Two-dimensional Quasi-Bessel beam synthesis and frequency-scanning leaky-wave launchers

M. Poveda-Garcia[1], **J.L. Gomez-Tornero**[1]

[1]Technical University of Cartagena, Cartagena (Spain)

Abstract — In this paper, a technique to synthesize a 2D frequency-scanned high order Bessel beam in a parallel plates waveguide (PPW) scenario is presented. This technique is based on a one dimensional beam launcher, rather than the more typical axicons or the more recent metasurfaces and antenna arrays for 3D Bessel beam synthesis. With this launcher, it is possible to generate a 2D Bessel beams whose direction can be controlled by changing the frequency of the feeding signal in the Ku band from 14.5 GHz to 15 GHz. It is demonstrated how this can be done by combination of two leaky waves (LW), which can be created by the same 1D structure, scanning at different angles and launched to the PPW region. The beams are generated in a triangular area, with one of its sides being the launcher itself and the other sides are defined by the direction of radiation of each of the forming leaky waves.

1. INTRODUCTION

Bessel beams are known to be a non-diffractive particular solution of the Helmholtz equation [1]. As an infinite aperture is required to generate an ideal Bessel beam, this is not possible in reality, so using a finite structure will lead to the generation of a Bessel beam in a limited region around the structure, in which the beamwidth keeps constant as it is propagating in the mentioned diffraction-less region. This kind of beams can be on much interest in applications such as medical imaging, wireless power transfer, sensing or even electromagnetic / quasi-optical trappers. To this aim, different approaches has been proposed in order to create a Bessel beam such as axicons [2], localized waves [3], holographic metasurfaces [4]–[6] or leaky-wave (LW) structures [7]–[10]. In this sense, apart from axicons and localized waves, two different ways of generating a Bessel beam are proposed, as depicted in Fig. 1a and Fig. 1b. The first one makes use of the interference between two forward propagating leaky-waves travelling in opposite directions, in which the non-diffractive region is triangular-shaped in one plane and cone-shaped in 3-D. The second one is very similar, but in this case backward leaky waves are used making the non-diffractive region to be diamond-shaped. In both cases, the direction of the Bessel beam is perpendicular to the surface of the launcher.

In this work, we propose the use of a 1-D launcher which creates a 2-D quasi-Bessel beam in a parallel plates waveguide (PPW) scenario, as sketched in Fig 1c. The principle is the same as in other works, i.e. the interference of two propagating leaky waves, but in this case the two leaky waves propagate in the same direction with different propagation angles. This way, the launcher can be easily fed by one side and can create a Bessel beam in a different direction rather than the perpendicular to the launcher. Also, as leaky waves are scanned with frequency [11], the direction of the Bessel beam can be frequency-scanned, as will be shown. In the case of the other solutions, a change in the frequency will lead to a longer beam, but will not change its direction. Synthesis of frequency steerable focal points has already been proposed in the literature in the same PPW scenario [12] and also to obtain broadbeams in the far field [13], but in this work the proposed modulation of the leaky mode is very different, allowing for the synthesis of the equivalent of two leaky-modes, which is key for the generation of Bessel beams. In [6], there is also frequency-scanning of a Bessel beam, but in that case the feeding network is very bulky and it is not integrated in the structure. The proposed structure includes the feeding network in the same PCB as the launcher is manufactured.

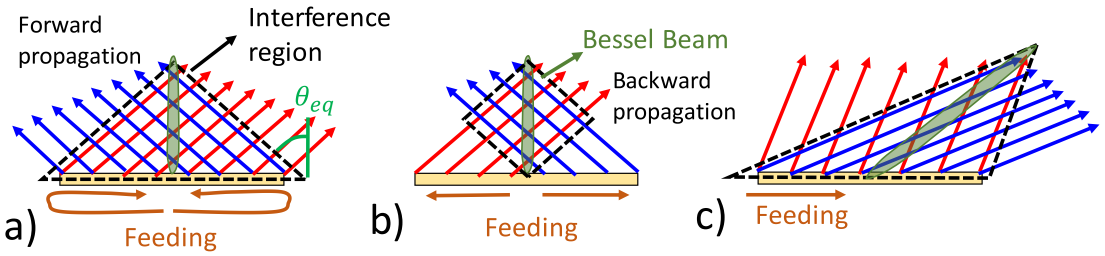

Fig. 1. a) Bessel beam generation by interference of forward propagating leaky waves in opposite directions. b) Bessel beam generation by interference of backward propagating leaky waves in opposite directions. c) Bessel beam generation by interference of forward propagating leaky waves in the same direction with different propagation angle.

2. GENERATION OF QUASI-BESSEL BEAM WITH 1-D LEAKY-WAVE LAUNCHER

A. 2D Scanned high order Bessel beams

As shown in Fig. 2a, the proposed launcher is fed by one side and the overlapping region for the two leaky waves is triangular. In this case, if the direction of the radiated fields for each LW can be independently controlled, the direction of the Bessel beam can also be controlled, as well as its depth, according to the dimensions of the triangle-shaped overlapping area.

The angles θ_1 and θ_2 are the scanning angles of the plane waves produced by each LW, θ_B is the quasi-Bessel beam scanning angle, L_A stands as the aperture length of the high order Bessel beam launcher and θ_{eq} is the equivalent to the axicon angle θ_a [10]. In Fig. 2b, an analogy between our proposal and the case of Fig. 1a is presented. The angles in Fig.2a can be related by (1):

$$\begin{cases} \theta_B = \dfrac{\theta_1+\theta_2}{2} \\ \theta_{eq} = \dfrac{\theta_2-\theta_1}{2} \end{cases} \tag{1}$$

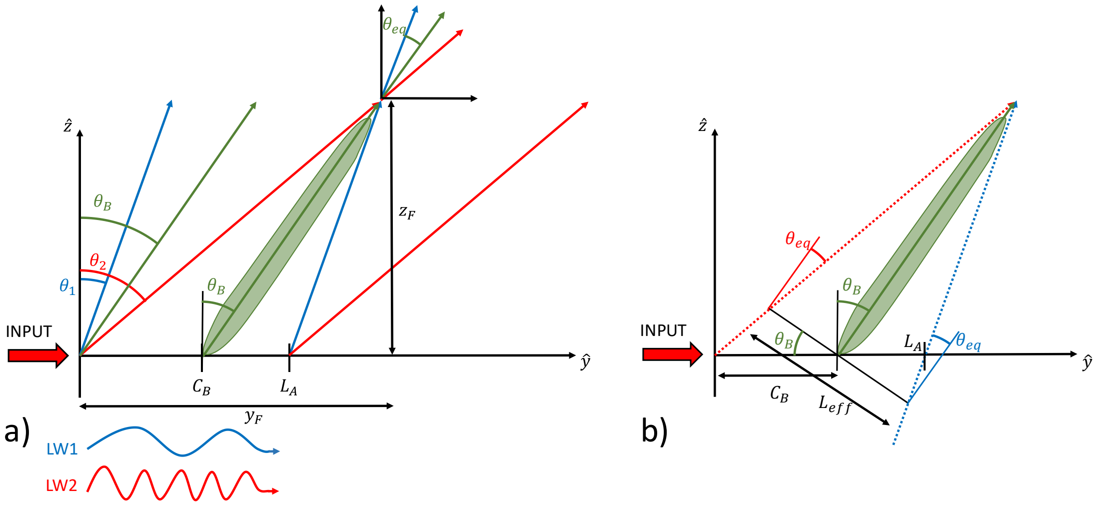

Fig. 2. a) Scheme of the different angles involved in the generation of a scanned high order Bessel beam by two plane waves. b) Equivalence with the angles involved in the case of Fig. 1a.

B. *Fields in the Aperture of the 1-D Launcher*

The field along the aperture to generate the scanned quasi-Bessel beam is the equivalent to the addition of two different field distributions to produce two plane waves radiating at different angles, this is, with different phase distribution. This can be written as follows:

$$E_{AP}(y) = Ae^{-j\beta_1 * y} + Ae^{-j\beta_2 * y} = Ae^{-j\sin(\theta_1) * k_0 * y} + Ae^{-j\sin(\theta_2) * k_0 * y} \tag{2}$$

where A is the amplitude of the field, which is uniform along the aperture and β_1 and β_2 are the phase constants of each field distribution and are related with the radiation angles, θ_1 and θ_2, by the next expression [11]:

$$\beta / k_0 = \sin(\theta) \tag{3}$$

with k_0 the free space wavenumber.

Using (1), (2) can be rewritten as:

$$E_{AP}(y) = A\left(e^{-jk_0 * \sin(\theta_1) * y} + e^{-jk_0 * \sin(\theta_2) * y}\right) =$$

$$= A\left(e^{-jk_0 * \sin(\theta_B - \theta_{eq}) * y} + e^{-jk_0 * \sin(\theta_B + \theta_{eq}) * y}\right) \tag{4}$$

since

$$\begin{cases} \sin(\theta_B + \theta_{eq}) = \sin(\theta_B) * \cos(\theta_{eq}) + \cos(\theta_B) * \sin(\theta_{eq}) \\ \sin(\theta_B - \theta_{eq}) = \sin(\theta_B) * \cos(\theta_{eq}) - \cos(\theta_B) * \sin(\theta_{eq}) \end{cases} \tag{5}$$

then,

$$E_{AP}(y) = A\left(e^{-jk_0 * \sin(\theta_B) * \cos(\theta_{eq}) * y} * e^{-jk_0 * \cos(\theta_B) * \sin(\theta_{eq}) * y} + e^{-jk_0 * \sin(\theta_B) * \cos(\theta_{eq}) * y} * e^{+jk_0 * \cos(\theta_B) * \sin(\theta_{eq}) * y}\right) \tag{6}$$

and rearranging the terms it can finally be written as:

$$E_{AP}(y) = A * e^{-jk_0 * \sin(\theta_B) * \cos(\theta_{eq}) * y} * 2 * \cos\left(k_0 * \cos(\theta_B) * \sin(\theta_{eq}) * y\right) \tag{7}$$

This aperture field has two main terms related to the amplitude and the phase. The amplitude term will be defined as follows:

$$|E_{AP}(y)| = 2A\left|\cos\left(k_0 * \cos(\theta_B) * \sin(\theta_{eq}) * y\right)\right| \tag{8}$$

The aperture illumination has a periodic behaviour along the whole aperture length, as it can be seen in Fig. 3a for the particular example of a high order Bessel beam generation with $\theta_B = 30°$ and $\theta_{eq} = 30°$ and a launcher with length $L_{AP} = 12\lambda_0$. On the other hand, the phase of the aperture can be written as:

$$\Psi\left(E_{AP}(y)\right) = -k_0 * \sin(\theta_B) * \cos(\theta_{eq}) * y + \Psi\left(\cos\left(k_0 * \cos(\theta_B) * \sin(\theta_{eq}) * y\right)\right)) \tag{9}$$

where $\Psi\left(\cos\left(k_0 * \cos(\theta_B) * \sin(\theta_{eq}) * y\right)\right)$ can be either 0 or π as the cosine is real. This means that when there is a change on the sign of the cosine, there will be a 180 degrees shift in $\Psi\left(E_{AP}(y)\right)$. This can be also seen in Fig. 3b.

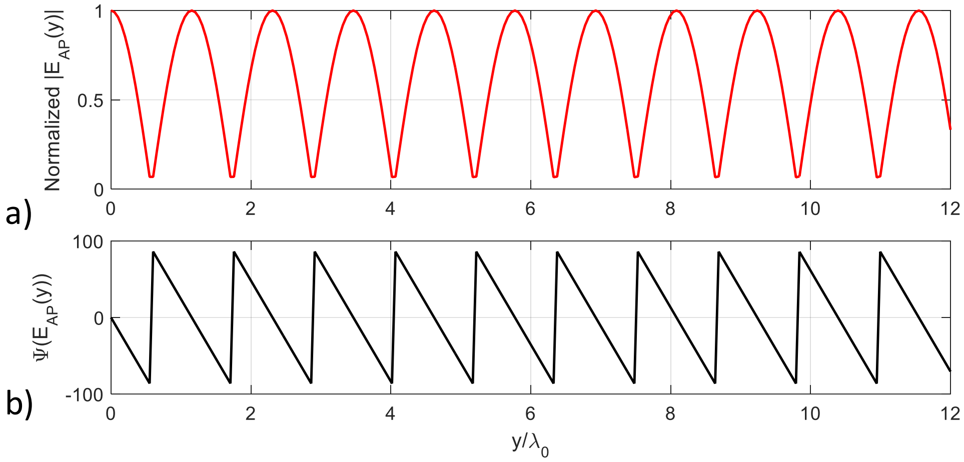

Fig. 3. a) Normalized amplitude of the electric field to generate a quasi-Bessel beam. b) Phase of the electric field to generate a quasi-Bessel beam.

C. Theoretical Bessel Beams

With an aperture illuminations similar to the one shown in Fig. 3, the near fields can be theoretically calculated as described in [14]. This is illustrated in Fig. 4, where different combinations of θ_B, θ_{eq} and L_A are used.

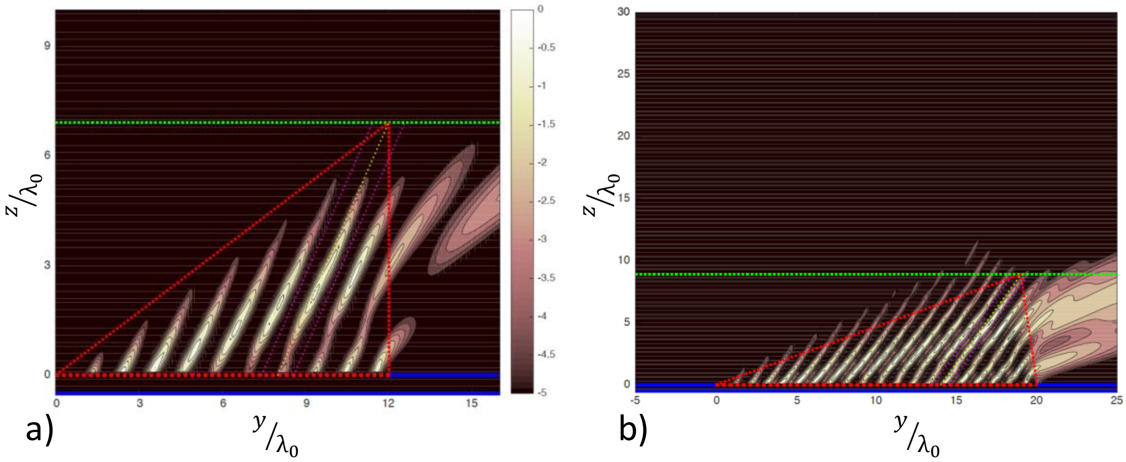

Fig. 4. a) Quasi-Bessel beam with $\theta_B = 30°$, $\theta_{eq} = 30°$ and $L_A = 12\lambda_0$. b) Quasi-Bessel beam with $\theta_B = 30°$, $\theta_{eq} = 35°$ and $L_A = 20\lambda_0$.

It is demonstrated how, using this technique, quasi-Bessel beams with different characteristics can be synthesized.

D. Leaky-Wave Complex Propagation Constant

Once we have the amplitude and the phase of the field along the aperture, we can relate them to the real and imaginary terms of the propagation constant of an equivalent leaky mode:

$$k_{eq}(y) = \beta(y) - j\alpha(y) \tag{10}$$

The imaginary part of the equivalent leaky mode along the antenna length is given by the next relation [11]:

$$\alpha(y) = \frac{1}{2}\frac{\overline{|E_{AP}(y)|}^2}{{}^1/_{\eta^*}\int_0^{L_A}\overline{|E_{AP}(y)|}^2 dy - \int_0^y \overline{|E_{AP}(y)|}^2 dy} \tag{11}$$

In this expression, η represents the radiation efficiency of the launcher, understood as the ratio between the power that is leaked from the launcher and the input power. The rest of the power is absorbed at the end of the launcher.

On the other hand, the real part of the equivalent leaky mode along the antenna length is given by the following expression:

$$\beta(y) = -\frac{\partial\Psi\big(E_{AP}(y)\big)}{\partial y} = k_0 * \sin(\theta_B) * \cos\big(\theta_{eq}\big) \tag{12}$$

Since the second term of $\Psi\big(E_{AP}(y)\big)$ is a constant, its derivative will be zero except for the points where there is a change in the value (0 or π). Thus, $\beta(y)$ will be constant along the launcher except for the points at which there is a 180-degree change of phase of the aperture illumination. In the next section, the proposed topology to achieve the kind of illumination previously obtained and to achieve this phase shift will be explained. For the moment, we will take $\beta(y)$ as a constant with no change along y. The complex propagation constant of the leaky mode corresponding to the illumination in Fig. 3 is plotted in Fig. 5.

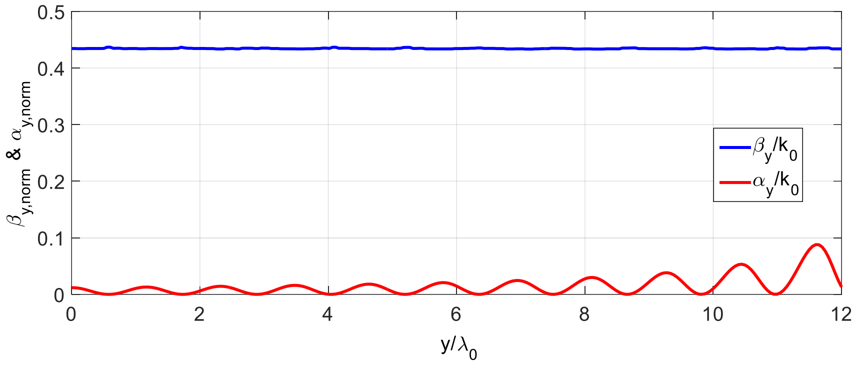

Fig. 5. Real (β) and imaginary (α) parts of the complex propagation constant of the equivalent leaky mode along the launcher aperture.

3. SUBSTRATE-INTEGRATED WAVEGUIDE BASED LEAKY-WAVE LAUNCHER

In order to achieve the illumination obtained in Section 2, a structure which allows for the independent control over $\alpha(y)$ and $\beta(y)$ in necessary. A good example is a leaky-wave antenna (LWA) based on substrate-integrated waveguide (SIW) technology as the one in [15] (also shown in Fig. 6). This structure consists on a SIW where the periodicity of one of its row of vias is increased to allow for some leakage. In this sense, the more separated the vias are, the stronger the radiation rate α is, while the radiation angle will be mainly controlled by the width of the SIW, showing that independent control over α and β is possible by changing two parameters: the width of the antenna W and the periodicity P of one of the rows of vias. The possible modulation of the complex propagation constant along the aperture length with this kind of structure was demonstrated in [16]. With this, it would be possible to synthesize the desired illumination, with the only drawback of the abrupt 180-degree phase shift at the longitudinal positions y where the cosine term in Eq. (7) changes sign. To achieve this, the novel proposed solution is to flip the radiating edge of the structure where the 180° phase shifts are necessary. As illustrated in Fig. 7 – which plots the fields in the transversal plane of the antenna-, the radiated fields flip polarity when the radiating edge is interchanged from one side to the opposite one, thus creating the necessary 180° phase shift.

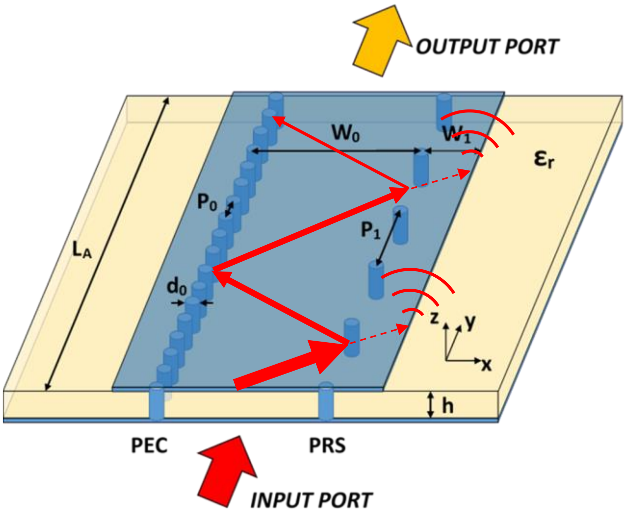

Fig. 6. Substrate-integrated waveguide leaky-wave antenna technology proposed for the 1-D quasi-Bessel beam launcher.

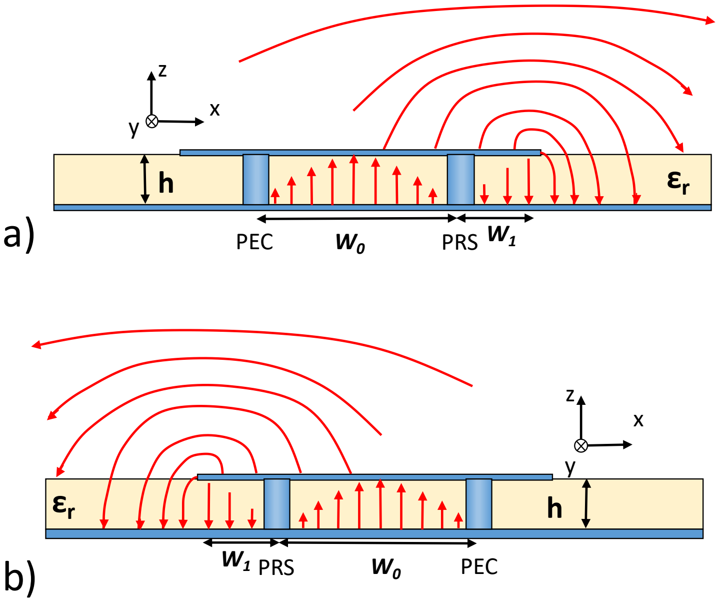

Fig. 7. Radiated field polarity flip by change of the radiating edge.

In Fig. 8 the structure to synthesize the illumination shown in Fig. 3 is depicted for a modulated SIW LWA launcher working at 15 GHz. It can be seen how both *W* and *P* are tapered along the antenna length to obtain the requested modulation of the leaky-wave phase and leakage rates shown in Fig.5, and also it can be observed the change in the radiating edge for the points which request 180 degrees shift in the illumination phase. The structure will radiate into a PPW region, where the high-order Bessel beam should be created. Fig. 9 shows the fields, obtained through full wave simulations, inside the PPW where the quasi-Bessel beam can be identified. To better compare with theory, the results in Fig. 9a are overlapped with the ones in Fig 4a, showing a good matching.

One of the main properties of LWAs is its inherent frequency-scanning behaviour, which means that the radiation angle of a LW, given a structure, will be increased when the frequency of the feeding signal increases, therefore, it is expected that the direction of the high-order Bessel beam also changes with frequency. This behaviour is observed in Fig. 10, where the fields inside the PPW for different frequencies are depicted.

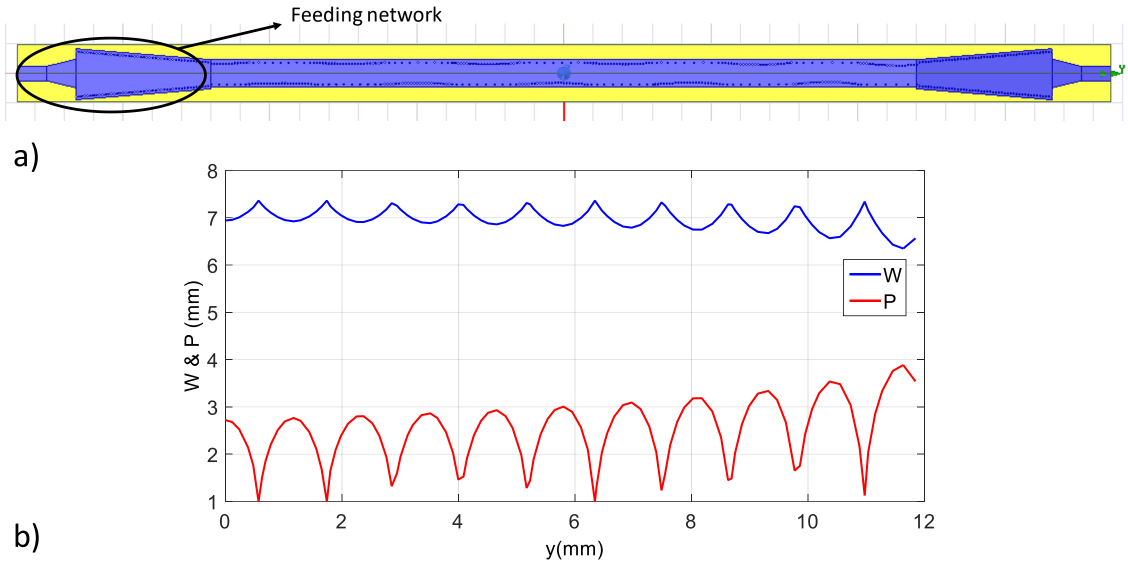

Fig. 8. a) Proposed 1-D quasi-Bessel beam launcher. b) Profile (width and vias periodicity) of the proposed launcher.

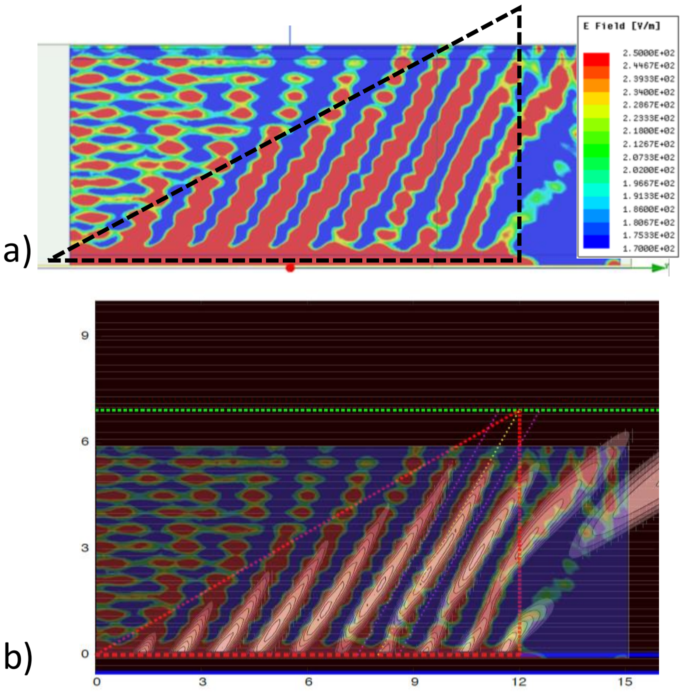

Fig. 9. a) Simulated fields inside the PPW region for a quasi-Bessel beam generation with $\theta_B = 30°$, $\theta_{eq} = 30°$ and $L_A = 12\lambda_0$. b) Comparison between theoretical and simulated results.

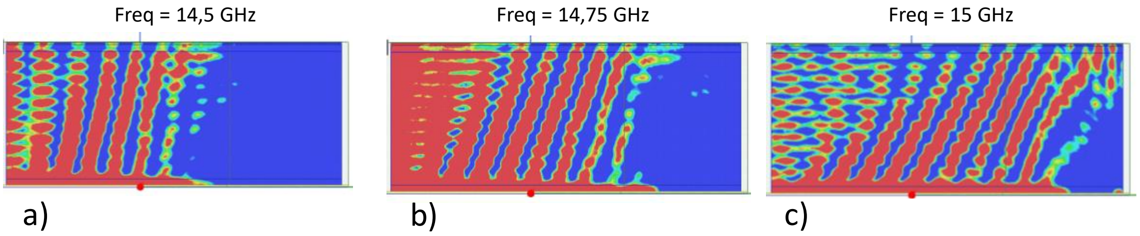

Fig. 10. Frequency-scanning behaviour of the generated quasi-Bessel beam.

4. CONCLUSION

It has been shown how, using a simple structure as a SIW LWA, with integrated feeding network, it is possible to generate frequency-scanned quasi-Bessel beams. The synthesis of a leaky mode which is equivalent to the combination of two propagating leaky waves is possible with a single structure thanks to the independent control over the real and imaginary parts allowed by the structure. Also, the abrupt 180-degress phase shift necessary in the aperture field if possible to achieve by flipping the radiating edge of the SIW LWA. In future works, the manufacturing and measurement of the structure will be performed to validate the design. Also, the number of beams and their width will be studied in detail.

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