Radiation analysis in the space domain of laterally shielded planar transmission lines:
2. Applications

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[1]   The second part of this two-piece paper applies the method developed in part 1 to show some interesting features for the practical analysis and design of leaky-wave antennas. Novel analytical formulas based on the propagation and radiation discrete Parallel-Plate Waveguide (PPW) spectrum coefficients described in the first part are presented, in order to calculate some interesting propagation and radiation aspects of leaky-wave modes. These formulas will be used in the design of a slot-line millimeter-wave band leaky-wave antenna, measuring parameters such as reflection and radiation losses, polarization purity, and coupling with undesired channel-guide leaky modes. Comparisons with previous works for slot-line leaky-wave modes are presented, together with novel and interesting results, showing the usefulness of the proposed approach for the design of practical leaky-wave laterally shielded planar printed antennas.   *INDEX TERMS:* 0609 Electromagnetics: Antennas; 0624 Electromagnetics: Guided waves; 0619 Electromagnetics: Electromagnetic theory; 0644 Electromagnetics: Numerical methods; *KEYWORDS:* leaky-waves antennas, modal method

1. Introduction

[2] As it was illustrated in the first part of this work [*Tornero and Melcón*, 2004b], many designs of leaky-wave antennas for microwave and millimeter-wave bands are based on nonradiative open transmission lines, in which an asymmetry is introduced to provide radiation, and a stub of parallel plates is located just before the top aperture to guide the energy. These antennas are of simple design so that they allow easy fabrication to be used in the millimeter-wave range, have low losses, and provide an easy interconnection and feeding mechanism with standard waveguide and planar circuitry technologies [*Oliner*, 1987]. Furthermore, the desirable electrical features involve the excitation of a leaky-wave mode in which both the phase constant and the attenuation constant of its complex propagation wavenumber can be controlled independently. In this way, the radiated beam direction and the beam width can be adjusted

separately by modifying different geometrical parameters of the antenna [*Oliner*, 1993].

[3] In Figure 1, a laterally shielded slot-line antenna is presented with its main structural dimensions. The bounded main mode of this open waveguide is represented in Figure 1a with its electric field. As explained in part 1 of the paper, the mode can be viewed as formed by a set of higher-order PPW modes ($m > 0$), all of them being below cutoff in the vacuum medium. This bounded mode becomes leaky due to some kind of asymmetry (in this case by placing the slit asymmetrically with respect to the side metallic walls), which is responsible for the excitation of the PPW $m = 0$ mode. This PPW $m = 0$ leaky mode is actually a TEM inhomogeneous plane wave which is not below cutoff in the stub, and therefore propagates between the parallel plates with no inclination in the X-Z plane ($m = 0 \rightarrow k_{xm} = 0 \rightarrow \phi_{RAD} = 0$), and can radiate at a given elevation angle θ in the outer free-space region, as shown in Figure 1b. Moreover, as it was demonstrated in part 1 of the paper, higher-order ($m > 0$) PPW modes contribute to both E^ϕ and E^y polarizations and have side lobes in the X-Z plane, while the first PPW $m = 0$ mode radiates only in a broadside beam with pure E^ϕ polarization. To achieve high radiation purity with
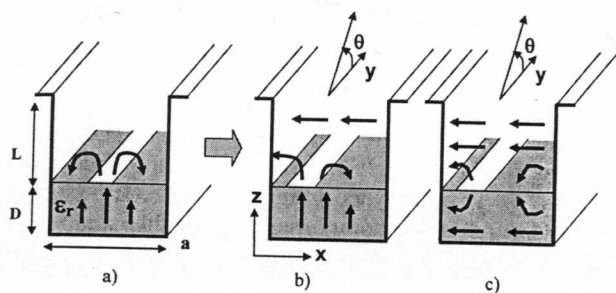
Figure 1. Slot-line antenna electric field for bounded mode in symmetrical structure. (a) Desired leaky-wave mode and (b) undesired channel guide leaky-wave mode (c) in asymmetrical structure.

only E^ϕ polarization, it would then be desirable that only the main PPW $m = 0$ mode could reach the top open aperture. That is why the stub of length L is located in the top of this type of leaky-wave antennas (see Figure 1). This stub is responsible for two main functions: the first one is to achieve that all higher-order PPW modes become below cutoff in the stub region, and the second one is to avoid that these evanescent higher-order PPW modes could excite the top aperture. The first task can be achieved by making the top stub narrower than the main waveguide, or by using a higher dielectric-constant substrate in the main waveguide, while leaving empty the top stub (which is the case of the slot-line antenna shown in Figure 1). The second task is obtained by making the height L of the stub long enough, so that the fields of higher-order PPW modes have decayed to negligible values at the top aperture.

[4] From this last point of view, the stub is necessary in order to obtain a pure E^ϕ polarized broadside beam, avoiding undesired grating lobes and cross-polarization [*Lampariello and Oliner*, 1987]. Although the finite length of this stub has been taken into account in the analysis of leaky-wave modes in various types of structures [*Shigesawa et al.*, 1986; *Lampariello et al.*, 1998; *Ma and Yamashita*, 1994], the stub has always been assumed to be so long that all higher PPW modes below cutoff are considered to have decayed to negligible magnitudes at the open end. Therefore, in previous works, only the main PPW $m = 0$ mode was responsible for the radiation mechanism, and the contribution of higher-order PPW modes were neglected. With the technique proposed in part 1 of this paper, higher-order contributions to radiation are calculated for the first time, and a novel quantitative measurement of the degree in which the radiation is purely due to the desired PPW $m = 0$ mode has been implemented (radiation purity with only E^ϕ component). The proposed method treats rigorously the slot-line discontinuity inside the Parallel-Plate Wave

guide by formulating a Magnetic Field Integral Equation which is solved using the Green's functions developed in part 1. Besides, the aperture at the end of the Parallel-Plate Waveguide can be modeled with the well-known approximated radiation impedance described in [*Marcuvitz*, 1951]. This equivalent circuit is valid in the range $a/\lambda_{zm} < 1$, being λ_{zm} the wavelength in the transverse direction for each PPW mode. This condition must be checked to maintain the accuracy of the results obtained.

[5] As it has been explained, the length of the stub L can be critical. One could think that making it long enough, all higher-order PPW modes would not contribute to radiation, and the desired broadside pattern in the X-Z plane with no cross polarization would be achieved without any difficulty. However, the finite length of the stub implies the appearance of a new type of leaky-wave modes, called channel guide leaky modes [*Shigesawa et al.*, 1986, 1994; *Lampariello et al.*, 1998]. The presence of this type of modes modifies the propagation and radiation properties of the desired leaky mode when both modes are coupled, thus corrupting the working radiation mechanism of the antenna. The desired leaky mode electric field is represented in Figure 1b, while in Figure 1c it can be seen the undesired channel guide field in the slot-line antenna. Both of them share a horizontal electric field pattern in the stub, which can couple under particular conditions, as it will be shown in this paper.

[6] The theory described in part 1 of this paper [*Tornero and Melcón*, 2004b] will be applied for the analysis and design of the slot-line millimeter-wave leaky-wave antenna shown in Figure 1, first studied by *Lampariello and Oliner* [1987]. In the context of the analysis of its propagation and radiation characteristics, new interesting features will be calculated in a simple and analytical way based on the discrete PPW propagation and radiation spectra defined in part 1 of the paper.

[7] First of all, the dispersion characteristic for the normalized phase constant will be computed, showing the undesired coupling effect with a channel guide leaky mode. It will be shown how this coupling can be avoided by varying the stub length L. Novel expressions will be computed to evaluate this undesired power coupling effect. Furthermore, we will evaluate what we call the radiation purity to show the trade-off which exists between these two important parameters. We understand by radiation purity the degree in which only the main PPW $m = 0$ mode contributes to radiation, avoiding higher-order PPW modes side lobes and cross-polarization effects. Also some other novel and interesting propagation and radiation characteristics of the desired leaky mode will be easily obtained, such as reflection and radiation losses as a function of the frequency, which are of much help in order to distinguish the practical frequency range of usage of the antenna. Comparisons with results and conclusions for slot-line antennas exist-
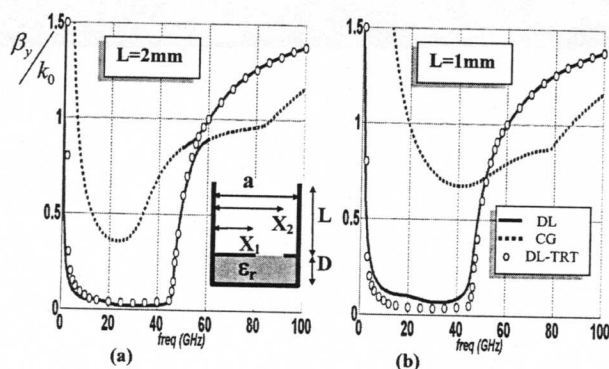
Figure 2. Normalized phase constant for desired (DL) and channel guide (CG) leaky modes for L = 2 mm and L = 1 mm. Results for DL analyzed with Transverse Resonance Technique [*Lampariello et al.*, 1990] (DL-TRT) with infinite stub are also presented.

ing in the past literature [*Lin and Sheen*, 1997] will be presented. As a result, the method developed in part 1 of the paper will show to be not only accurate and simple, but also very useful for the design of leaky-wave antennas for practical applications.

2. Coupling With Undesired Channel Guide Leaky Mode and Radiation Purity

[8] The millimeter-wave slot-line leaky-wave antenna first presented in *Lampariello and Oliner* [1987] will be studied with the proposed method. The antenna was designed to operate in the frequency band of 50 GHz, as was described in the original work. In this way the following dimensions according to Figure 2 were obtained: a = 2.2 mm D = 1.59 mm X1 = 1.1 mm X2 = 2.1 mm and ε_r = 2.56. In that work it was mentioned that with L = 2 mm an undesired coupling effect with a channel guide leaky mode appeared. It was also said that using L = 1 mm this coupling could be avoided. However, no calculations of the degree of coupling were computed. Also, it was not mentioned the effect of shortening the stub length from L = 2 mm to L = 1 mm in the radiation purity of the antenna.

[9] In Figure 2, the dispersion characteristics for the normalized phase constant are shown for two different lengths L of the stub: L = 2 mm (Figure 2a) and L = 1 mm (Figure 2b). The results obtained in *Lampariello et al.* [1990] are represented with round symbols (DL-TRT) to compare with our curves. In that work the Transverse Resonance Technique (TRT) was used assuming an infinite stub, so no channel guide leaky mode appeared. On the contrary, our method is able to find all the channel-guide leaky modes which can be excited in the structure. The propagation of these modes is very sensitive to the

length L of the stub, since they are basically parallel-plate modes slightly disturbed by the planar metallization, as was shown in Figure 1c.

[10] In Figure 2 it is shown the normalized phase constant for the desired leaky-wave mode (DL) with continuous line, while the undesired channel guide leaky mode (CG) is represented with dotted line. If a stub of length L = 2 mm is used in order to avoid the radiation of higher-order PPW modes, the dispersion characteristic curve of the desired leaky mode couples to that of an undesired channel guide leaky mode, as can be seen in the frequency range between 50 and 60 GHz in Figure 2a. This coupling effect must be avoided [*Shigesawa et al.*, 1994], since it deteriorates the antenna performance and the radiation mechanism cannot longer be controlled. By shortening the stub to a length of L = 1 mm, the channel guide leaky mode dispersion characteristics shift to higher frequencies, avoiding the undesired coupling of the dispersion curves, as can be seen in Figure 2b. However, since the stub length has been decreased, it will be more likely that higher-order PPW modes reach the aperture, deteriorating the desired broadside pure E^ϕ polarized radiation pattern of the antenna. This phenomenon can be graphically seen from Figure 3, where both the transverse power flux and the total radiation pattern are shown for the desired leaky-wave mode operating at the frequency of 50 GHz of the antenna, for both stub lengths.

[11] As predicted, by shortening the top stub from L = 2 mm to L = 1 mm, the radiation pattern of the leaky mode in the X-Z plane deteriorates, increasing the tilt of the angle of maximum radiation, ϕ_{RAD}. The angle of maximum radiation changes from ϕ_{RAD} = −14° if L = 2 mm is used, to ϕ_{RAD} = −29° when the stub length is only L = 1 mm. We also see from the transverse real power flux, that the aperture is not uniformly illuminated when L = 1 mm is used as the stub length, leading to a wider radiation pattern.

[12] From these last results, it is demonstrated qualitatively the influence of the stub length in both the coupling of the dispersion curves of desired and undesired channel guide leaky modes, and in the radiation purity of the desired leaky mode. However, it would be attractive to measure this propagation and radiation characteristic in a more quantitative way. For this purpose, the proposed analysis method presented in part 1 of the paper seems to be very adequate, since direct analytical formulas can be derived from the propagation and radiation discrete PPW spectra.

[13] The coupling between the dispersion curves of different modes leads to degeneracy points in which these curves intersect one to another, or to frequency bands, in which the modes interchange in terms of their field behavior. However, it does not necessary imply that these modes exchange their carried power. We are
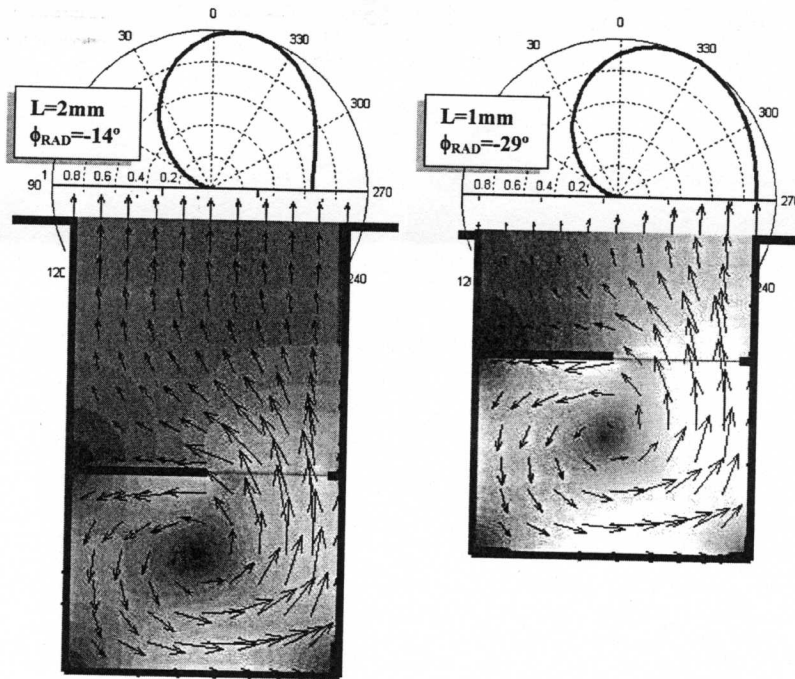
Figure 3. Radiation diagram and real transverse power flux for the desired slot-line leaky-wave mode at $f = 50$ GHz for different stub lengths.

interested in measuring the degree of power coupling between leaky-wave modes, since it is an indicator of the power carried by a pair of complex excited modes [*Rozzi et al.*, 1997]. In order to compute the power coupling between the desired leaky-wave mode (DL) and the unwanted channel guide leaky mode (CG), the following cross power density integral in the y-direction must be evaluated:

$$P_Y^{DL-CG} = \frac{1}{2} \cdot \int_{x=0}^{a} \int_{z=-D}^{z=+L} \left(\vec{E}_{DL} \times \vec{H}_{CG}^* \right) \cdot \hat{y} \cdot \partial x \cdot \partial z$$

$$= \frac{1}{2} \cdot \int_{x=0}^{a} \int_{z=-D}^{z=+L} \left(E_{Z_{DL}} \cdot H_{X_{CG}}^* - E_{X_{DL}} \cdot H_{Z_{CG}}^* \right)$$

$$\cdot \partial x \cdot \partial z \quad \text{(watts)} \tag{1}$$

This power coupling integral has been used in a great number of published articles [*Omar and Schünemann*, 1985; *Rozzi et al.*, 1997] to measure the power coupling between different complex modes. However, it has never been applied to leaky-wave modes in the context of the design of leaky-wave antennas. In past works, only the dispersion curves degeneracy regions were analyzed [*Shigesawa et al.*, 1994]. However, it is needed to explicitly compute (1) to measure, in a quantitative manner, the real interchange of power between two different leaky modes.

[14] Applying the PPW modes expansions to the DL mode and the CG mode, their transverse (X-Z plane) fields are described by the following expressions:

$$E_{X_{DL}}(x,z) = \sum_{m,p} A_{DL_m}^p \cdot \hat{V}_{DL_m}^p(z) \cdot e_{X_m}^p(x) \tag{2}$$

$$E_{Z_{DL}}(x,z) = \sum_{m} \frac{k_{DL_{cm}}}{j\omega\varepsilon} \cdot A_{DL_m}^{TM} \cdot \hat{I}_{DL_m}^{TM}(z) \cdot \phi_m^{TM}(x) \tag{3}$$

$$H_{X_{CG}}^*(x,z) = \sum_{m,p} A_{CG_m}^{*p} \cdot \hat{I}_{CG_m}^{*p}(z) \cdot h_{X_m}^{*p}(x) \tag{4}$$

$$H_{Z_{CG}}^*(x,z) = \sum_{m} \frac{k_{CG_{cm}}^*}{j\omega\mu} \cdot A_{CG_m}^{*TE} \cdot \hat{V}_{CG_m}^{*TE}(z) \cdot \phi_m^{*TE}(x) \tag{5}$$

Introducing equations (2)–(5) into (1), and taking into account the orthogonality properties between complex PPW modes described in *Tornero and Melcón* [2004a], the next analytical formulas are obtained:

$$P_Y^{DL-CG} = \left(P_Y E_Z^{DL-TM} H_X^{CG-TE*} + P_Y E_Z^{DL-TM} H_X^{CG-TM*} \right)$$

$$- \left(P_Y E_X^{DL-TE} H_Z^{CG-TE*} + P_Y E_X^{DL-TM} H_Z^{CG-TE*} \right) \tag{6}$$

$$P_Y E_Z^{DL-TM} H_X^{CG-TE*} = \frac{1}{2} \sum_m \frac{k_{DL_{cm}}}{j\omega\varepsilon} \cdot A_{DL_m}^{TM} \cdot A_{CG_m}^{*TE} \cdot \frac{a}{2} \cdot \delta$$
$$\cdot \int_{z=-D}^{z=+L} \hat{I}_{DL_m}^{TM}(z) \cdot \hat{I}_{CG_m}^{*TE}(z) \cdot \partial z$$

$$(7)$$

$$P_Y E_Z^{DL-TM} H_X^{CG-TM*} = \frac{1}{2} \sum_m \frac{k_{DL_{cm}}}{j\omega\varepsilon} \cdot A_{DL_m}^{TM} \cdot A_{CG_m}^{*TM} \cdot \frac{a}{2} \cdot \delta$$
$$\cdot \int_{z=-D}^{z=+L} \hat{I}_{DL_m}^{TM}(z) \cdot \hat{I}_{CG_m}^{*TM}(z) \cdot \partial z$$

$$(8)$$

$$P_Y E_X^{DL-TE} H_Z^{CG-TE*} = \sum_m \frac{1}{2} \frac{k_{CG_{cm}}^*}{j\omega\mu} \cdot A_{DL_m}^{TE} \cdot A_{CG_m}^{*TE} \cdot \frac{a}{2} \cdot \delta$$
$$\cdot \int_{z=-D}^{z=+L} \hat{V}_{DL_m}^{TE}(z) \cdot \hat{V}_{CG_m}^{*TE}(z) \cdot \partial z$$

$$(9)$$

$$P_Y E_X^{DL-TM} H_Z^{CG-TE*} = \sum_m \frac{1}{2} \frac{k_{CG_{cm}}^*}{j\omega\mu} \cdot A_{DL_m}^{TM} \cdot A_{CG_m}^{*TE} \cdot \frac{a}{2} \cdot \delta$$
$$\cdot \int_{z=-D}^{z=+L} \hat{V}_{DL_m}^{TM}(z) \cdot \hat{V}_{CG_m}^{*TE}(z) \cdot \partial z$$

$$(10)$$

$$\delta = \begin{cases} 2 \text{ for } m = 0 \\ \\ 1 \text{ for } m > 0. \end{cases}$$

$$(11)$$

As described in *Tornero and Melcón* [2004a], PPW leaky modes with different harmonic index values m are orthogonal, and therefore only the coupling between TE and TM PPW modes of the same order m must be calculated. In this way, the analytical expressions get much simpler, involving a single series instead of a double-summation. Four integrals for each value of m must be computed, in which the voltage and current functions in the equivalent transmission line network [*Tornero and Melcón*, 2004a] are employed. Since these are simple exponential functions which describe the progressive and the regressive equivalent waves in the z-direction, the integrals (7)–(10), can be easily computed analytically. To show that the computation of the power of a mode using equations (1)–(11) is correct, the characteristic impedance of the fundamental mode of a boxed microstrip line is plotted in Figure 4, computed using our method and two totally different techniques
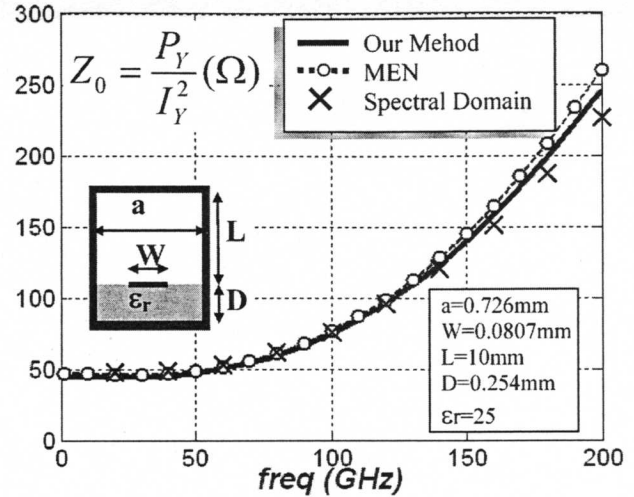
Figure 4. Validation of power formulation by comparison of the characteristic impedance of a boxed microstrip line with different approaches.

(the Multimode Equivalent Network (MEN), and the Spectral Domain method; these results can be found in *Guglielmi and Melcón* [1994, 1995]). The characteristic impedance is computed using the power identity ($Z_0 = \frac{P_Y}{I_Y^2}$), to show the correctness of our power calculations. As can be seen in Figure 4, excellent agreement is achieved between the three methods. The algorithm takes 2 s in a Pentium4 with 2 GHz CPU for the computation of 100 points in the complex variable k_y. Good convergence was observed with $N = 4$ basis functions and $M = 100$ PPW modes for the summation of the Kernel. Besides, equations (1)–(11) are computed analytically, and therefore the computation cost of the cross-power is very low. This postprocessing takes approximately 1 s per frequency point.

[15] Following this procedure, the next quantity can be evaluated to measure the normalized degree of coupling between the DL and the CG modes:

$$CROSS - COUPLING = \frac{|P_Y^{DL-CG}|}{\sqrt{|P_Y^{DL}| \cdot |P_Y^{CG}|}}$$

$$(12)$$

where P_Y^{DL} and P_Y^{CG} stand for the power in the y-direction which is carried by the DL and the CG modes, respectively, and which are computed following a similar procedure as with P_Y^{DL-CG}:

$$P_Y^{DL} = \frac{1}{2} \cdot \int_{x=0}^{a} \int_{z=-D}^{z=+L} (E_{DL} \times H_{DL}^*) \bullet \hat{y} \cdot \partial x \cdot \partial z \text{ (watts)}$$
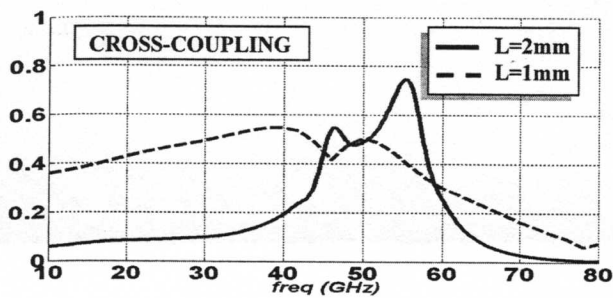
$$(13)$$

Figure 5. Cross-power coupling between desired and channel guide leaky modes for L = 2 mm and L = 1 mm.

$$P_Y^{CG} = \frac{1}{2} \cdot \int\limits_{x=0}^{a} \int\limits_{z=-D}^{z=+L} \left(E_{CG} \times H_{CG}^* \right) \bullet \hat{y} \cdot \partial x \cdot \partial z \quad \text{(watts)}$$

(14)

In Figure 5, the normalized cross-coupling coefficient (12) between the desired leaky-wave mode and the undesired channel guide leaky mode of the studied slot-line antenna is represented for $L = 2$ mm (continuous line) and $L = 1$ mm (dotted line).

[16] The qualitative behavior of the coupling with the variation of the stub length L is confirmed with this quantitative indicator. The coupling in the frequency range from 50 to 60 GHz has a peak of 70% when the stub length L is equal to 2 mm. When the stub is shortened to 1 mm, the relative coupled power decreases below 50% in this frequency range. Furthermore the frequency of maximum coupling is shifted to lower frequencies (40 GHz in this case). It is needed to compute the power coupling between CG and DL in the frequency range of operation of the antenna, since the excitation of a channel guide leaky mode should be avoided. Channel guide leaky modes radiation cannot be controlled by the slot-line, since their leakage is not created by the slot asymmetry. On the contrary, the desired leaky mode radiation is controlled by the slot width and offset [*Lampariello and Oliner*, 1987; *Lampariello et al.*, 1998].

[17] The other interesting topic to be studied is the radiation purity of the desired leaky mode. Again, the intrinsic PPW modal analysis given by the proposed technique in part 1 of the paper allows for a direct evaluation of the performance of the antenna. Since we are interested in computing the degree in which the main PPW TE $m = 0$ mode is responsible for the radiation with respect to the contribution of higher-order PPW modes, we can use the next factor of merit:

RADIATION − PURITY

$$= \frac{\left| B_{m=0}^{\phi^{TE}} \right|}{\sum\limits_{m,p} \left(\left| B_m^{\phi^{TE}} \right| + \left| B_m^{y^{TE}} \right| + \left| B_m^{\phi^{TM}} \right| + \left| B_m^{Y^{TM}} \right| \right)}$$

(15)

It can be observed from equation (15) that this factor is derived directly from the discrete PPW radiation spectrum coefficients, B_m, introduced in part 1 of the paper. If only the main PPW $m = 0$ mode reaches the top aperture, and none higher-order PPW mode contributes to radiation, the radiation-purity measurement will be equal to one. This indicates a purely E^ϕ polarized broadside beam in the X-Z plane. As this coefficient decreases, it indicates a stronger cross-polarization effect and a more tilted beam with side-lobes, as produced by higher-order PPW modes illuminating the top aperture of the antenna. In Figure 6, the normalized radiation-purity factor of the desired leaky-wave mode of the slot-line antenna is shown for L = 2 mm and L = 1 mm.

[18] From Figure 6, many interesting conclusions can be obtained. First, it can be seen that, as expected, the radiation is purer for a longer stub (about 15%), thus providing a better radiation pattern. Second, the behavior of this radiation purity factor is observed not to be constant in the entire frequency band of operation of the antenna. Particularly, as the frequency increases, the radiation seems to be purer. This result can be readily explained since, as the frequency increases, higher-order PPW modes are "more below cutoff" in the z-direction; that is, their attenuation factor is higher. This phenomenon is well known to happen in inhomogeneous wave-guides [*Balanis*, 1989], and can be depicted from the transverse propagation constant in the vacuum medium derived for each PPW mode in part 1 of this work:

$$k_{zm}^{VACUUM} = \sqrt{k_0^2 - \left(m\frac{\pi}{a} \right)^2 - k_y^2}$$

(16)

As the frequency increases, β_y tends to be greater than k_0 (as can be seen in Figure 2), resulting into a slower
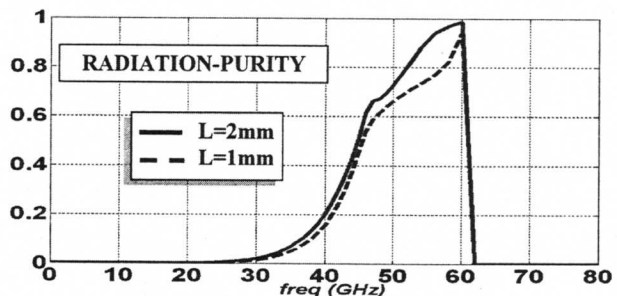

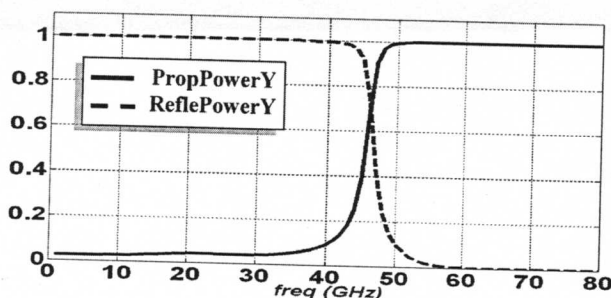**Figure 6.** Normalized radiation purity factor for L = 2 mm and L = 1 mm.

Figure 7. Normalized real and imaginary power carried by desired leaky mode (L = 1 mm).

propagating leaky-wave in the longitudinal direction of the antenna [Oliner, 1987]. As a result, k_{zm} tends to be below cutoff, and PPW modes do not propagate outside the dielectric slab. This effect of energy enclosed into the dielectric region is more pronounced for higher frequencies and higher-order PPW modes (greater values for the index m in equation (16)). Therefore, for higher frequencies, PPW higher-order fields decay more rapidly in their way to the top aperture, contributing in a weaker manner to the radiation of the desired leaky-wave mode. To the authors' knowledge, this is the first time these two features of a leaky mode (coupling with other leaky-wave modes and radiation purity) are computed in a quantitative manner.

3. Leaky-Wave Modes Below Cutoff

[19] At the frequency of 62 GHz, the leaky-wave mode becomes bounded, since above this frequency, even the main PPW $m = 0$ mode is below cutoff in the stub region (see equation (16) for $m = 0$ and $k_y > k_0$). This frequency point establishes the top frequency limit of operation of the antenna, and can be readily obtained from the dispersion curve of the leaky mode (Figure 2) when the solution becomes purely real ($k_y = \beta_y > k_0$, bounded or surface wave region). However, the lowest frequency at which the antenna can operate cannot be delimited so easily, since it is related to the frequency range in which the leaky-wave mode moves to its cutoff region.

[20] The cutoff region of a leaky-wave mode has been identified in past works [Oliner, 1987; Lin and Sheen, 1997; Lampariello et al., 1998] as the zone in which the imaginary part of the propagation constant of the leaky mode increases abruptly as the frequency decreases. This is due to the reflection losses associated to a mode below cutoff, whose energy cannot travel along the transmission line. These reflection losses must not be confused with the radiation losses, which are accounted by the imaginary part of the propagation constant of a leaky mode when it is radiating energy to the outer space.

[21] In this section, these phenomena will be computed and distinguished in an easy and analytical fashion, obtaining very interesting conclusions to determine the range of operation of a leaky-wave mode with practical applications to antennas. For this purpose, the same approach used by Lin and Sheen [1997] can be used. In that work, the radiation efficiency of a slot-line antenna was computed and measured by studying the real and the imaginary part of the power carried by the leaky-wave mode. Following this theory, we can compute the normalized propagated and reflected power in the longitudinal y-direction of the slot-line antenna as:

$$PROPAGT - POWER_Y = \frac{\Re eal(P_Y)}{|P_Y|} \quad (17)$$

$$REFLECT - POWER_Y = \frac{\Im mag(P_Y)}{|P_Y|} \quad (18)$$

In equations (17) and (18), the complex power in the y-direction carried by the leaky mode can be computed analytically as was described in equation (13). In Figure 7, the results are displayed for the desired leaky-wave mode of the slot-line antenna, with a length of the stub which avoids the coupling with the undesired channel guide mode ($L = 1$ mm).

[22] Lin and Sheen [1997] established as the cutoff frequency the frequency in which the power carried by the mode (real power) is equal to the reflected power (imaginary power). Following this definition, in our case the cutoff frequency would be 46.1 GHz, according to Figure 7. Also they represented the real and the imaginary part of the complex propagation constant of the leaky-wave mode, and determined that such cutoff condition occurred when $\beta = \alpha$. In Figure 8, the phase and the attenuation constants of the studied leaky-wave mode are plotted. We can confirm that at the frequency of 46.1 GHz this condition is fulfilled. Our results obtained for the slot-line of Figure 1 are therefore in agreement with the conclusions stated by Lin and Sheen. Further-
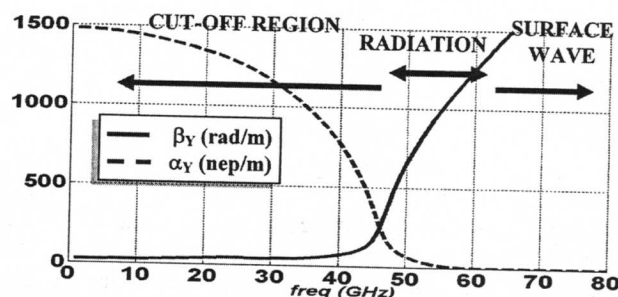

Figure 8. Phase and attenuation constants of desired leaky mode (L = 1 mm).

more, the method presented in part 1 of this paper allows for a fast and easy computation of the power carried by a leaky mode.

[23] From this last study, we can obtain the frequency point below which the reflected power becomes larger than the propagated power, by just monitoring the frequency response of the complex propagation constant. In this way, the operating range of the designed antenna would be between 46 GHz ($\alpha_Y = \beta_Y$) and 62 GHz ($\alpha_Y = 0$). Below this range, we obtain the cutoff zone of the leaky-wave, in which most of the injected power would be reflected. Finally, above this range the leaky-wave becomes a bounded mode with no radiation. Following this procedure, it can be easily established the different ranges of operation of the antenna, namely the cutoff region, the radiation region and the surface region, as illustrated in Figure 8.

[24] As it has been remarked, in previous works [Oliner, 1987; Lin and Sheen, 1997; Lampariello et al., 1998], the cutoff zone of a leaky mode was identified by the rapid rise of the imaginary part of its complex propagation constant (α_y). However, it is still not clear how much of the attenuation constant in the cutoff region is due to reflection losses or to radiation losses. In order to quantify these two aspects, we decompose the imaginary part of the complex propagation constant of a leaky-wave mode in a part due to storage of energy (reflection) and another due to radiation losses:

$$\alpha_Y^{TOTAL} = \alpha_Y^{REF} + \alpha_Y^{RAD} (\text{nep/m}) \qquad (19)$$

The part of the attenuation constant due to radiation (α_Y^{RAD}) can be calculated in a simple but rigorous manner, as illustrated next. The longitudinal power (P_Y) propagating at a point $y = y_0 + \Delta y$ can be expressed using the power at the point $y = y_0$ as follows:

$$P_Y(y = y_0 + \Delta y) = P_Y(y = y_0) \cdot e^{-2 \cdot \alpha_y \cdot \Delta y} (\text{watts}) \qquad (20)$$

Besides, the radiated power per unit length (Real P_Z) can be defined as:

$$\Re eal(P_Z)$$
$$= \lim_{\Delta y \to 0} \frac{P_Y(y = y_0 + \Delta y) - P_Y(y = y_0)}{\Delta y} \quad (\text{watts/m}) \qquad (21)$$

Introducing (20) into (21), and using the L'Hopital rule to solve the indetermination of the limit, the next expression can be easily obtained:

$$\Re eal(P_Z) = \lim_{\Delta y \to 0} \frac{P_Y(y = y_0) \cdot \left(e^{-2 \cdot \alpha_y \cdot \Delta y} - 1\right)}{\Delta y}$$
$$= -2 \cdot \alpha_y \cdot P_Y(y = y_0) \quad (\text{watts/m}) \qquad (22)$$

It must be noticed that in equation (22), the attenuation rate α_Y is due only to radiation losses. Therefore, from equation (22), it can be obtained a way to determine the value of the attenuation constant due to radiation α_Y^{RAD}, once the longitudinal power (P_Y) and the transverse radiated power density (Real P_Z) are known:

$$\alpha_Y^{RAD} = \frac{1}{2} \frac{\Re eal(P_Z)}{|P_Y|} (\text{nep/m}) \qquad (23)$$

The power propagated by a leaky-wave mode in the longitudinal direction of the slot-line antenna (P_Y) can be computed applying equation (13) to the leaky mode. The power density radiated in the transverse z-direction at the aperture plane (Real P_Z of equation (23)), can also be easily computed following the same procedure, but replacing (1) with the next formula:

$$P_Z = \frac{1}{2} \int_{x=0}^{a} (\vec{E} \times \vec{H}^*) \cdot \hat{z} \cdot \partial x$$
$$= \frac{1}{2} \int_{x=0}^{a} (\vec{E}_X \cdot \vec{H}_Y^* - \vec{E}_Y \cdot \vec{H}_X^*) \cdot \partial x \quad (\text{watts/m})$$
$$\qquad (24)$$

In this way, it has been computed separately for the first time the radiative and the reactive parts of the attenuation constant of a leaky-wave mode. The radiation losses constant α_Y^{RAD} is computed for the studied leaky-wave mode ($L = 1$ mm) using the expression (23), and plotted in Figure 9 together with the total attenuation constant, α_Y^{TOTAL}. Also, a more detailed plot in the radiation region of the leaky mode, is shown between 44 and 54 GHz. Above 48 GHz, the attenuation constant is due totally to radiation, and therefore $\alpha_Y^{TOTAL} = \alpha_Y^{RAD}$ as can be seen in Figure 9. However, below that frequency, α_Y^{TOTAL} is higher than α_Y^{RAD} due to the addition of reflection losses. The attenuation rate due to reflection losses can be computed from (19) as:

$$\alpha_Y^{REF} = \alpha_Y^{TOTAL} - \alpha_Y^{RAD} \qquad (25)$$

This attenuation rate due to reflection appears below 46 GHz and grows as the frequency decreases, as can be depicted from the rapid rise of α_Y^{TOTAL} in Figure 9, therefore being in perfect agreement with the predicted cutoff frequency. Moreover, below 46 GHz α_Y^{RAD} decreases, making the reflection losses of this leaky-wave mode to rise drastically. This behavior is expected, since the leaky-wave mode is now in the cutoff region.

[25] The knowledge of the rate in which the power is radiated and reflected is essential to determine the
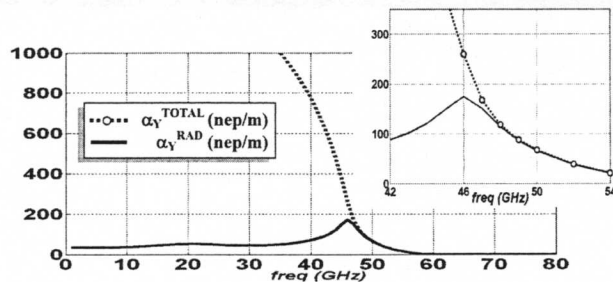
Figure 9. Total attenuation constant of desired leaky mode (L = 1 mm) and part due to radiation losses.

radiation efficiency of a leaky-wave antenna [*Lin and Sheen*, 1997]. The proposed method provides fast and accurate results also for this purpose. Also it has been developed an original way to distinguish the radiative and the reactive parts of the attenuation constant of a leaky mode, which are of much help for analyzing the radiation efficiency of a practical antenna.

4. Conclusions

[26] In the design of a leaky-wave antenna, the power injected by the feed must be radiated in the desired leaky-wave mode, avoiding any coupling with other modes and unwanted reflection effects due to cutoff mechanism. The proposed method provides a novel and original perspective for the analysis of leaky-wave modes. Simple analytical expressions have been derived using the discrete PPW spectrum propagation and radiation coefficients (A_m and B_m, respectively) derived in part 1 of this work. With these formulas, the power coupling between different leaky-wave modes has been computed to measure the degree of excitation of undesired channel guide modes. Furthermore, the radiation properties of the desired leaky-wave mode have been analyzed including for the first time all higher-order effects. In this way, the radiation purity has been defined as the degree in which the radiation is purely created by the main PPW $m = 0$ mode, and it has been straightforwardly computed. Finally, the propagation regimes of the desired leaky-wave mode have been analyzed, with special emphasis in the calculation of the degree in which the power is radiated and reflected to determine the cutoff zone of a leaky-wave mode. The radiation and reflection parts of the attenuation constant of a leaky-wave mode have been computed separately for the first time. All the results have been compared with previous works, obtaining analogous conclusions. The technique is accurate and the computation cost is very low, there-fore being very suitable for the design and optimization of laterally shielded planar leaky-wave antennas.

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