Spatial dispersion in Cournot Competition

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Abstract

Several papers suggest that Cournot competition in a spatial model, with a uniform distribution of consumers, agglomerate at the center of the market. In this paper we prove that this is not true under an hyperbolic demand function and concave transport cost. Moreover we prove that under the above assumptions in the center of the market the firms obtain minimum profit reaching the maximum when both firms locate at opposite places of the market.

Key words: Equilibrium, Cournot Competition, Spatial competition.

1 Introduction

Spatial competition was firstly developed in 1929 in the seminal work by Hotelling. The literature could be classified into two categories: location models with Bertrand competition and those with Cournot competition.

There are many papers on location theory that coincides in the following conclusion: firms will never agglomerate in a location-price game. A smaller set of papers deal with Cournot competition in spatial models, and one may wonder whether the Cournot or Bertrand assumption is more appropriate to model spatial competition. Although price setting firms seems to be more realistic than quantity setting firms the use of the Cournot model competition has been used in the energy market as can be seen in the paper of Salant (1982) in which propose a computerized Nash-Cournot model. The Cournot model can be justified on the basis of the results of Kreps and Scheinkman (1983). They showed that a Cournot competition in a two stages game, in which the first stage game they compete in quantities and in the second one in price, the perfect equilibrium outcome is the same as the outcome of the one stage Cournot game. Then as Anderson-Neven (1981) and T. Mayer (2000) pointed out, for quantity competition to be a good approximation of real economic decision making, we need the capacity of the plant to be inflexibly determined. On the other hand Bertrand competition will be more relevant where quantity decision are more flexible than price decisions.

The model of spatial competition that we present is motivated by the papers of Anderson-Neven (1981), Brnali Gupta et al. (1997) and T. Mayer (2000). Anderson and Neven stated that Competition between Cournot-Type duopolist which discriminate over the space leads to spatial agglomeration. They worked with linear demand function to enable them to simplify the proof and they con-

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jectured that their result would hold for a wider class of demand functions. T. Mayer also stated that the agglomeration occurs at the center when uniform production cost are assumed or when production cost are minimized in the center. Barnali Gupta et al. proved that agglomeration can occur if the population density is not too thin in that point and for symmetric distribution agglomeration can occur only at the center. In this paper we will prove that in the case of hyperbolic demand function and concave transport cost, perfect equilibrium yields spatial dispersion, obtaining the minimum profit when the firms locate at the center of the market.

The remainder of the paper is structured as follows: In Section 2 we present the model and compute the Cournot equilibrium for given firm locations. In Section 3 we solve the model. Finally in Section 4 give some conclusions and remarks.

2   The Model

We study a subgame perfect equilibrium with location choice as the first stage. Then it is first necessary to characterize equilibrium in the second stage for given locations. Let the two firms locate at x_1 and x_2. Without lost of generality we may assume that $x_1 \leq x_2$ and $x_i \in [0,1] = I$ for $i = 1, 2$. Each point $x \in I$ generates a inverse demand $P = \frac{b}{Q}$ where P is the price of the homogeneous product sold by the firms and arbitrage is not allowed.

Each firm has constant marginal cost which we may assume to be zero without loss of generality. The firms pay transport cost $c(|x - x_i|)$, $i = 1, 2$, to ship a unit of the product from its own location to consumer at point $x \in I$. The function c is assumed to be increasing , twice differentiable, concave (i.e $c'' \leq 0$) with $c(0) = 0$.

The whole market will always be served by both firms and we follow the Cournot assumption that firms compete in quantities at each point of I. The second stage Cournot equilibrium can be characterized by a set of independent Cournot equilibria, one at each point $x \in I$, provided that quantities set at different points by the same firm are strategically independent.

Under th above assumptions, the profit function at each point $x \in I$ by firm i is given by:

$$B_i(x) = (\frac{b}{Q(x)} - c(|x - x_i|))q_i(x)$$

Where $Q(x) = q_1(x) + q_2(x)$ and $q_i(x)$ is the $i's$ output offered for sale at x. After a few calculation yield the following unique Cournot-Nash equilibrium for all $x \in I$:

$$q_i(x) = \frac{bc(|x - x_j|)}{(c(|x - x_i|) + c(|x - x_j|))^2}$$

$$P(x) = c(|x - x_i|) + c(|x - x_j|)$$

$$Q(x) = \frac{b}{c(|x - x_i|) + c(|x - x_j|)}$$

And the resulting profit at any point $x \in I$ is given by

$$\Pi_i(x, x_1, x_2) = \frac{b(c(|x - x_j|)^2}{(c(|x - x_i|) + c(|x - x_j|))^2}$$

for $i, j \in \{1, 2\}$ with $i \neq j$.

In this model, as in the model developed by Anderson and Neven (1991) each firm will supply all positions in space. The distribution of output among firms at any point depends on their respective locations. At any $x \in I$ the firm

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which is closer will supply a larger share of output which decreases with the distance between the firm location and x as it is shown in the above formula.

3 The resolution of the model

The total profit function for the location game for the firm i is given by

$$\Delta_i(x_1, x_2) = \int_0^1 \Pi_i(x, x_1, x_2) dx \quad i = 1, 2$$

. Let us denote x_i^* the location equilibrium point for firm i. The first order conditions for the maximization of firm $1's$ profit is given by:

$$
\begin{aligned}
\frac{1}{b}\frac{\partial \Delta_1}{\partial x_1} = & -\int_0^{x_1} \frac{2c'(x_1-x)(c(x_2-x))^2}{(c(x_1-x)+c(x_2-x))^3} dx \\
& +\int_{x_1}^{x_2} \frac{2c'(x-x_1)(c(x_2-x))^2}{(c(x-x_1)+c(x_2-x))^3} dx \\
& +\int_{x_2}^1 \frac{2c'(x-x_1)(c(x-x_2))^2}{(c(x-x_1)+c(x-x_2))^3} dx
\end{aligned}
$$

Note that when $x_1 = x_2 = \frac{1}{2}$ the second term is zero and the first and third sum to zero as can be easily verified applying the change of variable $x = 1 - y$ in the third integral of the above expression.

The second derivative of the profit function is given by

$$
\begin{aligned}
\frac{1}{b}\frac{\partial^2 \Delta_1}{\partial x_1^2} = & -\int_0^{x_1} \frac{2c''(x_1-x)c(x_2-x)^2(c(x_1-x)+c(x_2-x))-6c'(x-x_1)^2c(x_2-x)^2}{(c(x_1-x)+c(x_2-x))^4} dx \\
& +\int_{x_1}^{x_2} \frac{-2c''(x-x_1)c(x_2-x)^2(c(x-x_1)+c(x_2-x))+6c'(x-x_1)^2c(x_2-x)^2}{(c(x-x_1)+c(x_2-x))^4} dx \\
& +\int_{x_2}^1 \frac{-2c''(x-x_1)c(x-x_2)^2(c(x-x_1)+c(x-x_2))+6c'(x-x_1)^2c(x-x_2)^2}{(c(x-x_1)+c(x-x_2))^4} dx
\end{aligned}
$$

which is ≥ 0 because $c'' \leq 0$. Then when both firms locate at the center of the market they obtain minimum profit.

Therefore as the total profit function Δ_1 is convex as a function of x_1 we have that the maximum is obtained when $x_1^* = 0$ or $x_1^* = 1$.

Similar calculations shows that the maximum for Δ_2 is also obtained for $x_2^* = 0$ or $x_2^* = 1$.

As we have assumed that $x_1 \leq x_2$ the unique locations equilibrium points are $(x_1^*, x_2^*) = (0, 0), (1, 1)$ or $(0, 1)$.

A straightforward computation shows that

$$\Delta_i(a, a) = \frac{b}{4}$$

for all $a \in I$, $i = 1, 2$. Therefore if $(0, 0)$ is the location equilibrium point, then $(1, 1)$ is too. Hence we obtain

$$\frac{b}{4} = \Delta_1(0, 0) = \Delta_1(1, 1) \geq \Delta_1(0, 1)$$
$$\frac{b}{4} = \Delta_1(0, 0) = \Delta_2(1, 1) \geq \Delta_2(0, 1)$$

then summing up the above expression we obtain

$$\frac{b}{2} \geq \int_0^1 \frac{b[c(1 - x)^2 + c(x)^2]}{[c(1 - x) + c(x)]^2}.$$

On the other hand $\frac{c(1-x)^2+c(x)^2}{[c(1-x)+c(x)]^2} > \frac{1}{2}$ for all $x \in I$. Therefore $\int_0^1 \frac{b[c(1-x)^2+c(x)^2]}{[c(1-x)+c(x)]^2} > \frac{b}{2}$ yielding a contradiction. Then the location equilibrium point is $(0, 1)$.

4 Conclusions

In this paper we have shown that for a demand function of elasticity 1, $P = \frac{b}{Q}$ the maximum profit in a Cournot competition is obtained when both firms locate at opposite places in the market, contrary to the assumption made by Anderson and Neven (1991). Therefore this is not only a question of the transport cost and production cost (T. Mayer 2000), neither population density (B. Gupta 1997) but the demand function. In any case, it would be an interesting open problem to investigate the equilibrium location point depending on the elasticity of the demand function.

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