

# Rigorous CAD of Rectangular Aperture Coupled Circular Waveguide Multi Mode Filters

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**Abstract** - A rigorous field theory analysis is presented for the accurate design of rectangular iris coupled circular waveguide single, double and triple mode filters. The multi mode coupling between the orthogonal modes is achieved by a new technique using additional asymmetric slots in the Aperture plane by multipoint coupling techniques. This avoids the traditional 45° coupling screws. The theory is based on the full-wave mode-matching method for the key-building block discontinuity circular waveguide to smaller rectangular waveguide. Arbitrary iris location, different cross sections, unequal  $E$ - and  $H$ -plane cross size and finite thickness are rigorously taken into account. The efficiency of the design method is demonstrated at two new filter examples; one rectangular-iris coupled one-resonator dual mode and one triple mode circular filter in the P-band (10-15 GHz). The theory is verified by comparison with measurements.

## I. INTRODUCTION

Multi-mode circular waveguide filters are commonly used in many microwave applications, i.e. narrow bandpass multimode filters [1]. The traditional way to couple the orthogonal modes in such Filters is by way of coupling screws. In the absence of good design tools numerous redesigns and additional tuning elements are necessary. Low-cost and mass-production requirements of modern filter design has led to this alternative solution, which allows the overall CAD of multi-mode filters without the necessity of tunable coupling elements. For rectangular structures, the analysis of irises [2], [4] and L-shaped waveguide sections for providing the required orthomode coupling have been investigated [8]. For circular structures, the rigorous analysis of centered [3]-[5] and asymmetric [6] rectangular irises as well as circular irises [4] is well-known.

The purpose of this paper is to combine these methods to come to a easy-to-manufacture coupling technique, which avoids the usual coupling elements and post assembly tuning procedures. These filters are rigorously designed by a mode-matching method based, fast and reliable design tool. The efficiency of this method is demonstrated at a dual-mode and a triple-mode filter at 12 GHz.

## II. THEORY

The multi-mode filter (Fig. 1a) with the iris element of finite thickness and additional stub holes is decomposed into the key-building block discontinuity *asymmetric waveguide transition* (circular to rectangular [6] or circular to circular [4]), the key-building block *waveguide N-furcation* [9] and *homogenous waveguide* sections (rectangular or circular) of finite length  $t$ .

These stub-waveguides may be shortened or open, if the electromagnetic fields are sufficiently decayed.

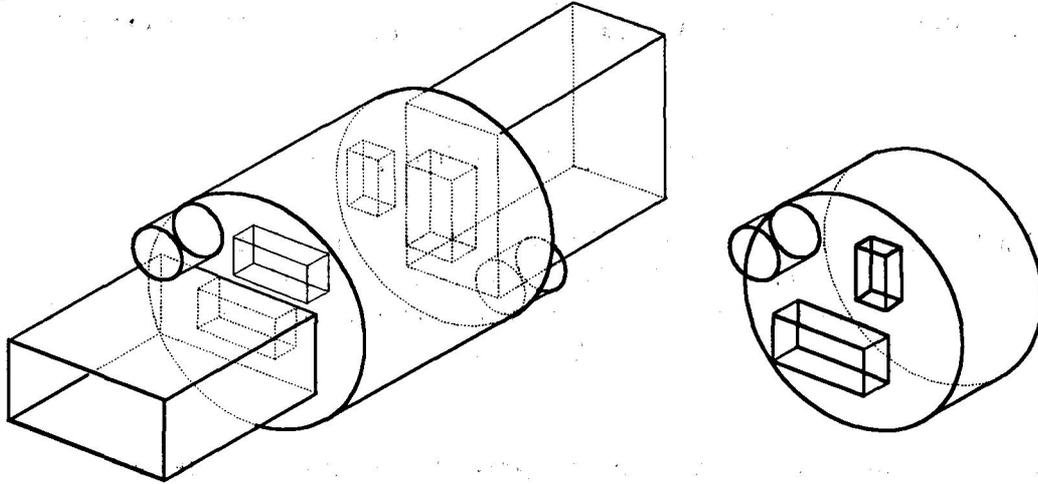


Fig. 1 a) Multi-mode filter structure

b) Furcation circular to circular and circular to rectangular waveguide

The mode-matching solution for the single key-building block discontinuities circular to smaller rectangular and circular waveguide have been presented in [6] and [4], so merely the basic steps are described here using the present notation.

In the circular region I and the circular or rectangular region II, the fields

$$\begin{aligned}\bar{E}^{\nu} &= \nabla \times (\bar{A}_h^{\nu}) + \frac{1}{j\omega\epsilon} \nabla \times \nabla \times (\bar{A}_e^{\nu}) \\ \bar{H}^{\nu} &= \nabla \times (\bar{A}_e^{\nu}) + \frac{1}{j\omega\mu} \nabla \times \nabla \times (\bar{A}_h^{\nu})\end{aligned}\quad \nu = \text{I, II} \quad (1)$$

are derived from the z-components of the electric (index *e*) and magnetic (index *h*) Hertzian vector potentials:

$$\begin{aligned}\bar{A}_e^{\nu} &= \sum_m \frac{1}{\sqrt{Z_{fe}^{\nu}}} T_{em}^{\nu} (a_{em}^{\nu} \exp(-\gamma_{em}^{\nu} z) - b_{em}^{\nu} \exp(+\gamma_{em}^{\nu} z)) \cdot \bar{e}_z \\ \bar{A}_h^{\nu} &= \sum_m \sqrt{Z_{fh}^{\nu}} T_{hm}^{\nu} (a_{hm}^{\nu} \exp(-\gamma_{hm}^{\nu} z) + b_{hm}^{\nu} \exp(+\gamma_{hm}^{\nu} z)) \cdot \bar{e}_z\end{aligned}\quad (2)$$

with the wave impedances

$$Z_{fe}^{\nu} = \frac{\gamma_{em}^{\nu}}{j\omega\epsilon_{\nu}} \quad Z_{fh}^{\nu} = \frac{j\omega\mu_{\nu}}{\gamma_{hm}^{\nu}} \quad (3)$$

and the propagation constants  $\gamma_{em}^{\nu}, \gamma_{hm}^{\nu}$  of the TM- and TE-modes respectively.  $T_{em}^{\nu}, T_{hm}^{\nu}$  are the normalized cross-section eigenfunctions of the corresponding circular (index *c*) or rectangular (index *r*) waveguide regions  $\nu = \text{I, II}$ .

$$\begin{aligned}
T_{h\eta v}^r &= \frac{2 \cdot \cos(k_x^r x) \cos(k_y^r y)}{k_c^r \sqrt{ab} \sqrt{1 + \delta_{0\eta}} \sqrt{1 + \delta_{0v}}} & T_{e\xi\xi}^r &= \frac{2 \cdot \sin(k_x^r x) \sin(k_y^r y)}{k_c^r \sqrt{ab}} \\
T_{h\eta v}^c &= \frac{J_\eta(k_{ch}^c \rho) \sin(\eta\varphi) - \cos(\eta\varphi)}{\sqrt{x_{\eta v}'^2 - \eta^2} J_\xi(x_{\eta v}')} \sqrt{\frac{2}{\pi(1 + \delta_{0\eta})}} & T_{e\xi\xi}^c &= \frac{J_\xi(k_{ce}^c \rho) \cos(\xi\varphi) \sin(\xi\varphi)}{x_{\xi\xi}' J_\xi'(x_{\xi\xi}')} \sqrt{\frac{2}{\pi(1 + \delta_{0i})}}
\end{aligned} \tag{4}$$

Matching the fields at the step discontinuity at  $z = 0$  leads to the coupling matrix  $\nu$  for a waveguide transition of any two waveguide cross sections, rectangular or circular [2], [6]. With the normalized voltage and current coefficient vectors  $\mathbf{u}_\nu = \mathbf{a}_\nu + \mathbf{b}_\nu$ ,  $\mathbf{i}_\nu = \mathbf{a}_\nu - \mathbf{b}_\nu$  we get the matrix equations:

$$\mathbf{u}_1 = \nu \cdot \mathbf{u}_2 \qquad \mathbf{i}_2 = \nu^T \cdot \mathbf{i}_1 \tag{5}$$

Together with the normalized modal admittance diagonal submatrices  $\mathbf{y}'_{\mu\nu}$  of the homogenous circular waveguide of finite length  $l$ , we arrive at the desired modal admittance matrix of the (P-1) - furcation circular to any of circular or rectangular waveguide (Fig. 1b):

$$\begin{pmatrix} \mathbf{i}_1 \\ \mathbf{i}_2 \\ \mathbf{i}_3 \\ \vdots \\ \mathbf{i}_P \end{pmatrix} = \begin{pmatrix} \mathbf{y}'_{11} & \mathbf{y}'_{12}\nu_{12} & \mathbf{y}'_{12}\nu_{13} & \cdots & \mathbf{y}'_{12}\nu_{1P} \\ \nu_{12}^T \mathbf{y}'_{21} & \nu_{12}^T \mathbf{y}'_{22}\nu_{12} & \nu_{12}^T \mathbf{y}'_{22}\nu_{13} & \cdots & \nu_{12}^T \mathbf{y}'_{22}\nu_{1P} \\ \nu_{13}^T \mathbf{y}'_{21} & \nu_{13}^T \mathbf{y}'_{22}\nu_{12} & \nu_{13}^T \mathbf{y}'_{22}\nu_{13} & \cdots & \nu_{13}^T \mathbf{y}'_{22}\nu_{1P} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \nu_{1P}^T \mathbf{y}'_{21} & \nu_{1P}^T \mathbf{y}'_{22}\nu_{12} & \nu_{1P}^T \mathbf{y}'_{22}\nu_{13} & \cdots & \nu_{1P}^T \mathbf{y}'_{22}\nu_{1P} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{u}_3 \\ \vdots \\ \mathbf{u}_P \end{pmatrix} \tag{6}$$

The matrices  $\nu_{ij}$  describe the various transitions from circular to circular or circular to rectangular waveguide according to equation (5). The derivation of this equation is given in [9]. Inserting the coupling matrices  $\nu_{ij}$  for the rectangular to rectangular waveguide transition leads to the well known rectangular waveguide N - furcation [9,10,11]. However equation (6) may be used for an arbitrary set of different waveguide transitions, i. e. for the circular to rectangular and the circular to circular step.

Together with shortened or open waveguide sections for the coupling holes, some examples of multimode filters were calculated using the GAM formulation. To get good convergence, the number of TE- and TM-modes taken into account in the several regions are estimated by the ratio of the geometrie on the two sides of the discontinuity. For the circular to rectangular step the ratio  $2r/a = 2$  means double number of modes in the circular waveguide than in the rectangular section. For a brief result or for optimization 40 modes (TE-, TM-modes, both cosine- and sine-types) in the circular waveguide may be sufficient.

### III. RESULTS

To verify the theory, first a very simple structure was fabricated and compared with the theory. Fig. 2 shows the insertion and return loss versus frequency of a rectangular-iris coupled one-resonator circular filter with two rectangular ports in the P-band. Excellent agreement may be stated.

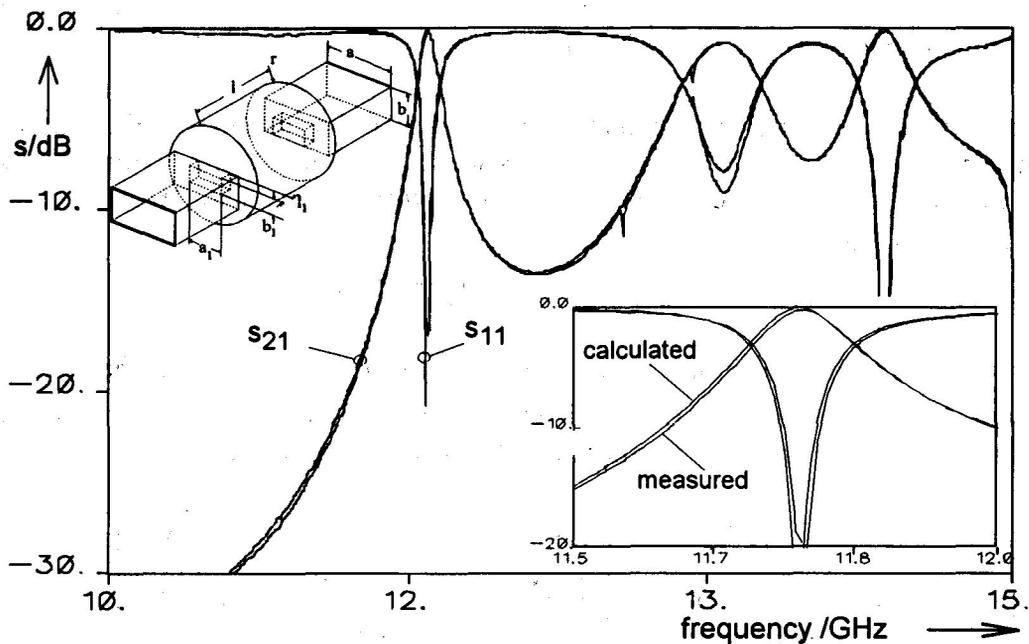


Fig. 2 One-resonator filter: measured and computed scattering parameters.  
 $a=19.05\text{mm}$ ,  $b=9.52$ ,  $l=45.0\text{mm}$ ;  $a_1=9.53\text{mm}$ ,  $b_1=2.95\text{mm}$ ,  $l_1=1.0\text{mm}$

Fig. 3 shows a one resonator dual-mode filter with rectangular irises and circular stubs for mode conversion. Input and output coupling is done by two orthogonal rectangular irises, which are coupling the TE<sub>113</sub> - cosine mode and the TE<sub>113</sub> - sine mode to the TE<sub>10</sub> - mode in the rectangular input and output waveguides respectively. To achieve the desired coupling between the orthogonal modes and to omit excitation of the TM<sub>01</sub> - mode in the Filter, two stubs are required in the input iris plane, located 45° and 225° referring to the input iris. These stubs cause a magnetic current distortion in the wall and so provide magnetic field coupling of the two modes.

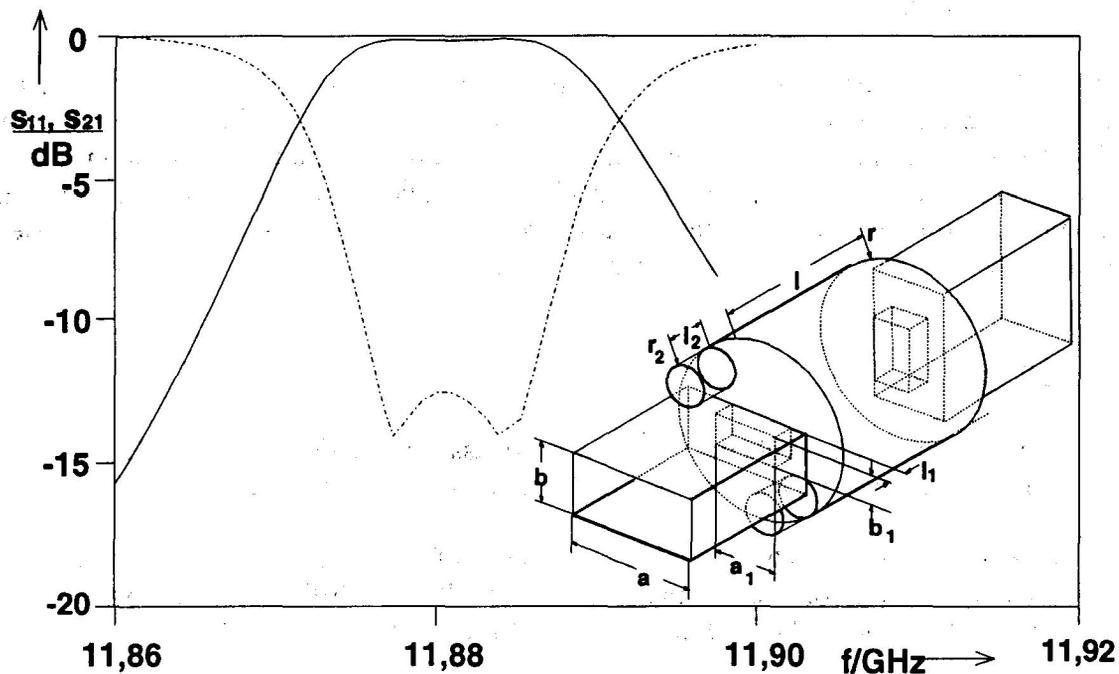


Fig. 3 Dual-mode filter with stub-coupling:  $a=19.05\text{mm}$ ,  $b=9.52$ ,  $l=45.0\text{mm}$ ;  
 $a_1=9.53\text{mm}$ ,  $b_1=2.95\text{mm}$ ,  $l_1=3.0\text{mm}$ ;  $r_2=2.75\text{mm}$ ,  $l_2=2.1\text{mm}$

A first result of a new type of triple-mode filter with a similar coupling mechanism as shown above is presented in Fig. 4. The TE<sub>113</sub> - cosine and - sine mode represent the first and third resonance respectively. Rectangular slots in the iris planes cause magnetic coupling to the TM<sub>012</sub> mode, being the second resonance of the filter. The coupling slots may be fabricated by milling techniques.

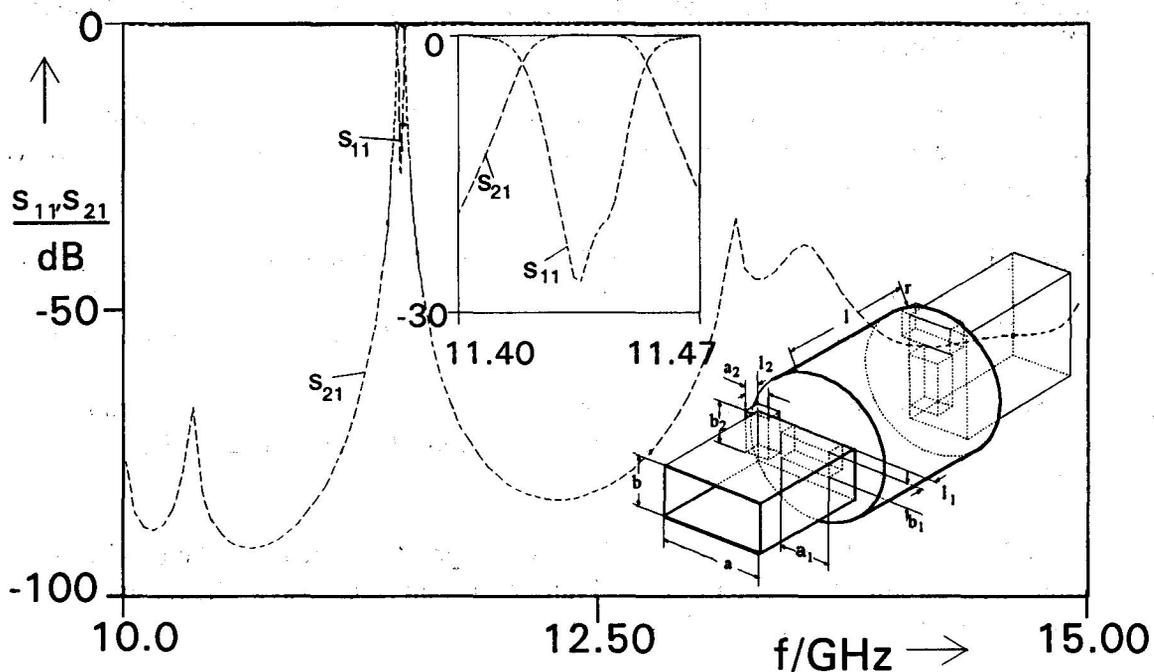


Fig. 4 Triple-mode filter with stub-coupling:  $a=19.05\text{mm}$ ,  $b=9.52$ ,  $l=52.0\text{mm}$ ,  $r=12.25\text{mm}$ ;  
 $a_1=7.6\text{mm}$ ,  $b_1=5.0\text{mm}$ ,  $l_1=3.0\text{mm}$ ;  $a_2=3.0\text{mm}$ ,  $b_2=8.6$ ,  $l_2=2.0\text{mm}$

#### IV. CONCLUSIONS

A new and complete formulation for arbitrary N - furcations from circular to both circular and rectangular waveguides was found. By this, a rigorous and efficient method for designing multi mode circular waveguide filters has been developed and applied to dual- and triple- mode resonators. All relevant design parameters, such as finite thickness of irises, higher order mode interaction, arbitrary position of irises and coupling structures, and multiple aperture irises are taken into account. Post assembly tuning is no more necessary.

#### V. REFERENCES

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