of the new proposed technique has been proved by computing the response of a practical E-plane waveguide transformer shown in Fig. 3. This device has been used as a benchmark for testing the multipactor phenomena in space communication applications [12].

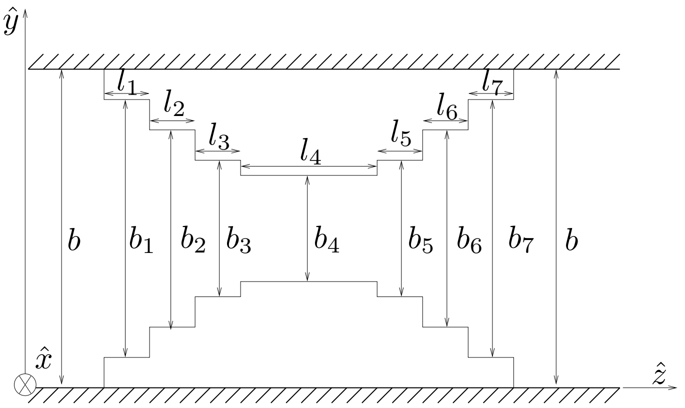

Fig. 3. E-Plane WR-75 waveguide ($a = 19.05$ mm, $b = 9.525$ mm) transformer. The dimensions of the device expressed in millimeters are: $l_1 = l_7 = 8.11$, $l_2 = l_6 = 8.03$, $l_3 = l_5 = 8.24$, $l_4 = 40.04$, $b_1 = b_7 = 5.46$, $b_2 = b_6 = 1.44$, $b_3 = b_5 = 0.44$, $b_4 = 0.32$.

The results provided by the new numerical technique compared to those given by the commercial software FEST3D© [13] and measurements show a very good agreement, as can be observed in Fig. 4 (measured results are shown in the single mode frequency band 10-15 GHz). The maximum number of modes used for computing the Green's functions presented in Section II-A is 30, whereas 500 basis functions have been enough for characterizing the unknown electric current yielding to an accurate response up to 30 GHz. The simulation time is less than 0.1 seconds per frequency point in a 64 bits computer with a 2.0 GHz clock.

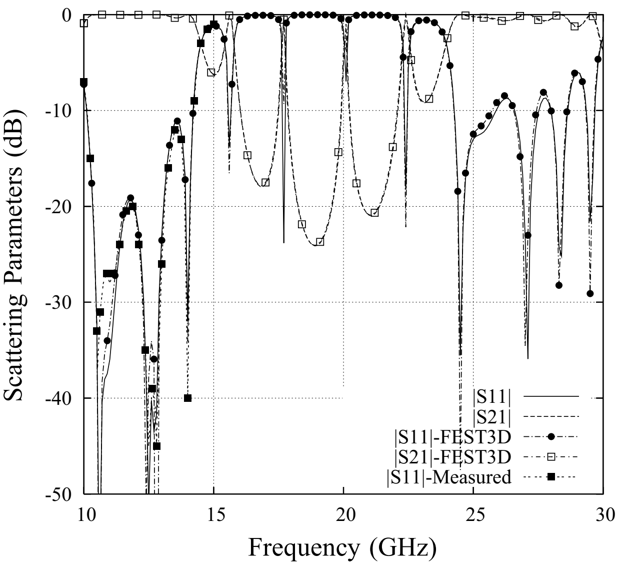

Fig. 4. Scattering parameters of the E-Plane converter shown in Fig. 3. The simulated results obtained with the new integral equation technique proposed in this paper are compared to the data provided by the commercial software tool FEST3D© [13] and to measurements (in the range 10-15 GHz).

To show the ability of the new technique to treat complex geometries, a second example of a capacitive structure composed of three conducting circular posts inside the same

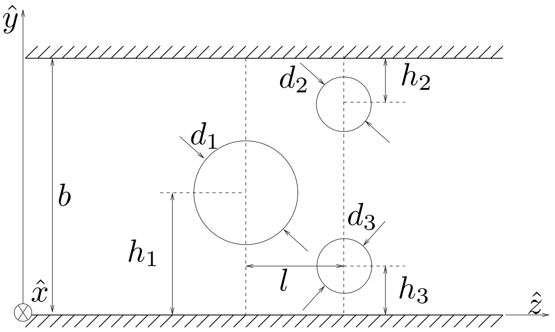

Fig. 5. Three circular capacitive conducting posts inside a WR-75 waveguide ($a = 19.05$ mm, $b = 9.525$ mm). The dimensions of the structure are: $h_1 = b/2$, $h_2 = h_3 = b/8$, $d_1 = b/4$, $d_2 = d_3 = b/8$ and $l = b/8$.

reference waveguide has been computed (see Fig. 5 for details). In this case, 200 basis functions have been employed for expanding the surface electric current induced on the posts. On the other hand, the maximum number of terms for evaluating the parallel plate Green's Functions is again 30. The simulated results obtained with the new technique have also been compared to the data provided by FEST3D[13], showing again a very good agreement, as indicated in Fig. 6. As an important difference with respect to the previous simulation example, part of the energy of the fundamental exciting mode is coupled to the TE^z_{12} mode after its cut-off frequency at 32.46 GHz, as can be seen in Fig. 6. Hence, this result also validates the accuracy of the new numerical technique for computing the out of band response of capacitive structures under multimode propagation. The simulation time has been of 0.03 seconds per frequency point in the same computer as before, demonstrating the efficiency of the new method.

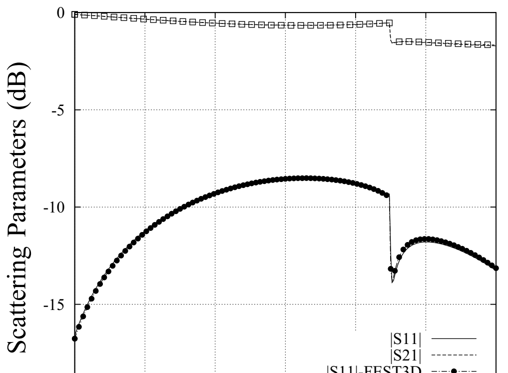

Fig. 6. Scattering parameters of the capacitive structure composed of three conducting posts shown in Fig. 5. The results computed by the new integral equation technique has been compared to those provided by the commercial software tool FEST3D©[13].

IV. CONCLUSIONS

A new integral equation technique has been presented for the analysis of arbitrarily shaped capacitive microwave waveguide circuits. For the first time, the scattering parameters of these kind of devices have been computed by formulating a 2D scattering problem with oblique incident angle. The boundary conditions of the original waveguide have been taken into account through the use of the parallel plate Green's functions. The technique can be easily extended for the treatment of dielectric posts by means of the application of the surface equivalence principle. Results are validated with measurements on a capacitive impedance transformer. The technique has shown its ability to treat complex geometries with the characterization of a waveguide structure containing three rounded capacitive posts. Simulation results compared to data from commercial software tools have shown the validity and accuracy of the new method, even for evaluating the out of band response under multimode operation.

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