

1 Filter design for folded canonical topologies based on  
2 equivalent circuit segmentation

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11 **Abstract**

This paper presents a methodology to design filters with folded canonical topologies, which implement cross couplings between non-adjacent resonators. The technique is based on segmenting the traditional coupling matrix in a step-by-step fashion. At each step, a subset of the whole physical structure is optimized to match the response of the corresponding segment of the coupling matrix. In the context of this design technique, in this paper we propose an efficient segmentation methodology of the coupling matrix based on multiport networks. The use of multiport networks allows to generate at each step several goal functions, which are simultaneously used during the optimization of the corresponding physical segment. These multiport networks allow to efficiently monitor the different paths of the signal, present in folded canonical topologies. It is shown that this strategy leads to a fast convergence of the step-by-step segmentation technique for the design of this type of coupling topologies. We apply the proposed methodology to the design of two filters using the quartet topology. The first filter has two transmission zeros placed at the real frequency axis, and the second one has two complex transmission zeros intended for group delay equalization. The results indicate that the proposed methodology is effective for the design of this type of coupling topologies, leading to initial dimensions for the filters that typically have less than 1% of error when they are compared with those obtained from a final global optimization.

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12 *Keywords:* Bandpass filters, Coupling Matrix, Optimization techniques,  
13 Design techniques, Rectangular waveguide filters

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## 14 **1. Introduction**

15 The design of microwave filters has traditionally been considered to be  
16 a very time consuming activity, mainly because full wave complex compu-  
17 tational software is usually needed in order to perform a large number of  
18 simulations. These simulations have high computational costs, and there-  
19 fore it is necessary to use efficient design techniques to minimize the time  
20 that is needed for the full production of this type of hardware. Although  
21 some techniques are available to obtain initial dimensions of filters [1, 2],  
22 they usually require a final optimization of the whole structure, until the  
23 required response is finally obtained. However, for complex filter topologies,  
24 optimization techniques may find difficulties in convergence, specially if the  
25 initial designs are far from the desired target specifications.

26 The classical technique presented in [3] models coupling elements and cav-  
27 ities separately. However, after assembling the filter, undesired interactions  
28 between resonators and coupling elements are usually present. Therefore,  
29 final global optimizations are required. The success of these optimization  
30 processes will mainly depend on the quality of the achieved initial design.  
31 This is specially true for optimization techniques based on gradient algo-  
32 rithms, which traditionally need initial dimensions of the filter very close to  
33 the optimum solution. Otherwise, they are easilly trapped into local minima,  
34 thus making the convergence to optimum solutions very problematic.

35 Another possibility is to use Genetic Algorithms (GA) for microwave  
36 filters design [4, 5, 6, 7]. They are based on selection mechanisms, which  
37 avoid the problem of local minima that is inherent to gradient algorithms.  
38 However, the high number of iterations required by these algorithms leads  
39 to a significant computational cost. Space mapping optimization techniques  
40 are also useful options for efficient filter design tasks [8, 9, 10, 11]. This  
41 kind of techniques connects two models, coarse and fine, and sometimes it is  
42 difficult to find a good mapping between them. This is especially true if a  
43 strong mis-alignment is present between the fine and coarse models. These  
44 techniques, therefore, rely in how good the coarse models are with respect to  
45 the corresponding fine models.

46 There are some alternative design techniques that are based on deriving  
47 equivalent circuits of the filter. For instance, in [12] a waveguide iris width

48 is computed to match the response of a circuit prototype in magnitude and  
49 phase. Even though it shows good convergence, it does not consider any of  
50 the couplings and interactions from nearby elements. Thus, after assembling  
51 all parts of the filter, the response usually presents significant deviations with  
52 respect to the target response. Also, authors in [13] use an equivalent circuit  
53 to design a dual-mode circular waveguide filter in quartet topology, with one  
54 cross coupling. The technique first adjusts individually the coupling irises  
55 and the resonators of the structure along the main path. As a final step, it  
56 proposes the adjustment of the iris along the cross coupling path with the  
57 help of an equivalent circuit. The technique results to be very accurate, since  
58 the final adjustment of the cross coupling is done with an equivalent circuit  
59 that takes into account the influence of nearby elements. However, it would  
60 be difficult to extend the proposed technique to higher order networks, or to  
61 topologies that include additional cross couplings.

62 A step by step procedure, presented in [14, 15], shows a different tech-  
63 nique to design waveguide filters. This procedure consists in obtaining an  
64 equivalent circuit that represents the complete microwave filter with the de-  
65 sired specifications. At each step, the circuit is segmented as needed. For  
66 inline filters, the process begins dimensioning the input coupling and first  
67 resonator, and continues adding coupling structures and resonators one by  
68 one. At each step, the physical sub-structure is optimized to match the re-  
69 sponse of the corresponding segment of the equivalent circuit. A reduced  
70 number of elements is optimized at each step (only two elements, namely  
71 the last coupling and resonator added to the structure). This technique has  
72 also been used, more recently, for the design of waveguide dual-mode filters  
73 in all inductive technology [16]. This is a complex topology that introduces  
74 cross-couplings between the two resonators hosted by one cavity with two  
75 other resonators placed in adjacent cavities.

76 In this paper, a similar strategy is employed for the design of complex  
77 filters using folded canonical coupling topologies. The main characteristic of  
78 these topologies is that they employ cross-couplings between non-adjacent  
79 resonators to implement transmission zeros in the filter response. Due to the  
80 introduction of these cross-couplings, the segmentation technique presented  
81 in [14, 15, 16] cannot be directly applied to this type of structures.

82 In this context, this paper extends for the first time the application of  
83 the segmentation technique proposed in [14, 15] to the design of microwave  
84 filters with folded canonical topologies. For this purpose, a new and efficient  
85 segmentation strategy for the equivalent circuit, including the cross-couplings

86 that are present in the topology, is proposed. The segmentation strategy  
87 is based on considering the cross-coupling paths at different stages during  
88 the design process. Multiport goal functions are for the first time proposed  
89 in connection to the segmentation design technique. The multiport goal  
90 functions are needed to monitor the signal along the main path, and along  
91 the cross-coupling paths, simultaneously. We want to emphasize here that a  
92 proper segmentation strategy is crucial to obtain an accurate initial design of  
93 the filter, in order to minimize subsequent optimization operations. This is  
94 because it is very important to take into account the influence of key elements  
95 of the structure, during the optimization of new nearby elements.

96 First, the theory for scaling the coupling matrix from the normalized  
97 coupling matrix using the physical parameters of a particular filter imple-  
98 mentation, will be reviewed. Since the theory for scaling the coupling matrix  
99 was presented before in [16], only a brief review will be presented in this  
100 work, together with final expressions needed for its practical implementa-  
101 tion. Although in [16] the work was developed using series resonators, the  
102 final expressions in this paper will be particularized for the case of parallel  
103 resonators, which are more appropriate to represent folded canonical topolo-  
104 gies. The novel strategy proposed for the segmentation of the equivalent  
105 circuit will then be illustrated with the design of a quartet topology. This  
106 includes how to employ multiport goal functions to take into account for  
107 cross couplings, to optimize the physical dimensions of the different seg-  
108 ments. Contrary to [13], the extension of the proposed strategy to the design  
109 of higher order folded canonical topologies, including those with additional  
110 cross couplings, is straightforward. The derived technique has been used for  
111 the first time for the design of two filters using the quartet topology. The first  
112 quartet filter exhibits transmission zeros in the real frequency axis, while the  
113 second is designed for complex transmission zeros, intended for group de-  
114 lay equalization. Results for both filters show that the new segmentation  
115 strategy is very efficient, and can be used for the fast design of this type of  
116 complex filter topologies. In fact, it is demonstrated in this paper that the  
117 initial designs obtained with the proposed technique are very close to fulfill  
118 with the desired target specifications.

## 119 **2. Theory**

120 The first step to apply the proposed technique is to find an equivalent  
121 circuit of the filter to be designed. Next, the equivalent circuit will be seg-

122 mented in several sub-circuits. The segments of the equivalent circuit have  
 123 electrical responses which must be similar to corresponding segments of the  
 124 physical structure. Therefore, the responses obtained from the sub-circuits  
 125 are used as goal functions to optimize the corresponding segments of the  
 126 physical structure. The segments are added one by one sequentially, until  
 127 the dimensions of the whole structure are found. This will produce initial  
 128 dimensions for the filter, with a response as close as possible to the desired  
 129 target specifications. If refinement is needed, a final global optimization can  
 130 be applied to the whole structure. This final optimization process can use,  
 131 in general, efficient gradient algorithms, provided that the starting point  
 132 performs closely enough to the target specifications.

133 To find the equivalent circuit of the filter, a scaled coupling matrix is  
 134 used. This scaled coupling matrix is obtained from the normalized coupling  
 135 matrix [3], using electrical parameters relevant to the technology that is going  
 136 to be employed during the filter implementation. In this work, waveguide  
 137 technology is used for the physical implementation, so the theory will be  
 138 illustrated as applied to rectangular waveguide resonators of cross section  
 139  $(a, b)$ .

140 The theory for the scaled coupling matrix was presented in [16]. However,  
 141 in [16] equivalent circuits based on series resonator and impedance inverter  
 142 were used. On the contrary, in this work, we use a dual equivalent circuit  
 143 based on parallel resonators and admittance inverters. We found that for  
 144 folded canonical topologies, containing cross-coupling paths, the representa-  
 145 tion with this dual version of the equivalent circuit is more convenient. This  
 146 is because it leads to more straightforward representations of the equivalent  
 147 circuit when the segments contain several coupling paths to account for cross-  
 148 couplings. In this section, the most relevant equations will be reviewed, for  
 149 this dual version of the equivalent network.

150 The coupling matrix is a circuit representation of a normalized lowpass  
 151 prototype network, as shown in Fig. 1(a). This figure shows a circuit com-  
 152 posed of parallel capacitors, whose values are normalized to 1 F, and also  
 153 contains constant susceptances of values  $(j B_m)$ . Couplings between the el-  
 154 ements of the network are characterized with admittance inverters of values  
 155  $(M_{mn})$ . As shown, the input/output admittance terminations are also nor-  
 156 malized to  $1 \Omega^{-1}$ . The coupling matrix represents this normalized network,  
 157 containing the constant susceptance values in the diagonal elements of the  
 158 matrix  $(M_{mm} = B_m)$ , and the admittance inverters  $(M_{mn})$  in the off-diagonal  
 159 positions of the matrix [3]. From this normalized prototype, a scaled version

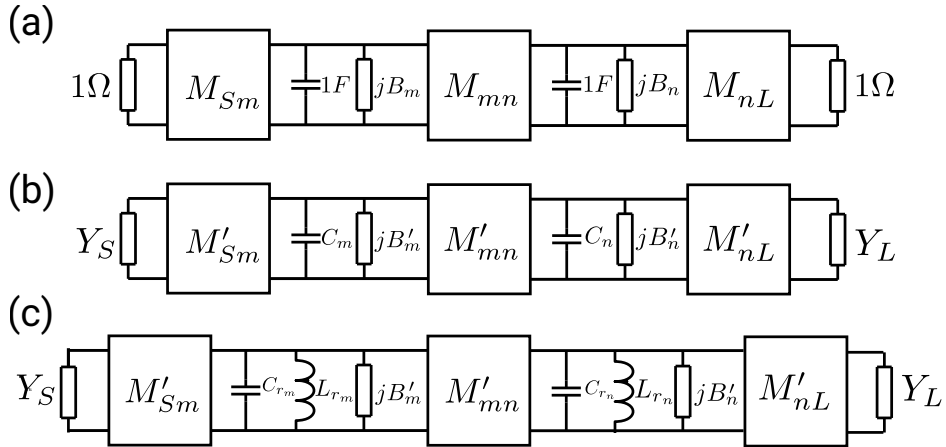


Figure 1: (a) Equivalent circuit of a normalized lowpass prototype network composed of parallel capacitors and constant susceptances, coupled by admittance inverters. (b) Lowpass filter with arbitrary input/output admittances, and scaled capacitors and constant susceptances. Admittance inverters are conveniently scaled to keep the same response as the original network. (c) Equivalent circuit of a bandpass filter obtained from (b) after applying a standard lowpass to bandpass transformation.

160 of the equivalent circuit can be obtained, as shown in Fig. 1(b). The input/161  
 162 output admittance terminations now have arbitrary values ( $Y_S, Y_L$ ). In the same way, constant susceptances ( $j B'_m$ ) and capacitors are scaled with 163  
 164 arbitrary constants ( $C_m$ ). It is well known that the response of this network does not change if the admittance inverters are also scaled to convenient 165  
 166 values ( $M'_{mn}$ ) [1]. Finally, applying a standard lowpass to bandpass transformation, each capacitor is transformed into a parallel resonator ( $C_{r_m}, L_{r_m}$ ), 167  
 168 as shown in Fig. 1(c). This last network represents the equivalent circuit of the bandpass filter that is going to be designed.

169 The key step in the described process, is to find a suitable scaling factor 170  
 171 ( $C_m$ ) to correctly represent the particular resonator that will be used in the physical implementation of the filter. To do this, we use the equivalence 172  
 173 shown in Fig. 2. Fig. 2(a) shows a half wavelength ( $\lambda_g/2$ ) open-ended transmission line resonator. In our case, this transmission line represents the resonant mode ( $TE_{101}$ ) in the rectangular waveguide resonator to be used 174  
 175 in the physical implementation of the filter. The physical resonator represented in this form will be made equivalent to the parallel lumped resonator 176  
 177 shown in Fig. 2(b). For both circuits to be equivalent, two conditions must be imposed. First, the resonant frequencies of both resonators must be the 178

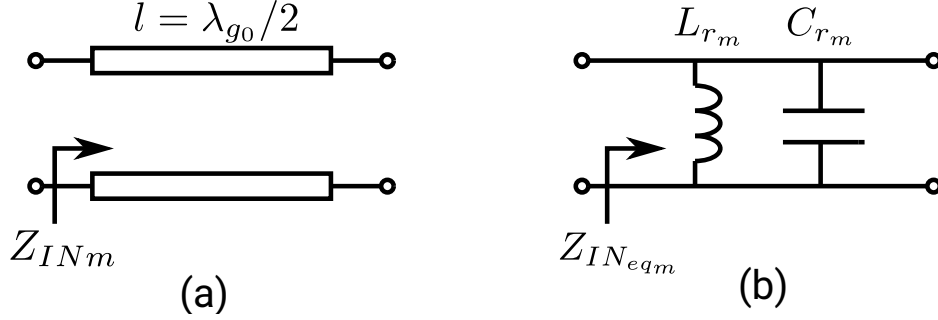


Figure 2: (a) Open ended transmission line resonator of  $\lambda_g/2$  length, representing the physical rectangular waveguide resonator (resonant mode  $TE_{101}$ ). (b) Lumped elements parallel resonator employed in the equivalent circuit.

179 same, and second the slope parameters of both resonators must be the same.  
 180 With the imposition of these two conditions, the value of the inductor and  
 181 capacitor for the lumped elements resonator can be computed as

$$C_{r_m} = \frac{\beta_1}{\omega_0 \omega_{c1} \mu_0} \tan(\beta_1 l) \left( \frac{\omega_{c1}}{\omega_0} - \frac{\omega_0}{\omega_{c1}} \right)^{-1}, \quad L_{r_m} = \frac{1}{\omega_0^2 C_{r_m}}, \quad (1)$$

182 being  $(\beta_1)$  the propagation constant of the mode ( $TE_{10}$ ) in the rectangular  
 183 waveguide evaluated at the lower ( $\omega_{c1}$ ) cut-off frequency of the passband, ( $l$ )  
 184 the physical length of the resonator, and ( $\omega_0$ ) the center angular frequency of  
 185 the filter. Note that since the resonators of the filter are implemented using  
 186 the ( $TE_{101}$ ) resonant mode, we will adjust the length to be

$$l = \frac{\lambda_{g0}}{2} = \frac{\pi}{\beta_0}, \quad (2)$$

187 where  $(\beta_0)$  is the propagation constant of the mode ( $TE_{10}$ ), but evaluated at  
 188 the center frequency of the passband

$$\beta_0 = \sqrt{\omega_0^2 \varepsilon_0 \mu_0 - \left(\frac{\pi}{a}\right)^2}, \quad \beta_1 = \sqrt{\omega_{c1}^2 \varepsilon_0 \mu_0 - \left(\frac{\pi}{a}\right)^2}. \quad (3)$$

189 Here we want to remark that for asynchronously tuned topologies, having  
 190 resonators tuned to different resonant frequencies, a similar process is applied,  
 191 but replacing the center frequency of the passband ( $\omega_0$ ) by the actual resonant  
 192 frequency of each resonator ( $\omega_{r_m}$ ).

193 Having computed the elements of the resonators, the values of the capacitors ( $C_m$ ) in the lowpass prototype of Fig. 1(b) are computed using, in the  
 194 inverse direction, the same lowpass to bandpass transformation, obtaining  
 195

$$C_m = F_B \omega_0 C_{r_m}, \quad F_B = \frac{\omega_{c_2} - \omega_{c_1}}{\omega_0}, \quad (4)$$

where ( $F_B$ ) is defined as the fractional bandwidth of the bandpass response, and ( $\omega_{c_2}$ ) is the upper cutoff angular frequency of the filter. These calculated lowpass capacitors actually represent the scaling factors needed to compute the scaled coupling matrix ( $M'$ ), representing the networks of Fig. 1(b) and Fig. 1(c). The elements of the scaled coupling matrix are finally calculated as

$$M'_{Sm}{}^2 = Y_S C_m M_{Sm}^2 \quad (5a)$$

$$M'_{mn}{}^2 = C_m C_n M_{mn}^2 \quad (5b)$$

$$M'_{nL}{}^2 = C_n Y_L M_{nL}^2. \quad (5c)$$

196 In these expressions, ( $M'_{Sm}$ ) represents the couplings from the input port to  
 197 internal resonators, while ( $M'_{nL}$ ) represents couplings from the output port  
 198 to internal resonators. All other inter-resonators couplings are determined  
 199 by ( $M'_{mn}$ ). A particular case occurs when ( $m = n$ ). In this case, the element  
 200 lies in the diagonal of the coupling matrix, and it gives the scaling factor of  
 201 the constant susceptances ( $M'_{mm} = B'_m$ ). This situation is covered with the  
 202 same equation (5b), shown above, which is still valid under the condition  
 203 ( $m = n$ ).

### 204 3. Design example

205 In this section, a fourth order filter with a quartet topology is going  
 206 to be designed to illustrate the design technique. This is a folded canonical  
 207 topology, which is synchronously tuned and has only one cross coupling  
 208 for symmetrical responses. The topology allows the implementation of two  
 209 transmission zeros in the insertion loss response of the filter. The coupling  
 210 topology is shown in Fig. 3(a), while a possible physical implementation using  
 211 folded rectangular waveguide technology is shown in Fig. 3(b). In Fig. 3(a)  
 212 input/output ports are indicated with dashed circles, while resonators are  
 213 shown with white circles marked as ( $R_m$ ,  $m = 1, 2, 3, 4$ ). The sketch also  
 214 shows the couplings along the main path with solid lines, and the cross



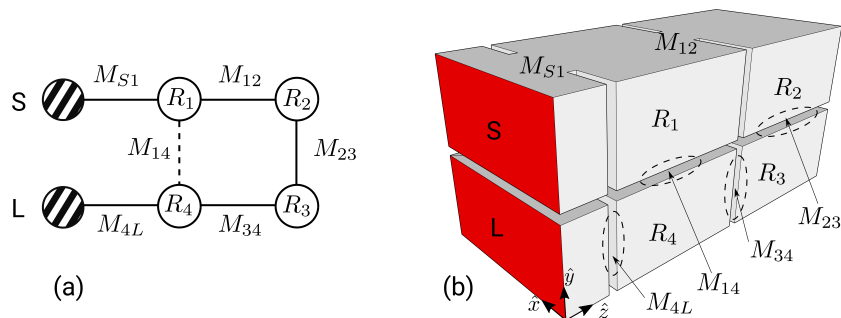


Figure 3: (a) Coupling topology of the folded canonical network selected for design. (b) Physical implementation of the filter using rectangular waveguide technology. Input ( $S$ ) and output ( $L$ ) ports are marked in red.

215 coupling with a dashed line. The correspondence between couplings with  
 216 window irises, and resonators with cavities in the physical implementation,  
 217 is indicated in the 3D view of Fig. 3(b). It can be observed that the physical  
 218 topology is folded in the E-plane. Consequently, the couplings ( $M_{23}$ ) and  
 219 ( $M_{14}$ ) are implemented through irises, which are open on the top and bot-  
 220 tom walls of the corresponding waveguides. The other couplings ( $M_{S1}$ ,  $M_{12}$ ,  
 221  $M_{34}$ ,  $M_{4L}$ ) are implemented with regular inductive irises.

222 The basic geometry of all the irises can be further explored in the top  
 223 and side views of the structure presented in Fig. 4. In these drawings, the  
 224 coupling ( $M_{23}$ ) is controlled with the iris width ( $a_3$ ), while the coupling ( $M_{14}$ )  
 225 is controlled with the iris width ( $a_4$ ). The figure also shows all the geometrical  
 226 parameters that will be optimized during the design process. For practical  
 227 reasons, the thickness of all walls ( $l_i$ ) is set to 1 mm. It is important to remark  
 228 that the folding of the structure along the E-plane allows to adjust the sign  
 229 of the vertical couplings. This fact makes possible the implementation of the  
 230 two transmission zeros of the topology, either in the real frequency axis or in  
 231 the complex plane.

232 For the physical implementation, we use a standard rectangular waveguide  
 233 WR-75 ( $a = 19.05$  mm and  $b = 9.525$  mm). The filter specifications are:  
 234 center frequency  $f_0 = 11$  GHz, bandwidth  $BW = 300$  MHz and minimum  
 235 return loss  $RL = 15$  dB. Also, the quartet topology has associated two trans-  
 236 missions zeros, that will be placed at  $f_{z_1} = 10.78$  GHz and  $f_{z_2} = 11.12$  GHz.  
 237 As it can be noticed, the two transmission zeros are symmetrically disposed  
 238 with respect to the center frequency of the filter.

239 The design process starts by obtaining the normalized coupling matrix

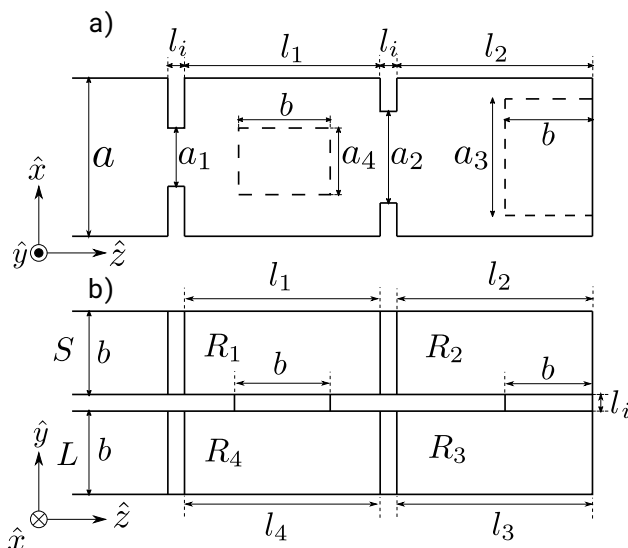


Figure 4: a) Top view of the waveguide filter, showing the relevant geometrical parameters involved in the design process. b) Side view of the waveguide filter. Due to practical considerations, the thickness of all waveguide walls is fixed to  $l_i = 1$  mm.

240 with the procedure shown in [3], with the desired specifications of the filter,  
 241 leading to

$$M = \begin{bmatrix} 0 & 0.9017 & 0 & 0 & 0 & 0 \\ 0.9017 & 0 & 0.7411 & 0 & 0.2483 & 0 \\ 0 & 0.7411 & 0 & -0.7576 & 0 & 0 \\ 0 & 0 & -0.7576 & 0 & 0.7411 & 0 \\ 0 & 0.2483 & 0 & 0.7411 & 0 & 0.9017 \\ 0 & 0 & 0 & 0 & 0.9017 & 0 \end{bmatrix}. \quad (6)$$

242 Note that this matrix has all the diagonal elements equal to zero. This means  
 243 that the topology is synchronously tuned, and all resonators are tuned at the  
 244 center frequency of the passband. As already mentioned, this is the case  
 245 when the frequency response to be synthesized is symmetrical with respect  
 246 to the center frequency. Therefore, in this case all constant susceptances  
 247 of the equivalent circuit will be equal to zero ( $M_{mm} = B_m = 0$ ). It is  
 248 also interesting to remark that one of the couplings of the matrix has a  
 249 negative sign. For the solution shown in (6) this is the coupling ( $M_{23}$ ).  
 250 As already mentioned, the folding of the structure along the E-plane allows  
 251 enough flexibility to adjust the sign of the couplings along the vertical path

252 ( $M_{14}$  and  $M_{23}$ ). Assuming that the signs of the couplings along the main path  
 253 are positive, the window ( $a_4$ ) placed at the center of the cavity as shown in  
 254 Fig. 4, will produce a coupling ( $M_{14}$ ) also positive. In order to reverse the  
 255 sign of the ( $M_{23}$ ) coupling, as required by the coupling matrix, the window  
 256 ( $a_3$ ) needs to be placed at one side of the cavity, as also shown in Fig. 4.

257 The next step in the design procedure is to calculate the scaled coupling  
 258 matrix ( $M'$ ), using as resonators the resonant mode ( $\text{TE}_{101}$ ) in the corre-  
 259 sponding rectangular waveguide cavities. We should emphasize that, since  
 260 all resonators employed in the physical implementation are the same, all the  
 261 resonators in the equivalent circuit will be identical, and the subindex ( $m$ ) can  
 262 be dropped from the notation ( $C_{r_m} = C_r$ ,  $L_{r_m} = L_r$ ). Using the propagation  
 263 constant ( $\beta$ ) of the mode ( $\text{TE}_{10}$ ) in our rectangular waveguide, at the center  
 264 frequency and at the lower cut-off frequency of the passband [ $\beta_0$  and  $\beta_1$ , see  
 265 (3)], equation (1) can be used to compute the lumped elements resonators, ob-  
 266 taining  $C_r = 0.086$  pF and  $L_r = 2.42$  nH. The use of (4) permits to calculate  
 267 the scaling factor in the lowpass domain, obtaining ( $C_m = C = 163.05$  nF).  
 268 Finally, for the source and load terminations we should use the information  
 269 of the real ports that excite the physical structure. As shown in the 3D  
 270 view of Fig. 3(b) (red areas), the excitation is formed with the  $\text{TE}_{10}$  mode  
 271 propagating in the standard WR-75 waveguides (waveguide ports). Using  
 272 the characteristic impedance of this mode, evaluated again at the center fre-  
 273 quency of the passband, we obtain:  $Z_S = 1/Y_S = 1/Y_L = 539.116 \Omega$ . With  
 274 these values, the scaled coupling matrix ( $M'$ ) can be calculated using (5a)-  
 275 (5c), obtaining

$$M' = 10^{-3} \cdot \begin{bmatrix} 0 & 0.4959 & 0 & 0 & 0 & 0 \\ 0.4959 & 0 & 0.1208 & 0 & 0.0405 & 0 \\ 0 & 0.1208 & 0 & -0.1235 & 0 & 0 \\ 0 & 0 & -0.1235 & 0 & 0.1208 & 0 \\ 0 & 0.0405 & 0 & 0.1208 & 0 & 0.4959 \\ 0 & 0 & 0 & 0 & 0.4959 & 0 \end{bmatrix}. \quad (7)$$

276 Once the scaled coupling matrix is known, the equivalent circuit of the  
 277 whole bandpass filter is directly obtained with the calculated values, as shown  
 278 in Fig. 5. In the step-by-step design technique, this equivalent circuit will  
 279 be divided in several sub-circuits. The process starts by considering just the  
 280 first admittance inverter ( $M'_{S1}$ ), as shown in Fig. 6, representing the input  
 281 coupling of the structure. The response of this circuit is very simple, as it  
 282 contains just one constant admittance inverter, as shown with red crosses

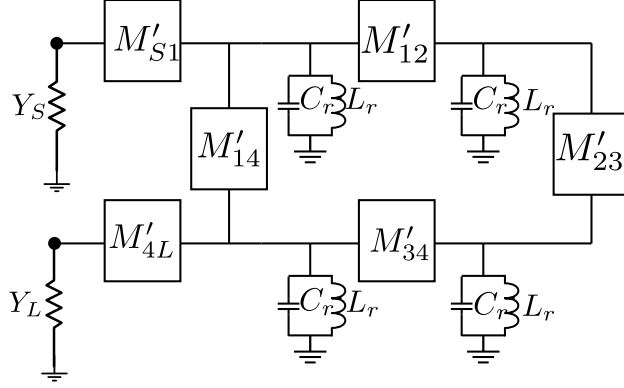


Figure 5: Equivalent circuit of the filter obtained from the scaled coupling matrix (7).

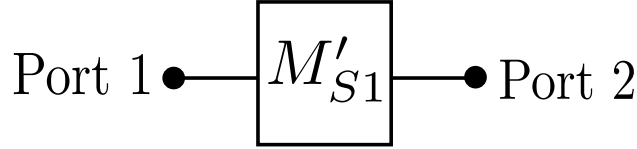


Figure 6: Equivalent circuit of the first segment of the filter. The response of this circuit is used to adjust the input iris of the filter ( $a_1$ ) in the first step of the technique.

283 in Fig. 7. This response is used to optimize the input iris of the physical  
 284 structure, composed of the inductive window ( $a_1$ ) shown in Fig. 4. The  
 285 part of the physical structure that corresponds to the first segment of the  
 286 equivalent circuit is shown in the panel of Fig. 7. We have to clarify that  
 287 in this structure the waveguide corresponding to the resonator  $R_1$  is simply  
 288 terminated by a waveguide port (Port 2 shown in the panel of Fig. 7). We  
 289 remark that the response of this iris is not constant with frequency, due  
 290 to the inherent dispersion of inductive irises. However, the iris width ( $a_1$ )  
 291 is adjusted until the right transmission level is retrieved at the center  
 292 frequency of the passband ( $f_0$ ), as shown in the plot. Here, full-wave  
 293 simulations are performed using FEST3D software [17] to evaluate the scattering  
 294 parameters of the different segments of the physical structure. In this first  
 295 step, only the width of the input iris ( $a_1$ ) is obtained, as shown in the  
 296 second column (Step 1) of Table 1.

297 For the second step of the design process, the admittance inverters ( $M'_{12}$ )  
 298 and ( $M'_{14}$ ) are added, together with the first resonator ( $R_1$ ), giving the  
 299 equivalent circuit shown in Fig. 8. It can be observed that the subcircuit is now

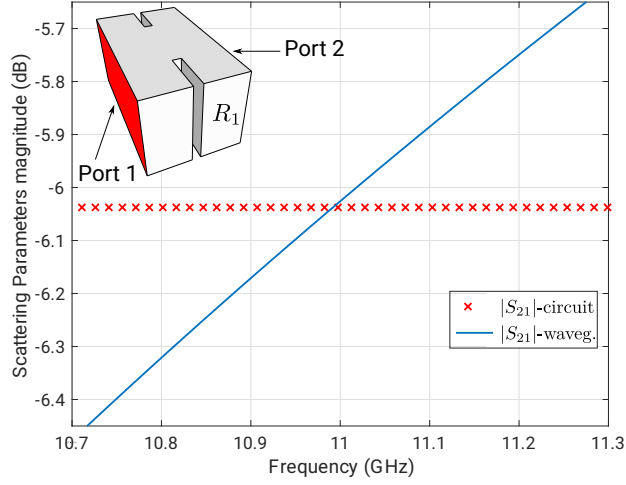


Figure 7: Transmission response ( $|S_{21}|$ ) of the equivalent circuit for the first segment shown in Fig. 6 (red crosses), and similar response of the input inductive iris ( $a_1$ ) after optimization. The panel shows a 3D sketch of the physical structure corresponding to the first segment.

	Step 1	Step 2	Step 3	Step 4	Final	Error (%)
$a_1$ (mm)	9.114	9.038	9.038	9.038	9.056	0.20
$a_2$ (mm)		5.574	5.517	5.517	5.548	0.60
$a_3$ (mm)			9.407	9.359	9.382	0.24
$a_4$ (mm)		3.923	3.923	3.923	3.929	0.20
$l_1$ (mm)		17.228	17.249	17.249	17.226	0.13
$l_2$ (mm)			18.565	18.553	18.542	0.05

Table 1: Values of all geometrical parameters of the filter obtained after each iteration, and after a global optimization of the whole structure. The relative errors obtained with the design technique proposed are also included in the last column.

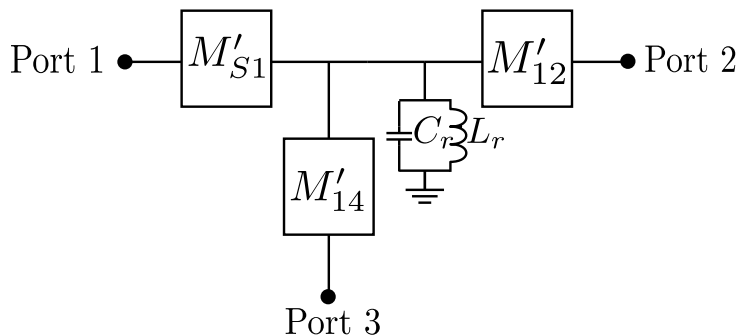


Figure 8: Equivalent circuit of the second segment of the filter. Note that three ports are included to take into account for the two paths of the signal. Port 2 is placed along the main path, while port 3 is placed along the cross-coupling path.

300 defined with three ports. In this way, it is possible to characterize how the  
 301 input power must be split between the direct and the cross coupling paths.  
 302 Therefore, for this second step, two main responses ( $|S_{21}|$ ) and ( $|S_{31}|$ ) will be  
 303 important to define the optimization operations. Both responses are shown  
 304 with symbols in Fig. 9, in the relevant bandwidth of the filter. These two  
 305 responses are simultaneously used to adjust the second segment of the phys-  
 306 ical structure, which is shown in the panel inside the figure. In the physical  
 307 structure, the coupling ( $M'_{12}$ ) is formed with the inductive iris ( $a_2$ ) shown in  
 308 Fig. 4. The cross coupling ( $M'_{14}$ ) is adjusted with the vertical iris ( $a_4$ ). Fi-  
 309 nally, the resonant frequency of the first resonator ( $R_1$ ) is adjusted with the  
 310 length ( $l_1$ ). As indicated in the panel of Fig. 9, the waveguide corresponding  
 311 to resonator  $R_4$  is terminated on the back side with a short circuit, and on  
 312 the front side with a waveguide port (Port 3). For the simulation, we have  
 313 selected a length for this waveguide section of ( $l_4 = l_1$ ). Also, the waveguide  
 314 corresponding to resonator  $R_2$  is terminated with a waveguide port (Port 2).

315 The main geometrical parameters involved in the optimization operations  
 316 for this second step of the design process are  $a_2$ ,  $a_4$  and  $l_1$  (see Fig. 4). The  
 317 three geometrical parameters are first optimized until the two responses are  
 318 simultaneously recovered, as shown with solid lines in Fig. 9. Here it should  
 319 be pointed out that, after a first optimization with these three variables, a  
 320 small refinement including the input iris ( $a_1$ ) is usually required. In this  
 321 way, the interactions and small couplings of all the elements of the struc-  
 322 ture are rigorously accounted for during the final calculation of the relevant  
 323 geometrical parameters.

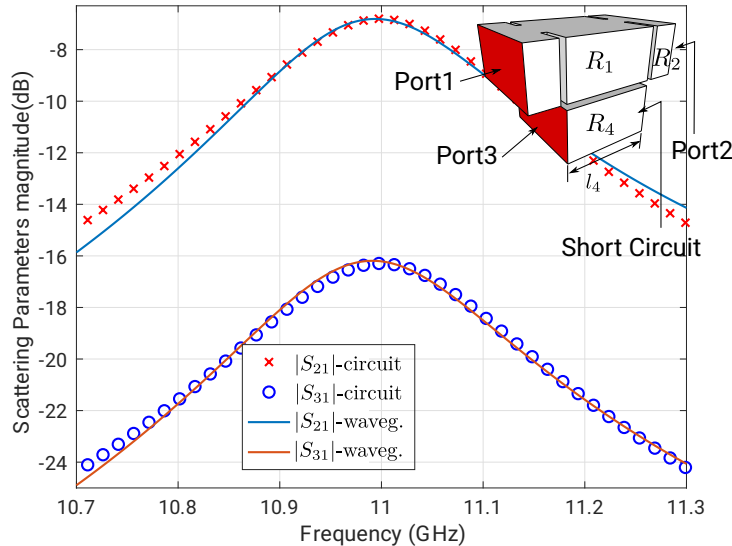


Figure 9: Transmission responses ( $|S_{21}|$ ) and ( $|S_{31}|$ ) of the second segment of the equivalent circuit during the design process (symbols). Solid lines refer to the responses obtained for the corresponding segment of the physical structure (shown in the panel), after optimization.

324 The geometrical parameters obtained, at this stage of the design process,  
 325 are collected in the third column of Table 1 (Step 2). In particular, the input  
 326 iris has been modified from  $a_1 = 9.114$  mm in Step 1 to  $a_1 = 9.038$  mm  
 327 in Step 2. This indicates that, although small, the interactions and loading  
 328 effects of the first resonator and the cross coupling on this iris are important,  
 329 resulting into a final variation of about 0.8%. The result also illustrates the  
 330 importance of taking into account for coupling effects and interactions from  
 331 neighboring elements, during the final adjustment of the different parts of  
 332 the structure.

333 In the third step of the design process, we propose to continue the moni-  
 334 torization of the signal into both main and cross coupling paths, but we  
 335 add a new coupling ( $M'_{23}$ ) and a new resonator ( $R_2$ ) into the main path of  
 336 the equivalent circuit, as shown in Fig. 10. It can be observed in the figure  
 337 that the equivalent circuit again contains three ports. This allows to monitor  
 338 at the same time the signal along the main path and along the cross cou-  
 339 pling path. Consequently, two transmission functions ( $|S_{21}|$  and  $|S_{31}|$ ) will  
 340 be used simultaneously in the optimization problem, as shown with symbols

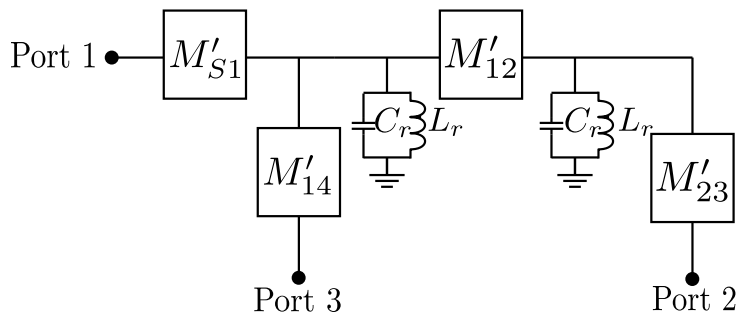


Figure 10: Equivalent circuit of the third segment of the filter. The signal along the main and the cross coupling paths are still monitored by using three ports. However, a new coupling ( $M'_{23}$ ) and the second resonator is added to the main path.

341 in Fig. 11.

342 In the physical structure, the new coupling is adjusted with the iris ( $a_3$ ),  
 343 while the new resonator is adjusted with the length ( $l_2$ ), as shown in Fig. 4.  
 344 As illustrated in the panel of Fig. 11, the waveguide associated to resonator  
 345  $R_4$  is again terminated on the back side with a short circuit and on the front  
 346 side with a waveguide port (Port 3). The waveguide associated to resonator  
 347  $R_3$  is terminated in the same way, having on the front side the waveguide  
 348 port (Port 2). For the simulations of this sub-structure, we have selected:  
 349  $l_4 = l_1$  and  $l_3 = l_2$  (see Fig. 4).

350 Following the same strategy as before, the new geometrical parameters  
 351 ( $a_3$  and  $l_2$ ) are first optimized to recover simultaneously the responses shown  
 352 in Fig. 11. After this first optimization process, a refinement is conducted  
 353 including also neighboring elements ( $a_2, a_4, l_1$ ). Note that at this stage, the  
 354 input iris ( $a_1$ ) does not need to be included in the optimization process. This  
 355 refinement operation allows to recover with high accuracy the multiport goal  
 356 functions of the equivalent circuit, as shown with solid lines in Fig. 11.

357 The geometrical dimensions obtained after this step are included in the  
 358 fourth column of Table 1 (Step 3). This shows that the important parameters  
 359 in the optimization are the new iris width ( $a_3$ ) and resonator length ( $l_2$ ). In  
 360 fact, the previous width ( $a_2$ ) and length ( $l_1$ ) vary little with respect to the  
 361 previous step, showing relative variations of 1% and 0.1%, respectively. The  
 362 iris width ( $a_4$ ), controlling the cross coupling, also shows a small variation of  
 363 0.15% with respect to the value obtained in the previous step. In any case,  
 364 these variations suggest again the importance of considering nearby elements  
 365 during the final adjustment of the different parts of the structure.



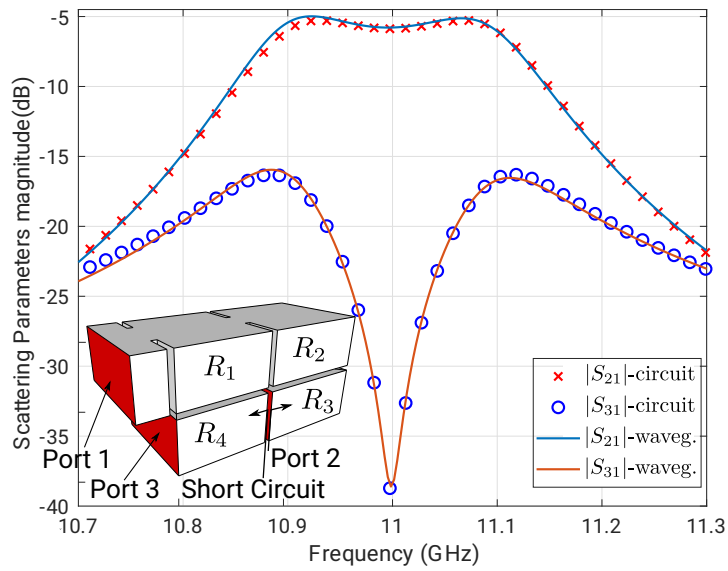


Figure 11: Transmission functions ( $|S_{21}|$  and  $|S_{31}|$ ) of the equivalent circuit proposed for the third step of the design process (symbols). Solid lines show the responses of the physical structure corresponding to the third segment (shown also in the panel), after optimization.

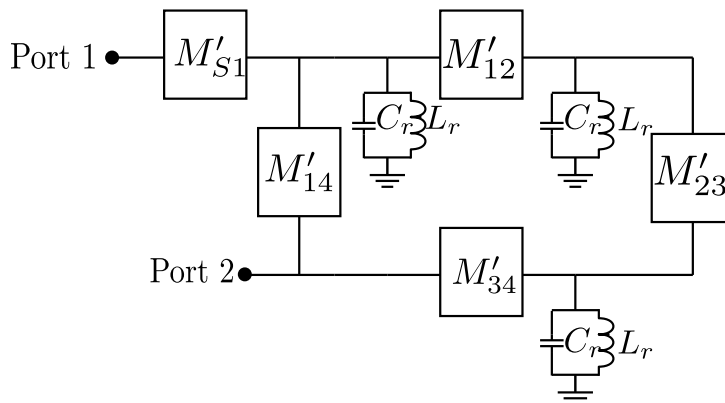


Figure 12: Equivalent circuit of the fourth segment of the filter. In this last step of the design process only two ports are used.

366 At this point, after three iterations, all the geometrical parameters cor-  
 367 responding to the first half of the filter have been calculated. However, it is  
 368 convenient to perform an additional step of the design procedure, in order to  
 369 take into account for nearby elements in the last components of the filter. In  
 370 this last step we will add the next resonator ( $R_3$ ) and coupling ( $M'_{34}$ ) along  
 371 the main path, obtaining the equivalent circuit shown in Fig. 12. It can be  
 372 observed that in this last step only two ports are considered in the equivalent  
 373 circuit. In fact, the main and the cross coupling paths are joined together  
 374 in the output port of the subcircuit (Port 2). Therefore, this time only one  
 375 objective function is used in the optimization problem ( $|S_{21}|$ ). The physical  
 376 segment corresponding to this subcircuit is shown in the panel of Fig. 13. It  
 377 can be observed that the waveguides corresponding to resonators ( $R_3$ ) and  
 378 ( $R_4$ ) are coupled by an additional inductive window, while the waveguide  
 379 corresponding to ( $R_4$ ) is used to place the output Port 2 of the segment.

380 In the optimization problem, we first adjust the last coupling window  
 381 and the length of the third resonator [ $l_3$  in Fig. 4(b)]. This first optimization  
 382 operation leads to a coupling window of 5.504 mm. However, due to the  
 383 symmetry of the structure, this value will be discarded when computing the  
 384 final filter geometry. The important task now is to refine the response shown  
 385 in Fig. 13 with symbols (circuit response) by including also the previous  
 386 coupling window ( $a_3$ ) and the previous resonator length ( $l_2$ ). By including  
 387 these two previous elements, the response of the subcircuit is retrieved with  
 388 great accuracy, as can be observed in Fig. 13 (solid line). It is interesting to

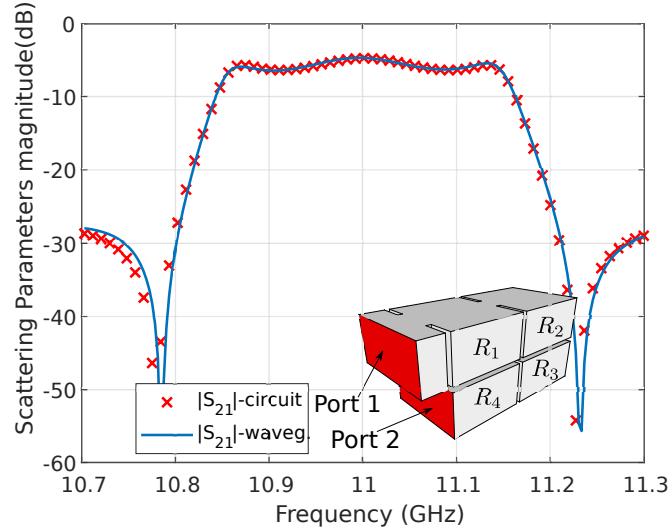


Figure 13: Transmission function ( $|S_{21}|$ ) of the equivalent circuit proposed for the fourth step of the design process (symbols). Solid line shows the response of the physical structure corresponding to the fourth segment (shown also in the panel), after optimization.

389 note that to obtain this response the window along the cross coupling path  
 390 ( $a_4$ ) was not modified. The accuracy obtained in the coupling ( $a_3$ ) and in  
 391 the length of the second resonator ( $l_2$ ) is increased, since the interactions  
 392 of additional nearby elements are included during the last optimization pro-  
 393 cess. The geometrical parameters obtained after this refinement operation  
 394 are included in the fifth column of Table 1 (Step 4). The table shows that  
 395 the coupling width ( $a_3$ ) and the length ( $l_2$ ) have slightly varied, with relative  
 396 errors of (0.51%) and (0.06%) with respect to the previous step.

397 The geometrical parameters shown in Table 1 for Step 4 are considered  
 398 to be the initial design, directly obtained from the proposed method. Fig. 14  
 399 shows the target response of the coupling matrix and the response of the  
 400 physical structure with these initial dimensions. In general, it can be observed  
 401 good agreement between both responses. Since the initial design obtained  
 402 with the proposed technique is very close to the target specifications, a global  
 403 optimization process using gradient techniques can be used to improve the  
 404 solution. In Table 1 we include the final dimensions obtained after the global  
 405 optimization of the structure, and the response is shown, for comparison,  
 406 in Fig. 14. It can be observed good agreement with respect to the target  
 407 response given by the coupling matrix. In particular the target return loss

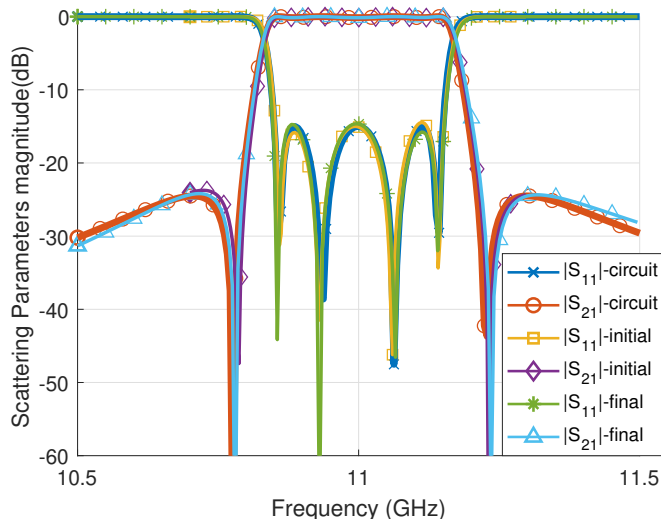


Figure 14: Comparison between the target response of the filter provided by the coupling matrix, and the response of the physical structure with the dimensions directly obtained from the proposed design technique. The response of the physical structure after applying a global optimization process is also shown.

408 level inside the passband of  $RL = 15$  dB has been obtained. In the last  
 409 column of Table 1, we further show the relative errors obtained in all the  
 410 geometrical parameters of the filter. Relative errors are defined between the  
 411 values obtained directly after the design process, and those obtained after  
 412 the global optimization. Results demonstrate high accuracy of the proposed  
 413 design strategy, as relative errors as low as 0.6 % are systematically obtained.

#### 414 4. Alternative design

415 The design technique illustrated in the previous section with a quartet  
 416 example exhibiting two transmission zeros in the real frequency axis, can  
 417 also be used for the design of a similar filter structure, but with complex  
 418 transmission zeros placed at the center of the passband. These complex  
 419 transmission zeros are used in filtering functions to achieve flat group delay  
 420 responses. Therefore, it is interesting to demonstrate that the design tech-  
 421 nique proposed in this paper is also valid for the design of filters exhibiting  
 422 this type of alternative transfer functions.

423 The in-band characteristics of the filter are the same as indicated for the

	Initial	Final	Error (%)
$a_1$ (mm)	9.154	9.189	0.38
$a_2$ (mm)	5.8089	5.819	0.17
$a_3$ (mm)	5.150	5.129	0.41
$a_4$ (mm)	4.312	4.358	1.06
$l_1$ (mm)	17.173	17.147	0.15
$l_2$ (mm)	19.306	19.308	0.01

Table 2: Dimensions of the filter in quartet topology with two complex transmission zeros. Dimensions obtained directly after the application of the design technique can be compared to those obtained after a global optimization. Last column shows the relative errors obtained between the initial and final optimized dimensions. The structure follows the same sketch of Fig. 4, but the iris ( $a_3$ ) is placed at the center of the cavity.

424 previous design ( $f_0 = 11$  GHz,  $BW = 300$  MHz,  $RL = 15$  dB). However,  
425 complex transmission zeros are placed at the center of the passband to reduce  
426 the group delay variation. After adjusting the positions of the transmission  
427 zeros in the complex plane, the normalized coupling matrix results to be

$$M = \begin{bmatrix} 0 & 0.9379 & 0 & 0 & 0 & 0 \\ 0.9379 & 0 & 0.8177 & 0 & 0.3120 & 0 \\ 0 & 0.8179 & 0 & 0.4899 & 0 & 0 \\ 0 & 0 & 0.4899 & 0 & 0.8177 & 0 \\ 0 & 0.3120 & 0 & 0.8177 & 0 & 0.9379 \\ 0 & 0 & 0 & 0 & 0.9379 & 0 \end{bmatrix}. \quad (8)$$

428 The design of this filter essentially follows the same steps as described in  
429 the previous section. The only relevant change is in the sign of the ( $M_{23}$ )  
430 coupling, which is now positive as shown in (8). This modification can be  
431 easily introduced in the physical structure, due to the fact that it is folded in  
432 the E-plane. The sign change is simply implemented by placing the vertical  
433 iris ( $a_3$  shown in Fig. 4) at the center of the second cavity [ $R_2$  indicated in  
434 Fig. 3(b) and Fig. 4(b)].

435 After following the same steps detailed in the previous section, the di-  
436 mensions obtained for the filter are collected in the second column of Table 2  
437 (Initial). The response obtained for this structure, as compared to the tar-  
438 get response of the coupling matrix (8) is presented in Fig. 15. It can be  
439 observed that the response obtained directly from the proposed design tech-  
440 nique agrees very well with respect to the reference response. Due to the

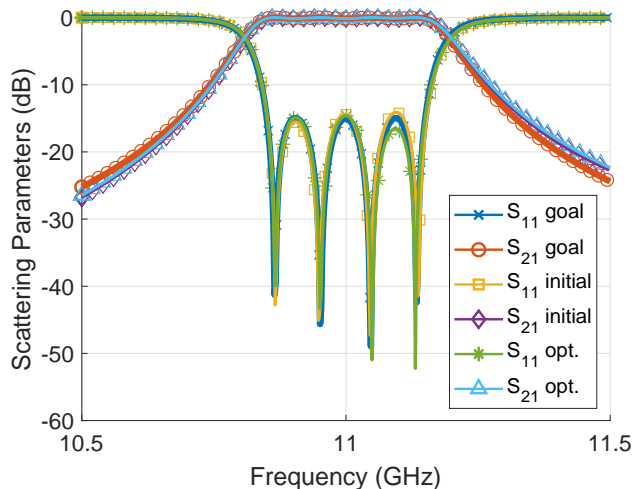


Figure 15: Comparison between the response obtained for the filter in quartet topology with two complex transmission zeros, and the reference response given by the coupling matrix (8). Results before and after performing a global optimization on the structure are shown.

441 high quality of the initial design, a global optimization using gradient tech-  
 442 niques converges easily, and allows to improve the response, as also shown in  
 443 Fig. 15. In Table 2 we also collect for completeness the dimensions obtained  
 444 after the final global optimization (Final). In the last column, we also give  
 445 the relative errors between the solution directly obtained from the proposed  
 446 design technique and the one obtained after the global optimization. In this  
 447 case, all relative errors are below 1.06%.

448 To show the effectiveness of this solution in the equalization of the group  
 449 delay, we present this electrical characteristic in Fig. 16. The figure shows  
 450 the group delay of the filter, and compares it to the reference solution given  
 451 by the coupling matrix (8). The group delays are given for the filter directly  
 452 obtained from the design technique, and for the filter after applying the  
 453 global optimization operation. For reference, the group delay of the filter  
 454 in quartet topology with the transmission zeros in the real frequency axis is  
 455 also included. Results clearly show that a significant reduction in group delay  
 456 variation can be achieved by placing the transmission zeros in the complex  
 457 plane, at appropriate locations. Results also show very good agreement of  
 458 the group delay of the designed filter with respect to the reference solution  
 459 provided by the coupling matrix. In any case, this second example also  
 460 demonstrates the usefulness of the proposed strategy, in the design of this

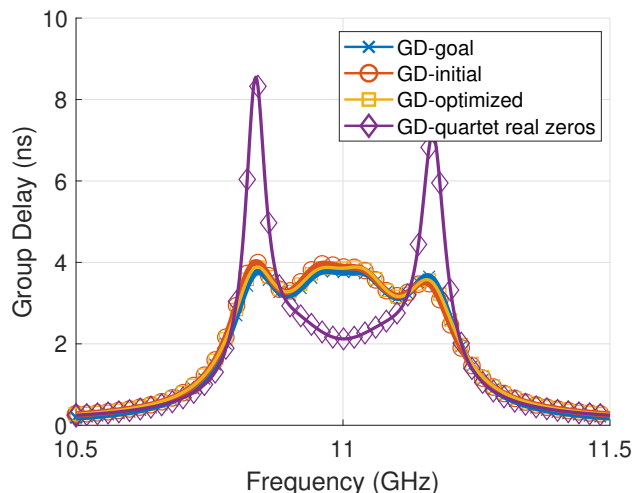


Figure 16: Group delay obtained for the filter in quartet topology with complex transmission zeros, as compared to the group delay given by the coupling matrix (8) (crosses). The group delay of the filter obtained directly after the design process (circles) and after a global optimization (squares) are shown. For comparison, the group delay of the filter in quartet topology with transmission zeros in the real frequency axis (diamonds) is also included.

461 type of folded canonical topologies implementing complex transmission zeros.

## 462 5. Conclusions

463 In this work, a strategy to design filters in folded canonical topologies  
 464 is presented. The technique is based on scaling the normalized coupling  
 465 matrix to find useful partial responses of the filter equivalent circuit. This  
 466 work proposes for the first time the optimization of the different segments  
 467 of the structure using multiport networks. This allows to consider several  
 468 goal functions in the equivalent circuit, which are simultaneously used in  
 469 the optimization problem of the different segments of the physical structure.  
 470 In this way, the signal can be monitored along the main path and along the  
 471 cross coupling paths, usually present in this type of topologies. The proposed  
 472 technique has been successfully applied to the design of two filters in quartet  
 473 topology, one with two transmission zeros in the real frequency axis, and  
 474 the other with complex transmission zeros. The results confirm that the  
 475 proposed technique leads to initial designs that are very close to the desired  
 476 target specifications. Similar strategy, as introduced in this paper, can be

477 applied to the design of other filters in folded canonical topologies of higher  
478 orders or containing additional cross couplings.

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