ISSN 0025-6455, Volume 45, Number 5

Meccanica (2010) 45: 657–669
DOI 10.1007/s11012-009-9263-6

SIMULATION, OPTIMIZATION & IDENTIFICATION

Design of reinforcement for concrete co-planar shell structures using optimization techniques

Antonio Tomás · Pascual Martí

Received: 18 February 2009 / Accepted: 26 October 2009 / Published online: 5 November 2009

Abstract The absence of universally accepted solutions for the design of reinforcement in plates and shells in the structural concrete codes, and the constant development of computers combined with powerful numerical methods, reveal the need for a standard procedure to calculate the required reinforcement in thin elements subject to membrane and flexural forces. In the present study, the amount of reinforcement is optimized locally for each finite element of the mesh that models the geometry of the problem. Some numerical examples are given and compared to the results provided by other authors, achieving significant savings in reinforcement.

Keywords Reinforcement · Concrete · Shell elements · Optimization · Co-planar shell structures

1 Introduction

Traditionally, the failure conditions of the materials in concrete shells have been applied to the stresses at some structure points. Even though most researchers have adopted this checking methodology, the basic hypotheses and methods to solve the problem are still open to debate. As proof of this, references in the design codes relating to the design of plates and shells are quite scarce and, in the majority of cases, these are dealt with rather superficially. Eurocode 2 [1] does not include any reference to shells, and only refers to plates that are loaded on their plane. The ACI 318 [2] code considers any design method that assures sufficient strength with equilibrium to be applicable. In the Model Code CEB-FIP 1990 (MC90) [3] these are dealt with in greater depth, given that design hypotheses are included which are based on the use of layer models, such as those developed by Marti [4], which resist the external membrane forces and the internal shear force. References in the technical literature are also scarce and normally focus on analysis methods (e.g. [5–9] among others) or on the design of particular structures (e.g. [10–13]) but not on design methods in concrete plates and shells.

Nevertheless, there are design methods that use plate or shell elements as dimensioning units resisting their nodal forces. The objective is to reach the equilibrium between the external and internal forces due to the contributions of the reinforcement and the concrete. To this end, calculus algorithms are used to provide the quantity of reinforcement at the outer layers of the element in two orthogonal directions [14]. Some strategies were developed to obtain more rational reinforcement distributions, with less weight in the complete concrete plate or shell [15].

The development of optimization techniques has been strongly boosted by the tremendous increase in

A. Tomás (✉) · P. Martí
Department of Structures and Construction, Technical University of Cartagena, Campus Muralla del Mar, 30202 Cartagena (Murcia), Spain
e-mail: antonio.tomas@upct.es

computational and graphical capacities. These techniques represent an effective means of obtaining alternative reinforcement distributions which comply with the design conditions (stress constraints, construction prescriptions, etc.) in an optimal way (minimum weight, minimum stress level, etc.). They could become a standard procedure for designing elements subject to membrane and flexural forces [16]. However, it is convenient to bear in mind that this design process may result in a reduction of the residual strength of the element, which may arise from unexpected increases in the loading (such as due to heavier loads: locomotives, lorries, new industrial machinery, etc.).

Tomás and Martí [17] optimized the amount of reinforcement locally for each finite element of the mesh that models the geometry of the problem. Starting from the equilibrium between applied and internal forces, it leads to an indeterminate system of nonlinear equations solved using optimization techniques. The formulation of the method includes the biaxial behaviour of the concrete and the different lever arms of the reinforcement, assuming ideal plastic behaviour for both materials. The ANSYS finite element program was used to analyse the structure and to obtain the forces in the shell elements. Furthermore, the formulation was implemented by means of user routines within the optimization module of the program. The objective function is the summation of the tensile forces in the reinforcement. A numerical example was given and compared to the results provided by other authors.

In the present study, extensive use of the aforementioned formulation is provided, namely:

(i) details of all the parameters are included for the different cases according to the necessity for reinforcement;
(ii) the influence of variations of some of the assumptions for the analysis is commented; and
(iii) more extensive application to case studies has been achieved by adding a second example of a three-span continuous deep beam reinforcement computation.

2 Formulation and solution of the optimal structural design problem

The aim of the optimal structural design is to obtain a design, a set of values for the *design variables*, which minimizes an *objective function* and complies with the *constraints* that depend on the variables.

The *design variables* of a structure can be properties of the cross-section of the elements (surface areas, thicknesses, inertia moments, etc.); structural geometry parameters; structural topology parameters (element densities in the range from 0 to 1) [18]; and properties of the material of the structure. The type of optimization carried out depends on the type of variables being considered. Traditionally, the design of minimum weight structures has been sought, which has led to the fact that the most common *objective function* is the weight of the structure. Nevertheless, the weight is not the determining factor in other applications, and other objective functions are used, such as cost, reliability, stiffness, etc. The *constraints* are the conditions that the design must comply with in order to be regarded as valid.

The optimum design problem was formulated as follows

To find the variable vector of design $\mathbf{x}$ which

$$\begin{array}{lll} \text{minimizes} & f(\mathbf{x}) & \\ \text{subject to} & h_j(\mathbf{x}) = 0 & j = 1, 2, \ldots, m_i \\ & g_k(\mathbf{x}) \geq 0 & k = 1, 2, \ldots, m_d \\ & x_i^L \leq x_i \leq x_i^U & i = 1, 2, \ldots, n \end{array} \tag{1}$$

where $\mathbf{x}$ is the n-dimensional vector of the design variables; $f(\mathbf{x})$ is the objective function; $h_j(\mathbf{x})$ is the jth equality design constraint; $g_k(\mathbf{x})$ is the kth inequality design constraint; m_i is the number of equality constraints; m_d is the number of inequality constraints; n is the number of variables; and x_i^L (x_i^U) is the lower limit (upper limit) of the variable i (e.g. [19] among others).

This problem was solved by mathematical programming using the optimization module in ANSYS [20], which has a conventional first-order method using the first derivatives of the objective function and constraints with respect to the design variables. The module converts the optimization problem with constraints into an unconstrained problem by adding penalty functions to the objective function. For each iteration, gradient calculations, which employ a steepest descent or conjugate direction method, are performed to determine a search direction. A line search strategy is adopted to minimize the objective function of the unconstrained optimization problem.

3 Formulation of the optimal design of reinforcement in shell elements

3.1 Description of the material

Concrete is a complex material that requires a large number of parameters in order to provide a complete description of its constitutive equations. In fact, concrete should be viewed as a quasi-brittle material having a size-dependent behaviour [21]. Nevertheless, in the adopted design model [22] concrete is considered to be a rigid-plastic material, which is characterized by a single parameter, f_c, the uniaxial concrete compressive strength measured on cylinders. This material consideration is sufficient to meet the proposed aim. With respect to the general case of the design, the choice of f_c is a delicate matter, so the values suggested in MC90 [3] for the design strength of concrete were adopted:

Non-cracked zones are given by

$$f_{cd1} = 0.85\left[1 - \frac{f_{ck}}{250}\right] f_{cd} \tag{2}$$

Cracked zones, where the compressive strength may be reduced by the effect of transverse tension from the reinforcement and by the need to transmit force through the cracks, are given by

$$f_{cd2} = 0.60\left[1 - \frac{f_{ck}}{250}\right] f_{cd} \tag{3}$$

where f_{ck} is the characteristic cylinder compressive strength of the concrete, f_{cd} is the design cylinder compressive strength of the concrete, and f_{cd1} and f_{cd2} are the design strength of concrete for noncracked zones and for cracked zones, respectively, the units are MPa.

The strength for concrete that is subject to biaxial compression state increases: it is possible to reach an increase of approximately 16% under a stress state of the same intensity in both directions [23]. This increase, due to the confinement effect, can be formulated by multiplying the compressive strength by a coefficient K

$$K = \frac{1 + 3.65\alpha}{(1+\alpha)^2}, \quad \alpha = \frac{\sigma_2}{\sigma_1}, \ 0 \le \alpha \le 1 \tag{4}$$

where σ_1 and σ_2 are the stresses referring to the principal directions.

As is normal when dealing with the design of concrete structures, neither concrete tensile strength nor aggregate interlocking phenomena are taken into account. The tensile strength is a small fraction, less than 10%, of the compressive strength. If it is considered in the optimization process, a slightly better optimum design of the reinforcement would be obtained. However, there are two disadvantages: (i) the code implementation is complex and the reinforcement savings are minimal; and (ii) the optimization may result in further reductions in the residual strength.

A rigid-plastic behaviour was considered for reinforcement steel, with a maximum stress equal to the yield limit. The reinforcement only resists uniaxial forces, and the dowel action of the bars is not taken into account. The contribution of the reinforcement in compression resistance was discarded due to its negligible effect in comparison to the surrounding concrete. Therefore, the reinforcement is designed solely to resist tensile forces. The effects related to the adhesion and to the anchoring of the bars were not taken into account. The orthogonal distribution of the reinforcement was considered.

3.2 General approach

The proposed design method is based on the lower-bound theorem of the theory of plasticity [24]. The flexural and membrane forces, which act on the sides of a shell element (Fig. 1), must be in equilibrium with the internal compression forces in the concrete and the tensile forces in the reinforcement. In general, the principal directions of the membrane and flexural forces do not coincide. Applying this theorem, it may be assumed that the solution obtained is sufficiently safe, although a deformation verification must be carried out later to guarantee that the performance is within the serviceability limit state.

Figure 2 shows the model used for a shell element, the reinforcement set parallel to the x- and y-axes, and the acting internal forces. The tensile forces of the reinforcement are identified as N_{sxt}, N_{syt}, N_{sxb}, N_{syb} with the subscripts x and y referring to the axes, and the subscripts t and b referring to the top and bottom layers, respectively. Different lever arms of the reinforcement (Fig. 2a) were included in the formulation ($h_{yt}, h_{yb}, h_{xt}, h_{xb}$). A vertical failure plane parallel to the direction of the cracks on the top layer is assumed, the normal vector of which forms an angle θ_t, with the x-axis and is contained in the xy-plane.

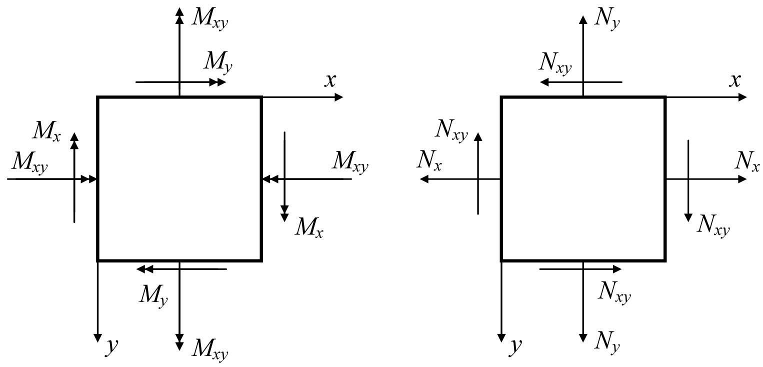


Fig. 1 Flexural and membrane forces acting on a shell element (per unit length)

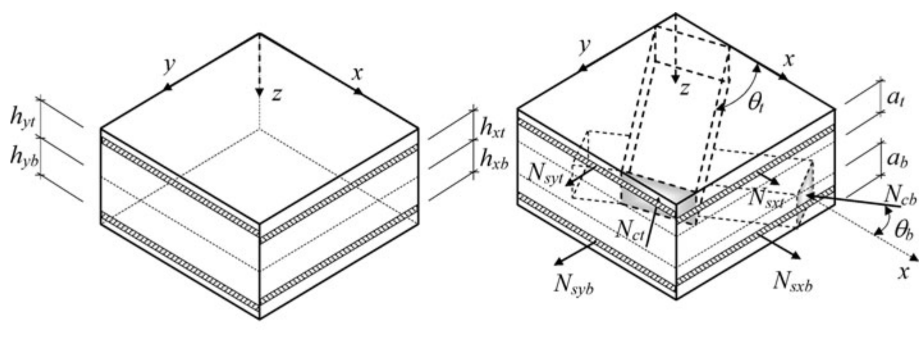


Fig. 2 Model of a shell element

The thickness of the compressed top layer is represented by a_t. Uniform stress distribution (rectangular stress block represented by the dashed line) is assumed within this layer, N_{ct} with its resultant force being parallel to that normal vector. In the same way, with respect to the bottom layer, θ_b refers to the normal vector of the failure plane, a_b to the thickness of the uniform stress distribution, and N_{cb} to its resultant force. The stresses within the concrete layers can be assumed to have other distributions (triangular, trapezoidal, parabolic, etc.) although the differences between the reinforcements obtained for each distribution are minimal, all the more so if the thickness of the layers is small.

This type of formulation was originally developed by Nielsen [25] for plates and slabs subject to the membrane state with symmetrical reinforcement in two directions. Later, the method was extended for orthogonal reinforcement [26] and generalized for random directions [27]. Subsequently, the membrane reinforcement of plates was extended to shells [28, 29] and Gupta [30] discussed the case of reinforcement necessary in the top and bottom layers of a shell. Finally, this formulation was widened to cover three further cases: reinforcement necessary only in the bottom layer; reinforcement necessary only in the top layer; and reinforcement not necessary [14, 22].

Even though the phenomenon described is relatively simple, this is not the case with the mathematical solution. The formulation required leads to an indeterminate system of nonlinear equations for each case. The equilibrium equations used come from [14], [22] and [30] and are listed below in the appropriate form for use in this work.

3.2.1 Reinforcement necessary in top and bottom layers

The sum of reinforcement forces in x and y directions and the moments are

$$N_{sx} = N_{sxt} + N_{sxb}; \qquad N_{sy} = N_{syt} + N_{syb} \tag{5}$$

$$M_{sx} = -N_{sxt} h_{xt} + N_{sxb} h_{xb};$$

$$M_{sy} = -N_{syt}h_{yt} + N_{syb}h_{yb} \tag{6}$$

Forces and moments in the concrete are

$$N_{ct} = -a_t f_{cd2}; \qquad N_{cb} = -a_b f_{cd2} \tag{7}$$

$$M_{ct} = -\frac{1}{2}(h-a_t)N_{ct}; \qquad M_{cb} = \frac{1}{2}(h-a_b)N_{cb} \tag{8}$$

Equations (5) to (8) express the internal forces and moments in terms of the defined parameters and must be in equilibrium with the external membrane/bending forces. The system of equations is

$$N_x = N_{sx} + N_{ct}\sin^2\theta_t + N_{cb}\sin^2\theta_b \tag{9}$$

$$N_y = N_{sy} + N_{ct}\cos^2\theta_t + N_{cb}\cos^2\theta_b \tag{10}$$

$$N_{xy} = -N_{ct}\sin\theta_t\cos\theta_t - N_{cb}\sin\theta_b\cos\theta_b \tag{11}$$

$$M_x = M_{sx} + M_{ct}\sin^2\theta_t + M_{cb}\sin^2\theta_b \tag{12}$$

$$M_y = M_{sy} + M_{ct}\cos^2\theta_t + M_{cb}\cos^2\theta_b \tag{13}$$

$$M_{xy} = -M_{ct}\sin\theta_t\cos\theta_t - N_{cb}\sin\theta_b\cos\theta_b \tag{14}$$

Combining (5) to (14) produces a system of six equilibrium equations with eight unknowns (N_{sxt}, N_{sxb}, N_{syt}, N_{syb}, a_t, a_b, θ_t and θ_b). In order to solve the system two of these must be set as design variables.

3.2.2 Reinforcement necessary in the bottom layer

N_{cxt}, N_{cyt} are the forces in directions x, y, and N_{cxyt} is the transverse shear, all in the top layer of concrete. This layer is under biaxial compression while reinforcement is necessary in the bottom layer. The internal forces are

$$N_{sx} = N_{sxb}; \qquad N_{sy} = N_{syb} \tag{15}$$

$$M_{sx} = N_{sxb}h_{xb}; \qquad M_{sy} = N_{syb}h_{yb} \tag{16}$$

$$N_{cb} = -a_b f_{cd2}; \qquad M_{cb} = \frac{1}{2}(h-a_b)N_{cb} \tag{17}$$

The equilibrium equations are

$$N_x = N_{sx} + N_{cxt} + N_{cb}\sin^2\theta_b \tag{18}$$

$$N_y = N_{sy} + N_{cyt} + N_{cb}\cos^2\theta_b \tag{19}$$

$$N_{xy} = N_{cxyt} - N_{cb}\sin\theta_b\cos\theta_b \tag{20}$$

$$M_x = M_{sx} + M_{cxt} + M_{cb}\sin^2\theta_b \tag{21}$$

$$M_y = M_{sy} + M_{cyt} + M_{cb}\cos^2\theta_b \tag{22}$$

$$M_{xy} = M_{cxyt} - M_{cb}\sin\theta_b\cos\theta_b \tag{23}$$

where

$$M_{cxt} = -\frac{1}{2}(h-a_t)N_{cxt};$$
$$M_{cyt} = -\frac{1}{2}(h-a_t)N_{cyt}; \tag{24}$$
$$M_{cxyt} = -\frac{1}{2}(h-a_t)N_{cxyt}$$

The compressive top layer thickness is

$$a_t = \frac{N_{ct,\max}}{K f_{cd1}} \tag{25}$$

where K is obtained by (4), with the principal stresses (forces per unit length, in this case) being

$$N_{c1,c2,t} = \frac{N_{cxt}+N_{cyt}}{2} \pm \sqrt{\left(\frac{N_{cxt}-N_{cyt}}{2}\right)^2 + N_{cxyt}^2} \tag{26}$$

Combining (15) to (23) and (25) produces a system of six equilibrium equations with seven unknowns (N_{sxb}, N_{syb}, a_b, θ_b, N_{cxt}, N_{cyt} and N_{cxyt}). In order to solve the system one of them must be set as a design variable.

3.2.3 Reinforcement necessary in the top layer

The equations for this case are the same as those for the previous case except that all "b" subscripts are replaced by "t" and vice versa.

3.2.4 Reinforcement not necessary

In this case, both layers are under biaxial compression and there is no need for reinforcement. Unknowns are the concrete forces, these being N_{cxt}, N_{cyt}, N_{cxyt} for the top layer and N_{cxb}, N_{cyb}, N_{cxyb} for the bottom layer.

The equilibrium equations are

$$N_x = N_{cxt} + N_{cxb} \tag{27}$$

$$N_y = N_{cyt} + N_{cyb} \tag{28}$$

$$N_{xy} = N_{cxyt} + N_{cxyb} \tag{29}$$

$$M_x = M_{cxt} + M_{cxb} \tag{30}$$

$$M_y = M_{cyt} + M_{cyb} \tag{31}$$

$$M_{xy} = M_{cxyt} + M_{cxyb} \tag{32}$$

where

$$M_{cxt} = -\frac{1}{2}(h - a_t)N_{cxt}; \quad M_{cxb} = -\frac{1}{2}(h - a_b)N_{cxb} \tag{33}$$

$$M_{cyt} = -\frac{1}{2}(h - a_t)N_{cyt}; \quad M_{cyb} = \frac{1}{2}(h - a_b)N_{cyb} \tag{34}$$

$$M_{cxyt} = -\frac{1}{2}(h - a_t)N_{cxyt}; \quad M_{cxyb} = \frac{1}{2}(h - a_b)N_{cxyb} \tag{35}$$

In addition, it is necessary to add the layers' thicknesses equations for the concrete

$$N_{c1,c2,t} = \frac{N_{cxt} + N_{cyt}}{2} \pm \sqrt{\left(\frac{N_{cxt} - N_{cyt}}{2}\right)^2 + N_{cxyt}^2} \tag{36}$$

$$N_{c1,c2,b} = \frac{N_{cxb} + N_{cyb}}{2} \pm \sqrt{\left(\frac{N_{cxb} - N_{cyb}}{2}\right)^2 + N_{cxyb}^2} \tag{37}$$

$$\alpha_t = \frac{N_{c2t}}{N_{c1t}}; \qquad \alpha_b = \frac{N_{c2b}}{N_{c1b}} \tag{38}$$

$$K_t = \frac{1 + 3.65\alpha_t}{(1 + \alpha_t)^2}; \qquad K_b = \frac{1 + 3.65\alpha_b}{(1 + \alpha_b)^2} \tag{39}$$

$$a_t = \frac{N_{ct,\max}}{K_t f_{cd1}}; \qquad a_b = \frac{N_{cb,\max}}{K_b f_{cd1}} \tag{40}$$

Combining (27) to (32) and (36) to (40) produces a system of eight equilibrium equations with eight unknowns, thus only one solution exists.

3.3 Objective function

The objective function is the sum of the tensile forces of the reinforcement. This expression depends on the case being optimized. The four equations are then

(1) reinforcement necessary in top and bottom layers

$$N_{tot} = N_{sxt} + N_{syt} + N_{sxb} + N_{syb} \tag{41}$$

(2) reinforcement necessary only in the bottom layer

$$N_{tot} = N_{sxb} + N_{syb} \tag{42}$$

(3) reinforcement necessary only in the top layer

$$N_{tot} = N_{sxt} + N_{syt} \tag{43}$$

(4) reinforcement not necessary.

On optimizing the sum of the tensile forces, the reinforcement cross-section area is also optimized, given its relationship to the steel design strength f_{yd}, and therefore, to the weight of the steel.

The objective function was normalized by dividing it by the initial value adopted in the optimization process (obtained by means of adopting any initial value for a_t, a_b between 0 and h and solving the system of equations). Thus, it takes on values approaching unity, having a similar range of values to those of the design variables that are stated in Sect. 3.4 (but in radians instead of degrees). This technique provides good results when nonlinear mathematical programming is used in order to solve the optimum design problem.

A tolerance value of 10^{-4} was used in order for the optimization process to be stopped in either of the following two cases: (a) whenever the variation in the objective function in two consecutive iterations is lower than the tolerance; or (b) whenever the variation in the objective function between this iteration and the minimum value obtained in the previous iteration is less than the tolerance.

3.4 Design variables

3.4.1 Reinforcement necessary in top and bottom layers

The difficulty of the problem lies in trying to solve the equilibrium equation system. Even though the unknowns that we are interested in are the tensile forces of the reinforcement (N_{sxt}, N_{syt}, N_{sxb} and N_{syb}), there are a further four unknowns (a_t, a_b, θ_t and θ_b). Thus, there is a system of six equilibrium equations with eight unknowns. This requires searching for θ_t and θ_b values in order for the total amount of reinforcement to be minimal, thus satisfying the equilibrium equations. Therefore, the θ_t and θ_b angles were used as the design variables.

The MC90 states that by adopting 45° for the compressed truss of the concrete, a minimum of the local reinforcement is obtained. Therefore, the values $\theta_t = \theta_b = \pm 45°$ (phase shifted by 360°) were used as the initial values for the design variables in the validation examples of the procedure, with satisfactory results being obtained. The phase shift is used as a strategy to avoid negative or zero values in the design variables, since the ANSYS program does not allow for their use in its optimization module [20]. In this way, the variation ranges are

$$270^\circ \le \theta_b \le 360^\circ \tag{44}$$

$$360^\circ \le \theta_t \le 450^\circ \tag{45}$$

The design variables are forced to adopt the extreme values of the variation intervals when they are too close to them (an angle of 10–15°). This is done for two basic reasons. The first is concerned with taking into consideration the recommendation in MC90 that the tensile and compression forces must be separated by at least 15°. For two very near directions, the tension in the reinforcement and the compression in the concrete could conflict, and this would infringe upon the displacement compatibility condition. The second reason is to ensure the convergence of the optimization process in the solution of a nonlinear equation system. We can assume that for θ_t and θ_b values which are close to 0 or $\pm 90^\circ$, the direction of the compression force in the concrete tends to be parallel to any of the reinforcement directions, leading to a compression force in the reinforcement. This is unacceptable according to the proposed formulation, and so the fact implies that this reinforcement is not needed, thus enabling convergence to be achieved.

The design variables were normalized by dividing them by the initial values chosen in the optimization process. These normalized values of the design variables are used by the optimization algorithm, for which reason we also refer to them as optimization variables.

Finally, a tolerance value of 10^{-6} was used in order for the optimization process to come to a halt when the variation in the design variables in two consecutive iterations is below this value.

3.4.2 Reinforcement necessary only in the bottom layer

In this case, the necessary unknowns are the tensile forces of the reinforcements N_{sxb} and N_{syb}. Nevertheless, there are another six unknowns, such as a_t, a_b, θ_b, N_{cxt}, N_{cyt} and N_{cxyt}, so there is a system of seven equations (the six equilibrium equations and a check equation of the maximum compressive stress in the top layer) with eight unknowns. This implies looking for the θ_b values in order for the total amount of reinforcement to be minimal and to satisfy the seven-equation system. Therefore, the θ_b angle was used as an optimization variable.

$\theta_b = 315^\circ$ is adopted as the initial value of the design variable, i.e. -45° phase shifted by 360° for the reasons given in the previous section. The range of variation is that adopted in (44). In the same way, for values of θ_b close to 270° or 360°, the direction of the compressive force in the concrete is very close to the direction of the reinforcement, and it thus becomes unnecessary. The variation tolerance of the design value takes the value 10^{-6}.

3.4.3 Reinforcement necessary only in the top layer

This case is similar to the previous one for the bottom layer. Now the angle θ_t was used as an optimization variable, and $\theta_t = 405^\circ$ was adopted as the initial value of the design variable, i.e. 45° phase shifted by 360°. The variation range is the same as that adopted in (45). In the same way, θ_t values that are close to 360° or 450° mean that the reinforcement is not necessary. Once again, the tolerance adopted is 10^{-6}.

3.5 Constraints

As with the objective function and the design variables, the constraints were normalized so they adopt similar values which are close to 1, thus making the mathematical programming algorithm evolve satisfactorily [31]. A tolerance value of 10^{-6} was used, for which the optimization process comes to a halt whenever it finds a lower variation in the constraints between two consecutive iterations.

Equality constraints. The equilibrium equations between applied forces and internal design-strength-of-materials forces and the thicknesses of the top and bottom layers were used as equality constraints. These thicknesses must comply with the expressions suggested in MC90, ($a_{j,N}$), to be able to resist the compressive force in the layer

$$a_{j,N} = -\frac{N_{cj}}{K f_{cd1}} \tag{46}$$

Then, the constraint can be formulated as follows

$$h_j^a = a_j - a_{j,N} = 0 \tag{47}$$

and be normalized as

$$h_j^a = \frac{a_j - a_{j,N}}{D_{a,\max}} = 0 \tag{48}$$

where j is t or b, and $D_{a,max}$ refers to the greatest difference between the initial a_j and $a_{j,N}$ values.

Strictly speaking, the constraint on the thickness is not an equality constraint. The convergence of a_j towards the value $a_{j,N}$ is considered to be reached when a certain tolerance T, set by the designer, is satisfied. This tolerance also has a bearing on the decreasing of the values achieved for the objective function. Therefore, the equality constraints can be formulated as follows

$$0 \le h_j^a \le T \tag{49}$$

It is also necessary to normalize the tolerance by dividing it by $D_{a,\max}$. To adopt a hundred-thousandth part of the thickness h of the element, $T = 10^{-5}h$ is deemed acceptable.

Inequality constraints. To be consistent in terms of the behaviour of the materials and for the different cases of reinforcement considered, the inequality constraints used consist in limiting the tensile forces of the reinforcement (N_{sxt}, N_{syt}, N_{sxb} and N_{syb}) so that they do not take on negative values. That would imply unacceptable compressive forces in the reinforcement, according to the formulation proposed. Thus, the constraints are

$$N_{sij} \ge 0 \tag{50}$$

where N_{sij} is the tensile force of the reinforcement in the i (x or y) direction in the j (t top or b bottom) layer. These constraints can be normalized as follows

$$g_{ij}^N = \frac{N_{sij}}{N_{sij,\max}} \ge 0 \tag{51}$$

where $N_{sij,\max}$ refers to the greater initial tensile force.

3.6 Description of the finite element

A reliable analysis method is crucial for accuracy of the optimum. Locking phenomena leads to an error in the optimization process, which is difficult to detect, and may have consequences for the case of stress and displacement constraints [32]. In these cases, it is possible that the optimal design obtained is not even in the 'real' feasible domain, that is to say that the considered displacement or stress violates the given constraint. Consequently, the solution obtained from the optimization procedure is not even within the group of admissible solutions. Therefore, it is necessary to use elements in the model with a formulation prepared to avoid locking.

The finite element used in this research is the *Shell*93 in ANSYS program. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. The deformation shapes are quadratic in both in-plane directions. The element has plasticity, stress stiffening, large deflection, and large strain capabilities. The material property matrix for the element includes a formulation to avoid shear locking [33].

4 Examples

Two examples were carried out: the simply supported slab; and the deep beam.

4.1 Simply supported slab

The concrete plate used by Lourenço and Figueiras [22] was used as the example to compare their results with those obtained using the method stated in the present study. It is a rectangular plate simply supported at the four edges, 0.15 m thick and has a span of 5.0 × 6.0 m. It is subject to a vertical load p of 15.0 kN/m^2 distributed uniformly, including its own weight. It is also subject to a membrane uniform load q around the edges of the plate (Fig. 3).

Three q values were used in our study, supposing a simple flexural state in the plate ($q = 0$ kN/m) [34, 35] and supposing a compressive and flexural state with q adopting a value of 150 kN/m and 250 kN/m. The materials are 20 MPa compressive strength concrete and 400 MPa tensile strength steel. The distance from the centre of gravity of the reinforcements to the exterior plate fibers is 0.025 m.

The finite element model of the plate is shown in Fig. 4. A convergence study using four mesh sizes ((1) $5 \times 6 = 30$; (2) $11 \times 13 = 143$; (3) $21 \times 25 = 525$;

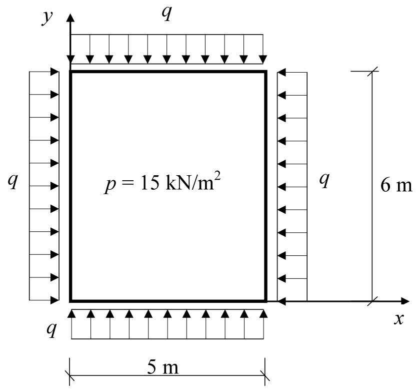


Fig. 3 Geometry and loads for the simply supported plate

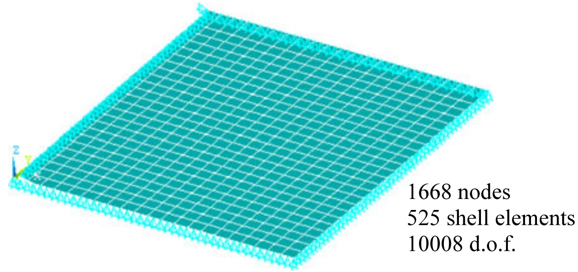
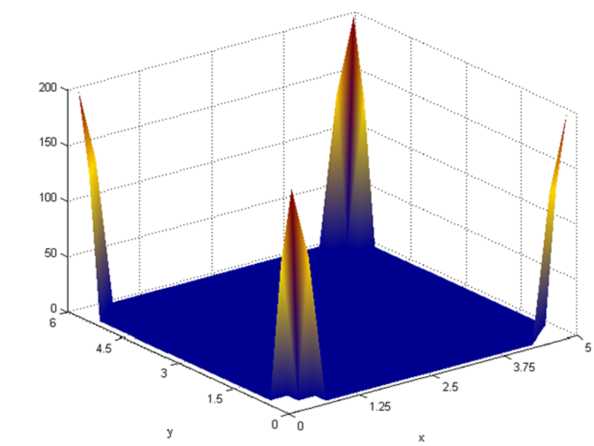
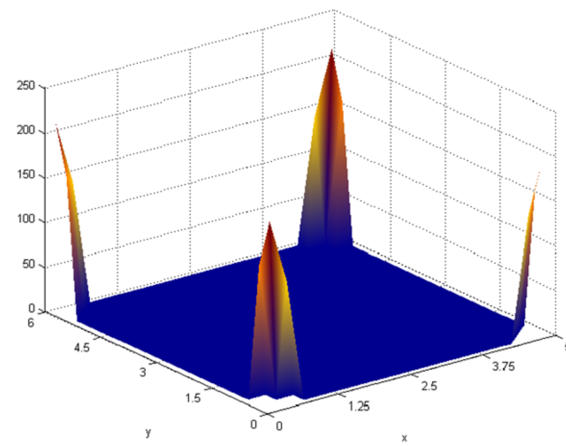


Fig. 4 Finite element model of the plate

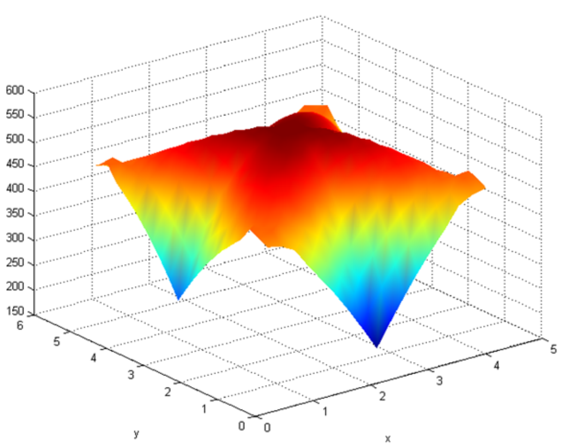
(4) $41 \times 49 = 2009$ elements) was carried out to determine the mesh to use for this example [36]. The most appropriate mesh, combining solution time and accu-



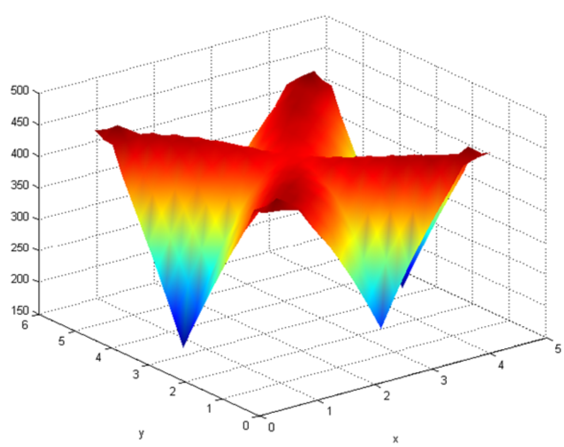
(a) Top layer. *X*-axis



(b) Top layer. *Y*-axis

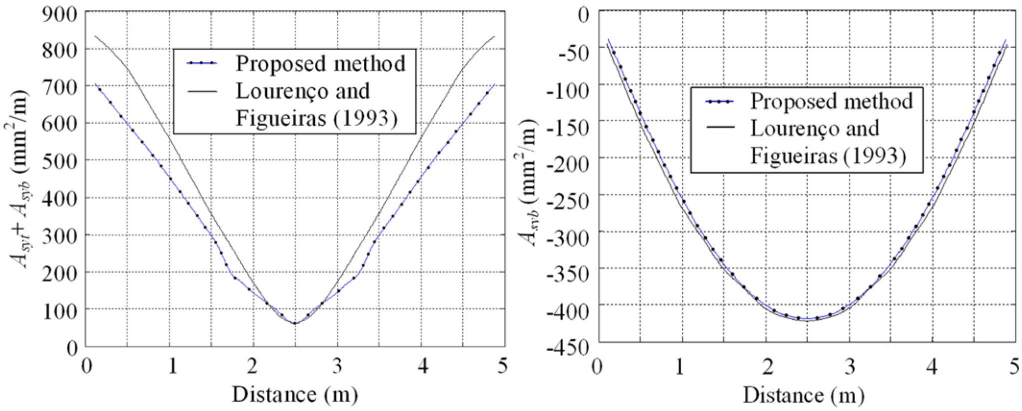


(c) Bottom layer. *X*-axis



(d) Bottom layer. *Y*-axis

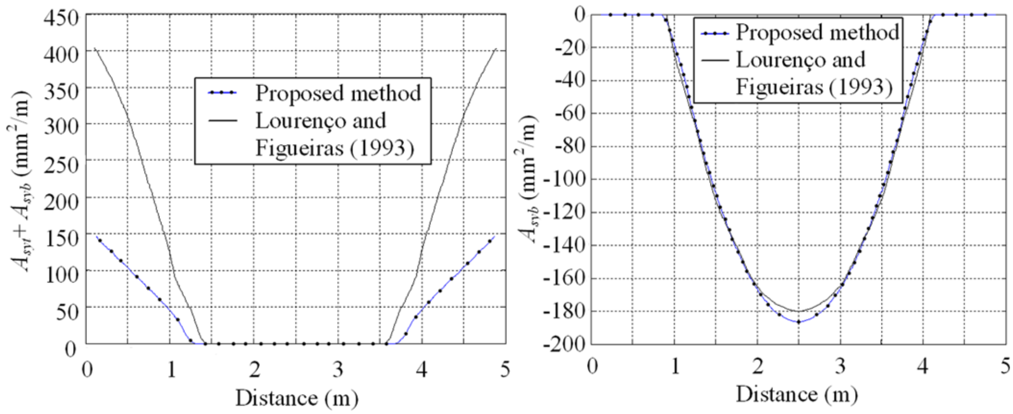
Fig. 5 Density reinforcement surfaces ($q = 0$ kN/m)



(a) Cross-section close to the supports

(b) Cross-section at mid-span

Fig. 6 Total y-reinforcement in a cross-section parallel to x-axis ($q = 0$ kN/m)



(a) Cross-section close to the supports

(b) Cross-section at mid-span

Fig. 7 Total y-reinforcement in a cross-section parallel to x-axis ($q = 150$ kN/m)

racy, was mesh 3, which was therefore employed in this study.

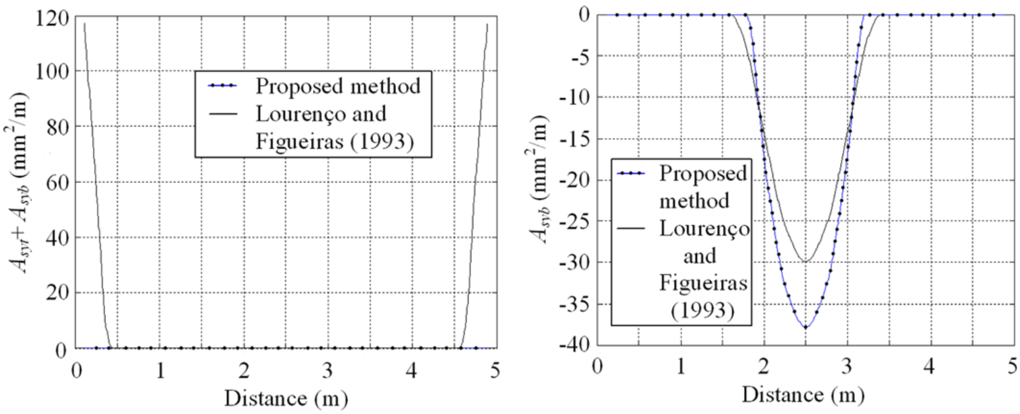
The density reinforcement surfaces in both directions for each layer are shown in Fig. 5, for the simple flexural state. The units are expressed in mm^2/m.

Figure 5 reveals how the reinforcement densities are grouped following the principal stress directions. They are nonexistent for most of the top layer because the fitting is not needed. The quantities in the reinforcement according to the x-axis are greater than those with respect to the y-axis, because of the difference in the length of their sides. Comparisons between the total reinforcement values are given in Figs. 6–8. The values obtained in [22] resolve a nonlinear equation system and the optimum reinforcement is in accordance with the method being proposed for the three cases, with differing values of q.

For the simple flexural state (Fig. 6) the average reinforcement savings are 18% in the section close to the supports of the plate and 3% in the section at mid-span.

For the compressive and flexural state with $q =$ 150 kN/m (Fig. 7), a percentage of 66% less reinforcement is a higher average than that obtained in the section adjacent to the supports and there is practically no difference in the central section.

Finally, for the case of high membrane load (Fig. 8), there is no need for reinforcement at the section close to the supports, with the proposed method. However, according to the results provided by Lourenço and Figueiras [22], there is some need for reinforcement.



(a) Cross-section close to the supports

(b) Cross-section at mid-span

Fig. 8 Total y-reinforcement in a cross-section parallel to x-axis ($q = 250$ kN/m)

The average loss of reinforcement in the section at mid-span is 19%. In general, the lesser need for reinforcement is quite logical in elements subject to a compressive and flexural stress state compared to those subject to simple bending.

4.2 Deep beam

This example consists of a three-span continuous deep beam used in reference [14] and validated in reference [37]. The geometry of the structure and the loads are shown in Fig. 9. The two cantilevered spans have a cavity. The beam has four vertical loads applied to the four loading columns.

The values are of 800 kN and 1000 kN as shown in Fig. 9, but in the finite element model they are applied as distributed loads (Fig. 10). The dimension of the cross-section of the four columns is 0.30 × 0.25 m. The beam is supported on two columns with a cross-section dimension of 0.4 × 0.4 m. The concrete has a compressive strength of 20 MPa.

A convergence study was carried out to determine the mesh to use for this example [36]. The most appropriate mesh, combining solution time and accuracy, was a mesh of 3360 elements, which was therefore employed in this study (Fig. 10).

The reinforcement density is shown in Fig. 11. The results show that reinforcement is required in the region of maximum bending and around the cavities where the greatest tensile stresses appear. These results are similar to those obtained by [14].

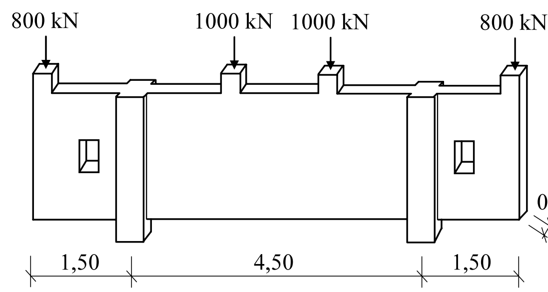


Fig. 9 Geometry and loads for the deep beam

5 Conclusions

Traditionally, computers have been used to analyse the response of a user-defined structure and to check its safety for given applied loads. The use of optimization techniques in the design of structures widens the application of computers and allows the user to obtain optimum designs for stated design conditions.

This paper presents an optimal reinforcement design formulation for concrete shell elements. A finite element program was used and the formulation was implemented by means of user routines within the program optimization module. Based on the results obtained, it may be concluded that the differences observed between the amount of reinforcement obtained by using optimization techniques and by means of traditional methods, may be considerable. It is possible to achieve average values in the savings of reinforcement for the simply supported slab of approximately 12% (for flexural and small membrane forces), 24%

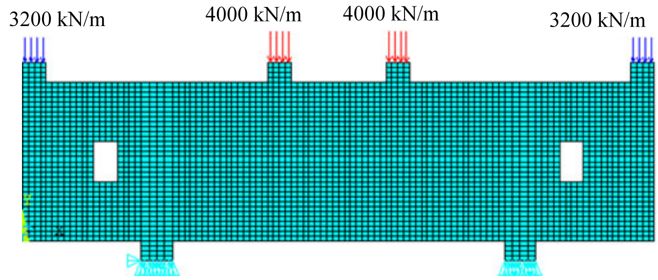
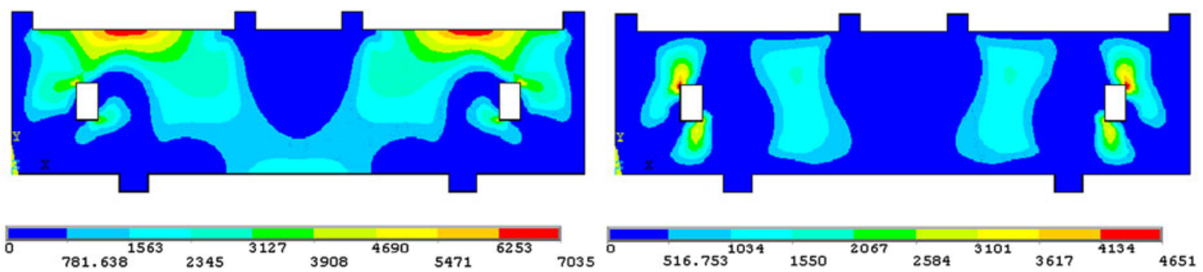


Fig. 10 Finite element model of the deep beam



(a) A_{sx} (b) A_{sy}

Fig. 11 Reinforcement density [mm^2/m]

(for flexural and medium membrane forces) and 44% (for flexural and large membrane forces).

References

1. Technical Committee CEN/TC250 (2004) Eurocode 2: Design of concrete structures—Part 1-1: General rules and rules for buildings. European Committee for Standardization CEN, Brussels
2. ACI Committee 318 (2008) ACI 318-08: Building code requirements for structural concrete and commentary. American Concrete Institute, Farmington Hills
3. Comite Euro-International Du Beton (1993) CEB-FIP Model code 1990: Design code. Thomas Telford, London
4. Marti P (1991) Dimensioning and detailing, IABSE colloquium on structural concrete. IABSE Rep 62:411–43
5. Polak MA (1998) Shear analysis of reinforced concrete shells using degenerate elements. Comput Struct 68(1–3):17–29
6. Lim L, Buchanan A, Moss P, Franssen JM (2004) Numerical modelling of two-way reinforced concrete slabs in fire. Eng Struct 26(8):1081–91
7. Roca P, Marí AR (1993) Nonlinear geometric and material analysis of prestressed concrete general shell structures. Comput Struct 46(5):917–29
8. Mura I (2006) Limit analysis of reinforced concrete rectangular plates with free edges. In: Topping BHV, Montero G, Montenegro R (eds) Proc 8th Int Conf Computational Structures Technology. Civil-Comp Press, Stirlingshire, paper 129
9. Ramm E, Mehlhorn G (1991) On shape finding methods and ultimate load analyses of reinforced concrete shells. Eng Struct 13(2):178–98
10. Hajek P (1991) Frangopol DM, Optimum design of shear-wall systems. Comput Struct 38(2):171–184
11. Lakshmy TK, Bhavikatti SS (1995) Optimum design of multiple barrel shells. IASS Bull 36(117):53–63
12. Tan GH, Thevendran V, Das Gupta NC, Thambiratnam DP (1993) Design of reinforced concrete cylindrical water tanks for minimum material cost. Comput Struct 48(5):803–10
13. Min CHS (1999) Design and ultimate behavior of reinforced concrete hyperbolic paraboloid saddle shell. In: Astudillo R, Madrid AJ (eds) Proc 40th IASS congr, vol I. CEDEX, Madrid, pp A89–A98
14. Lourenço PB, Figueiras JA (1995) Solution for the design of reinforced concrete plates and shells. J Struct Eng 121(5):815–23
15. Tabatabai SMR (1996) Finite element-based elasto-plastic optimum reinforcement dimensioning of spatial concrete panel structures. Ph.D. Thesis, Institute of Structural Engineering, Swiss Federal Institute of Technology, Zurich
16. Martí P, Tomás A, Torrano MS (2001) Application of optimization techniques in structural concrete teaching (in Spanish). In: García A (ed) Proc 1st ACHE conf structural concrete teaching. ACHE, Madrid, pp 203–209
17. Tomás A, Martí P (2006) Reinforcement design in concrete plates and shells using optimization techniques. In: Topping

BHV, Montero G, Montenegro R (eds) Proc 8th Int Conf Computational Structures Technology. Civil-Comp Press, Stirlingshire, paper 224
18. Bendsoe MP, Sigmund O (2003) Topology optimization: theory, methods and applications. Springer, Berlin
19. Arora JS (2004) Introduction to optimum design. Elsevier, London
20. ANSYS (2004) Advanced analysis techniques guide. ANSYS, Canonsburg
21. Carpinteri A, Lacidogna G, Niccolini G, Puzzi S (2008) Critical defect size distributions in concrete structures detected by the acoustic emission technique. Meccanica 43(3):349–63
22. Lourenço PB, Figueiras JA (1993) Automatic design of reinforcement in concrete plates and shells. Eng Comput 10(6):519–41
23. Hofstetter G, Mang HA (1995) Computational mechanics of reinforced concrete structures. Vieweg, Braunschweig
24. Park R, Gamble WL (2000) Reinforced concrete slabs. Wiley, New York
25. Nielsen MP (1964) Yield conditions for reinforced concrete shells in the membrane state. In: Olszak W, Sawczuk A (eds) Proc IASS symp non-classical shells problems. North-Holland, Amsterdam, pp 1030–1040
26. Nielsen MP (1971) On the strength of reinforced concrete disks. Acta Polytechnica Scandinavica, Civil Engineering and Building Construction Series, Report 70, Copenhagen
27. Braestrup MW (1980) Dome effect in RC slabs: rigid-plastic analysis. ASCE J Struct Div 106(6):1237–53
28. Gupta AK (1981) Membrane reinforcement in shells. ASCE J Struct Div 107(1):41–56
29. Medwadowski SJ (1989) Multidirectional membrane reinforcement. ACI Struct J 86(5):563–9
30. Gupta AK (1986) Combined membrane and flexural reinforcement in plates and shells. ASCE J Struct Div 112(3):550–7
31. Gill PE, Murray W, Saunders MA, Wright MH (1985) Model building and practical aspects of nonlinear programming. In: Schittkowski K (ed) Computational mathematical programming. Springer, Berlin
32. Camprubí N, Bischoff M, Bletzinger KU (2004) Shape optimization of shells and locking. Comput Struct 82(2930):2551–2561
33. ANSYS (2004) Theory reference. ANSYS, Canonsburg
34. Tomás A, Martí P (2002) Optimum design of reinforcement in concrete plates and shells. In: Mang HA, Rammerstorfer FG (eds) Proc 5th world cong computational mechanics. Vienna University of Technology, Vienna
35. Tomás A, Martí P (2003) Finite element-based optimum reinforcement dimensioning of concrete plates and shells. In: Cinquini C, Rovati M, Venini P, Nascimbene R (eds) Proc 5th world cong structural and multidisciplinary optimization. Schonenfeld & Ziegler-Italian Polytechnic Press, Milan
36. Tomás A (2007) Design optimization of thin-walled concrete structures (in Spanish). Ph.D. Thesis, Department of Structures and Construction, Technical University of Cartagena, Cartagena
37. Figueiras JA, Póvoas RHC Genésio ML (1990) Aplicaçao de modelos nao lineares à análise e dimensionamento de estructureas laminares de betao (in Portuguese). IX Cong Ibero Latinoam Met Comp Eng, COPPE-UFRJ, Rio de Janeiro