# Radiation analysis in the space domain of laterally shielded planar transmission lines:
# 1. Theory

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[1] In this contribution, the propagation and radiation nature of laterally shielded top-open planar waveguides (LShPW) is studied by using a discrete spectrum of parallel-plate waveguide (PPW) $TE^z$ and $TM^z$ modes. This formulation allows for a simple but precise analysis of the propagation and radiation characteristics associated with the leaky-wave modes which exist in such planar transmission lines. An approximate novel interpretation of the radiation mechanism is also given by describing each PPW mode as an inhomogeneous plane wave. Comparisons with previous works grant the accuracy of the proposed method. Besides, the analytical and rigorous nature of this approach makes it very suitable for the practical analysis and design of leaky-wave antennas, as will be discussed in the second part of this work. *INDEX TERMS:* 0609 Electromagnetics: Antennas; 0619 Electromagnetics: Electromagnetic theory; 0624 Electromagnetics: Guided waves; 0644 Electromagnetics: Numerical methods; *KEYWORDS:* leaky-waves, discrete and continuous spectrum, numerical method

## 1. Introduction

[2] Planar transmission lines with open boundaries have been widely studied to describe their radiation leakage properties [*Oliner*, 2000]. In the past literature, the Spectral Domain Approach (SD) [*Oliner*, 1987; *Tsuji et al.*, 1991; *Bagby et al.*, 1993; *Di Nallo et al.*, 1998; *Villegas et al.*, 1999, 2003] has been used to analyze this type of structures. The SD Approach was needed to express the Green's functions in such open structures and analyze the behavior of the constituent leaky-wave modes.

[3] For more closed scenarios, such as open waveguides and laterally shielded planar structures, the Space Domain approach can be used, since side metallic walls support the propagation of parallel-plate waveguide (PPW) modes (whose order is indicated by the index $m$ in this paper). In this way, leaky-wave line sources have been widely studied [*Oliner*, 1993; *Lampariello*, 1997] to obtain versatile and low-loss leaky-wave antennas for microwave and millimeter-wave ranges. In Figure 1, the sketch of these antennas, based on open waveguides and planar technology, is illustrated. It can

be seen that all of them share a parallel-plate stub in their top region.

[4] The principle of operation of all these leaky-wave antennas is the same: PPW $TE^Z$ m = 0 mode is excited due to any type of asymmetry introduced in the original nonradiative waveguide. This PPW m = 0 mode is actually a TEM wave propagating between the PPW, and therefore can radiate at a given angle in the outer free-space region, provided the axial propagation constant is in the "fast-wave" region ($\beta < K_0$) [*Oliner*, 1987]. The parallel-plate stub which appears at the top of all these structures is needed, so higher-order (m > 0) PPW modes, which are below cutoff, cannot reach the top aperture. In this way only the first PPW m = 0 mode contributes to radiation, avoiding undesired cross-polarization and grating lobes effects.

[5] Many different techniques have been used to analyze the leaky-wave modes which can propagate in these open structures. The offset groove-guide (Figure 1a) and the stub-loaded rectangular waveguide (Figure 1b) were studied in *Lampariello et al.* [1987, 1998], respectively. The analysis was carried out by using both a single-mode Transverse Equivalent Network for the main m = 0 PPW mode, and a full-wave Mode Matching technique. Also by applying a Transverse Resonance procedure, the complex propagation properties of a Slitted Asymmetric
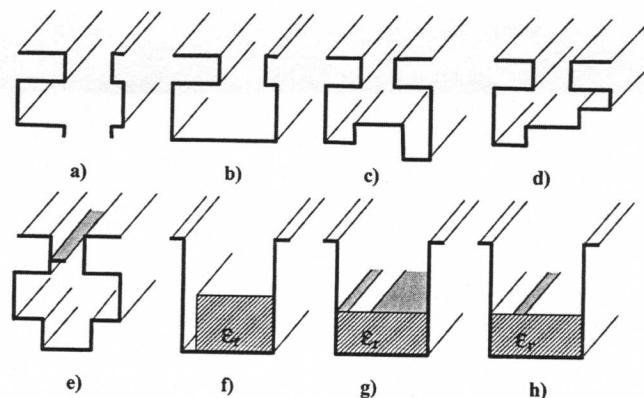
**Figure 1.**  Leaky-wave line sources for antenna applications with parallel-plate stub.

Ridge Waveguide (Figure 1c) [*Frezza et al.*, 1989] and a Stepped Leaky-Wave antenna (Figure 1d) [*Di Nallo et al.*, 1997] were studied.

[6] The groove guide with an asymmetrical conductor strip (Figure 1e) was first studied in *Lampariello and Oliner* [1985] using a small obstacle approximation to obtain a Transverse Equivalent Network. Only the first three PPW modes were considered. The second PPW $TE^Z$ and $TM^Z$ modes (m = 1) are excited in the main groove guide bounded-mode, while the first PPW $TE^Z$ mode (m = 0), created by the coupling strip, is responsible for the radiation. A more accurate full-wave method, based on the Mode Matching procedure, took into account all higher order PPW modes [*Ma and Yamashita*, 1994a], obtaining more precise results for this leaky-wave antenna. Also the Nonradiative Leaky-Wave antenna (Figure 1f) was studied with a multimode generalized Equivalent Transverse Network, in which the Mode Matching procedure was used to obtain the Equivalent Admittance Matrix at the discontinuity [*Shigesawa et al.*, 1986].

[7] Printed circuit versions of this type of leaky-wave antennas have not been studied with so much detail. The slot-line leaky-wave antenna (Figure 1g) was presented by the first time in 1987 [*Lampariello and Oliner*, 1987]. The properties of the main leaky-wave mode were studied with more detail [*Lampariello et al.*, 1990], by using the Transverse Resonance procedure, which neglected higher-order PPW modes effects. The Mode Matching technique for planar laterally shielded transmission lines, like the laterally shielded microstrip line (Figure 1h), has also been used to study higher-order leaky-wave modes [*Ma and Yamashita*, 1994b].

[8] As it can be seen, two main techniques are traditionally used in the analysis of leaky-wave modes in laterally shielded stub-loaded structures: the single-mode Transverse Resonance Procedure (TRP) and the full-wave Mode Matching (MM) technique. The last one, although more time-consuming and costly, is more accurate, since it

does not depend on the precision of the Equivalent Networks developed to model the discontinuities (usually single-mode).

[9] Nevertheless, in the past literature, the length of the PPW stub has been assumed to be so long, that all higher PPW modes below cutoff are understood to have decayed to negligible values at the open end. Therefore, only the main PPW m = 0 mode has been considered to be responsible for the radiation. Although this is the desired working principle in this kind of antennas, it is often necessary to evaluate the impact in the electrical characteristics of the antenna, when higher-order PPW modes contribute to the radiation. Moreover, the radiation of these higher order PPW modes can be used for novel antenna applications, such as circular-polarized leaky-wave antennas.

[10] In this work, the parallel-plate waveguide (PPW) which supports the laterally shielded multilayered planar transmission line, allows to readily expand the total electromagnetic fields of the Laterally Shielded Planar Waveguide (LShPW) by means of a series of PPW modes. In this way, the full-wave Space Domain Green's functions for LShPW can be computed as described in *Gómez and Melcón* [2003b], and an Electric Field Integral Equation (EFIE) can be solved by using the Method of Moments (MoM). In the equivalent transmission lines used in this formulation, all the PPW modes, including higher-order ones, contribute to radiation. Therefore, to the authors' knowledge, it is the first time that the radiation effects of higher-order PPW modes have been taken into account.

[11] In part 1 of this two-part paper, the PPW expansion theory will be reviewed, since it is the base for the Green's functions treatment in the Space Domain developed in the proposed method. Comparisons with previous results for a laterally shielded microstrip leaky-mode [*Lampariello et al.*, 1990; *Baccarelli et al.*, 2002] will be presented to validate this technique. Then, the definitions of what we call propagation and radiation discrete PPW

spectra for a LShPW leaky-mode will be introduced, and the propagation and radiation power flux for a laterally shielded suspended microstrip leaky-wave mode will be shown. With these results, the radiation of LShPW leaky-wave modes will be predicted from an original and novel perspective, which will be explained with detail. The theory of inhomogeneous plane waves [*Tamir*, 1973; *Machác and Zehentner*, 2002] will also be used to clarify the radiation phenomenon for each PPW leaky-wave mode. This theory will lead to very interesting and novel conclusions, which describe how higher-order PPW modes contribute to radiation.

[12] In part 2 of this paper, some analytical expressions based on the discrete PPW spectrum will be presented to measure many interesting propagation and radiation features of the leaky-wave LShPW modes, such as reflection losses (leaky-wave modes below cutoff [*Oliner*, 1987]), radiation efficiency [*Lin and Sheen*, 1997], and polarization purity and coupling with undesired channel-guide leaky-wave modes [*Shigesawa et al.*, 1994]. Following the theory developed, results will be presented for a slot-line leaky-wave antenna, making comparisons with preceding works [*Lampariello et al.*, 1990; *Lin and Sheen*, 1997]. In addition, we will also show the practical usefulness of the theory previously developed with the analysis and design of a laterally shielded planar leaky-wave slot-line antenna.

## 2. PPW Modes Expansion

[13] A laterally shielded top-open suspended microstrip line is shown in Figure 2 as an example of a LShPW, together with the reference axes and the coupled equivalent transverse networks which model the multi-layered cross-section, as described in *Gómez and Melcón* [2003b].

[14] The Space Domain Green's functions for this geometry can be expressed by a discrete spectrum of PPW modes. Using this expansion, an Electric Field Integral Equation (EFIE) can be imposed to the LShPW electric field on the metal strips. This EFIE can be discretized by using the Method of Moments (MoM) to express the currents on the planar metallization, obtaining a linear homogeneous system of equations. To ensure nontrivial solution, the determinant of the MoM matrix must be equal to zero. This determinant is a function of the unknown propagation constant in the axial direction of the open transmission line, $k_Y$. Leaky-wave LShPW modes are characterized by a complex longitudinal propagating constant as follows:

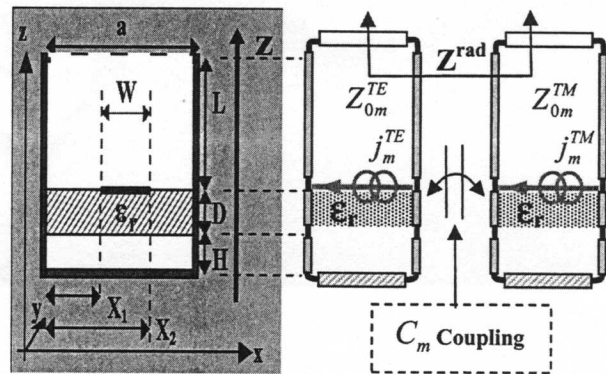$$k_y = \beta_y - j\alpha_y \tag{1}$$



**Figure 2.** Equivalent model for the open waveguide cross section.

By searching for the zeros of the determinant of the MoM matrix as described in *Gómez and Melcón* [2003a], LShPW leaky-modes can be easily found.

## 3. Discrete Propagation and Radiation PPW Spectra

[15] Once the complex $k_y$ solution in equation (1) for a LShPW leaky-mode has been found, its electromagnetic fields can be recovered from the PPW modes expansion as follows:

$$\vec{E}_{LShPW}(x,y,z) = \left[ \sum_{m,p} A_m^p \cdot \hat{V}_m^p(z) \cdot \vec{e}_m^p(x) + \frac{k_{cm}}{j\omega\varepsilon} \right.$$
$$\left. \cdot \sum_m A_m^{TM} \cdot \hat{I}_m^{TM}(z) \cdot \vec{\phi}_m^{TM}(x) \right] \cdot e^{-jk_y y} \tag{2}$$

$$\vec{H}_{LShPW}(x,y,z) = \left[ \sum_{m,p} A_m^p \cdot \hat{I}_m^p(z) \cdot \vec{h}_m^p(x) + \frac{k_{cm}}{j\omega\mu} \right.$$
$$\left. \cdot \sum_m A_m^{TE} \cdot \hat{V}_m^{TE}(z) \cdot \vec{\phi}_m^{TE}(x) \right] \cdot e^{-jk_y y} \tag{3}$$

PPW $TE^Z$ and $TM^Z$ modes are described by the next vector-modal functions, in which a transverse-longitudinal notation with respect to the z-direction has been used to easily obtain the equivalent transmission lines shown in Figure 2:

$$\vec{e}_m^{TE}(x) = \frac{-jk_y \cos(k_{xm}x)\hat{x} + k_{xm}\sin(k_{xm}x)\hat{y}}{N_m}$$
$$\vec{e}_m^{TM}(x) = \frac{+k_{xm}\cos(k_{xm}x)\hat{x} - jk_y\sin(k_{xm}x)\hat{y}}{N_m} \tag{4}$$

$$\vec{h}_m^{(p)}(x) = \hat{z} \times \vec{e}_m^{(p)}(x) \tag{5}$$

$$\vec{\phi}_m^{TE}(x) = \frac{-k_{cm}\cos(k_{xm}x)}{N_m}\hat{z} \qquad \vec{\phi}_m^{TM}(x) = \frac{-k_{cm}\sin(k_{xm}x)}{N_m}\hat{z}$$
(6)

$$k_{xm} = m\frac{\pi}{a}(rad/m) \quad m = 0,1,2\dots$$
(7)

$$k_{cm} = \sqrt{k_{xm}^2 + k_y^2}$$
(8)

where $m$ stands for the PPW mode index, which determines the discrete $k_{xm}$ wavenumber along the x-axis (equation (7)), and $p$ discriminates the TE$^z$ and TM$^z$ polarizations. Furthermore, $V_m(z)$ and $I_m(z)$ are the voltage and current functions in the equivalent coupled-transmission lines described in *Gómez and Melcón* [2003b] and shown in Figure 2, and $N_m$ are normalization factors for the PPW vector-modal functions.

[16] In can be seen in Figure 2 that the top aperture is modeled in the Equivalent Transmission Line network with a modal radiation impedance $Z_m^{RAD}$. In the analysis, the radiation impedance derived by *Marcuvitz* [1951] has been used. An interest feature of the technique proposed, is that the Marcuvitz radiation impedance is applied not only to the dominant PPW m = 0 mode, but also to higher-order PPW modes. This is accomplished by first computing the angle of propagation of each PPW mode inside the Parallel-Plate Waveguide, and then applying the radiation impedance of Marcuvitz for oblique incidence [*Marcuvitz*, 1951]. This approximation for modeling the top aperture has shown to be accurate for the structures studied. More rigorous results can be obtained by solving a more complex problem in which the top aperture is modeled by equivalent magnetic currents [*Joubert and Malherbe*, 1997].

[17] The coefficients $A_m$ used in equations (2) and (3) can be computed from the currents on the strips once the MoM homogeneous system has been solved. These coefficients $A_m$ form what we call "Discrete Propagation PPW spectrum" since they account for the weight in which each excited PPW mode contributes to the total LShPW leaky-mode.

[18] Moreover, these PPW modes excited in the strip can create an equivalent magnetic current at the top aperture. This magnetic current, in turn, is responsible for the radiation according to the Equivalence Principle [*Balanis*, 1982]. As a result, the total equivalent magnetic current creates the radiation pattern of the LShPW leaky-mode in the outer free-space region. Since each PPW mode contributes to the radiation with its corresponding equivalent magnetic current, the radiation pattern of the LShPW leaky-mode can be decomposed as a sum of far-field contributions of each PPW mode. The radiated patterns of each PPW mode can be obtained from the equivalent magnetic current created by the PPW

electric field on the top aperture. Using the Green's functions for an infinite wire of magnetic current over a ground plate (Hankel function), the next analytical expressions are obtained to describe the far-field radiation pattern in the X-Z plane for a LShPW leaky-mode:

$$D(\phi) = G \cdot \left[\left|\sum_{m=0}^{\infty} D_m^y(\phi)\right|^2 + \left|\sum_{m=0}^{\infty} D_m^{\phi}(\phi)\right|^2\right]$$
(9)

$$D_m^{\phi}(\phi) = B_m^{\phi}(\phi) \cdot \sin(\phi) \cdot R_m(\phi) \cdot e^{-j\beta_\rho\frac{a}{2}\sin(\phi)}$$
(10)

$$D_m^y(\phi) = B_m^y(\phi) \cdot \cos(\phi) \cdot R_m(\phi) \cdot e^{-j\beta_\rho\frac{a}{2}\sin(\phi)}$$
(11)

$$B_m^{\phi}(\phi) = \frac{-C_m^{\phi} \cdot 2j\beta_\rho}{k_{xm}^2 - \beta_\rho^2\sin^2(\phi)} \qquad B_m^y(\phi) = \frac{C_m^y \cdot 2k_{xm}}{k_{xm}^2 - \beta_\rho^2\sin^2(\phi)}$$
(12)

$$C_m^{\phi} = -A_m^{TE} \cdot \hat{V}_m^{TE}(L) \cdot \frac{jk_y}{N_m} + A_m^{TM} \cdot \hat{V}_m^{TM}(L) \cdot \frac{k_{xm}}{N_m}$$
(13)

$$C_m^y = +A_m^{TE} \cdot \hat{V}_m^{TE}(L) \cdot \frac{k_{xm}}{N_m} - A_m^{TM} \cdot \hat{V}_m^{TM}(L) \cdot \frac{jk_y}{N_m}$$
(14)

$$R_m(\phi) = \begin{cases} -j\sin\left(\beta_\rho\frac{a}{2}\sin\phi\right) & ; m \text{ even} \\ +\cos\left(\beta_\rho\frac{a}{2}\sin\phi\right) & ; m \text{ odd} \end{cases}$$
(15)

where G is a constant given by

$$G = \frac{2\pi f\frac{k_0}{|k_0|}}{P_{RAD_{XZ}}}$$
(16)

Moreover, $P_{RADXZ}$ is the total radiated power in the X-Z plane, and $k_\rho$ is the free-space transverse wavenumber given by

$$k_\rho = \sqrt{k_0^2 - k_y^2} = \beta_\rho + j\alpha_\rho$$
(17)

It can be seen that the whole radiation pattern of a leaky-mode can be obtained by adding in complex form the contribution of all PPW modes as shown in equation (9). To each individual PPW mode there is the corresponding individual radiation pattern for both $E^{\phi}$(10) and $E^y$(11) polarizations. In Figure 3, the radiation patterns for the first four PPW modes are illustrated.

[19] As it can be seen, the main PPW m = 0 mode radiates a purely $E^{\phi}$-polarized broadside beam without side lobes. On the contrary, higher-order PPW modes radiate with side lobes and in both polarizations. The interference between the contributions of each PPW mode will determine the total LShPW leaky-wave mode radiation pattern. From expressions (9)–(12), it can be inferred that the associated coefficients $B_m^{\phi}$ and $B_m^Y$ in
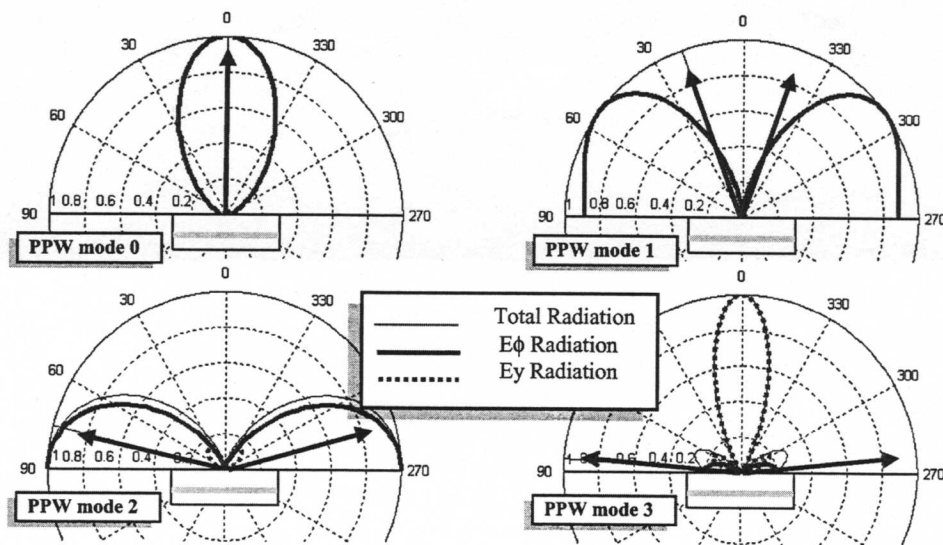
**Figure 3.** Radiation diagrams for the first four PPW modes.

equation (12), particularized at the point $\phi = \pi/2$, contain the weight with which each PPW mode contributes to the total radiation. Therefore, these $B_m(\phi = \pi/2)$ coefficients form the so called "Discrete Radiation spectrum" and can give us useful information on the nature of the radiation mechanism of the structure, as it will be illustrated in the next sections.

## 4. Inhomogeneous PPW Leaky-Mode Plane Waves

[20] For each PPW radiated mode (for each discrete value of $m$), equations (10)–(15) represent a continuous superposition (continuous spectrum) of cylindrical leaky waves [*Ip and Jackson*, 1990; *Rozzi and Mongiardo*, 1997]. They can propagate both in the y axis (see $k_y$ in equation (1)) and in the radial $\rho$ direction according to the transverse propagation factor shown in equation (17).

[21] Following the discrete nature of the radiation pattern (it is formed by adding the contributions of all excited PPW modes), the radiation characteristics of a LShPW leaky-wave mode can also be approximated by considering that it is produced by a set of inhomogeneous plane waves [*Machác and Zehentner*, 2002]. Using this point of view, each PPW mode inside the waveguide can be decomposed into two inhomogeneous plane waves. From the basic PPW modal expressions (equations (4)–(8)), and taking into account the characteristic impedance in the equivalent transmission lines, it can be obtained one of these plane waves for each PPW

mode (the other wave would be identical but interchanging $+k_{xm}$ by $-k_{xm}$ in all the expressions):

$$\vec{E}^{(p)}_m{}_{PPW} = \vec{E}^{(p)}_{0m} \cdot e^{-j\vec{k}\cdot\hat{r}} = \vec{E}^{(p)}_{0m} \cdot e^{-\vec{\alpha}\cdot\hat{r}} \cdot e^{-j\vec{\beta}\cdot\hat{r}} \quad (18)$$

$$\vec{H}^{(p)}_m{}_{PPW} = \vec{H}^{(p)}_{0m} \cdot e^{-j\vec{k}\cdot\hat{r}} = \vec{H}^{(p)}_{0m} \cdot e^{-\vec{\alpha}\cdot\hat{r}} \cdot e^{-j\vec{\beta}\cdot\hat{r}} \quad (19)$$

$$\vec{E}^{TE}_{0m} = \left[-jk_y\hat{x} + jk_{xm}\hat{y}\right]\frac{\omega\mu}{k_{zm}} \quad (20)$$

$$\vec{H}^{TE}_{0m} = \left[-jk_{xm}\hat{x} - jk_y\hat{y} + j\frac{k^2_{cm}}{k_{zm}}\hat{z}\right] \quad (21)$$

$$\vec{E}^{TM}_{0m} = \left[+k_{xm}\hat{x} + k_y\hat{y} - \frac{k^2_{cm}}{k_{zm}}\hat{z}\right] \quad (22)$$

$$\vec{H}^{TM}_{0m} = \left[-k_y\hat{x} + k_{xm}\hat{y}\right]\frac{\omega\varepsilon}{k_{zm}} \quad (23)$$

We are interested to know which is the propagation direction of these waves inside the parallel-plate stub. For this purpose, the complex wave number in the z-direction can be easily derived from $k_y$ and $k_{xm}$:

$$k_{zm} = \sqrt{k^2_0 - (k^2_{xm} + k^2_y)} = \beta_{zm} + j\alpha_{zm} \quad (24)$$

When computing the propagation direction of these plane waves, much care must be taken, since these plane waves are nonuniform due to their complex propagation vector [*Tamir*, 1973]. In our case, from equations (1), (7), and (24) we obtain:

$$\vec{k} = k_{xm}\hat{x} + k_y\hat{y} + k_{zm}\hat{z} = \vec{\beta} - j\vec{\alpha}$$
$$= \left(k_{xm}\hat{x} + \beta_y\hat{y} + \beta_{zm}\hat{z}\right) - j\left(\alpha_y\hat{y} - \alpha_{zm}\hat{z}\right) \quad (25)$$

Consequently, it must be distinguished a propagation or phase vector $\vec{\beta}$ and an amplitude or attenuation vector $\vec{\alpha}$. Furthermore, the complex Poynting Vector of each PPW mode inhomogeneous plane wave can be computed as:

$$\vec{S}_m^{(p)}{}_{PPW} = \frac{1}{2}\left[\vec{E}_m^{(p)} \times \vec{H}_m^{(p)*}\right] = \vec{P}_m^{(p)}{}_R + j\vec{P}_m^{(p)}{}_I \quad (26)$$

From expressions (18)–(23), the power propagation direction of the plane waves which form the PPW modes can be obtained by extracting the real part vector $\vec{P}_{mR}$. This direction is in general different from the phase-front propagation direction given by $\vec{\beta}$. The objective, then, is to obtain the direction of the radiated power associated to each PPW mode from $\vec{P}_{mR}$. Applying basic theory of inhomogeneous plane wave propagation, the angle in which the electromagnetic energy of each PPW mode propagates can be computed. The associated $TE^z$ and $TM^z$ PPW inhomogeneous plane waves propagate at an inclination angle $\phi_{INSm}$ inside the waveguide (see Figure 4) given by

$$\tan\left(\phi_{INSm}^{TE}\right) = \frac{real\left(k_{xm}\frac{\omega\mu}{k_{zm}} \cdot \frac{\left(k_{xm}^2 + k_y^2\right)^*}{k_{zm}^*}\right)}{real\left(\frac{\omega\mu}{k_{zm}} \cdot \left(k_{xm}^2 + \left|k_y\right|^2\right)\right)}$$

$$\tan\left(\phi_{INSm}^{TM}\right) = \frac{real\left(k_{xm}\frac{\omega\varepsilon}{k_{zm}^*} \cdot \frac{\left(k_{xm}^2 + k_y^2\right)}{k_{zm}}\right)}{real\left(\frac{\omega\varepsilon}{k_{zm}^*} \cdot \left(k_{xm}^2 + \left|k_y\right|^2\right)\right)} \quad (27)$$

It should be noted that these inhomogeneous plane waves exist only inside the waveguide. Once they reach the top aperture, the radiation of each PPW mode is actually created by a continuous spectrum of cylindrical waves in the outer region as given by (10) and (11). This continuous spectrum in the outer-region can be approximated by two inhomogeneous plane waves, as can be seen in Figure 4, leading to an approximated angle of radiation $\phi_{RAD}$.

[22] This approximation allows the understanding of the relation between the guided discrete spectrum and the radiated continuous spectrum in a novel manner. In the past literature, some efforts [*Rozzi and Mongiardo*, 1997;
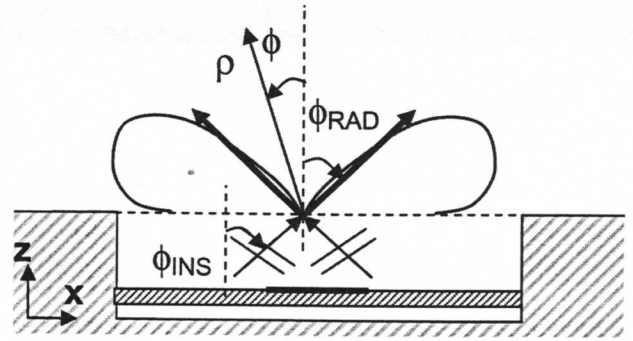


**Figure 4.** Inhomogeneous plane waves associated to each PPW mode inside the waveguide and continuous spectrum of cylindrical waves in the outer region.

*Tzuang*, 1999] have been made to treat the waveguiding and radiation problems from a unifying point of view. The theory derived in this paper is an important step towards this objective.

[23] The equivalent outgoing plane wave direction computed using equation (27) is shown by an emerging arrow superposed for each PPW radiation pattern in Figure 3. As expected, the direction does not exactly agree with the rigorous maximum radiation direction derived from (9–15). In fact, as the order of the PPW mode increases, the approximation is less valid. This is due to the appearance of side lobes from a higher diffraction effect in the corners of the aperture, as it can be seen in the radiation pattern of PPW m = 3 mode in Figure 3. However, the theory is useful to easily compute an approximate value of the radiation direction for each mode.

[24] Moreover, another important parameter is the equivalent reflection coefficient at the top aperture for each PPW mode, $\rho_m^{(p)}$. This reflection coefficient depends on both the radiation impedance $Z_m^{RAD}$ and the characteristic impedance $Z_{0m}^{(p)}$, which are computed for each $TE^z$ and $TM^z$ polarized PPW mode with the procedure described in *Gómez and Melcón* [2003b]. As mentioned, the equivalent radiation impedance is obtained from *Marcuvitz* [1951], but modified accordingly to the angle of propagation of each PPW mode given by equation (27). In this way, this radiation impedance is easily extended for higher-order PPW modes, and the equivalent modal reflection coefficient can be computed as:

$$\rho_m^{(p)} = \frac{Z_m^{RAD} - Z_{0m}^{(p)}}{Z_m^{RAD} + Z_{0m}^{(p)}} \qquad m = 0, 1..p = TE^z, TM^z \quad (28)$$

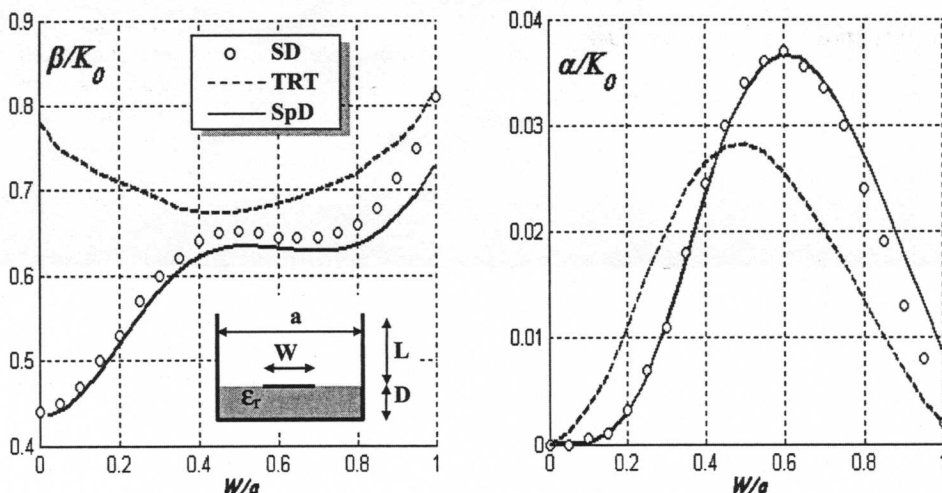This reflection coefficient gives very important information of the degree in which the energy of each PPW

**Figure 5.** Comparisons on the accuracy between our implementation and the other two techniques.

inhomogeneous plane wave is radiated or reflected by the top discontinuity at the aperture.

## 5. Results

[25] For validation purposes of the theory presented in this paper, we present in Figure 5 results for the normalized phase constant ($\beta/K_0$) and the normalized attenuation constant ($\alpha/K_0$) as a function of the ratio W/a, for a millimeter-wave microstrip higher-order leaky-mode. The microstrip structure was analyzed at the frequency of 50 GHz, with the next dimensions: a = 4, 5mm, D = 1, 59mm, $\varepsilon_r$ = 2, 56. The width of the centered strip "W"

was varied to compare the accuracy between three different techniques: a Transverse Resonance Technique (TRT) based on a single-mode Transverse Equivalent Network [*Lampariello et al.*, 1990], a full-wave Spectral Domain (SD) method [*Baccarelli et al.*, 2002], and our full-wave Space Domain (SpD) technique.

[26] As mentioned before, the TRT is based on an Equivalent Tee-Network representation of the transverse discontinuity for PPW m = 0 mode. This approximate model is more accurate for large W/a values, and gives wrong results for small strip widths (see Figure 5). It can be observed that our method (with 100 PPW modes in the fields' expansion) agrees very well with the Spectral
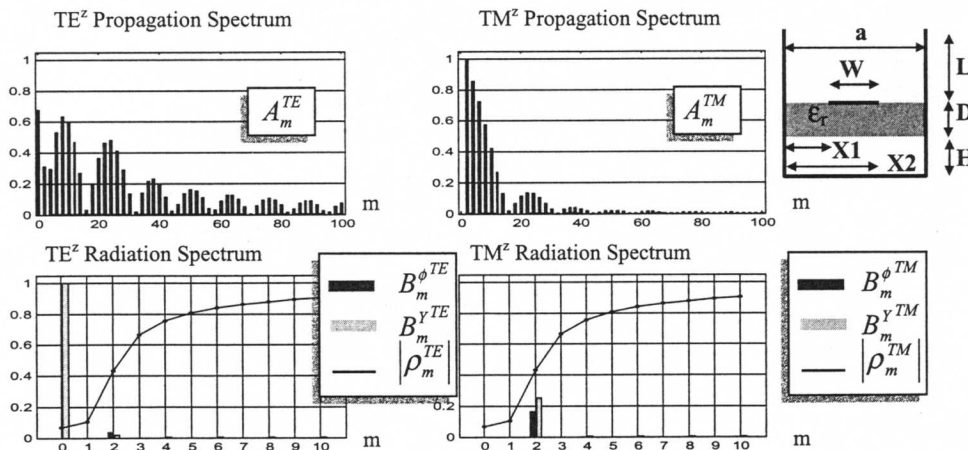


**Figure 6.** Propagation and radiation discrete PPW spectra for centered microstrip leaky-wave mode.
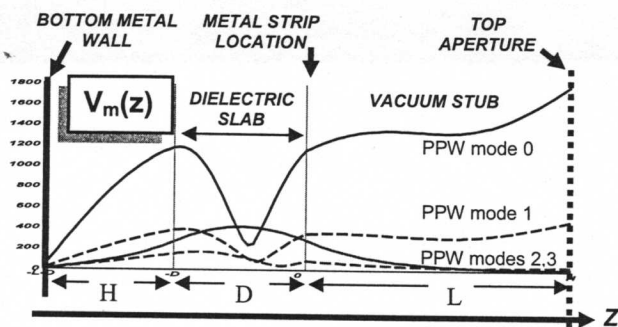
**Figure 7.** PPW modes behavior in z-axis, Vm(z), from the strip to the aperture.

Domain results presented. However, some differences between both results can be observed in Figure 5 for large values of $W/a$. This is due to the fact that we take into account the finite length $L$ of the stub. As will be explained with more detail in part 2 of this paper, by assuming an infinite stub, the effects of the reflected wave in the top aperture are no taken into account. Also the coupling effects with channel-guide leaky-modes are not considered in the SD approach, while they are accurately modeled with our technique.

[27] Next, a leaky mode of a laterally shielded suspended microstrip line is studied. The dimensions according to Figure 6 are as follows: a = 0.508mm, L = 0.254mm, D = 0.127mm, H = 0.127mm, $\varepsilon_r$ = 5, and W = 0.08mm. The strip is centered, and therefore X1 = 0.214mm and X2 = 0.294mm. The frequency of analysis is 490 GHz. In Figure 6, its propagation ($A_m$) and radiation ($B_m$) discrete spectra are shown, together with the modulus of the equivalent reflection coefficient for each PPW $TE^Z$ and $TM^Z$ mode.

[28] As it can be seen, although the propagation spectrum $A_m$ needs a high amount of PPW $TE^z$ and $TM^z$ modes (convergence is achieved with 100 modes), only a few of them (2 modes) contribute to the electromagnetic radiation according to the discrete radiation spectrum coefficients $B_m$. The reason is that only these few modes can excite an equivalent magnetic current in the top-aperture, located at a distance $L$ mm far from the metal-strip where they are excited. In Figure 7 it can be observed the behavior of the equivalent modal voltage function $V_m(z)$. Higher-order PPW modes are below cutoff in the vacuum medium, and do not contribute to radiation since they do not excite the top aperture.

[29] It is worth mentioning a key effect which plays a fundamental role in the determination of the weight in which each PPW mode contributes to radiation ($B_m$ coefficients). PPW leaky-wave modes are known to be inhomogeneous waves with an "improper" or "nonspectral" behavior in the transverse direction [*Tamir,*

1973], which in our case corresponds to the z-axis. This definition is due to the apparently anomalous behavior of a wave whose amplitude grows as it travels from the source to the top aperture, as can be depicted from the transverse $k_{zm}$ wavenumber in equation (24). However, this outgoing wave finds the aperture discontinuity, and a reflected inhomogeneous plane wave which grows from the aperture to the strip is created. The amplitude of this reflected wave depends on the value of the reflection coefficient for each PPW mode, $\rho_m$, defined in (28). If the absolute value of $\rho_m$ is closed to one, the standing inhomogeneous wave which is created in the stub will have the shape of a customarily wave below cutoff, as can be seen for PPW modes 2 and 3 in Figure 7. On the contrary, the "improper" growing behavior will be more evident if the reflected coefficient $\rho_m$ is low. In this situation only the growing incident wave is presented in the stub, as can be seen with PPW m = 0 mode of Figure 7.

[30] The absolute value of the reflection coefficient ($|\rho_m|$) is represented also in Figure 6. It can be seen how it tends to one as the PPW mode order $m$ is higher. In this way, it is also confirmed that higher-order PPW modes do not contribute to radiation, since they are totally reflected back at the top aperture. It is also confirmed that higher-order PPW leaky modes are below cutoff in
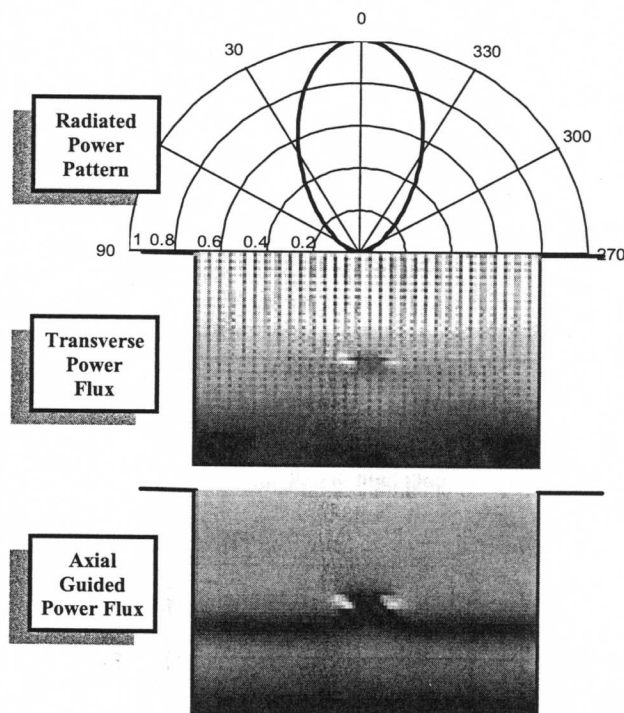


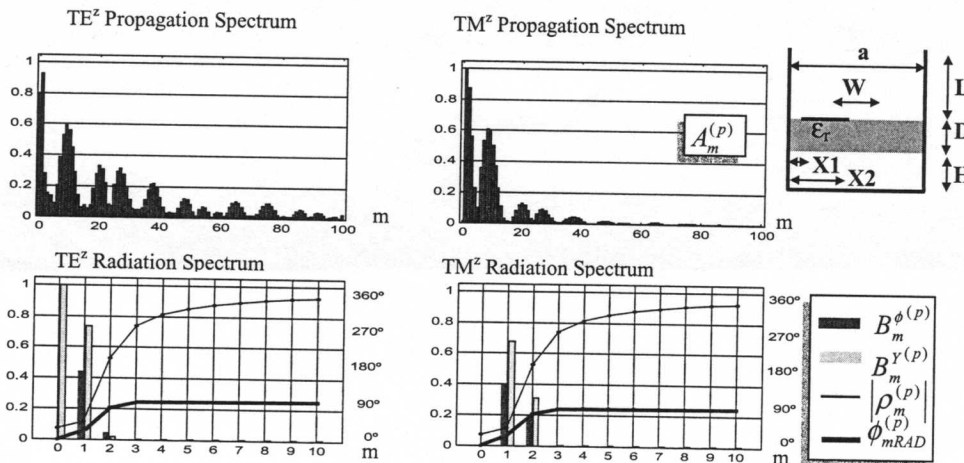**Figure 8.** Guided and radiated power patterns for centered strip.

**Figure 9.** Propagation and radiation discrete PPW spectra for not centered microstrip leaky-wave mode.

the stub region. Only the few first PPW modes are responsible for the radiation mechanism, since the equivalent reflection coefficient is not so high, and a part of their energy can travel through the top-aperture towards the outer free-space.

[31] The total radiation pattern of this LShPW leaky mode can be observed in Figure 8, where also a 2-D plot of the longitudinal and transverse Poynting Vector is shown. As it can be derived from the radiation spectrum (Figure 6), TE-polarized PPW $m = 0$ mode is the one which contributes the most to the radiation, followed by TM-polarized $m = 2$ mode. The individual normalized radiation patterns of these two modes were shown in Figure 3. As it can be seen, the radiated power beam of the LShPW is broadside directed and $E\phi$ polarized, since the main radiated PPW mode 0 has these characteristics. The influence of PPW mode 2 is very small, due to its low radiation coefficient $B_m$ for $m = 2$.

[32] In Figure 9, it can be observed the PPW modes propagation and radiation discrete spectrum for the same Leaky Wave as before, but with the metal strip placed closed to the left side wall, with $X1 = 0.014$mm and $X2 = 0.094$mm. Now there are three PPW modes which contribute to the radiated field. Higher-order PPW modes contribute to both $E^\phi$ and $E^y$ polarizations, but the fundamental TE $m = 0$ mode can radiate only in $E^\phi$ polarization. Also in Figure 9, the approximated radiation angle ($\phi_{RAD}$) is plotted for each PPW mode using equation (27). It can be checked how, as the order of the mode increases, the outgoing angle tends to 90 degrees. The PPW modes which do not radiate have indeed a power direction of 90°, since all the power is guided in the y-direction and none is radiated. This nonradiation phenomenon for higher-order PPW modes is in accor-

dance with the behavior of the reflection coefficient ($|\rho_m|$) previously mentioned.

[33] Therefore, it can be concluded that higher-order PPW modes provide less radiation as the order $m$ increases, since they are totally reflected in the aperture
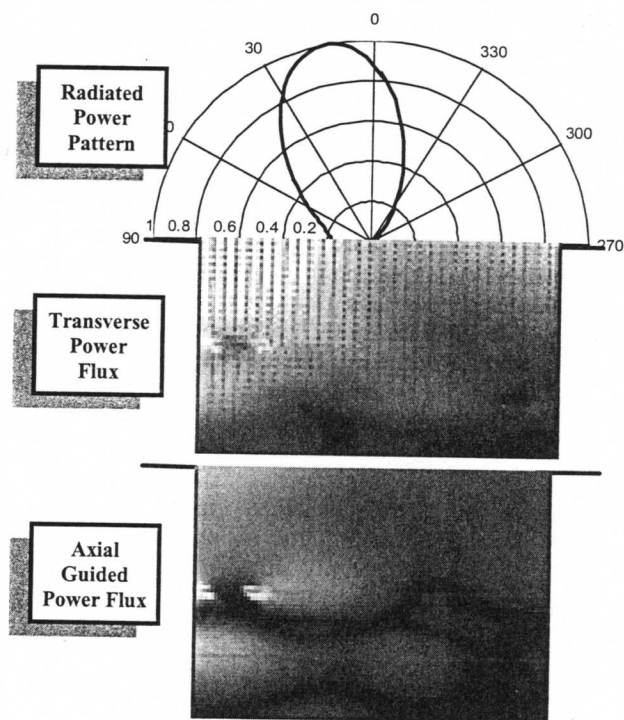


**Figure 10.** Guided and radiated power patterns for displaced strip.

and propagate longitudinally. These physical effects create the same field pattern of a standard mode below cutoff, although PPW leaky-modes are nonspectral growing inhomogeneous waves. To the authors' knowledge, previous works have never taken into account the correct reflection coefficient at the aperture for higher-order modes [*Shigesawa et al.*, 1986; *Ma and Yamashita*, 1994a, 1994b].

[34] In Figure 10, the radiated and guided power patterns for this displaced Leaky Wave Mode are shown. The total radiation pattern has been tilted due to the asymmetrical interference of the second PPW mode (m = 1) radiated fields. The transverse Poynting vector inside the open waveguide also shows this asymmetry to the left. As it can be seen, the novel point of view based on the contribution of each discrete PPW mode can give much information on the propagation and radiation mechanism of leaky-modes in laterally shielded structures. In part 2 of this paper, novel and useful applications are derived for the analysis and design of practical leaky-wave planar antennas.

## 6. Conclusions

[35] In this contribution, the propagation and radiation characteristics of Leaky-Wave Modes in laterally shielded planar waveguides are studied by a novel approach based on the Space Domain PPW modes expansion. This method introduces the discrete PPW propagation and radiation spectra for each leaky-wave mode, and allows a direct interpretation of the radiation phenomenon by a superposition of modal PPW radiated fields. Each PPW mode far-field radiated pattern is created by a continuous spectrum of cylindrical inhomogeneous waves. However, some interesting features, as the maximum radiation direction or the total reflection phenomena for nonradiative PPW higher-order modes, can be approximated by decomposing each PPW mode by two inhomogeneous plane waves. With this theory, the discrete PPW radiation spectrum can be related directly to the continuous radiation spectrum for this type of open-waveguide leaky-modes, in an attempt to unify the theory of guiding and radiation phenomena. Useful results are obtained for a laterally shielded top-open suspended microstrip waveguide, but this novel analysis method is extensible to any laterally shielded multilayered and multiconductor open transmission line.

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