Mathware & Soft Computing 7 (2000)

Fuzzy Multicriteria Decision Making Applied to the Strategic Plan of Valencia

Ana M Nieto Morote (1), Francisco Ruz Vila (2)

(1) Dpto. de Electronica, Tecnologia de Computadoras y Proyectos.
(2) Dpto. de Ingeniera Elctrica
Universidad Politecnica de Cartagena.
Paseo Alfonso XIII. n 48, 30203 Cartagena.
Ana.Nieto@upct.es, Paco.Ruz@upct.es

Abstract

We present a fuzzy multicriteria decision making to get the ranking of several projects presented to the major council of Valencia whose final aim is to define the future urbanistic structure of the city. This technique allows us to deal with such problems that are defined by linguistic (and vague) terms, like the case mentioned below.

Keywords: multicriteria decision making, vagueness, application.

1 Introduction

Every decision making problem involves the choice of one alternative as a result of the required (or desirable) properties evaluation. We get the solution as an agreement among the different degrees of fulfilment of the properties. We define this kind of situations as multicriteria decision making problems.

Some of the basic elements that take part in decision making problems are:

- The set of alternatives.
- The set of factors (or atributes), that are the properties of the alternatives.
- The set of criteria. When we add to the factors some information about our preferences, we obtain the selection criteria.

It is not so difficult for decisors to define binary relationships between factors and alternatives. In classical methods, the decision matrix includes the numerical values that evaluate the relationship between one alternative and one factor.

But classical method does not consider that in the mind of decisors could exist some criteria that are in conflict among them: all the criteria does not have the same importance when we consider them to choose the best altenative. To deal with this question it is necessary to define the way to evaluate the importance of the criteria, because it is not so easy for decisors to give a numerical value to do it. The problem becomes more difficult when the number of factors and criteria is high.

In this paper we show a way to solve this difficulty using linguistic terms to evaluate the set of criteria.

2 Strategic Plan of Valencia

At the end of 1993, the major council of Valencia began to develop the strategic plan of the city, to desing its future urbanistic structure to prepare it to the competitiveness and modernity challenges that the new world-wide scene demands.

A group of experts in different subjects, was designated to guide this complex plan. Their first work was to study the socioeconomic reality of the city and its environment, ponting the wake and strong points, threats and chances. After a long (and hard) process of study and discussion, they proposed 47 initiatives considering some criterias as innovation, singularity and sinergy [1]. These initiatives were grouped in 7 strategic lines: developement of communications, environment improvement, cualification and competitivity of the productivity sectors, cualification of tourist, commercial and cultural sectors, cualification of the human resources, improvement of well-being and foreing hold.

Because of the limitation of the available resources,it was necessary to stablish the order in wich the initiatives considered must be ejecuted. This is a very complex problem due to the great ammount of criteria that are involved, even with a lot of contradictions between the underlying interests that this kind of decissions implicates. Moreover,we have to remark that the definiton of the alternatives was vague and it was so difficult to determine a set of objective and cuantificable criteria, with regard to be able to decide the order of the project executions. Then, these characteristics lead us to choose fuzzy decision making techniques to solve this problem.

3 Fuzzy Multicriteria Decision Making Model

The discussion among the experts resulted in a list of soft fragments of decision criteria. For example, they said that:

- If the alternative considered improves the quality of life, it generates employement, it does not harmful for the environment, it improves the city image and it is important for the foreing-hold of the city then the the alternative would be more than satisfactory.
- If the alternative considered does not improve the quality of life or it is not important for the foreing-hold of the city then the the alternative would be unsatisfactory.

¿From the analysis of the twelve fragments we form a set $F = f_i$ ten factors:

f_1.- It improves the quality of life.
f_2.- It generates employement.
f_3.- From the legal point of view, it is feasible.
f_4.- It is technoloicaly feasible.
f_5.- It does not harmful for the environment.
f_6.- It improves the city image.
f_7.- It is economically cost-effective
f_8.- Its starting and operation are independent from the others initiatives.
f_9.- It is feasible with a few resources.
f_{10}.- It is important for the foreing-hold of the city.

But it was so difficult to find natural scales to measure these variables. As one of the aims of this work is to compare the results obtained with the corresponding calculated with the non-fuzzy method developed in our reseach group [1], we have used the same information that they had. Then, we use a numerical value in [0,1] to measure of the importance of each criteria with regard to each element of the set of alternatives $A = a_m$. Then, the single-factor evaluation for alternative a_m is a fuzzy mapping from F to A. We use this mapping as the input variables.

The output variable will be the qualification of the initiatives with regard to factors listed above.Then we define a linguistic variable S, the satisfaction degree, and its term-set with the following linguistic values [2]:

- Satisfactory.
- More than satisfactory.
- Much more than satisfactory.
- Very satisfactory.
- Perfect.
- Unsatisfactory.

To compute these linguistic values we use the functions on the set X={0, 0.1 ,0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1} listed below [3]:

Satisfactory:

$$V_1(x) = x \quad , \quad x \in X \tag{1}$$

More than satisfactory:

$$V_2(x) = x^{3/2} \quad , \quad x \in X \tag{2}$$

Much more than satisfactory:

$$V_3(x) = x^2 \quad , \quad x \in X \tag{3}$$

Very satisfactory:

$$V_4(x) = x^3 \quad , \quad x \in X \tag{4}$$

Perfect:

$$V_5(x) = \begin{cases} 1 & if \quad x = 1 \\ 0 & if \quad x \neq 1 \end{cases} \quad x \in X \tag{5}$$

Unsatisfactory:

$$V_6(x) = 1 - x \ , \ x \in X \tag{6}$$

The decision criteria are stated by the relations between input and output variables that are described by inference rules with a formal appearance $IF \ c_j \ THEN \ S = V_i$. For our problem, we have defined 12 inference rules listed below, by translating the soft fragments of decision criteria:

If the initiative $f_1 \wedge f_2 \wedge f_3 \wedge f_4 \wedge f_5 \wedge f_6 \wedge f_7 \wedge f_8 \wedge f_9 \wedge f_{10}$ then $S = V_5$
If the initiative $f_1 \wedge f_2 \wedge f_3 \wedge f_5 \wedge f_6 \wedge f_7 \wedge f_9 \wedge f_{10}$ then $S = V_4$
If the initiative $f_1 \wedge f_2 \wedge f_4 \wedge f_5 \wedge f_6 \wedge f_7 \wedge f_9 \wedge f_{10}$ then $S = V_4$
If the initiative $f_1 \wedge f_2 \wedge f_5 \wedge f_6 \wedge f_7 \wedge f_8 \wedge f_9 \wedge f_{10}$ then $S = V_4$
If the initiative $f_1 \wedge f_2 \wedge f_5 \wedge f_6 \wedge f_7 \wedge f_9 \wedge f_{10}$ then $S = V_3$
If the initiative $f_1 \wedge f_2 \wedge f_5 \wedge f_6 \wedge f_7 \wedge f_{10}$ then $S = V_2$
If the initiative $f_1 \wedge f_2 \wedge f_5 \wedge f_6 \wedge f_9 \wedge f_{10}$ then $S = V_2$
If the initiative $f_1 \wedge f_2 \wedge f_5 \wedge f_6 \wedge f_{10}$ then $S = V_2$
If the initiative $f_1 \wedge f_2 \wedge f_5 \wedge f_{10}$ then $S = V_1$
If the initiative $f_1 \wedge f_2 \wedge f_6 \wedge f_{10}$ then $S = V_1$
If the initiative $f_1 \wedge f_5 \wedge f_6 \wedge f_{10}$ then $S = V_1$
If the initiative is not: $f_1 \vee f_{10}$ then $S = V_6$

The inference is mathematically defined,in these cases, by the Lucksiewicz rule [3], that integrates both sides of the logic rule:

$$d_j(m,l) = \min(1, (1 - \tilde{c}_j(a_m) + V_k(x_l)) \tag{7}$$

The antecedent of the logic rule, $\tilde{c}_j(a_m)$, represents the valuation of the decision criteria c_j with regard to each initiative a_m. We obtain this value from the matrix R_{47x10}, being each position of it, $r_{m,l}$, the membership degree of the initiative m with regard to the characteristic l. The mathematical expressions to calculate $\tilde{c}_j(a_m)$ are the following:

$$\begin{aligned}
&\tilde{c}_1(a_m) = r_{m,1} * r_{m,2} * r_{m,3} * r_{m,4} * r_{m,5} * r_{m,6} * r_{m,7} * r_{m,8} * r_{m,9} * r_{m,10}\\
&\tilde{c}_2(a_m) = r_{m,1} * r_{m,2} * r_{m,3} * r_{m,4} * r_{m,5} * r_{m,6} * r_{m,7} * r_{m,9} * r_{m,10}\\
&\tilde{c}_3(a_m) = r_{m,1} * r_{m,2} * r_{m,4} * r_{m,5} * r_{m,6} * r_{m,7} * r_{m,9} * r_{m,10}\\
&\tilde{c}4(a_m) = r_{m,1} * r_{m,2} * r_{m,5} * r_{m,6} * r_{m,7} * r_{m,8} * r_{m,9} * r_{m,10}\\
&\tilde{c}_5(a_m) = r_{m,1} * r_{m,2} * r_{m,5} * r_{m,6} * r_{m,7} * r_{m,9} * r_{m,10}\\
&\tilde{c}_6(a_m) = r_{m,1} * r_{m,2} * r_{m,5} * r_{m,6} * r_{m,7} * r_{m,10}\\
&\tilde{c}_7(a_m) = r_{m,1} * r_{m,2} * r_{m,5} * r_{m,6} * r_{m,9} * r_{m,10}\\
&\tilde{c}_8(a_m) = r_{m,1} * r_{m,2} * r_{m,5} * r_{m,6} * r_{m,10}\\
&\tilde{c}_9(a_m) = r_{m,1} * r_{m,2} * r_{m,5} * r_{m,10}\\
&\tilde{c}_{10}(a_m) = r_{m,1} * r_{m,2} * r_{m,6} * r_{m,10}\\
&\tilde{c}_{11}(a_m) = r_{m,1} * r_{m,5} * r_{m,6} * r_{m,10}\\
&\tilde{c}_{12}(a_m) = (1 - r_{m,1}) + (1 - r_{m,10})
\end{aligned}$$

The consequence of the rule, $V_k(x_l)$, is the valuation function associated to each linguistic term used to describe the degree os satisfaction of the criteria that is defined for each rule.

When we apply Lucksiewicz inference to each rule we will obtain for each one, a matrix d_i of fuzzy values whose rows represent the fuzzy degree of satisfaction of one altenative to the criteria c_j.

All the fuzzy mappings associated to the alternatives that imply fuzzy relations, must be integrated to obtain only one fuzzy matrix. In order to avoid too much information, the aggregation rule used to get this fuzzy matrix is [4]:

$$D = \prod_{j=1}^{12} d_j(m,l) \tag{8}$$

The elements of the m row of the global decision matriz D, form a fuzzy subset D_m representing the degree of global satisfaction of the alternative a_m with regard to the whole set of factors defined.

If we want to order the alternatives to determine priorities, we need to compare the fuzzy sets D_m as follows.

Each D_m has an α-level set, $D_{m\alpha}, \alpha \in [0,1]$. For each $D_{m\alpha}$, we calculate the mean value of the elements in D_m:

$$H_l(D_{m\alpha}) = \frac{1}{N_\alpha} \sum_{n=1}^{N_\alpha} Z_n(\alpha) \qquad (9)$$

Where α is the level of the level set, $Z_n(\alpha)$ is the element in $D_{m\alpha}$, and N_α is the cardinality of the finite set $D_{m\alpha}$.

Finally we estimate $S(m)$, the point value of D_m that we consider as one measure of the degree of satisfaction of the alternative a_m [6]:

$$S(m) = \frac{1}{\alpha_{\max}} \sum_{l=1}^{11} H_l(D_{m\alpha}) \cdot \Delta\alpha_l \qquad (10)$$

Where α represents the ordered values of D_m, $\alpha_{\max}$ is its maximum, $\Delta\alpha_l = \alpha_l - \alpha_{l-1}$ and $\alpha_0 = 0$

With the set of values S, order the alternatives to determine the priorities mentioned above.

4 Application of the Method to the Strategic Plan of Valencia

In this case, the initial data are the elements of R_{47x10}. We must remember that $r_{m,l}$ is the membership degree of the initiative m with regard to the characteristic l, and we define it with a number $r_{m,l} \in [0,1]$. For instance, the following matrix shows the valuations that the group of experts have given for thr membership degree of the characteristics that we have defined for 3 initiatives:

$$R' = \begin{bmatrix} 0.81 & 0.61 & 0.58 & 0.76 & 0.40 & 0.60 & 0.80 & 0.70 & 0.80 & 0.90 \\ 0.88 & 0.61 & 0.63 & 0.79 & 0.50 & 0.60 & 0.80 & 0.70 & 0.80 & 0.80 \\ 0.66 & 0.61 & 0.60 & 0.81 & 0.40 & 0.50 & 0.80 & 0.60 & 0.60 & 0.70 \end{bmatrix}$$

With this matrix, we calculate $\tilde{c}_j(a_m)$for each alternative a_m and each inference rule. The matrix CR_{47x12} contains all of these parameters. For the same 3 initiatives we have the following values:

$$CR' = \begin{bmatrix} .032 & .057 & .074 & .071 & .010 & .120 & .110 & .136 & .224 & .291 & .220 & .239 \\ .049 & .088 & .099 & .097 & .125 & .152 & .145 & .176 & .260 & .316 & .288 & .254 \\ .018 & .032 & .044 & .038 & .054 & .066 & .067 & .082 & .149 & .164 & .133 & .600 \end{bmatrix}$$

Then, for each rule, we obtain a matrix $d_{j,47x11}$. Here, we reproduce the corresponding values for the initiatives considered:

$$d_1' = \begin{bmatrix} .968 & .968 & .968 & .968 & .968 & .968 & .968 & .968 & .968 & .968 & 1.00 \\ .951 & .951 & .951 & .951 & .951 & .951 & .951 & .951 & .951 & .951 & 1.00 \\ .982 & .982 & .982 & .982 & .982 & .982 & .982 & .982 & .982 & .982 & 1.00 \end{bmatrix}$$

$$d_2' = \begin{bmatrix} .864 & .895 & .953 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .824 & .856 & .941 & .988 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .918 & .950 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$d_3' = \begin{bmatrix} .864 & .895 & .953 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .824 & .856 & .941 & .988 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .918 & .950 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$d_4' = \begin{bmatrix} .864 & .895 & .953 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .824 & .856 & .941 & .988 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .918 & .950 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$d_5' = \begin{bmatrix} .864 & .895 & .953 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .824 & .856 & .941 & .988 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .918 & .950 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$d_6' = \begin{bmatrix} .864 & .895 & .953 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .824 & .856 & .941 & .988 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .918 & .950 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$d_7' = \begin{bmatrix} .864 & .895 & .953 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .824 & .856 & .941 & .988 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .918 & .950 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$d_8' = \begin{bmatrix} .864 & .895 & .953 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .824 & .856 & .941 & .988 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .918 & .950 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$d_9' = \begin{bmatrix} .776 & .876 & .976 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .740 & .841 & .940 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .851 & .951 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$d_{10}' = \begin{bmatrix} .710 & .810 & .915 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .685 & .785 & .855 & .985 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .836 & .936 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$d_{11}' = \begin{bmatrix} .780 & .880 & .980 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .712 & .812 & .912 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ .868 & .968 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

$$d_{12}' = \begin{bmatrix} 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & .962 & .862 & .762 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & .946 & .846 & .746 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & .900 & .800 & .700 & .600 & .500 & .400 \end{bmatrix}$$

The next step is to calculate the matrix D_{47x11}. The results for the three alternatives are:

$$D' = \begin{bmatrix} .206 & .337 & .958 & .850 & .952 & .968 & .968 & .968 & .931 & .834 & .762 \\ .134 & .226 & .410 & .730 & .873 & .951 & .951 & .951 & .900 & .805 & .746 \\ .409 & .369 & .884 & .950 & .982 & .884 & .785 & .687 & .589 & .941 & .400 \end{bmatrix}$$

With every row of D matrix, we obtain $D_{m\alpha}$. As an example to show how to obtain it, for the first row we have:

$$D_{1\alpha} = \begin{bmatrix} .206 & .337 & .958 & .850 & .952 & .968 & .968 & .968 & .931 & .834 & .762 \\ 0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1.0 \end{bmatrix}$$

Then, $H_l(D_{m\alpha})$ is obtained. For $D_{1\alpha}$, the results are:

L	*Rango de* α	$D_{1,\alpha}$	$H_l(D_{1\alpha})$	$\Delta\alpha_l$
1	$0 \leq \alpha \leq 0.206$	{ 0, .1, .2, .3, .4, .5, .6, .7, .8, .9, 1.0 }	0.50	0.206
2	$0.206 \leq \alpha \leq 0.377$	{ .1, .2, .3, .4, .5, .6, .7, .8, .9, 1.0 }	0.55	0.131
3	$0.377 \leq \alpha \leq 0.439$	{ .2, .3, .4, .5, .6, .7, .8, .9, 1.0 }	0.60	0.102
4	$0.439 \leq \alpha \leq 0.521$	{ .2, .3, .4, .5, .6, .7, .8, .9 }	0.55	0.082
5	$0.521 \leq \alpha \leq 0.598$	{ .2, .3, .4, .5, .6, .7, .8}	0.50	0.077
6	$0.598 \leq \alpha \leq 0.618$	{ .3, .4, .5, .6, .7, .8}	0.55	0.030
7	$0.618 \leq \alpha \leq 0.715$	{ .3, .4, .5, .6, .7}	0.50	0.097
8	$0.715 \leq \alpha \leq 0.812$	{ .3, .4, .5, .6}	0.45	0.097
9	$0.812 \leq \alpha \leq 0.850$	{ .3, .4, .5}	0.40	0.038
10	$0.850 \leq \alpha \leq 0.909$	{ .4, .5}	0.45	0.059
11	$0.909 \leq \alpha \leq 1$	{ .4}	0.40	0.043

Finally, we calculate $S(m)$ whose decreasing order gives the priority of the alternatives (Figure 2).As an example, we show the corresponding expression for $S(1)$:

$$\begin{aligned} S(1) = \tfrac{1}{0.952} \cdot \{&(0.55 \cdot 0.206) + (0.55 \cdot 0.131) + (0.6 \cdot 0.102) + (0.55 \cdot 0.082) + \\ &+(0.5 \cdot 0.77) + (0.55 \cdot 0.3) + (0.5 \cdot 0.97) + (0.45 \cdot 0.47) + (0.4 \cdot 0.087) + \\ &+(0.45 \cdot 0.059) + (0.4 \cdot 0.043)\} = 0.5785. \end{aligned}$$

5 Conclusions

In decision making problems, the selection criteria and the proposed alternatives are sometimes vague in their definitions. The fuzzy model used in this work seems to be appropriate to solve this kind of problem. The calculation techniques are very simple and the correct definition of the decision criteria, based on the desirable characteristics that the alternatives must have, is the most complicated step when we try to stablish the priorities in the project execution of these alternatives.

One of the adventages of the tested method is the stability of the results, if we compare it with other multicriteria decision making models, even when the variations are applied to decision criteria definitions.

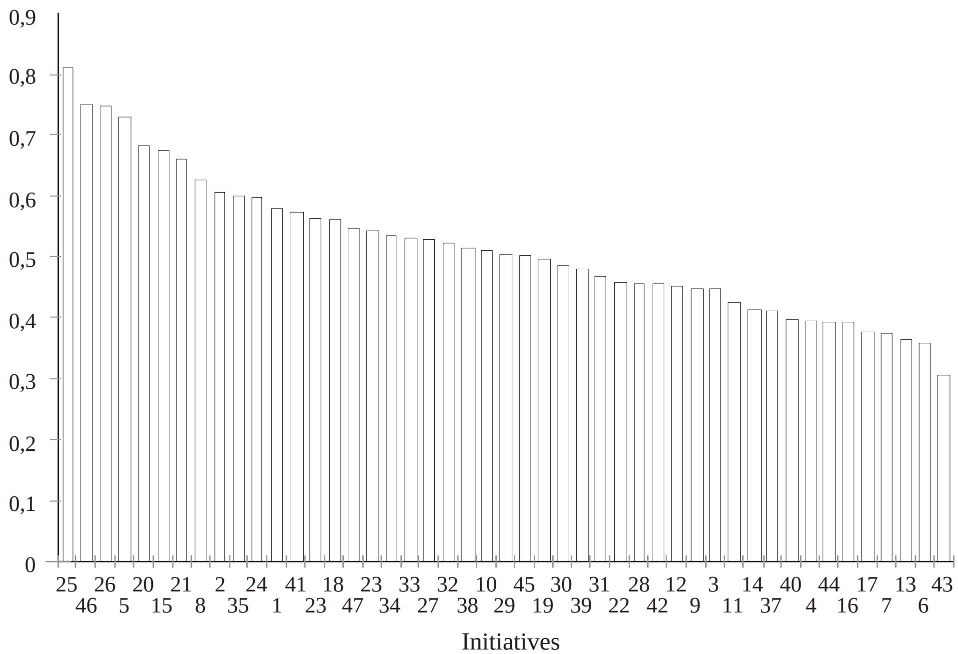

Figure 1: Priority of the alternatives

References

[1] Aragones,P. *Aproximacion a la Toma de Decisiones Multicriterio en Proyectos. Implementacion de una Metodologa Multicriterio y Multiexperto: PRES II*. Tesis Doctoral. Universidad Politecnica de Valencia, 1997.

[2] Zadeh, L. *A theory of approximate reasoning*. Memo UCB/ERL M 77/58, University of California, Berkeley, 1977.

[3] Yagger, R. *Multicriteria decisions with soft information: An appplication of fuzzy set and Posibility Theory (I) & (II)*. Fuzzy Math Vol.2. (I) pp 21-28. (II) 7-16. 1982.

[4] Feng, C. *Fuzzy Multicriteria Decision-Making in Distribution of Factories: an Application of Approximate Reasoning*. Fuzzy Sets and Systems, Vol.71. pp 195-205. 1995.

[5] Klir G; Yuan B. *Fuzzy set and fuzzy logic. Theory and applications*. Prentice Hall. 1995.

[6] Yagger, R. *A procedure for ordering fuzzy subsets of the unit interval*. Technical Report RRY 79-12, Iona College, New Rochelle. 1979.