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RESEARCH ARTICLE

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Numerical simulation of seepage maps under dams with sheet piles on their ends

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Seepage maps formed by both stream and equipotential lines, emerging under dams with sheet piles on their ends, can be determined by simulating the Laplace conjugate equations using a numerical technique such as the network method. Based on these maps, engineers can immediately deduce the amount of water circulating under the structure and design the sheet piles depth to safe values that allow to limit risks such as siphoning or erosion of the base of the dam. For a fix depth of the upstream sheet pile, seepage maps are shown for different configurations of the downstream sheet pile, in a 2D scenario with finite depth and with large extensions both upstream and downstream of the dam.

KEYWORDS

equipotential lines, seepage, sheet piles, stream lines

1 | INTRODUCTION

Seepage is a phenomenon that takes place when pressures in the surface regions of the ground, upstream and downstream of a dam, have different values.[1-5] Groundwater, which flows from high pressure to low pressure regions, distributes through the ground conditioned by the problem properties and geometry. This phenomenon is commonly represented by a net of stream and equipotential lines, from which can be obtained both the total infiltrated flow and the zones where the maximum pressure gradients are located (sometimes, resulting in siphoning, which occurs when the effective pressure is zero and the soil loses its structure, and/or erosion of the dam base and its subsequent failure). To prevent these risks, engineers build one or more sheet piles fixed to the base of the dam in order to change the flow net, in such a way that high gradients are avoided and, in addition, the total infiltration is reduced to safe values. In general, sheet piles force the water path to be longer, offering greater resistance to water movement, reducing the total infiltrated flow to the minimum, and making the streamlines below the structure to separate in comparison with a dam without sheet piles.

Seepage is ruled by steady-state Laplace equation, so that analytical solutions can always be derived.[6,7] The potential dependent variable is the total piezometric head (h). However, due to the complexity of the geometry in real scenarios, according to the sheet piles location, these solutions are complex. Therefore, numerical or graphical methods are currently used by engineers for the design of these retaining structures. These methods can be applied in heterogeneous soils with isotropic hydraulic conductivity.[8] The use of the stream function (Ψ) allows to introduce a new governing equation (also a Laplace equation) in terms of such variable. This gives a direct visual information of the water flow through the domain. Potential and stream function lines are perpendicular to each other and the maps containing both lines provide a complete information of the solution of the seepage problem.

In this communication, seepage maps are represented by the data obtained from the numerical simulation of the decoupled governing (Laplace) equations in terms of the piezometric head and the stream function. Simulations are carried out by the network method,[9] a numerical tool extensively used in a large variety of problems in physics and engineering, such as heat and mass transfer,[10] elasticity,[11] soil consolidation,[12] inverse problems,[13] and many other fields.

This study starts from a typical 2D homogeneous and isotropic scenario, with large extension upstream and downstream of the dam and a finite constant depth along the domain. For the first sheet pile, located at the left border of the dam, a constant depth is assumed, while for the second are considered three different depths.

2 | PHYSICAL MODEL AND GOVERNING EQUATIONS

The physical model is shown in Figure 1. A dam or retaining water structure placed on the soil surface separates two regions of water with different piezometric head. Water flows through the porous soil from the region of higher potential to that of lower potential in a quantity that depends on the soil hydraulic conductivity.

The governing equation, in terms of the piezometric head (h), comes from the water conservation law in a volume element, given by

$$\frac{\partial \mathrm{v_x}}{\partial \mathrm{x}} + \frac{\partial \mathrm{v_y}}{\partial \mathrm{y}} = 0. \tag{1}$$

This, together with the boundary conditions for variable h, sets the mathematical model that writes as

$$\mathrm{k_x}\left(\frac{\partial^2 \mathrm{h}}{\partial \mathrm{x}^2}\right) + \mathrm{k_y}\left(\frac{\partial^2 \mathrm{h}}{\partial \mathrm{y}^2}\right) = 0 \tag{2}$$

$$\mathrm{h}_{\left(\mathrm{y=p},\ \mathrm{x}\le -\frac{a}{2}\right)} = \mathrm{h_1} \tag{3a}$$

$$\mathrm{h}_{\left(\mathrm{y=p},\ \mathrm{x}\ge \frac{a}{2}\right)} = \mathrm{h_2} \tag{3b}$$

$$\frac{\partial \mathrm{h}}{\partial \mathrm{y}}_{\left(\mathrm{y=0},\ -\frac{a}{2}\le \mathrm{x}\le \frac{a}{2}\right)} = 0 \tag{3c}$$

$$\frac{\partial \mathrm{h}}{\partial \mathrm{x}}_{(\mathrm{x}=-\infty \text{ and } \mathrm{x}=+\infty)} = 0. \tag{3d}$$

Equations (3a) and (3b) represent Dirichlet or first-type conditions, while Equations (3c) and (3d) are homogeneous second-type conditions that represent impermeable surfaces.

On the other hand, in terms of the stream function variable Ψ, connected with the velocity components in the form

$$\mathrm{v_x} = \frac{\partial \Psi}{\partial \mathrm{y}}, \mathrm{v_y} = -\frac{\partial \Psi}{\partial \mathrm{x}}, \tag{4}$$

the governing equation and boundary conditions can also be written as

$$\mathrm{k_x}\left(\frac{\partial^2 \Psi}{\partial \mathrm{x}^2}\right) + \mathrm{k_y}\left(\frac{\partial^2 \Psi}{\partial \mathrm{y}^2}\right) = 0 \tag{5}$$

$$\Psi_{(\text{dam and sheet piles contour})} = \Psi_1 \tag{6a}$$

$$\Psi_{(\text{impervious contour})} = 0 \tag{6b}$$

$$\frac{\partial \Psi}{\partial \mathrm{y}}_{(\text{upstream and downstream soil surface})} = 0, \tag{6c}$$

where Ψ_1, a constant value, is equal to the flow infiltrated downstream of the dam. This can be obtained, among other options, from the resolution of the problem in terms of the variable h.

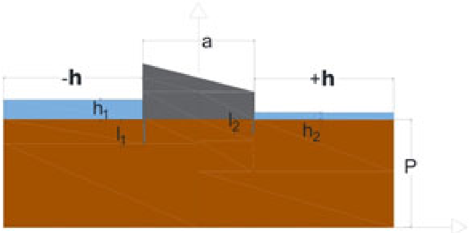

FIGURE 1 Physical model

In this way, by solving the set of previous equations, the solutions to the h and Ψ fields are obtained, thus allowing to represent the flow net.

3 | SIMULATIONS AND RESULTS

According the nomenclature of Figure 1, the scenario to be solved has the following parameters: P = 20 m (total depth), a = 20 m (dam length), $l_1 = 5$ m (upstream sheet pile depth), and $l_2 = 2.5$, 5, 10 m (downstream sheet pile depth), respectively, for each of the three cases raised. Both sheet piles are assumed of negligible thickness. The lengths of the upstream and downstream stretches are assumed infinite, so that these parameters do not influence the flow net. As for the values of the piezometric head, the difference between the upstream and downstream sections has been assumed to be $\Delta h = h_1 - h_2 = 7$ m. Finally, a value of $k_x = k_y = 8 \cdot 10^{-5}$ m/s has been taken for the hydraulic conductivity. Once the above parameters have been introduced into the governing equations, the numerical network model is designed, following the rules of the network simulation method,[9] and its computation is carried out in a circuit resolution code such as Ngspice.

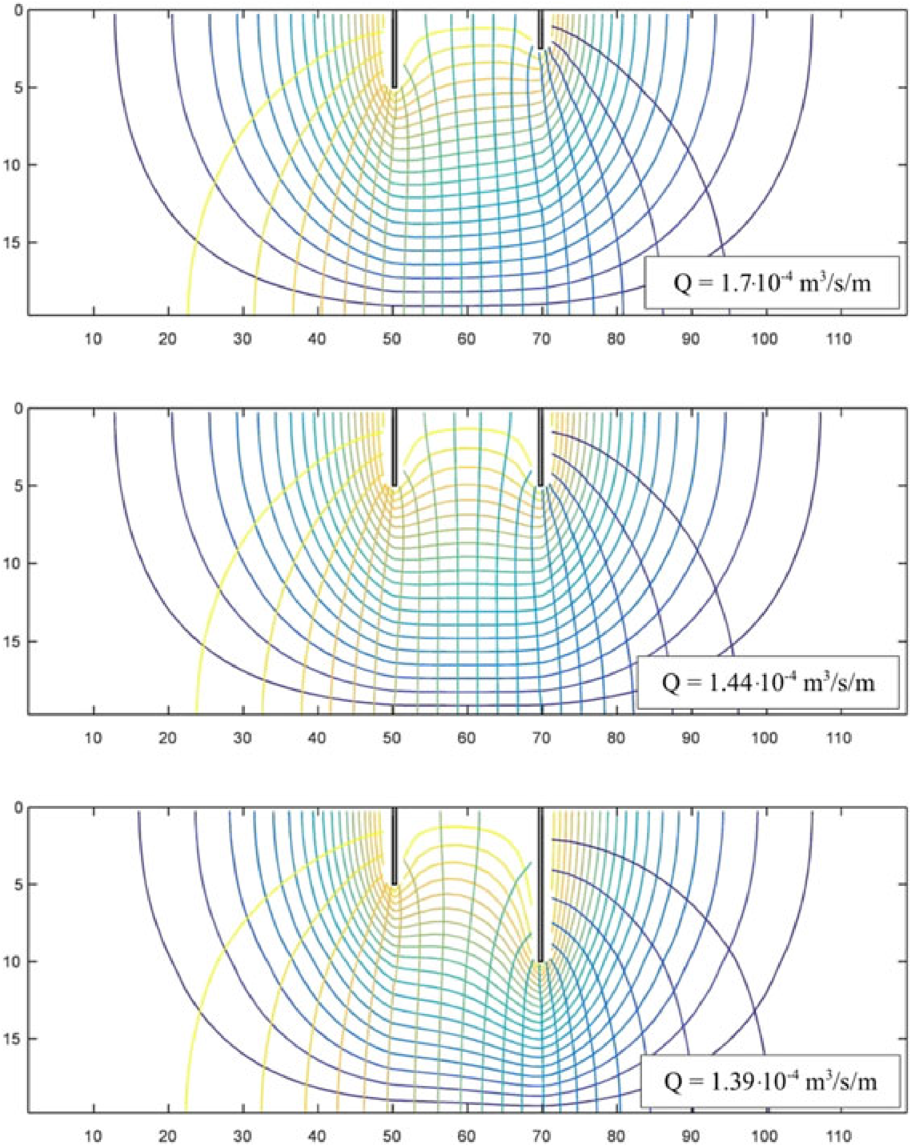

FIGURE 2 Seepage maps for scenarios I (top), II (middle), and III (bottom)

Figure 2 shows the seepage maps (functions h and Ψ) for the three scenarios described above: I, II, and III. A detail in the dam surroundings in order to better appreciate the potential gradients and the separation between streamlines (which report the amount of circulating water) is shown in Figure 3, for the aforementioned scenarios.

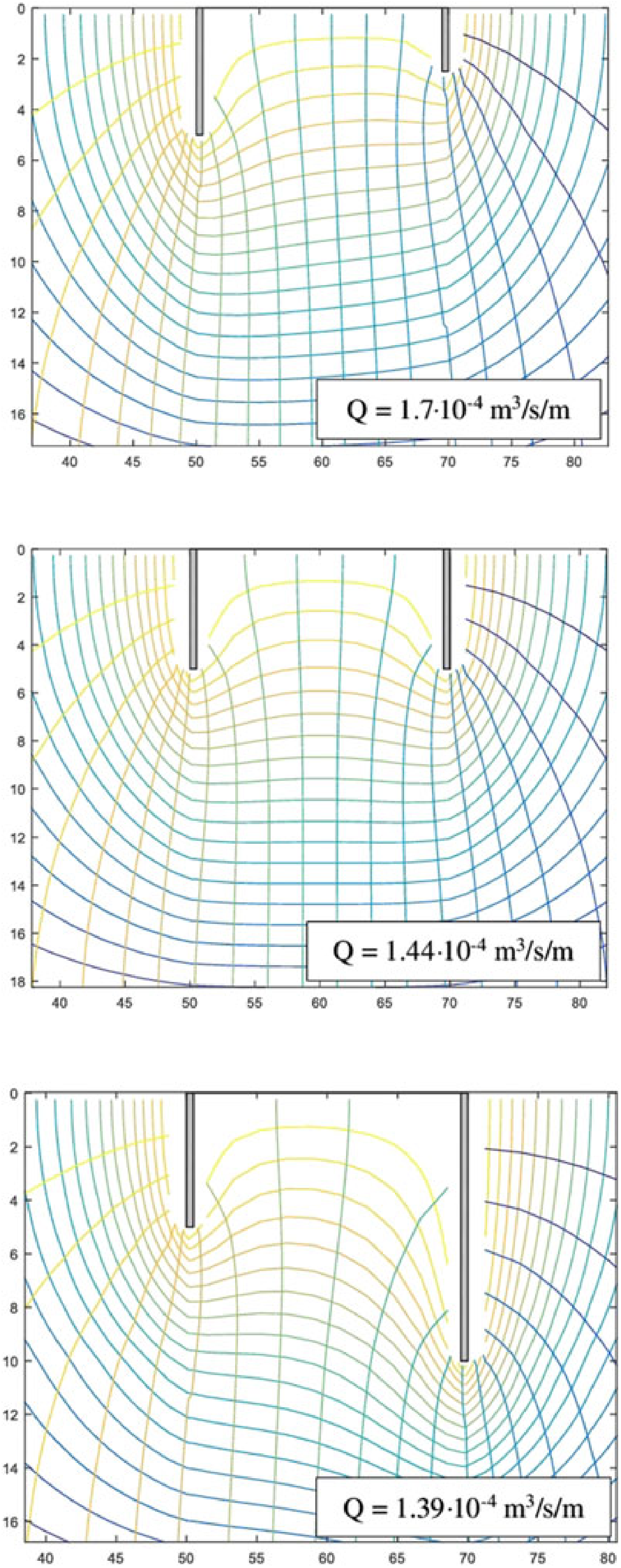

FIGURE 3 Detail of seepage maps around and under the dam for scenarios I (top), II (middle), and III (bottom)

In view of the results, it is easy to verify that, as the depth of the downstream sheet pile increases, the flow that circulates under the base of the dam decreases (thus reducing the risk of erosion in that zone). In the same way, the values of the hydraulic gradient are reduced for that area, which is observed in the greater spacing of the equipotential lines (reducing, in this case, the risk of siphoning due to high gradients).

4 | CONCLUSIONS

By combining the conservation equation with Darcy's law and by defining a potential stream function from which the velocity components can be derived, the seepage problem of groundwater under dams with sheet piles can be solved numerically, both in terms of the piezometric head as of the stream function. This allows to represent the complete flow net, both the equipotential lines and the streamlines, which facilitates the interpretation of the resulting seepage maps. By varying the penetration depths of the upstream and downstream sheet piles, it is possible to modify the seepage flow net until a configuration is reached that guarantees the safety of the structure, in terms of avoiding various risks such as erosion and siphoning.

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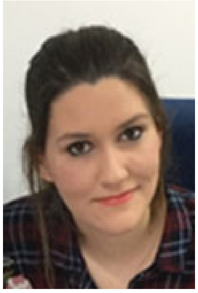

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