A CLASS OF MODELS FOR LOAD MANAGEMENT APPLICATION AND EVALUATION REVISITED

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KEYWORDS: DEMAND SIDE MANAGEMENT, STOCHASTIC LOAD MODELING

ABSTRACT

The problem of load modeling for Demand Side Management (DSM) purposes is addressed in this paper. The proposed load models rely on information about both the physical characteristics of elemental load devices at the distribution level, and usage statistics of these devices.

Although the class of models discussed here has been previously proposed in the literature, its suitability for DSM purposes is definitely established by showing how the models can be a tool for real DSM actions evaluation. Some results are shown.

1.- INTRODUCTION

The use of Demand-Side Management (DSM) alternatives is gaining adepts between utilities and distribution companies in order to achieve a better operation of the Electric Power System.

Two different approaches may be used to cope with the growth of the demand in an Electric Power System. The first one is to expand the Power System so that the new energy requirements can be met (Supply-Side policy). The second one is to try to influence the electric energy consumption so as to reduce the investment requirements (Demand-Side policies).

Demand Side Management has been defined as those activities oriented to influence customer uses of electricity in ways that will produce the desired changes in the load shape [1]. We will refer to the Control actions directly performed upon the customer loads as Load Management (LM) actions.

The reason for considering the possibility of influencing the customer uses must be found in the continuous rise in the cost of electricity and equipment, the availability of the required technology, more severe environmental constraints on power system generation, transmission and expansion, and the necessity to offer new options to the customer.

92 WM 131-3 PWRS A paper recommended and approved by the IEEE Power System Engineering Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1992 Winter Meeting, New York, New York, January 26 - 30, 1992. Manuscript submitted July 18, 1990; made available for printing November 20, 1991.

The consequences of DSM for the utility are a better use of its Power System, and hence a deferral of the need of new investments, whereas for the customer they represent the possibility of benefiting from reduced fares.

Typical DSM objectives include Peak Clipping, Valley Filling, load Shifting and Strategic Conservation and Growth.

Voltage reduction is a typical LM action that has been traditionally used by the utility for power peak consumption reduction.

Some other actions need to be considered as potential LM control actions, mainly those related to the possibility of end-user load shedding: load interruption and load cycling.

Obviously, the possibility of performing these kinds of actions upon the consumers must be attached to a flexible rates policy.

One of the most critical problems when considering the application of DSM by the utility is to be able to assess whether this policy is going to produce the desired effects or not. Thus, in order to evaluate the DSM policies, it is necessary to have load models that can fulfill at least two objectives: First they should provide the necessary information to evaluate the benefits obtained through the use of the DSM and, secondly, they must allow the evaluation of every control action from the end-customer side, for example, through the evaluation of some "comfort index".

These comfort indices, in conjunction with a proper rates structure, can become very important in securing a high level of acceptance of DSM policies among the customers.

The load models we are about to discuss in this paper have appeared earlier elsewhere in the literature [6], [7], [8], [9] and [12]. However, due to their relative mathematical sophistication, their potential practical usefulness has remained largely unsuspected. The main goal of this paper is an attempt at correcting the above situation.

These models are being tested by the authors with encouraging results. However we chose not to report them in this paper because lack of space.

The paper is organized as follows: constraints on the load modeling problem are analyzed in section 2. The model building approach to be used is reviewed and compared with other proposed methodologies in section 3. Section 4 is devoted to the application of the models in LM. Numerical results are shown in section 5. Finally, in section 6, conclusions are drawn and directions for further research proposed.

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2.- CONSTRAINTS ON THE LOAD MODELING PROBLEM

Two different kinds of models can be considered for the electric load consumption:

Demand Models, where the object is to model the load behavior with respect to time.

The load demand modeling problem has been traditionally approached, for Power System purposes, by the use of large amount of past information filtered through some statistical techniques (Time Series Analysis) [2], [3].

The main field of application of short term demand models has been Automatic Generation Control (AGC), where highly diversified load aggregates are modeled.

Response Models, where the object is to characterize the behavior of the load when some changes in the external inputs to the load (such as voltage, frequency, operating state,etc) are to be considered.

The main field of application has been stability studies.

Both types of models need to be considered jointly for the purposes of this paper. Indeed, it is necessary to know how the demand is going to evolve during the period in which the control actions are to be performed. Also, the way in which the load is going to react against a given DSM action is basic for the evaluation and selection of that particular action.

Two special characteristics are specific to the modeling problem considered here:

1.- The aggregation level is very low. Only several hundreds of KW and KVAR are to be grouped in a control group.

2.-The transient behavior of the control groups cannot be neglected. In fact, it is essential that the model accounts for that behavior with reasonable accuracy.

Although some important attempts [4] have been made to include some parameters in the Time Series approach, such a methodology cannot be applied to solve the modeling problems discussed in this case mainly for the following reasons:

1.- As the aggregation level is quite reduced, typical ARMA (Auto Regressive Moving Average) models will not work very well.

2.- Since LM actions take the power system outside its "ordinary" state, regression analysis based models which have to rely on "ordinary" load data are inadequate . As a result, no identification of the result of control actions can be carried out, unless one sets up specific experiments to do so. Even if this were possible, the results would be valid only under the weather conditions of the experiments

3.- The model structures developed under such approaches are not necessarily exportable to other distribution environments.

3.- BASIC LOAD MODEL

The most promising avenue for handling the problem of load modeling for DSM purposes is thus to consider **Physically Based** [5], [6] modeling methodologies, where the problem is decomposed into two subproblems: Modeling loads, at the elemental level, and subsequently devising schemes to aggregate these elemental load models efficiently.

The load modeling scheme used in this paper is based in the one proposed by Chong and Debs [7], and subsequently developed and improved by R. Malhamé and Chong [8], [9] and Malhame [12].

There has been considerable (and relatively recent) activity in the field of physically-based analytical load modeling methodologies for Load Management purposes. An excellent survey which unifies various modeling viewpoints can be found in Mortensen and Haggerty [11]. The approach recommended by these authors is the one closest in spirit to ours. They have formulated some concerns about the numerical complexity of our models. Thus, after a brief review of our own work, and while appreciating their insight into the problems we shall attempt to respond to these concerns, before we mention some of the advantages of our models which we feel may be lost in their approach.

3.1 Elemental models.

The basic elemental models on which our work is based were first proposed by Chong and Debs [7]. The originality of their insight was in recognizing that for certain types of devices (typically devices associated with an energy storage capability), there was a dissociation between service demand by the customer and the operating or "functional" state of the device as they chose to call it.

Thus while a demand for electricity appears simultaneously with the turning on of lights by a customer, an electric water heater may be off while a customer is drawing water from the tank. Consequently and for devices associated with energy storage, it is essential to model the dynamics of the functional state properly. Inputs to the functional model could be weather variables, service demand, as well as power to the device. The output would be the operating state of the device.

Typical devices falling within this class are electric space heaters, air conditioners and water heaters. The dynamics of the functional models associated with the first two, "as seen by the thermostat" can be adequately modeled by the following hybrid-state stochastic differential equation (a paradigm for so called weakly-driven functional models):

Continuous State:

$$dx(t)= -a(\ x(t)-x_a(t)\ )dt+R(V)m(t)b(t)dt+dv(t) \qquad (3.1.a)$$

where:

- a : Thermal resistance, that accounts for the heat loss through the floors, walls, ceiling, etc. of the dwelling.
- $x(t)$: internal temperature
- $x_a(t)$: ambient temperature.
- $R(V)$: proportional to the rate of power supply. This parameter depends on: the voltage (V) of the power supply and both internal and external temperature.
- $m(t)$: the operating state of the device (1 for ON and 0 for OFF).
- $b(t)$: control action (1 for ON and 0 for OFF)
- $v(t)$: a Wiener process of variance parameter σ^2 simulating unaccounted for processes of heat gain or heat loss (fluctuating number of people in the residence, doors, windows being opened and closed, refrigerators, cooking etc).

Discrete State: The evolution of the discrete state $m(t)$ is governed by a thermostat with temperature $x\_$ and dead band (x_+,x_-). $m(t)$ switches from 0 to 1 when x(t) reaches x_- and from 1 to 0 when x(t) reaches x_+. No switching occurs otherwise.

A simplified model of electric water heater operating state was also proposed by Chong and Debs [7]. It is based on a linearized energy balance analysis and assumes that the devices are thermostat-controlled. The model comprises a continuous state m(t) to account for thermostat action, and is a member of a class of piece wise-deterministic Markov processes (a paradigm for so called strongly-driven functional models):

$$C\frac{dx}{dt} = -a(x(t)-x_a(t))-q(t)(x_d-x_{in}(t)) + R(V)m(t)b(t) \quad (3.1.b)$$

where :

- C : Tank thermal capacity
- $x_a(t)$: Ambient temperature at time t
- x_d : Desired outlet water temperature (depends on the customer)
- $x_{in}(t)$: Inlet water temperature at time t.
- a : Thermal resistance of tank walls (a function of the water heather insulation).
- R(V) : Power rating of heating element
- m(t) : Thermostat control (1 for "on" and 0 for "off")
- b(t) : The on-off control applied by the utility in a load management program (1 for "on" and 0 for "off")
- q(t) : Hot water rate of extraction

The modeling of the customer-driven hot water demand process q(t) is a difficult step. It is basically a non-stationary (piecewise-constant) random process [14]. However, it could be considered stationary during the control period (up to four hours) of interest, although other type of processes can be considered. As a first step, we have considered the following demand model : a two state (A - 0) Markov jump process where A is the constant rate of water demand, when present. The switching of q(t) is characterized by the following time-invariant transition probability, for h a small time increment:

$$Pr(q(t+h)=A \mid q(t)=0) = \alpha_0 h$$
$$Pr(q(t+h)=0 \mid q(t)=A) = \alpha_1 h$$

where α_0, α_1 are positive constants.

Finally m(t) behaves as in (3.1.a).

Note that more complex models of noise and demand for water in (3.1.b) can be easily incorporated in (3.1.a).

3.2. Aggregation.

Given that within a load management program by device control, it is not wise to send the same control signal to dwellings with different dynamics, we consider the aggregation problem for homogeneous or near homogeneous control groups (HCG), i.e. devices described by models (3.1.a) or (3.1.b) with nearly identical parameters and subjected to the same control by the utility. For an HCG, the aggregation problem consists of describing approximately the expected value of the total power demand due to the HCG. Note that this is tantamount to determining the expected value of the discrete state m(t), or equivalently the approximate fraction of devices that are in the "on" state at any time t, once the total number of connected devices in the HCG is known, as well as the common individually absorbed power when the device is "on".

The aggregation problems for (3.1.a) and (3.1.b) were solved by Malhamé and Chong [8] and Malhamé [9], respectively. We review briefly here the results for (3.1.a). They are the basis for our modeling of aggregate heating or cooling loads. The dynamics of $\overline{m}(t)$ are described by the interaction of two coupled Fokker-Planck partial differential equations. Each Fokker-Planck equation describes one of the two "hybrid" probability density functions $f_1(x,t)$, $f_0(x,t)$ defined by :

$$\int_{-\infty}^{x_o} f_1(\lambda,t)d\lambda = Pr\left[x(t)\le x_o \text{ and } m(t)=1 \right] \quad (3.2.a)$$

$$\int_{-\infty}^{x_o} f_0(\lambda,t)d\lambda = Pr\left[x(t)\le x_o \text{ and } m(t)=0 \right] \quad (3.2.b)$$

Thus $f_1(x,t)$ characterizes the distribution of temperatures for the population of devices in the "on" state, and $f_0(x,t)$ that for the population of devices in the "off" state.

The Fokker-Planck equations are as follows:

$$\frac{\delta f_1}{\delta t} = \frac{\delta}{\delta x}\left[r_1(x,t)\ f_1(x,t) \right] + \frac{\sigma^2}{2}\ \frac{\delta^2}{\delta x^2} f_1(x,t) \quad (3.3.a)$$

$$\frac{\delta f_0}{\delta t} = \frac{\delta}{\delta x}\left[r_0(x,t)\ f_0(x,t) \right] + \frac{\sigma^2}{2}\ \frac{\delta^2}{\delta x^2} f_0(x,t) \quad (3.3.b)$$

where:

$$r_1(x,t) = R(V)b(t) - a(x(t) - x_a(t))$$
$$r_0(x,t) = -a(x(t) - x_a(t))$$

They are coupled through a certain number of boundary conditions at thermostat dead band edges x_- and x+ (see [12] for furthers details). A pictorial representation of the model dynamics is shown in Fig 1.

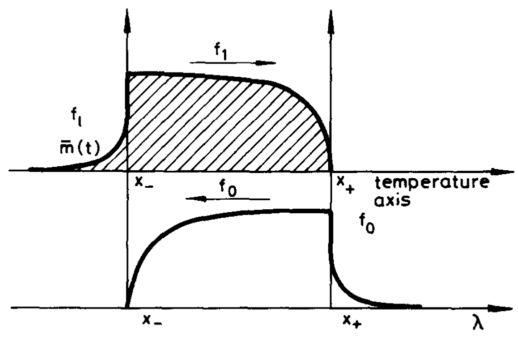

Fig. 1. Geometric Representation of the Aggregate Load Dynamics

Notice that :

$$\overline{m}(t) = \int_{-\infty}^{x+} f_1(x,t)dx \qquad (3.4)$$

A few remarks are in order here:

* The complexity of (3.3.a), (3.3.b) coupled through boundary conditions may seem formidable at first sight. In reality, we show in appendix A the result of a difference numerical approximation scheme for the partial differential equations. It can be characterized as formed of two linear systems of algebraic equations of the forms:

$$A_1 F_1^{n+1} = D_1^n \qquad (3.5.a)$$

$$A_0 F_0^{n+1} = D_0^n \qquad (3.5.b)$$

where A_1, A_0 are matrices the entries of which are directly expressible in terms of the parameters in equation 3.1.a and external temperature. These entries are constant for constant ambient temperature. F_0^{n+1}, F_1^{n+1} are vectors corresponding to the values of $F_0(x,t)$, $F_1(x,t)$ on a discrete temperature grid, respectively, and t is the discretized value of time corresponding to iteration step n+1, where:

$$F_0(x,t) = \int_{+\infty}^{x} f_0(\lambda,t)d\lambda \qquad (3.6.a)$$

$$F_1(x,t) = \int_{-\infty}^{x} f_1(\lambda,t)d\lambda \qquad (3.6.b)$$

Finally D_1^n and D_0^n are vectors the entries of which are known at the end of iteration step n. The two linear systems of equations are approximately decoupled. Thus a dimensionality reduction is possible.

Note that $\overline{m}(t) = F_1(x_+,t)$ (for heating loads), and $\overline{m}(t) = 1 + F_0(x_-,t)$ (for cooling loads).

The following remarks are in order:

1.- As shown in Appendix A, matrices A_1 and A_0 in (3.5) are tridiagonal. This shows that the insight of Mortensen and Haggerty in [11] was indeed correct. They hypothesize a tridiagonal Markov Chain structure to characterize the evolution of $\overline{m}(t)$. Thus our numerical model has the same level of simplicity as theirs. However because we remain close to the physics of the switching process, we can easily accommodate time-varying external temperature, but more importantly we can compute control dependent comfort indices to evaluate the effects of control actions (interruptions or cycling) from the point of view of the customers.

2.- By going directly to a Markov chain model, one loses interesting analytical results that one can obtain for the probability densities of "on" and "off" durations. Indeed, based on these results, and if utilities are willing to gather individual "on" and "off" thermostat durations, it is possible to devise surprisingly efficient on-line parameter estimation schemes (see [13]).

3.- Through parameter R in equation (3.1.a) and (3.1.b), the electrical model of the energy conversion device can be taken into account. The dependence of these parameters versus the electrical supply parameters (voltage and frequency) or temperature can be found experimentally

4.- Finally note that the total number of connected devices (not devices which are "on") will in general vary with time. We propose a regression analysis based model to predict that number.

In the next section, we demonstrate the kind of information that can be extracted from (3.5), within a load management program.

4.- LM APPLICATION

Although the models described in the above section can be applied to DSM programs that consider end-use loads that can store energy, we are interested here only in Load Management applications.

Three different LM options are considered: voltage reduction, load interruption and load cycling.

The influence of a specific control action in the behavior of an HCG will depend very much on the nature of the loads integrating this HCG.

LM control actions should be considered for HCG formed by loads with and without energy storage.

4.1 Voltage control.

The effect of voltage reduction for loads without energy storage capacity will depend only on the electrical model of the individual load components integrating the HCG.

To study the effect of this LM action, the dependence of the real and reactive power absorbed by the elements should be established through testing. The service demand will inform us about the number of devices connected to the distribution system during the control period.

In order to evaluate the effect of voltage reduction in loads with energy storage, not only the electric model has to be considered, but also the functional influence. Indeed, in case of a voltage variation, the rate of heat/cool extracted from the energy storage area will change, and so will the time that a specific device is on or off.

This effect can be completely studied with the basic model described in section 3. The parameter R(V) in (3.1.a) is voltage dependent, and the effect of the voltage can be taken into account through the determination of the relation between R and the input voltage.

It can be assumed, as a first approximation, that the reduction in the real power absorbed by the electromechanical converter will be equal, in % rate, to the reduction experienced in R.

In that case, for one typical air conditioning unit, the relation between increments in R and increments in the input voltage (ΔV) is as follows:

$$\Delta R(V) = k_1 * \Delta V + k_2 * \Delta V^2 \qquad (4.1.a)$$

where k_1 and k_2 are constant for a given voltage, frequency and temperature. Some results are shown in section 5.

4.2 Load interruption and cycling.

With respect to LM actions such as load interruption and cycling, only HCG's formed by loads associated with energy storage capacity have to be considered. The reason is that a load interruption or cycling will not mean the total interruption of the service as some inertia is provided by the dynamic behavior of the system.

We consider only weakly driven loads (space heating/cooling) in this section. Nevertheless, the same analysis can be carried out for strongly driven loads (water heater) using the models described in section 3.

In model (3.1.a), an interruption of power supply to the energy converter can be easily simulated by setting discrete control variable b(t) to 0.

Considering first the load interruption, it is obvious that, after the interruption, the internal temperature will evolve steadily approaching the external temperature.

The dynamic behavior of the HCG can be obtained from the model, and some interesting comfort indices can be computed to evaluate the quality of the control action.

The longer the interruption lasts, the more uncomfortable the LM action can become for the customer. It is clear on the other hand that the utility will be interested in having the freedom to choose interruptions which are as long as possible.

In order to evaluate the quality of the control action, a quality index, q_x, is proposed here. This quality index is defined as:

> The maximum probability, during the control period, of reaching a temperature x degrees lower (for heating systems) or higher (for Air Conditioning systems) than the temperature setting of the thermostat in any residence or buildings belonging to the HGC under consideration.

The temperature dynamics can be obtained from the model. So can the reconnection transients; this is very important when using this kind of models in Cold Load Pick-up studies.

q_x can be easily obtained from the model used in this paper in the following way:

For heating loads:

$$q_x = \int_{-\infty}^{x\_-x} f_1(\lambda,T)d\lambda = F_1(x\_-x,T) \qquad (4.2.a)$$

where f_1 and F_1 have been defined in the previous section and T is the time corresponding to the end of an interruption (at T, b(t) switches from 0 to 1), while x_ is the thermostat setting :

For cooling loads :

$$q_x = \int_{x\_+x}^{\infty} f_0(\lambda,T)d\lambda = F_0(\infty,T) - F_0(x\_+x,T) \qquad (4.2.b)$$

Thus q_x can be easily computed. In fact, it is one entry of the F_1 vector in (3.5.b). Comfort index q_x is essential in assessing the effect of the control policy on the customer.

A LM policy with bad quality indices will not be popular at all and, presumably, will not be tolerated by the customers.

5.- RESULTS

The model described in section 3 has been applied, according to section 4, to some simulated situations. The simulated situation refers to an Air Conditioning system for residential use.

To do this simulation, real data from AC devices have been obtained from testing in a laboratory specially designed and built in the Department of Electrical Engineering of the Universidad Politécnica de Valencia, Valencia, Spain.

The results that are going to be discussed are based in the aggregation of individual loads whose basic characteristics are the following: AC unit rate, 5,600 Btu/h; room thermal capacity, 300000 J/°C; Loss coefficient, 120 W/°C.

As the model equation (3.1.a) is normalized by dividing by the thermal capacity C of the dwelling, "a" parameter has to be computed by dividing the loss coefficient by the thermal capacity. R is the quotient between the AC unit rate and the thermal capacity (in the proper units).

The service demand has been considered as a white noise with variance parameter 0.01. This corresponds to an uncertainty in the model of about 15% (see Appendix B).

A homogeneous control group is formed with these types of loads so that the number of elements can be considered large enough.

The connection process for this HCG is shown in figure 2, with an external temperature of 34 °C and a setting for the thermostat of 24 °C. It can be observed that, after a transient period, the aggregate operating state settles at a constant value of 64%. It can be observed that all the units are connected to the supply about half an hour after the devices are reconnected.

The effect of a 10% reduction in the input voltage is found by testing and corresponds to a reduction of 5% in the real power absorbed by the AC.

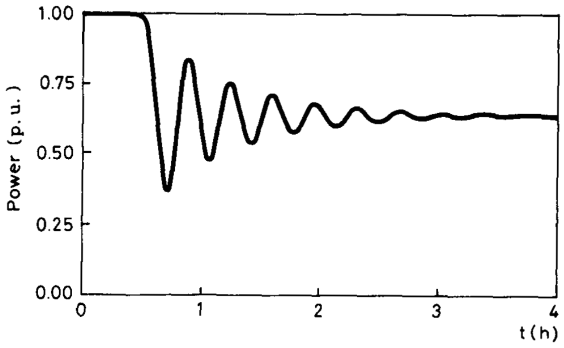

Fig. 2. HCG Connection transient

Assuming the same reduction (5%) in the heat extraction rate, an increment of 3% in the aggregated operating state is found through the model. As a result, no effective energy saving is achieved in this type of loads through voltage reduction.

The effect of a 10 minute load interruption action is shown in Figure 3, for external temperatures 34 °C (solid) and 38 °C (discontinuous). The associated quality indices, q_3 for these situations are evaluated at 2.2% for 34 °C and 17.1% for 38 °C.

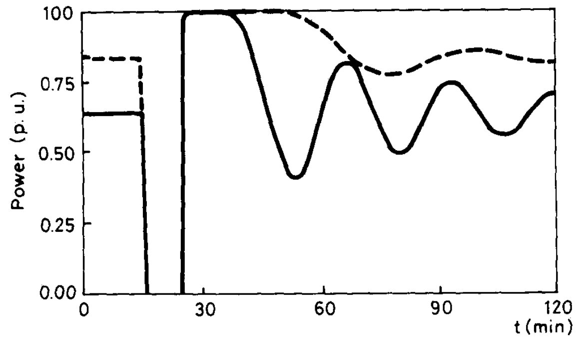

Fig. 3. An HCG Interruption Transient

With respect to load cycling, two different situations have been considered. First, as in the previous case, only one HCG is considered; subsequently a set of several HCG's of similar characteristics are considered in a distribution feeder.

When considering the response of an HCG to cycling LM control actions, special consideration is devoted to the quality of the control action as measured by the comfort indices.

A cycling control action will be referred as T_{OFF}/T_{ON} where T_{OFF} is the time in minutes the HCG is power interrupted and T_{ON} is the time, also in minutes, the group is energized.

Figures 4, 5 and 6 show the behavior of a single control group for the actions 10/10, 15/10 and 10/5 respectively. The LM control period is 7 hours, at external temperature of 34 °C.

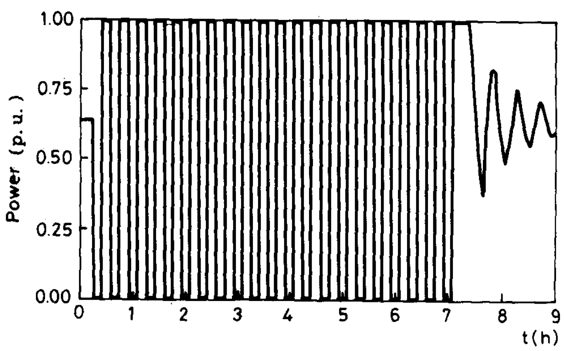

Fig. 4. 10/10 HCG cycling control.

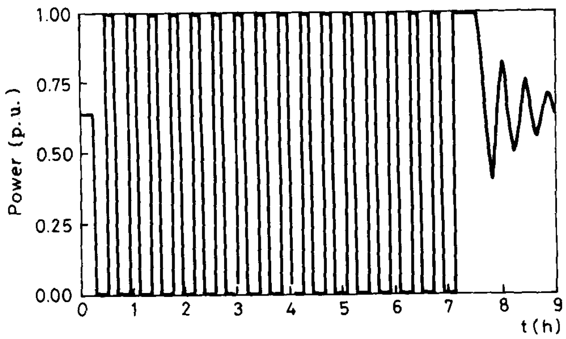

Fig. 5. 15/10 HCG cycling control.

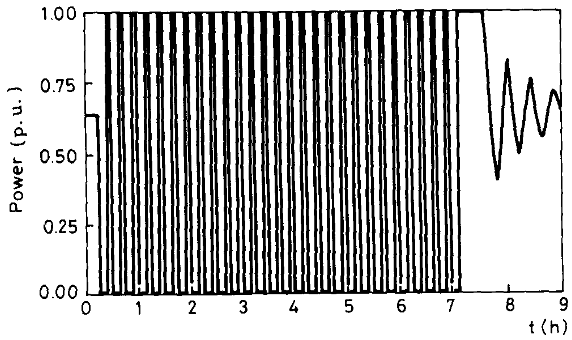

Fig. 6. 10/5 HCG cycling control.

Table 1 shows the associated energy reduction and quality indices obtained through these cycling actions.

TABLE 1. Cycling Control Parameters

LM Cycle T_{OFF}/T_{ON} (min)	Energy Reduction %	Quality q_4	q_5
10/10	21.8	2.9	0.0
15/10	37.5	89.7	26.0
10/5	47.9	99.9	70.0

Obviously, these savings are not very important unless the disconnection time is quite large with respect to the connection time.

To study the effect of the HCG control in a distribution system, consider a simulated distribution feeder where 4 HCG's of the same characteristics of those studied previously can be found. The four HCG 's amount to 25% of the rated power of the feeder.

The control cycles (15/10) have been conveniently staggered so that the equivalent aggregated operating state is 0.4 all the time (Figure 7).

For evaluation purposes, an actual load curve for the Valencia area has been used, and the total feeder load curve with (I) and without (II) control is shown in Figure 8 The reconnection transient once the control period is finished can be minimized through a more sophisticated control action, i.e. allowing a longer connection vs disconnection time as the end of the control period approaches.

It can be observed that over 10% power peak load saving can be obtained.

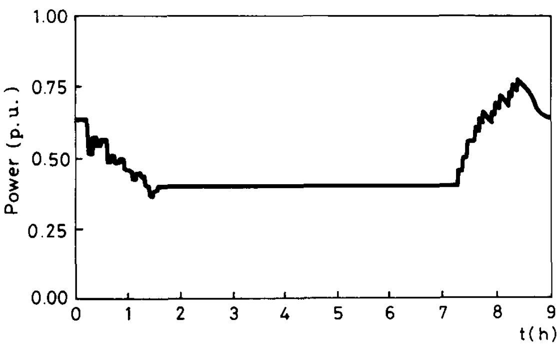

Fig. 7. Feeder HCG's 15/10 cycling control.

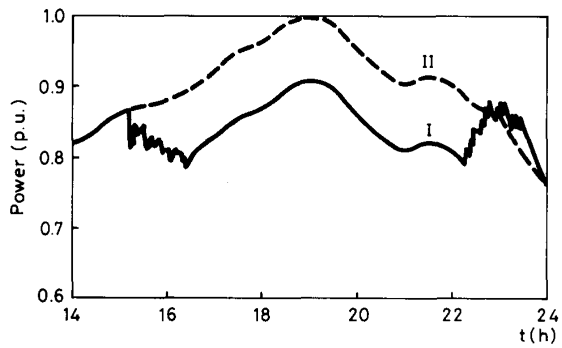

Fig. 8. Feeder load profile

6.- CONCLUSIONS

It can be concluded from this paper that a previously proposed methodology can be successfully adapted to the study of the load response evaluation for Demand Side Management control actions, cold load pick-up, etc.

This is a physically based load modeling methodology that allows the independent consideration of individual load components use and response models and the evaluation of the dynamic behavior of their aggregates.

Although these models were previously proposed in the literature, this paper shows some ideas of how to make them useful for the above mentioned purposes. Although mathematically sophisticated, the computer implementation of these models is quite simple, as shown in Appendix A.

The power of the models is demonstrated through some simulation results, where different control actions are simulated and the response of the loads obtained. Also the actions are evaluated in terms of both the utility and the end user convenience. This last feature is quite new.

More research is needed in order to model more realistic service demand functions and in the real life validation of these models for Load Management applications as described in this paper.

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Appendix A. discretization of the Coupled Fokker-Planck Equations Model.

The following numerical difference approximation scheme has been developed in [12] for equations (3.3) and the corresponding boundary conditions. Heating loads were considered.

$$A_1 F_1^{n+1} = D_1^n \qquad \forall\, n > 0 \qquad \text{(A.1.a)}$$

$$A_0 F_0^{n+1} = D_0^n \qquad \forall\, n > 0 \qquad \text{(A.1.b)}$$

where in (A.1.a,b) :

$$A_1 = \begin{vmatrix} b_1(2) & c_1(2) & 0 & \cdots & \cdots & 0 \\ a_1(3) & b_1(3) & c_1(3) & 0 & \cdots & 0 \\ 0 & a_1(4) & b_1(4) & c_1(4) & & \vdots \\ \vdots & & & & & \vdots \\ \vdots & & & & & c_1(J_1-1) \\ 0 & \cdots & \cdots & 0 & a_1(J_1) & b_1(J_1) \end{vmatrix}$$

$$A_0 = \begin{vmatrix} b_0(1) & c_0(1) & 0 & \cdots & \cdots & 0 \\ a_0(2) & b_0(2) & c_0(2) & 0 & \cdots & 0 \\ 0 & a_0(3) & b_0(3) & c_0(3) & & \vdots \\ \vdots & & & & & \vdots \\ \vdots & & & & & c_0(J_0-2) \\ 0 & \cdots & \cdots & \cdots & a_0(J_0-1) & b_0(J_0-1) \end{vmatrix}$$

$$F_1^{n+1} = \begin{vmatrix} F_{12}^{n+1} \\ F_{13}^{n+1} \\ \vdots \\ F_{1J_1}^{n+1} \end{vmatrix} \qquad F_0^{n+1} = \begin{vmatrix} F_{01}^{n+1} \\ F_{02}^{n+1} \\ \vdots \\ F_{0(J_o-1)}^{n+1} \end{vmatrix}$$

$$D_1^n = \begin{vmatrix} d_1^n(2) \\ d_1^n(3) \\ \vdots \\ d_1^n(J_1) \end{vmatrix} \qquad D_0^n = \begin{vmatrix} d_0^n(1) \\ d_0^n(2) \\ \vdots \\ d_0^n(J_0-1) \end{vmatrix}$$

where vectors F_1^{n+1}, F_0^{n+1} have already been defined in section 3 of the paper. More precisely, if k is the discrete time step and h is the discrete temperature step :

$$F_{1i}^n = F_1(x_1 + (i-1)h\ ,\ nk) \quad ;\ i = 1,..,L_1,\ ..,\ J_1 \quad n=0,1,..$$

$$F_{0i}^n = F_0(x_0 + (i-1)h\ ,\ nk) \quad ;\ i = 1,..,L_0,\ ..,\ J_0 \quad n=0,1,..$$

Furthermore, in the above, x_1 is the lowest expected temperature (with or without interruptions) in the "on" population, and:

$$x_- = x_1 + (L_1 - 1)h$$
$$x_+ = x_1 + (J_1 - 1)h$$
$$x_+ = x_- + (L_0 - 1)h$$
$$x_0 = x_- + (J_0 - 1)h$$

where x_0 is the highest expected temperature (with or without interruptions) in the "off" population.

In addition, let :

$$\mu = \frac{\sigma^2 k}{2 h^2} ,$$

$$r_{1i}^n = r_1(x_1 + (i-1)h,\ nk)\ ,\ r_{0i}^n = r_0(x_- + (i-1)h,\ nk)$$

Then

$$a_1(i) = -\mu + \frac{k}{2h} r_{1i}^{n+1} \qquad \text{for } i = 2,\ \ldots,\ J_1-1$$

$$a_1(J_1) = -2\mu$$

$$b_1(i) = 1 + 2\mu \qquad \text{for } i = 2,\ \ldots,\ J_1-1$$

$$c_1(i) = -\mu - r_{1i}^{n+1} \frac{k}{2h} \qquad \text{for } i = 2,\ \ldots,\ J_1-1$$

while ,

$$a_0(i) = -\mu + \frac{k}{2h} r_{0i}^{n+1} \qquad \text{for } i = 1,\ \ldots,\ J_0-1$$

$$b_0(i) = 1 + 2\mu \qquad \text{for } i = 1,\ \ldots,\ J_0-1$$

$$c_0(1) = -2\mu$$

$$c_0(i) = -\mu - r_{0i}^{n+1} \frac{k}{2h} \qquad \text{for } i = 2,\ \ldots,\ J_0-1$$

Finally :

$$d_1^n(i) = F_{1i}^n \qquad \text{for } i= 2 \text{ to } L_1-1$$

$$d_1^n(L_1) = F_{1L_1}^n + S_0^{n+1} \left(\mu + r_{1L_1}^{n+1} \frac{k}{2h} \right)$$

$$d_1^n(i) = F_{1i}^n + 2\mu\, S_0^{n+1} \qquad \text{for } i= L_1+1,\ \ldots, J_1$$

$$d_0^n(i) = F_{0i}^n + 2\mu\, S_1^{n+1} \qquad \text{for } i= 1\ ,\ \ldots, L_0-1$$

$$d_0^n(L_0) = F_{0L_0}^n - S_1^{n+1} \left(-\mu + r_{0L_0}^{n+1} \frac{k}{2h} \right)$$

$$d_0^n(i) = F_{0i}^n \quad \text{for } i = L_0+1, \ldots, J_0-1$$

Also :

$$S_1^{n+1} = F_{1(J_1-1)}^{n+1} - F_{1J_1}^{n+1}$$

$$S_0^{n+1} = F_{02}^{n+1} - F_{01}^{n+1}$$

Note that all the entries in D_1 and D_0 are known at time n except for S_1^{n+1} and S_0^{n+1}. We set :

$$S_1^{n+1} \simeq S_1^n$$

$$S_0^{n+1} \simeq S_0^n$$

This yields two decoupled tridiagonal systems (A.1.a), (A.1.b) which can be solved separately at each time step.

The discretization in the case of cooling loads is similar except that the "geometries" of the "on" and "off" temperature distributions are inverted with respect to the case of heating loads. Thus one should interchange the indices "1" and "0", associating 0 for "on" and 1 for "off". In this case :

$$\overline{m}(t) = - F_0(x,t)$$

Appendix B. Practical Estimation of the Noise Parameter.

As we show in this appendix, the level of the noise variance parameter σ in Equation (3.1.a) is directly correlated with is energy content. Consider an "on" cycle of duration T_1. Then it is a well known property of the Brownian motion (the process corresponding to integrated white noise) that the corresponding total root mean square energy contribution of the noise is given by $C\sigma\sqrt{T_1}$, where C is the thermal capacity of the dwelling. Thus the longer the "on" cycle, the more energy (cooling or heating) is contributed by the noise. Now T the "on" duration is a random variable. However its mean is given approximately by Δ/p, where Δ is the thermostat dead band and p is the average cooling rate of the dwelling with the thermostat in "on", i.e.:

$$p \simeq R + a(\overline{x} - x_a) \qquad (B.1)$$

$$\text{where } \bar{x} = \frac{x_- + x_+}{2}$$

Over an "on" cycle, the net energy gained by the dwelling is given by CΔ. If we now consider that the noise root mean square energy over that "on" cycle is a fraction, say 15% of the net energy gained by the dwelling, we can write approximately:

$$C\,\sigma\sqrt{\Delta/p} = (\,C\Delta\,)(0.15) \qquad (B.2)$$

Thus :

$$\sigma = \sqrt{p\Delta}\ (0.15) \qquad (B.3)$$

(B.3) yields for our homogeneous group a value of approximately 0.01 deg/min

(B.3) used with a reasonable estimate of the relative noise mean square energy, can became a useful rule of thumb for estimating σ, in general. Notice however, that all parameters in Equation (3.1.a) can be directly estimated from thermostat "on"-"off" durations [13].

ACKNOWLEDGEMENT

The collaboration between the two research groups involved in this paper has been supported by the **"NATO Collaborative Research Grants Programme"**.

The testing facilities required for this work have been financed by the **"Dirección General de Investigación Científica y Técnica"**, of the Bureau for Educational Affairs, of Spain.

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