INVESTIGACIONES ECONÓMICAS. vol. XXIX (1), 2005, 149-179

THE TIMING OF WORK IN A GENERAL EQUILIBRIUM MODEL WITH SHIFTWORK

ANTONIO GARCÍA SÁNCHEZ
MARÍA DEL MAR VÁZQUEZ MÉNDEZ
Universidad Politécnica de Cartagena

This paper presents a model in which the work schedule results from the equilibrium between the firm's decision and workers' preferences. A technology which organises the inputs in shifts has been considered for this objective, and the workers' preferences are defined over the different feasible work-days. In this context, the treatment given to the concepts of shiftwork and workday are key innovations. The existence or not of capital constraints, the "fatigue" effect, and the preferences regarding leisure determine the amount of inputs utilized as well as the timing of work.

Keywords: Capital utilization, general equilibrium, shiftwork, work schedule.

(*JEL* D24, E24, J22)

1. Introduction

Working hours for the employees of a company usually coincide both for economic, social, cultural, and environmental (light, temperature, etc.) reasons, and to take advantage of positive externality that arises from the potential for communication and co-ordination among employees who work at the same time. In the United States, 80% of full-time workers begin their work-day between 7 and 9 in the morning and end between 4 and 6 in the evening. Only 26% work on weekends, see Weiss (1996). However, work shifts have allowed the increase in operational time of installed capital and have had a great influence on the increase in productivity of factors, see Foss (1997).

Our interest centres on analysing the determination of not only *how much* work is performed but also *when* it is performed. For this, we must take into consideration the *timing* of the workday. We shall tackle this issue from the work system based on shifts, as this form of work organisation implies the existence of several work schedules within the same period. For this we shall introduce two new elements in the literature concerning these issues: the definition of workday and the concept of *shift*. Consequently, a workday features a starting time and duration (compared with the usual concept of a workday as a number of hours). Concerning the concept of *shift*, we shall define it as a group of workers who work the same workday and who jointly use a capital stock. This allows us to consider the existence of workdays that are different not only due to their duration but also based on the moment they start (an 8 hour shift is not equally productive nor has the same effect on the disutility of workers if it begins early in the morning or at night) and, furthermore, this gives rise to different *shift* systems concerning the number of workers, the capital stock used, and the type of workday. This is an extension of the definition of *team* used by Fitgerald (1998a), where work time was a variable of quantitative importance, in the sense that workdays differed in the number of hours worked, while, in the present model, a qualitative aspect is introduced by incorporating another variable: the starting time of the workday. Furthermore, salaries will depend on the type of workday, both concerning its duration and its time of commencement.

Although, in the model, we consider that there are a great variety of types of workday, there must be a finite number of types, which lies within the framework of the indivisible work concept of Rogerson (1988) and Hansen (1985), i.e., the agents offer work for one workday with a pre-established duration or, alternatively, they do not work. This leads to the fact that there cannot be variations in the hours worked by an individual. In our model, the worker can choose from among different working hours, which are established previously by the firm to try to take advantage of the externalities that arise from the co-operation and co-ordination between workers. An attempt is made to show that the flexibility of the workday is limited by issues that are sometimes of a non-economic type, such as biological (circadian rhythm), cultural, familial, etc. In equilibrium, each agent will work for one of the said workdays, or he will not work. Indivisible work implies the non-convexity of the set of consumption possibilities and of the set of production possibilities, although convexity is achieved with the introduction of employment lotteries. Following the approach used by Hornstein and Prescott (1993) and Fitgerald (1998a) of representing the economy in a McKenzie (1959) type context of general equilibrium, the set of production possibilities becomes a convex cone and, furthermore, with identical agents it allows us to obtain a result where some work and others do not; and, among those who do work, not all perform the same type of workday. Our main contribution consists in defining and introducing, in this context, the concept of *shift*, as defined above.

In the economic literature, the work schedule has received little attention. Some exceptions are Hamermesh (1998, 1999) and Stemberg (1991). Interest has centred more on issues related with overtime (establishment, bonuses, etc.) and, concerning shifts, references are not very abundant: Calmfors and Hoel (1989), Bosworth (1991), Mayshar and Halevy (1997), Dupaigne (2001), and Hornstein (2002). In this literature we find models where the relevant variable is the number of shifts, defined as the quotient between the time the capital is used and the duration of the workday (Calmfors and Hoel (1989)), and models that propose the existence of two shifts within the production function, Mayshar and Halevy (1997) and Hornstein (2002). In these cases, firm output is the production obtained using capital and work during the usual workday (which is usually defined as the instantaneous output multiplied by the number of hours of the said workday), plus the output that results from assigning work to the same capital stock in a second shift (in turn, instantaneous production multiplied by the hours of that shift), therefore, the workday is extended when optimal. In Bosworth (1991), an empirical study on the incidence of shifts in the United Kingdom was performed and, in Dupaigne (2001), the number of shifts extends the operational time on which the disutility of work is defined. In neither of them does the definition of shift coincide with our proposal.

The pioneering work by Sargent (1978) considers the difference in the total costs of a company caused by increasing labour on a normal workday (which the text sometimes calls *first shift*) compared with the alternative of increasing labour through overtime (called *second shift*). It is a dynamic model where each period is a *day* and the daily output is the sum of what is obtained based on the workers assigned to performing a normal work-day, which is pre-established, and the workers assigned to performing overtime, which is also given. The adjustment costs between periods of workers who work normal workdays are greater than the adjustment costs of the workers who work overtime. In Mayshar and Halevy (1997), employment varies along different margins: the total number of employees, their distribution on two discrete shifts, and the duration of the work-day. In the case of fluctuations in the price of output, labour will be assigned to one shift or to two depending on labour productivity and costs. Hornstein (2002) analyses the use of the production capacity and its relation with the fluctuations observed in the time over which capital is used and in the time over which labour is used. For this, he builds a model where the production function allows two shifts, but the capital-labour ratio of the first shift cannot be modified. Therefore, the response to productivity shocks takes place by means of changes in employment in the second shift.

The concept of shift is approached in a different way by Calmfors and Hoel (1989). They study, in a static model, the effect of the elasticity of substitution between the productive factors when the firm faces a reduction in the workday imposed exogenously and the utilization time of capital may increase through an increase in the number of shifts. In their model, the decision variables are the total employment of the firm and the number of shifts, which is defined as the quotient between the time over which capital is used and the duration of the workday. The wage is the same for all employees, although its level will vary depending on the number of shifts (it is supposed that all employees rotate in the shift system). In papers with a less theoretical turn, especially applied to the automobile industry, one of the company decision variables is the introduction or not of new shifts, see Bresnahan and Ramey (1994), Aizcorbe (1992). Empirical research about capital operating hours has been carried out in Foss (1997) and in Anxo (1995). They both contain an elaborate analysis of the recourse to shiftwork and its long-term trends.

On the other hand, the determination of work hours has been widely treated in the literature from different approaches. Business cycle models that centre on the endogenous determination of employment and hours are provided in Kydland and Prescott (1991), Hornstein and Prescott (1993), and Cho and Cooley (1994). Models that analyse the co-ordination between workers as a determining factor for work-days and employment are presented in Lewis (1969), Weis (1966), in a partial equilibrium approach. Furthermore, models for determining the workday in a context of general equilibrium with heterogeneous work appear in Fitzgerald (1998b) and, also co-ordinated in teams, Fitzgerald (1998a). One question that has aroused interest throughout the last decade has been the effect of the reduction of the workweek on job creation. Among the extensive literature on this issue, the reduction is imposed exogenously in Fitzgerald (1998b), Marimon and Zilibotti (2000), Fitzroy, Funke and Nolan (2002) and Rocheteau (2002), or is the result of a certain tax on overtime, as in Osuna and Ríos-Rull (2003).

The model that we propose explains the determination both of the number of shifts and their schedule as a result of the joint decisions by the firm and its workers. Therefore, it is a general equilibrium model and it introduces the preferences of workers defined over the work schedule and the technology of the firm that organises the factors into shifts. Following the economic literature about these questions, we suggest diverse specifications for the functions that represent the workers' utility and the output of a shift, given that the model could be developed under different assumptions. The levels of employment and output are also determined, and an analysis of how they depend on the length of the workday, in the face of changes in the preferences concerning the work schedule, and the importance of the *fatigue* effect or other variables affecting technology. All in all, our concept of shift and, consequently, of workday, is a first approximation to the idea of a flexible workday. We consider that the traditional system of working time organisation, with a workday that begins and ends at the same time for most employees, is changing in response to production needs of new goods and services, technological developments, or even to the appearance of new forms of social and family life organisation. Further developments of the model will allow us to analyse these issues.

The paper is organized as follows: in Section 2 we provide some evidence on work schedules. Section 3 describes the economy. Then we determine, in Section 4, within the framework of a static competitive equilibrium, the optimal number shifts and their features based on the existing capital in the economy. In Section 5, we analyse the wages and the employment required by each shift. Next, we examine how preferences and technology interact to determine the optimal work schedule. Section 6 analyses the importance of the available capital in the determination of working time. Section 7 concludes.

2. Some evidence on work schedules

Hamermesh (1998) indicates, among his conclusions, the enormous quantity of information available on the amount of work and leisure in western economies compared with the scarce knowledge on timing. He bases his information on the only source that he considers appropriate in the case of the United States (US), which is the data on the usual hours when people start and finish work and which, for the years 1973, 1978, 1985 and 1991, were provided by the Current Population Survey. With this, he elaborates indicators on the fraction of workers who are working at each hour of the day and offers a more wide-ranging study of the results in Hamermesh (1999). The main conclusion he reaches is that work in afternoon and night shifts decreased considerably between the 1970s and the 1990s and, nevertheless, work performed around the normal work-day (from 9 a.m. to 5 p.m.) increased, more specifically, from 6 a.m. and after 6 p.m. This reflects a tendency to increase the work interval of the normal workday.

Bosworth (1991) analyses the evolution of the incidence of shifts in the United Kingdom between 1970 and 1989. Throughout the period, this system is used more in the manufacturing industry than in the rest of the economy, and more so among manual workers than among non-manual workers. The data for the economy as a whole oscillate between 11.2% of workers on shifts in 1973 and 13.5% in 1989. The highest figure is 15.3% in 1983.

Cette (1995) studies the evolution of shifts in France from 1957 to 1990, which is explained, in part, by the pro-cyclical nature of this variable. The percentage of manual industrial workers who work on shifts in 1990 was 34.1.

Foss (1997) elaborates a study about shiftwork and the changes in weekly hours of capital use over long periods of time in US manufacturing industry. Manufacturing plants were working about 25% more hours per week in 1976 than in 1929. From 1976 to 1988, weekly operating hours increased by 4.1%. The rise in hours of capital use has come about mainly because of an increase in shiftwork. In fact, changes in average weekly plant hours over an extended period are a reflection of new technology, a continuous process. However, that change occurred earlier in the past century and some new technological developments are associated with shorter average weekly capital hours. In his study, Foss corroborates, as a cursory examination of the statistics suggests, that plants in capital-intensive industries work the longest hours, and that plants in labour intensive industries work the shortest[1]. On the other hand, shifts are cyclical since fluctua-

[1] In spite of its limitations, the measure of capital intensity used was the ratio of kilowatt-hours of all electricity consumed to wage earner man-hours.

tions in demand may cause firms to alter their plants' hours by adding or dropping shifts. Premium pay is usually required for second and third shifts because workers dislike working at other than conventional daytime hours. But other variable costs, besides labour, and institutional and social regulations may be included. In sum, one of the Foss' study conclusions is that, other things equal, the more rapid the rate of technological change, the greater the incentive to use up plant and equipment in the physical sense in a given time period.

In an international comparative study, Anxo (1995) examines the recourse to shiftwork in France, Germany, Norway, Sweden, the UK and the US. With regard to long-term trends, the analysis reveals a noticeable increase in the incidence of shiftwork. The results of national studies on the determinants of shiftwork stress both the close connection between the recourse to shiftwork and capital intensity, and the significance of firm size.

In the case of Spain, the *Encuesta de Coyuntura Laboral* (ECL) (Survey on the Labour Situation) offers information on the percentage of workers who work on shifts. The average percentage of workers whose companies use shift systems, between 1998 and 2001, is of 33.5; and among all workers, an average of 24.6% worked on shifts during the period studied. The survey differentiates between morning shift, afternoon shift and night shift, and the distribution of workers between those shifts was constant for all years: approximately half for the morning shift, one third for the afternoon shift and one seventh for the night shift. By sectors, the incidence of shifts is greater in the industrial sector and almost inexistent in the construction sector. Specifically, the average percentage for the 1998-2001 period of workers whose companies used shift systems was 48.3 in the industrial sector, 33.7 in the service sector and 3.3 in the construction sector. Furthermore, the larger the size of the company, the higher the percentage of workers who work on shifts, reaching 45.6 in the case of companies with over 250 workers (the average for 1998-2001). On the other hand, the ECL classifies workers based on the type of measures that would be adopted in their company in the event of an increase in demand and by sector. In this sense, according to data from 2001, the highest percentage of workers work in companies that would opt for *Contracting new workers* (74.8). The firms that would opt for *Improved production capacity* represent 9.8% of workers; while the *Increase in overtime hours* would affect 1.8% of workers. Alternatively, the *Encuesta de Población Ac-*

tiva (EPA) (Labour Force Survey) includes, in the second quarter of 2001, a special module called *Relaciones laborales especiales y condiciones y horarios de trabajo* (Special labour relationships and work conditions and schedules), which states that 17.5% of wage-earners work in a team that uses a shift system[2] [3]. Their distribution among the different shift systems, as defined in this module, is: fixed shift, i.e., the employee does not change shifts (34.6%); morning, afternoon and night shifts every day (20.8%); morning, afternoon and night shifts from Monday to Friday (7.9%); morning and afternoon from Monday to Friday (12.1%); day and night from Monday to Friday (0.8%); or another type[4] (23.9%). More information on the work schedule is given by the distinction made in this module between fixed entrance and exit hours, which comprises 87.8% of wage-earners, and flexible entrance and exit hours[5], which applies to 5.3% of them; the rest are included in the category of *another type,* as in the definition of the shift system (see footnote 3).

Finally, it is interesting to note, also for Spain, that the question of workday flexibility is dealt with by the *Encuesta de Calidad de Vida en el Trabajo* (Survey on Quality of Life at Work) through three issues: possibility of resting during the workday, modification of entrance and exit hours, and the possibility of taking a non-recoverable day off. In 2001, 33.8% of workers answered that they could never or hardly ever take a rest during the work-day, 60.3% could never or hardly ever decide the hour they start and end their work-day, and 45.6% lacked the possibility of taking a day off without losing money or holidays.

[2] According to the EPA, work in shifts is defined as a system where employees belong to a group of people who replace each other to perform the same tasks at the same place.

[3] The employees referred to in the module differ in number from the employees according to the corresponding EPA, due to the lack of response of the module, and they differ from the workers in the ECL as the definitions of the two surveys do not coincide.

[4] This category includes: variable weekly schedule established by the employer, schedule established by agreement with the employer, schedule established by the worker and other non-described types.

[5] In accordance with the EPA methodology, this category means that the entrance and exit hour are freely determined, as long as a certain period of the day is covered, which is compulsory, and the total number of hours per week is worked. This is the case of many workers in the public sector.

3. The shiftwork economy

Let us now describe the model. This economy lasts for one period and it is populated by a continuum of people with measure 1. All individuals are identical and each person has a time endowment of 1 that can be allocated to either work, h, or leisure, $l = 1 - h$, and $\overline{k} > 0$ units of capital.

Preferences are defined over consumption and working time, which can be performed with different timing. The production is organized in shifts consisting of a group of workers, using capital during one type of workday. We represent this economy in McKenzie-type general equilibrium language. Let a commodity point x be an element of the Euclidean space L. For an agent the consumption set X is a subset of the commodity space L. Preferences over consumption bundles in X are represented by the utility function $U : X \to R$. Production is described by some aggregate production possibility set Y, which is a convex cone in L. As all agents are identical and with measure 1, an allocation $[x, y]$ is feasible if $x \in X, y \in Y$, and $x = y$.

3.1 Technology

Shiftwork production is modelled in the following manner. Output is produced by a large number of production teams which can operate in different shifts. This allows the extension of capital utilization for a longer time period than a corresponding workday. We consider that different shifts can operate different workdays, and workdays are distinguished by the number of hours and by the moment at which work starts. So, we define a workday and a shift:

Definition 1: A workday, s, is characterized by a pair (t, h) where t is the moment at which work starts and h is the length.

Definition 2: A shift is a group of workers, e, working during a workday, $s = (t, h)$, with k units of capital. A shift is characterized by a four-tuple (t, h, k, e).

This way of defining a shift is an extension of the *plant* in Hornstein and Prescott (1993), where both the length of time over which a plant can be operated and the number of workers operating it can be varied, and a plant is characterized by (h, k, e). In comparison with this model, our shiftwork technology allows, in addition, a plant to be operated for more than a workday, and different plants can be operated during the same workday. This logically implies that more than a shift can use the same stock of capital if their workdays do not overlap. Concerning the output of a shift, we distinguish between the instantaneous production and the total production. The instantaneous production function is $f(k, e)$, which displays constant returns to scale, but the resulting output depends on the workday type that the shift operates. Both the length and the moment of starting matter because we could consider, for example, that the productivity in a 8-hour workday is not the same during days and nights. So, the output of a type (t, h, k, e) shift is: $F(t, h, k, e) = f(k, e)\ g(t, h)$, where $g(t, h)$ measures the effective working time of a workday starting at t and ending at $t+h$. If the set of feasible workdays is denoted by S, then $g : S \rightarrow R_{+}$, multiplies the instantaneous output of the shift that operates the workday (t, h). Therefore, although both capital and labour are homogeneous, they become different inputs depending on the shift they work.

3.2 Traded commodities

The model is in continuous time. However, it is simplified by assuming that there is a large but finite number of possible workday lengths, and consequently the number of possible starting times is also finite. In this case, the set of possible workday lengths is denoted by H, where $H \subset [0, 1]$, $H = \{h_0, h_1, ...h_{N_H}\}$, with $h_0 = 0$ and $h_{N_H} = 1$[6]. The lengths determine the possible starting times, so there is a set $T = \{t_0, t_1, ...t_{N_T}\}, T \subset [0, 1[$, of possible starting times, with $t_0 = 0, t_1 = h_1, t_2 = h_2$ or $t_2 = 2h_1$ depending on $h_2 < 2h_1$ or $2h_1 < h_2$, and so on, so that, apart from 0, each time a workday is finished another can start with the same or different length, except at the end of the period. Therefore, the set of feasible workdays, S, is finite, $S \subset TxH$, and is given by:

$$S = \{(t, h)/\ t \in T, h \in H;\ t + h \leq 1\ ;\ and\ \ h = 0\ \Rightarrow t = 0\}$$

Thus the set contains the following conditions: the length is restricted by the duration of the period and the workday $(0, 0)$ which implies the length 0 is included. The number of feasible workdays is N_S and it is greater than or equal to the number of feasible starting times N_T.

Introducing a finite number of different workdays creates an indivisibility because people cannot work, for example, 1/3 of an 8-morning-hour

[6] Considering the existence of working time regulation would imply defining the longest workday as a number less than 1, that is $h_{N_H} < 1$.

workday and 2/3 of a 6-evening-hour workday. An analytically useful strategy for working with economies that have indivisibilities is to allow people to randomize over different workdays using lotteries, following Rogerson (1988). In this way, people supply a lottery contract that specifies the probability of working different workdays, and they will work only one workday depending of the lottery's outcome.

The commodity space, L, is $R^2 \times M(S)$, where $M(S)$ denotes the set of signed measures on the Borel sigma algebra of S. An element of L is given by (c, k, n), where c is the consumption good, k denotes the services of the capital stock, and n is a measure over labour workdays. One unit of capital produces one unit of capital services. When S is a finite set, n is a vector and $n(s)$ is the measure of type s workday (with start at t and length h). The agent who chooses a point in L receives c units of the consumption good in exchange for providing k units of capital[7] and some measure n over labour workdays.

3.3 Production possibility set

Let $\mathbf{K}$ and $\mathbf{E}$ be finite sets, and let J be Sx$\mathbf{K}$x$\mathbf{E}$, the set of feasible shifts, with generic element (s, k, e) and cardinality N_J. We can index shift types by $j = \{1, 2, ..., N_J\}$. A production plan organizes the distribution of inputs across shifts of different types, given that workers are available for certain workdays, while capital is available at the beginning of the period, and each time a shift finishes the capital utilized is available for another shift. Let z_j denote the measure of type j shift operated, then the production plan is a vector of N_J numbers, $z = \{z_1, z_2, ..., z_{N_J}\}, z \in R_+^{N_J},\ z_j \geq 0$, that describes how the inputs are allocated across shifts of different types. The production possibility set, Y, is defined as:

[7] The component k of the commodity space is not the same as the element k of a shift because the latter is an element of a finite set. Thus they are denoted differently.

$$Y \equiv \{\{C, K, N\} : \textit{there exists a production plan } z \in R_+^{N_J} \textit{ such that}$$

$$C \leq \sum_j z_j\, f(k_j, e_j)\, g(t_j, h_j)$$

$$\sum_{\{j:\, t_j \leq t < t_j + h_j\}} z_j\, k_j \leq K, \qquad \textit{all } t \in T \qquad [1]$$

$$\sum_{\{j:\, h_j = h,\, t_j = t\}} z_j\, e_j \leq N(s), \qquad \textit{all } s \in S\ \}$$

The first constraint says that the total amount of the consumption good is less than or equal to the total output produced by all shift types. The second constraint states that, for each feasible starting time, the capital allocated across all the shift types with this starting time or with the previous starting time but not finished yet is less than or equal to the total capital available. The third constraint states that the amount of type s workdays allocated across all shift types is less than or equal to the total amount of type s workdays available. It is immediate that Y is a convex cone.

3.4 Preferences

The specification of this economy is completed by presenting individuals' preference ordering and their feasible consumption bundles. The utility of a person choosing the commodity point $x = (c, \mathrm{k}, n)$ is given by:

$$U(x) = u(c) - \sum_s n(s)\, v(s) \qquad [2]$$

where $v : S \longrightarrow R_+$ represents the disutility of working the workday s, and : $u : R \longrightarrow R$; $v(0,0) = 0$, and $\lim_{c \to 0} u'(c) = \infty$. The function $u(c)$ is assumed to be continuously differentiable and strictly concave. Notice that $\sum_s n(s)\ v(s)$ is the expected disutility of working. The consumption possibility set of an agent is:

$$X(\,\overline{k}\,) = \left\{ (c, \mathrm{k}, n) : \mathrm{k} \leq \overline{k},\ c \geq 0,\ \mathrm{k} \geq 0, \sum_{s \in S} n(s) = 1,\ n(s) \geq 0 \right\} \qquad [3]$$

which contains the standard nonnegativity constraints and the conditions that capital services are restricted by the capital stock endowment, and n is a probability measure.

4. Competitive equilibrium

The commodities traded are given by $x = (c, \mathrm{k}, n)$. Prices are in terms of the consumption good. The rental price of capital is r. The wage is a function w mapping signed measures into R. With a finite set of possible workdays, w is a vector of prices, where $w(s)$ is the price of the type s workday. That is, if a person works the workday s with probability 1, $w(s)$ units of the consumption good are received.

4.1 The firm's decision problem

The firm rents capital, employs workers for workdays of different types and decides how to allocate these resources across all the shifts. On hiring workers, the firm buys lottery contracts that specify the probability of a person working workdays of different types, possibly including a workday of length 0. The firm, which buys a large number of these contracts, faces no uncertainty as to the number of workers who will work different workdays. As all the individuals are identical, all will sell the same lottery contract, but this does not mean that all people work the same workday. In fact, each workday will be worked by a measure of agents similar to the probability specified by the contract of working that workday. Hence, given prices (r, w), the firm chooses quantities (C, K, N) to solve:

$$Max \;\; C - rK - \sum_{s} w(s)\; N(s) \qquad [4]$$

$$s.t. : (C, K, N) \in Y \qquad [5]$$

where $N(s)$ is the measure of workdays of type s.

4.2 Individuals' decision problems

In this economy individuals purchase the consumption good and sell capital and labor services to firms. The labor services are supplied in the shape of lottery contract that specifies the probability of working different workdays. The amount an individual receives for a given lottery contract does not depend on the lottery's outcome, that is, on the type of workday the individual works ex post, but the probabilities of work supplied. The individual decision problem is:

$$Max\ \ u(c) - \sum_s n(s)\, v(s) \qquad [6]$$

$$s.t. : (c, \mathrm{k}, n) \in X(\overline{k}) \qquad [7]$$

$$c \leq r\mathrm{k} + \sum_s w(s)\, n(s) \qquad [8]$$

4.3 Definition of equilibrium

A competitive equilibrium for this economy is an allocation (x^*, y^*) and a price system (r, w) such that:

i) x^* maximizes $U(x)$ subject to $x \in X(\overline{k})$ and the budget constraint (8).

ii) y^* maximizes (4) subject to $y \in Y$.

iii) $x^* = y^*$.

In this economy with one agent type, all individuals choose the same commodity point, although this does not imply that all of them work the same workday, since the chosen commodity point will involve randomizing over different workdays. Also, the anonymous competitive equilibrium and the anonymous Pareto-optimal allocation coincide. Therefore, and as in Hornstein and Prescott (1993), we can study the properties of the anonymous Pareto optima of this economy to establish properties of competitive equilibrium allocations. The social planner's problem is:

$$Max\ \ U(x) \qquad [9]$$

$$s.t. : x \in X(\overline{k}),\ y \in Y,\ x = y$$

The characteristics of an anonymous Pareto-optimal allocation are derived by analyzing a simpler equivalent problem. For this version, we define $\mathbf{k} = k/e$ as the capital-labour ratio in a shift, therefore we no longer distinguish between the organization of production, vector z, and the supply of workdays, vector n. So, the event j is characterized by a triplet $(t, h, \mathbf{k})$ which is interpreted as a shift operating the workday of length h, with starting time at t, and using $\mathbf{k}$ units of capital per worker. There is a finite number, N_J, of $(t, h, \mathbf{k})$ triplets indexed by $j = \{1, 2, ...N_J\}$. The Pareto-optimal allocation then solves:

$$\underset{c,\, n \geq 0}{Max} \;\; u(c) - \sum_j n_j \, v(s_j) \qquad [10]$$

$$s.t.: c \leq \sum_j n_j \; f(\mathbf{k}_j, 1) \; g(s_j) \qquad [11]$$

$$\sum_{j:\, t_j \leq \, t \, < t_j + h_j} n_j \; \mathbf{k}_j \leq \overline{k} \qquad all \; t \in T \qquad [12]$$

$$\sum_j n_j \leq 1 \qquad [13]$$

Given that $f(k, e)$ displays constant returns to scale, the solution to this problem has the same measure of agents working each workday (t, h) on shifts with $\mathbf{k}$ units of capital per worker, and the same c as does the solution to the original problem.

The constraint set defined by [11]-[13] is closed, bounded and non-empty ($\overline{k} > 0$), and the objective function in [10] is continuous in c and n; consequently, a solution exists.

We divide the Pareto problem into two subproblems: one that is a linear program and one that is a nonlinear problem. Let the function $V : R^{N_J} \rightarrow R$ be defined as:

$$V(n) = -\textstyle\sum_j n_j v(s_j)$$

This function returns the disutility of work associated with the working plan n. Notice that $V(n)$ is linear in n, and for a given value of c, the constraint set defined by [11]-[13] is a convex polyhedral set. Then, according to Int, and if we establish the following assumption:

Assumption 1: The slope of the contours of the function $V(n)$ does not equal the slope of any of the bounding faces of the convex polyhedron defined by constraints [11]-[13], for any given value of c,

we state the following:

Proposition 1: there is only one distribution of workers across workdays of different types which maximizes the welfare of the population, because the solution to maximizing [10] subject to [11]-[13] is unique. The number of triplets $(t, h, \mathbf{k})$ receiving strictly positive mass is less than or equal to $N_T + 2$.

PROOF: Given the division of the Pareto problem, it is possible to consider:

$$W(C) = \max_{n \geq 0} V(n) \tag{P1}$$

$$s.t. : C \leq \sum_j n_j \, f(\mathbf{k}_j, 1) \, g(s_j) \tag{i.1}$$

$$\sum_{j:\, t_j \leq t < t_j + h_j} n_j \, \mathbf{k}_j \leq \overline{k} \qquad all \; t \in T \tag{i.2}$$

$$\sum_j n_j \leq 1 \tag{i.3}$$

Let $C_{\max}$ denote the solution to $\max_{n \geq 0} C$ subject to (i.1)-(i.3). $W(C)$ is the smallest sum of the disutilities of working associated with producing C units of output, and $C_{\max}$ is the maximum feasible value of C which can be produced. Linear programming results guarantee that for $0 \leq C \leq C_{\max}$ there exists a solution to (P1) which has at most a number of nonzero unknowns equal to the number of constraints, that is to say $N_T + 2$. Furthermore, assumption 1 guarantees that this solution is unique. So, the original problem can be rewritten as:

$$Max_{c \geq 0} \;\; u(c) + W(c)$$

$$s.t : c \leq C_{\max}$$

where it is straightforward that the solution is $C_{\max}$ given the continuity and strict concavity of $u(c)$ and the concavity of W. Associated with this unique value of c is the unique n that solves (P1), which has at most $N_T + 2$ nonzero elements.

Hence, the Pareto optimal allocation (which must be an equilibrium allocation) will have people working at most $N_T + 2$ different shifts, a number of shifts normally less than those feasible. The remaining shifts will not be traded. The number of different shifts that will start to run and the measure of workers allocated to these, in the problem (P1), depends on the coefficients both in the objective function and in the constraints, as well as the parameter values in the constraints. That is to say, it is a question of $\mathbf{k}_j$, $f(\mathbf{k}_j)$, $g(t_j, h_j)$, $v(t_j, h_j)$, and $\overline{k}$.

We can see this through the dual constraints or first-order conditions with respect to n_j:

$$-v(t_j, h_j) + f(\mathbf{k}_j)g(t_j, h_j)\ \lambda_0 - \mathbf{k}_j\Big(\sum_{i=t_j}^{i=t_{\bullet}-1} \lambda_{1_i}\Big) - \lambda_2 \leq 0$$

$$\forall\ (t_j, h_j, \mathbf{k}_j) \in J \qquad [14]$$

where λ_0 is the Lagrange multiplier associated with the constraint (i.1); each λ_{1_i} is the multiplier on the constraint of the capital corresponding to the starting time t_i, that is, the constraints denoted (i.2). Then, the condition [14] corresponding to a type j shift, with starting time t_j and ending time $t_p = t_j + h_j$, includes the multipliers associated with the starting times from t_j to t_{p-1}, given that, at the moment t_p the capital allocated to this shift can be used for another shift. The multiplier λ_2 is associated with the labour supply constraint. Equation [14] must hold with equality if n_j is strictly positive.

5. Determination of timing patterns

Now, we are interested in examining what type of workdays are determined by the interaction of the individuals' preferences and shift-work technology. Individuals' preferences over workdays determine the shape of the wage schedules and the firm, then, chooses the shifts that are going to be operated. Therefore, first we look at the equations [6]-[8], which show the individual's maximization problem. From the necessary conditions for a solution we obtain:

$$\overline{\mu}_1 v(s) + \mu_2 \geq w(s)\ for\ all\ s \in S$$

$$\overline{\mu}_1 v(s) + \mu_2 = w(s)\ if\ n(s) \geq 0 \qquad [15]$$

$$\mu_2 n(0,0) = 0 \qquad [16]$$

where μ_1 is the multiplier on the budget constraint and $\overline{\mu}_1$ is its inverse. μ_2 is the multiplier on the constraint that an individual cannot place more than one unit of probability across different workdays. The multiplier μ_2 is nonnegative and equals zero if a positive weight is placed on working a workday of length 0. That is, μ_2 is zero if not everybody is working in equilibrium. The condition in [15] holds with equality if a lottery contract is traded that has a strictly positive probability of working a workday s. Then, individuals will supply their work only on the workdays that the wage reaches the level indicated by the left-hand side of the equations in [15]. Given that $\overline{\mu}_1$ and μ_2 are determined in equilibrium, these levels can be interpreted as the supply reservation wages of the different workdays, and are defined as

$$w^s(s;\overline{\mu}_1^*,\mu_2^*)=\overline{\mu}_1^* v(s)+\mu_2^* \qquad [17]$$

where $\overline{\mu}_1^*$ and μ_2^* are equilibrium values. The supply reservation wages are the wages the firm must pay to attract workers at various workdays.

Now, we shall take a closer look at how workdays are determined in the firm's decision problem given in [4] and [5]. If we bear in mind that the restriction of capital is applied to all starting times t, and that the amount of work available to each workday will be employed in it (the constraint about $N(s)$ will hold with equality), the necessary conditions for a solution are:

$$f(k_j,e_j)g(s_j)-w(s_j)e_j-k_j\sum_{i=t_j}^{i=t_{\bullet}-1}\lambda_i\leq 0 \; for\; all\; j=(s_j,k_j,e_j)\in J$$

$$=0\; for\; all\; j\; with\; z_j>0 \qquad [18]$$

$$r=\sum_{i=t_0}^{t_{N_T}}\lambda_i \qquad [19]$$

where each λ_i is the multiplier on the constraint that the capital is restricted in the starting time t_i and, if the shift type j lasts until $t_p=t_j+h_j$, then $\lambda_{t_{\bullet}-1}$ is the multiplier on the constraint corresponding to the previous starting time (as shown in the condition [14]). The condition in [18] states that no shift type earns strictly positive profits in equilibrium, and the shift types that are operated generate zero profit. The condition in [19] states that the cost of a unit of capital utilized sums up the *costs* resulting from every moment that this unit is utilized. Each λ_i is the unit price of capital utilized by the shifts that start or are being operated at the moment t_i. Therefore, as is shown in [19] the unit cost of capital utilized by a shift is a proportion of the total unit cost of capital, and depends on its operating time, during which it is not available to other shifts that could overlap.

So, from [18] we can infer that the firm will operate the shifts in which the production is equal to the cost, including salary cost and utilized capital cost. Since the left-hand side of [18] is homogeneous of degree one in (k,e), only the ratio k/e (denoted $\mathbf{k}$) is determined for the workday type (t,h) among the shift types that effectively work this workday in equilibrium. That is, in equilibrium, if more than one shift operates during the same workday, the rate k/e is the same for them given the constant returns to scale in function f. Define the profit function:

$$\Pi(t,h,\mathbf{k}) = f(\mathbf{k})\ g(t,h) - w(t,h) - \mathbf{k}\ r_k \qquad [20]$$

where $r_k = \sum_{i=t}^{t+h-1} \lambda_i$. In this profit function we can substitute the wage for the supply reservation wage given in [17] and from the necessary condition in [18] we can state that any shift type (t^*, h^*, k^*, e^*) that is operated in equilibrium must satisfy:

$$\begin{gathered} \{t^*, h^*, k^*/e^*\} \in \ \arg\ \max\ \Pi(t,h,\mathbf{k}) \\ s.t.: (t^*, h^*, k^*, e^*) \in J \end{gathered} \qquad [21]$$

Then, in order to gain insight into the equilibrium patterns of timing that arise in this model it is necessary to specify both the function $v(t,h)$, that describes individuals' preferences over workdays, and the function $g(t,h)$, that measures the effective working time of a workday starting at t and ending at $t+h$.

5.1 Description of the preferences over workdays. The function v(t,h).

The function $v(t,h)$ measures the disutility of working the workday (t,h). Dupaigne (2001) provides a function $v(\cdot)$ that describes preferences regarding leisure on a continuous time-scale, as a means of modelling the idea that the value of leisure varies throughout the day. Besides, in his article, the length of the working time is a constant given by H, and the workers choose the time at which they start to work, τ^0. Then, Dupaigne defines the value of foregone leisure during this working time as: $\int_{\tau^0}^{\tau^0+H} v(\tau)d\tau$. We consider that preferences are defined over both the beginning and the length of the workday, and following Dupaigne, the function $v(t,h)$ sums up the instantaneous value of leisure between t and $t+h$, which is measured by the function $v(\tau)$:

$$v(t,h) = \int_t^{t+h} v(\tau)\ d\tau \qquad [22]$$

Now, we can assume different types of preferences over leisure and consequently different preferences over workdays depending on the functional form of $v(\tau)$. That is, we propose the following:

CASE 1: The value of leisure is constant through out the day. The worker cares only about the number of hours spent at work and does not care how it is distributed over time. Moreover, the disutility is linear in hours of work. We assume: $\upsilon(\tau) = \overline{\upsilon}$, where the parameter $\overline{\upsilon}$ represents the degree of aversion to work. A similar assumption is considered in Weiss (1996), which analyses the cyclical pattern of work.

CASE 2: The value of leisure at each instant τ is increasing and convex through out the day. This case is consistent with the most usual form of defining preferences over leisure in the literature, for instance in Fitzgerald (1998a) and Fitzgerald (1998b), Hornstein and Prescott (1993) and Kydland and Prescott (1991), although in all of them the relevant variable is the length of the leisure time and not its distribution. If the function $v(t, h)$ is defined by:

$$v(t,h) = \int_0^h \upsilon(\tau, t)\, d\tau \quad \text{and } \upsilon(\tau, t) = \gamma(1 - \tau - t)^{\sigma-1}$$

where $\gamma \geq 0, \sigma < 1 (\sigma \neq 0)$, when we take $t = 0$ all the feasible workdays start at the beginning of the day, as in the articles quoted above. Then the resulting disutility of working the workday $(0, h)$ is:

$$v(h) = -\gamma[(1-h)^\sigma - 1]/\sigma$$

that is the same functional form for preferences used in FIta. On the other hand, if $t \neq 0$, it is straightforward to show that $v(t, h)$ is:

$$v(t,h) = -\gamma[(1-h-t)^\sigma - (1-t)^\sigma]/\sigma$$

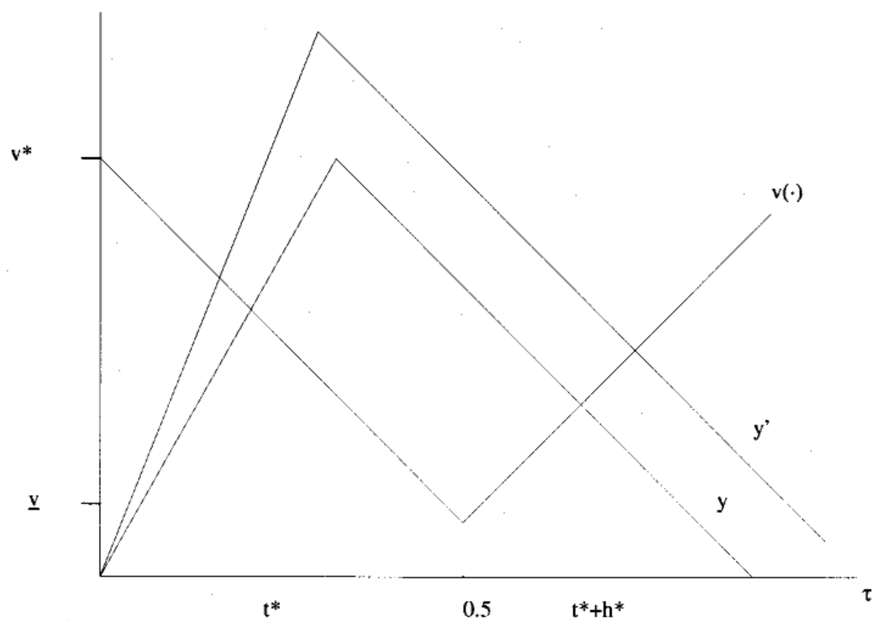
which is increasing both in h and t.

CASE 3: The value of leisure is at its minimum in the middle of the day and at its maximum in $t = 0$ and $t = 1$. The function $\upsilon(\tau)$ is symmetric over the interval [0,1], with $\upsilon(0.5) = \underline{\upsilon}$ and $\upsilon(0) = \upsilon(1) = \upsilon^*$. This is the proposal in Dupaigne (2001), implying that one unit of leisure at night increases the agent's welfare more than one unit of leisure during the day. People prefer to distribute the workday symmetrically around midday, so that the disutility of work is minimized if an eighthours-workday starts at 8 a.m. and finishes at 4 p.m, for example. We assume:

$$v(\tau) = \begin{cases} v^* - \frac{(v^* - \underline{v})}{0.5}\tau & if \quad \tau < 0.5 \\ \underline{v} + \frac{(v^* - \underline{v})}{0.5}(\tau - 0.5) & if \quad \tau \geq 0.5 \end{cases} \qquad [23]$$

which is shown in Figure 1 for the parameter values: $v^* = 1, \underline{v} = 0.1$.

FIGURE 1
The timing of the workday



5.2 Effective working time. Function g(t,h)

The function $g(t, h)$ measures the effective working time of a workday starting at t and ending at $t + h$, and jointly with the instantaneous production, determines the total production of a shift, that is: $F(t, h, k, e) = f(k, e)\ g(t, h)$. In other studies about working time the length of the workweek usually enters in a Cobb-Douglas production function, and the effect on the production obviously depends on its coefficient in the function. Some of these studies deal with a technology that is linear in hours and workers are not subject to fatigue (Fitzgerald (1998), Hornstein and Prescott (1993)). Others consider that total hours and not the breakdown into hours per worker and employment is what matters (Osuna and Ríos Rull (2003) treats both cases). In addition, some of them include a variable measuring the set up time, Hansen (1985) describes it as *warm up time,* and this set up time, therefore, reduces the effective working time corresponding to whichever workweek[8]. On the other hand, Mulligan (1998) provides a model of labour supply such that individuals choose which *work session* to work and how long each will last. Then, Mulligan defines an instantaneous productivity $y(\tau)$ function which is negatively related to a function that can be interpreted as the *fatigue* caused by extended intervals of work, and depends on the history of time used up to that instant. And with respect to effective working time too, Booth and Ravallion (1993) define an efficiency hours index where the number of efficiency hours obtained from a given number of clock hours is a strictly concave function of hours worked, thus there is a *warm-up* period (the function increases) followed by a *fatigue* period (the function decreases).

Considering all these questions, we define the effective working time in a general form that allows the introduction of different assumptions. Although the possible starting times take place at discrete intervals: $t_0, t_1, t_2, \ldots$, an indicator of the effective working time that wears on from t until $t+h$ in continuous time is:

$$g(t,h) = \int_{t}^{t+h} y(\tau)\, d\tau \qquad [24]$$

where $y(\tau)$ measures the instantaneous behaviour of the indicator. Then, following Fitzgerald (1998a) or Hornstein and Prescott (1993), $y(\tau)$ must be constant and equal to 1, in order to get a linear technology in hours; following Osuna and Ríos-Rull (2003) or Fitzgerald (1998b), in fact, $g(t,h)$ should be $g(h) = h^{\psi}$, since the variable t is not considered and different values of ψ can be interpreted as a different elasticity of labor services with respect to hours. However, we are interested not only in the number of hours but also in their distribution. There are studies on the health and safety problems associated with shiftwork. These problems stem from the fact that working irregular hours can be in conflict with biological rhythms and could affect perfor-

[8] Kydland and Prescott (1991), Fitzgerald (1998b) and Osuna and Ríos-Rull (2003) assume that the set up time enters the utility function rather than affecting labour productivity. They are in this way modeling the commuting costs.

mance too (see Folkard and Monk (1996)). We propose the following as an appropriate specification of $y(\tau)$:

$$\begin{aligned} y(\tau) &= 1 - G(\tau), \quad G(\tau) \in [0,1] \\ G'(\tau) &\leq 0, \quad G''(\tau) \leq 0, \quad 0 \leq \tau \leq \underline{\tau} \\ G'(\tau) &\geq 0, \quad G''(\tau) \leq 0, \quad \underline{\tau} \leq \tau \leq 1 \end{aligned} \qquad [25]$$

where $G(\tau)$ represents the *fatigue* which is increasing with τ from $\underline{\tau}$, but falls from 0 to $\underline{\tau}$. We assume that $\underline{\tau}$ is the usual starting time. Then, the effective working time differs from clock hours because of the fatigue effect, which is intensified by the length of the workday and by starting times different from $\underline{\tau}$. To illustrate, the function $y(\tau)$ is shown in Figure 1. The plot is for the function:

$$y(\tau) = \begin{cases} -\tau^2 + y_1\tau & if \quad \tau < \underline{\tau} \\ \tau^2 - y_2\tau + y_3 & if \quad \tau \geq \underline{\tau} \end{cases} \qquad [26]$$

with the parameter values: $y_1 = 3.36, y_2 = 2.8225, y_3 = 1.8225$ and $\underline{\tau} = 1/3$.

5.3 The optimal workday

The characteristics of the workdays that arise in the competitive equilibrium are analysed from the profit function [20]. Conditional on k/e, we are looking for the solution to the problem of maximizing $\Pi(t, h, \mathbf{k})$ by choosing of t and h. That is:

$$(t^*, h^*) \in S, \quad \textit{such that}\ \Pi(t^*, h^*) \geq \Pi(t, h) \quad \forall (t, h) \in S$$

and by substitution of $v(t, h)$ and $g(t, h)$ for [22] and [24] respectively, the funtion $\Pi(t, h)$ results:

$$\Pi(t, h) = f(\mathbf{k}) \int_t^{t+h} y(\tau)\, d\tau - \overline{\mu}_1^* \int_t^{t+h} \upsilon(\tau)\, d\tau \qquad [27]$$

The set of feasible workdays S is finite. Then, in order to propose the necessary conditions for an optimal workday we define, first, what are the necessary conditions if $T = [0, 1)$ and $H = [0, 1]$, and the set S is a rectangular subset of R^2. Secondly, the existence of a workday belonging to S that satisfies these conditions. With a continuum of feasible workdays, the necessary conditions are:

$$\begin{aligned} \overline{\mu}_1^*\, \upsilon(t) &= f(\mathbf{k})\, y(t) \\ f(\mathbf{k})\, y(t+h) &= \overline{\mu}_1^*\, \upsilon(t+h) \end{aligned} \qquad [28]$$

that is, the optimal moment to start is the instant in which the marginal utility of leisure, weighted with the marginal utility of consumption, coincides with the marginal productivity at that instant, and the same concerning the optimal moment to finish.

If the preferences over leisure are constant through out the day, as we define in case I, the optimal instant to start to work is when $f(\mathbf{k})y(t) = \overline{\mu}_1^* \, \overline{v}$ (assuming $t < \underline{\tau}$, that is, when $y(t)$ is increasing). The workday finishes when $f(\mathbf{k})y(t+h) = \overline{\mu}_1^* \, \overline{v}$ (assuming $t+h > \underline{\tau}$).

When we consider the functions v and y defined in [23] and [26] respectively, the optimal workday is between the points t^* and $t^* + h^*$ shown in Figure 1, because both the condition [28] and the second order conditions are satisfied in those points (in the plot it is assumed $\overline{\mu}_1^* = f(\mathbf{k}) = \mathbf{1})$). Then, if the workday (t^*, h^*) that results from [28] belongs to S, the profit is maximized in that shift, and the other shifts are ranked below.

However, since the set of feasible workdays is finite and following linear programming results, it can be shown that an equilibrium in this economy is characterized as follows: the measure of the each workday j that will be worked is $z_j = \frac{n(j)}{e_j}$, the stock of capital used in it will be $n(j)\frac{k_j}{e_j}$, and output will be $n(j)f(k/e,1)g(j)$. In addition, it is inferred that the existence of one or more than one workdays, that is the existence of different working schedules in the same period, depends on the amount of capital available in the economy. Then:

a) There is only one shift type $(t^*, h^*, \mathbf{k}^*)$, with $n(t^*, h^*) = 1$ if:

a.i) $\Pi\ (t^*, h^*, \mathbf{k}^*) > \Pi(t, h, \mathbf{k}) \quad \forall (t, h, \mathbf{k}) \in J$

a.ii) $\overline{k}/\mathbf{k}^* \geq 1$

b) There are more than one shift type $(t^*, h^*, \mathbf{k}^*), (t', h', \mathbf{k}'), ...$, with $n(t^*, h^*)$, $n(t', h')... < 1$ if:

b.i) $\Pi\ (t^*, h^*, \mathbf{k}^*) > \Pi(t', h', \mathbf{k}') > ...\Pi(t, h, \mathbf{k})\ \forall (t, h, \mathbf{k}) \in J$

b.ii) $\overline{k}/\mathbf{k}^* < 1,\ \ \overline{k}/\mathbf{k}' < 1, ...$

where workdays can be partially overlapped.

It follows that the existence of either only one shift or more than one is a question of the relationship between the available capital and the capital-labour ratio. Next, we analyse how this ratio is chosen, although this implies altering the problem slightly.

6. The economy without shifts

In this modified environment, output is produced by plants in which both the workday can be performed and the capital-labour ratio utilized can be varied. A plant type is defined by (t, h, k), and the set of feasible plant types, J, is a rectangular subset of R^3, where $k \in [0, K'], t \in [0,1], h \in [0,1]$. However, it is not possible to run shifts, in the sense that each unit of capital can be utilized only in a plant and during one workday, and when that workday finishes the corresponding capital cannot be utilized again. We let z be the measure of people who work in each plant type; so, the social planner's problem is:

$$\underset{c,\,z}{Max} \;\; u(c) - \int_J v(t,h)dz \tag{P2}$$

$$s.t.: c \leq \int_J f(k)g(t,h)dz$$

$$\int_J k \; dz \leq \overline{k}$$

$$\int_J dz \leq 1$$

which contains a nonlinear problem and a linear program. Although this is a semi-infinite linear program, an optimal measure assigns positive mass to no more points than there are constraints[9]. From the necessary conditions, the plants with $z > 0$ (which can be, at most, three) must satisfy:

$$-v(t,h) + \lambda_0 f(k)g(t,h) - \lambda_1 k - \lambda_2 = 0$$

a similar condition to [14] except that there is only one multiplier associated to the sole constraint on capital. If we define:

$$V(t,h,k) = -v(t,h) + \lambda_0 f(k)g(t,h) - \lambda_1 k - \lambda_2$$

to maximize $V(t,h,k)$ by choice of t and h, this implies satisfying the conditions shown in [28], so that $\lambda_0 = 1/\mu_1^*$, and from those conditions we get: $t^*(k), h^*(k)$. With the functions $\upsilon(\tau)$ and $y(\tau)$ given in [23] and [26] the optimal workday will be:

$$t^*(k) = \begin{cases} \frac{y_1}{2} + \frac{\left[[\lambda_0 f(k)y_1 + \vartheta]^2 - 4\lambda_0 f(k)v^*\right]^{1/2} + \vartheta}{2\lambda_0 f(k)} & if \quad t < \underline{\tau} < 0.5; \; k \geq \underline{k} \\ 0 & if \quad\quad k < \underline{k} \end{cases}$$

[9] Hornstein and Prescott (1993).

$t^*(k) + h^*(k) =$

$$\begin{cases} \frac{y_2}{2} - \frac{\left[[\lambda_0 f(k) y_2 + \vartheta]^2 - 4\lambda_0 f(k)(\lambda_0 f(k) y_3 - \vartheta')\right]^{1/2} - \vartheta}{2\lambda_0 f(k)} & if\ (t+h) > 0.5\ ;\ k \geq \underline{k} \\ 0 & if \qquad k < \underline{k} \end{cases}$$

where: $\underline{k} > 0$ depends on the parameter values, $\vartheta = \frac{(v^* - \underline{v})}{0.5}$ and $\vartheta' = 2\underline{v} - v^*$.

In those expressions for t^* and $(t^* + h^*)$, it is proved that $\frac{\partial t}{\partial k} < 0$, $\frac{\partial (t+h)}{\partial k} > 0$, that is, the optimal workday is increasing in k, in the sense that it starts earlier and finishes later. Figure 1 shows the conditions in [28] for two values of k, where y denotes $f(k)y(\tau)$, with $f(k) = 1$, and y' denotes $f(k_1)y(\tau)$, being $k_1 > k$.

Now, we define: $W(k) = V[t^*(k), h^*(k), k]$ so that the value of k which maximizes $W(k)$ is allocated to the plants with $z > 0$. The question is that if the value of k maximizing $W(k)$ is less than or equal to $\overline{k}$, the constraint over capital in (P2) is not binding, and all the mass will be placed on a single plant $j_1 = (t_1, h_1, k_1)$, $z_{j_1} = 1$. We can show a sufficient condition that ensures the solution to (P2) has positive mass on two points:

PROPOSITION 2: if the following condition

$$f''(k) g(t(k), h(k)) + f'(k) \frac{\partial g}{\partial k} > 0 \qquad [29]$$

is satisfied for the value of capital $\overline{k}$, then the equilibrium places positive mass on two points.

PROOF[10]. Under the conjecture of all the mass being placed on a single point, the value of k_1 must be $\overline{k}$. If the condition [29] is satisfied the second derivative of $W(k)$ is positive, and this contradicts $\overline{k}$ maximizing W. But we have already shown that points receiving positive mass must maximize W. This establishes the result.

The left hand side in [29] contains the derivative of the instantaneous marginal productivity of capital, $f''(k)$, (which is negative) multiplied by the effective working-time, plus the instantaneous marginal productivity of capital, $f'(k)$, multiplied by the effect of capital on the optimal workday, $\frac{\partial g}{\partial k}$. Then if for the total amount of capital available in the economy, the increasing effect on the working-time compensates for

[10]The argument utilized in proposition 2 and its proof is based on Hornstein and Prescott (1993), although the condition is obviously different.

the decreasing effect on the marginal productivity, the capital-labour ratio chosen, k^*, will be greater than $\overline{k}$. Therefore, the measure allocated to that plant , z^*, will be less than 1, and the other feasible point is $t = 0, h = 0, k = 0$, given that $V(0,0,0)$ is a local maximum of V. In this type of solution, not all the agents will be working because of the gains in production generated by the utilization of capital-intensive technology.

Now, if we translate this result into the economy with shiftwork, and under the condition defined in [29], the capital-labour ratio utilized during the optimal workday is such that $\frac{\overline{k}}{k^*} < 1$. Therefore, the economy is included in case b, there is more than one shift type, and people will perform more than one workday type.

7. Concluding remarks

The traditional coincidence of working time, like the regular workday, the standard workweek (Monday-to-Friday), or else the summer holidays in August that we observe for most workers obey multiple reasons. The fact that most people are working simultaneously has influenced the economic, social, and environmental conditions to the point where employees and employers prefer to continue this pattern. Working during evenings or nights is generally considered unpleasant. The analysis of the timing of work requires the definition of a technology in which not only the *amount* of work to be undertaken, but also *when,* is relevant. For this purpose, the shiftwork technology we have described seems suitable. In addition, the context of general equilibrium adopted in the model allows us to examine the interaction of workers' preferences over work schedules and that shiftwork technology.

The existence of one shift or more than one, and the corresponding workdays, is conditioned by the available stock of capital and the capital-labour ratio utilized. In the appendix we show an easy example of the model, which ilustrates these relations. Effectively, when the capital-labour rate increases more workdays are performed and the capital utilization time is extended. With respect to the timing of these workdays, it is a question of workers' preferences and the production function. The framework presented is a first approximation to the determination of the work schedules. We have proposed a broad enough model which can be useful for addressing some other issues related to the timing of work like commuting cost, adjustment cost to implement new shifts, and overtime taxation. All these issues could affect the equilibrium of workdays. Moreover, the change of preferences over schedules driven by new life-styles (for example to reconcile the family and working life), or the change of the production function can change the notion of a regular workday. In this sense, a shorter workday or, if the period considered is a week, a 35-hours workweek could be the equilibrium result.

Appendix 1

Through an easy example, we show the results that the model predicts and the relevance of some key parameters. In order to obtain the numerical simulation we assume that the preferences over workdays are given by the functions defined in subsection 5.1 and shown in [22] and [23]. The function $g(t,h)$ is given by the expressions [24] and [26] in subsection 5.2. In addition, the instantaneous production function is Cobb-Douglas, $f(k,e) = Ak^{\alpha}e^{(1-\alpha)}$, and the utility function is $u(c) = \log c$, both of them standard functional forms and used in FIta too. The parameter values are: $v^{*} = 1, \underline{v} = 0.1, y_1 = 3.36, y_2 = 2.82, y_3 = 1.82, \underline{\tau} = 1/3, A = 3, \alpha = 0.36$.

The set of workday lengths is predetermined and limited to three elements: $H = \{0, 1/4, 1/3\}$, that is, the standard length, 1/3 (8 hours), and the other is more reduced, 1/4. The set of feasible moment to start is derived from H, and is given by $T = \{0, 1/4, 1/3, 1/2, 7/12, 2/3, 3/4\}$. In this scenario, if we solve the planner's problem, we get the results listed in Table 1. The length of the workday chosen is always 1/3, but if there are no capital constraints, the optimal workday starts at 1/4, that is, earlier than the standard workday (1/3, 1/3). For values of capital-labour ratio higher than $\overline{k}$, which are utilized if Proposition 2 is satisfied, more workdays are performed. It is a matter of defining the feasible workdays in a suitable way that allows increasing employment and output.

TABLE A.1
Results from the numerical example

	Workdays	Employment	Wages per workday	Wages per hour	Output
$\overline{k}$/k ≥ 1	(1/4, 1/3)	1.0	0.07	0.22	0.78
$\overline{k}$/k = 0.5	(0, 1/3)	0.5	0.17	0.53	0.76
	(1/3, 1/3)	0.5	0.06	0.19	
$\overline{k}$/k = 0.1	(0, 1/3)	0.1	0.07	0.21	0.31
	(1/3, 1/3)	0.1	0.02	0.07	
	(2/3, 1/3)	0.1	0.07	0.21	

References

Aizcorbe, A. (1992): "Procyclical labour productivity, increasing returns to labour and labour hoarding in car asembly plant employment", *Economic Journal* 102, pp. 860-873.

Anxo, D., G. Bosch, D. Bosworth, G. Cette, T. Sterner and D. Taddei (eds.) (1995), *Work Patterns and Capital Utilisation,* Kluwer Academic Publishers, Dordrecht.

Booth, A. and M. Ravallion (1993): "Employment and length of the working week in a unionized economy in which hours of work influence productivity", *Economic Record* 69, pp. 428-436.

Bosworth, D. (1991): "Shiftwork and working times in the UK", *Economies et Sociétés, Série Economie du travail* AB, pp. 95-112.

Bresnahan, T. and V. Ramey (1994): "Output fluctuations at the plant level", *Quarterly Journal of Economics,* pp. 593-624.

Calmfors, L. and M. Hoel (1989): "Work sharing, employment, and shiftwork", *Oxford Economic Papers* 41, pp. 758-773.

Cette, G. (1995): "Capital operating time and shiftworking in France", in Anxo, D., G. Bosch, D. Bosworth, G. Cette, T. Sterner and D. Taddei (eds.), *Work Patterns and Capital Utilisation,* Kluwer Academic Publishers, pp. 149-177.

Cho, J. and T. Cooley (1994): "Employment and hours over the business cycle", *Journal of Economic Dynamics and Control* 18, pp. 411-432.

Dupaigne, M. (2001): "Capital utilization and work schedules: the welfare cost of shiftworking", *Economics Letters* 73, pp. 195-200.

Fitzgerald, T. (1998a): "Work schedules, wages and employment in a general equilibrium model with team production", *Review of Economic Dynamics* 1, pp. 809-834.

Fitzgerald, T. (1998b): "Reducing working hours: a general equilibrium analysis", Working Paper 9801, Federal Reserve Bank of Cleveland.

Fitzroy, F., M. Funke and M. Nolan (2002): "Working time, taxation and unemployment in general equilibrium", *European Journal of Political Economy* 18, pp. 333-344.

Folkard and Monk (1996): "Shiftwork", *Applied Ergonomics* 27, pp. 17-23.

Foss, M. (1997), *Shiftwork, Capital Hours and Productivity Change,* Kluwer Academic Publishers, London.

Hamermesh, D. (1998): "When we work", *American Economic Review Papers and Proceedings* 88, pp. 321-325.

Hamermesh, D. (1999): "The timing of work over time", *Economic Journal* 109, pp. 37-66.

Hansen, G. (1985): "Indivisible labor and the business cycle", *Journal of Monetary Economics* 16, pp. 309-327.

Hornstein, A. (2002): "Towards a theory of capacity utilization: shiftwork and the workweek of capital", *Economic Quarterly* 88, Federal Reserve Bank of Richmond.

Hornstein, A. and E. Prescott (1993): "The firm and the plant in general equilibrium theory", in R. J. R. Becker, M. Boldrin and W. Thomson (eds.), *General Equilibrium and Growth: The Legacy of Lionel McKenzie,* Academic Press, San Diego, pp. 393-410.

Intriligator, M. (1971), *Mathematical Optimization and Economic Theory,* Prentice-Hall, Englewood Cliffs, NJ.

Kydland, F. and E. Prescott (1991): "Hours and employment variation in business cycle theory", *Economic Theory* 1, pp. 63-81.

Lewis, H. (1969): "Employer interests in employee hours of work", mimeo, University of Chicago.

Marimón, R. and F. Zilibotti (2000): "Employment and distributional effects of restricting working time", *European Economic Review* 44, pp. 1291-1326.

Mayshar, J. and Y. Halevy (1997): "Shiftwork", *Journal of Labor Economics* 15.

McKenzie, L. (1959): "On the existence of general equilibrium for a competitive Market", *Econometrica* 27, pp. 54-71.

Mulligan, C. (1998): "Microfoundations and macro implications of indivisible labor", Discussion Paper 126, Federal Reserve Bank of Minneapolis.

Osuna, V. and J. Ríos-Rull (2003): "Implementing the 35 hour workweek by means of overtime taxation", *Review of Economic Dynamics* 6, pp.179-206.

Rocheteau, G. (2002): "Working time regulation in a search economy with worker moral hazard", *Journal of Public Economics* 84, pp. 387-425.

Rogerson, R. (1988): "Indivisible labor, lotteries, and equilibrium", *Journal of Monetary Economics* 21, pp. 3-16.

Sargent, T. (1978): "Estimation of dynamic labor demand schedules under rational expectations", *Journal of Political Economy* 86, pp. 1009-1044.

Stemberger, G. (1991): "Working time reduction and a-typical employment in Austria", *Economies et Sociétés, Série Economie du Travail* AB, pp.137-153.

Weiss, Y. (1996): "Synchronization of work schedules", *International Economic Review* 37, pp. 157-179.

Resumen

Este estudio presenta un modelo de determinación del horario de trabajo como resultado del equilibrio entre las decisiones de la empresa y las preferencias de los trabajadores. Para ello se considera una tecnología ya que organiza los factores de producción en turnos de trabajo, y se definen las preferencias de los trabajadores acerca de trabajar las diferentes jornadas posibles, definiendo los conceptos de turno y jornada de forma novedosa en este contexto. La existencia o no de restricciones de capital, el efecto "fatiga" y las preferencias por el ocio determinan la cantidad de factores empleada y en qué horario.

Palabras clave: Utilización del capital, equilibrio general, trabajo por turnos, horario de trabajo.

Recepción del original, febrero de 2003
Versión final, junio de 2004