From geometric rules to ribbed vaults procedural modeling
Parametric tools to study star vaults geometry in St-Eustache

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Abstract
The definition of adaptive parametric models to automate modelling architectural elements is one of the main goals for heritage documentation, dissemination and management in HBIM process. Parametric objects library of historical architectural elements is a very important step in more than one process, starting from 3D Point Clouds segmentation with Deep Learning (DL) techniques to 3D modelling from Point Cloud. Following the state of the art we have defined an adaptive parametric model for ribbed vaults modelling that allow us to generate a 3D model based on geometric rules and/or from point cloud data. Starting from geometric rules we defined the main parameters to use to generate different “ideal models” and the specific parameters from point cloud data to use to generate a “reality based model”. We compared the different 3D models based on geometric rules from literature, 3D reality based model with mesh model from point cloud data to evaluate the process, to improve it and to verify the design hypothesis. We are going to test this workflow on a case study: Saint-Eustache church in Paris. There are some different kind of star vaults in Saint-Eustache, in this paper we show the parametric model definition process from French literature for the five keystone vault samples.

Keywords
Ribbed vaults, procedural modeling, computational geometry, star vaults, five keystone vaults.

Figure 1. Parametric model for five keystones generation. From treatises geometric rules to reality based modelling.
1. Introduction

In the past decade, 3D modelling has been increasingly used in heritage documentation, analysis, preservation and especially in HBIM process or in 3D virtually reconstruction based on the concept of shape grammar. 3D modelling techniques for Cultural Heritage documentation is based on two main different approaches: reality based modelling and parametric object oriented modelling. Using one of the reverse modelling tools we are able to convert point cloud data into a polygonal mesh surface, in this way we obtain a mesh model that can be covered with a texture using a series of photographs taken during the data acquisition process. This 3D model is a digital twin, very close to the real shape of the vault, that is the result of deformations suffered during the time and/or construction errors but it is not able to describe the design geometric rules.

The spread of Point Cloud semantic segmentation using Deep Learning techniques and generative tools is the basis of the processes that aim to define a workflow for Point Cloud big data interpretation. Based on the current researches in this field our aim is to test an hybrid method in order to study the gothic vault geometry and to define a parametric model to generate a 3D model that we can use for cultural Heritage dissemination and/or in HBIM process.

A parametric object oriented modelling consists in the use of pre-modelled objects collected in libraries. This is the typical approach for BIM software, in the case of HBIM we have to generate parametric objects library that we can change in relation to point cloud data to recognize the geometric rule. Interactive parametric objects representing architectural elements are generally constructed using a set of geometric rules from historical data. Starting from different approaches that involve from parametric objects libraries based on historical literature and architectural samplers (Murphy 2013), to parametric model based on geometric rules and some specific point cloud data (Dore 2015), our main goal is to define an algorithm tool to study the star vaults geometry.

Starting from the main principle of gothic vault tracing and some geometric specific rules from French treaties we defined the main parameters to use to generate the parametric object oriented model and the reality based model. In this process the reality based model is a model that we can generate using some specific data from point cloud (Angjeliu 2019). We are going to evaluate two different methods: the first is based on use of intrados line from point cloud, the second is based on intrados line geometric construction using geometric rules and some specific data from point cloud (Webb 2019).

Overlapping the three models (the mesh model, the parametric object oriented model and the reality based model) we can understand the design geometry and we are able to generate the 3D model that best fit the real vault.

2. Methodological path

This paper is a part of a wider research about Gothic churches in Paris from early gothic to late gothic. Our aim is to compare a great number of gothic vault systems, from different periods, with geometric rules from literature. In this paper we are going to deal different geometric rules used for five keystones vault (Palacios Gonzalo 2009) based on building gothic methods (Palacios Gonzalo 2015). We are going to test this methodological path using some case studies from gothic vault starting from French samples.

The workflow based on the computational approach that we are testing in our research includes the following activities: 1. vault parametric models based on treaties rules; parametric object oriented model; 2. data acquisition (point cloud from laser scanning or photogrammetry); 3. mesh model generation from point data; 4. overlapping of parametric object oriented model to mesh model; 5. critical interpretation of data (geometric configuration hypothesis); 6. construction of the reality based model; 7. construction of interoperable geometric model to be used both as an interface to a 3D communication system and as part of a HBIM system.

Generally this process is based on knowledge and analytical and critical abilities of individual scholars, the idea is to increase the efficiency of the system through collaborative workflow forms that allow us to optimize the processes through effective knowledge management actions (Capone 2019).

3. Parametric tools to study gothic vault geometry

Our parametric model is based on historical sources. Starting from Villlet-le-duc Dictionaire and the interpretation of Villard rule we compared Philibert de l’Orme (1567), Amedée-François Frézier, (1737-1739), François Derand, Claude Milliet- Dechaux, treaties in order to define different set of geometric rules (López, 2016). The workflow for parametric model definition can be summarised in these main steps: 1) star vault plan definition; 2) extrusion lines definition based on treaties geometric rules, wireframe 3d model; 3) parametric voussoirs parametric model; 4) Tas de charge parametric model; 5) 3D model generation using extrusion lines from literature; 6) Web surfaces generation.

The first step for ribbed vault design is star vault plan definition, the projection on the horizontal plane of the ribs. These segments are the traces of the vertical planes.
where the ribs are located. There are some different geometric rules to trace plan ribs projection (Rabasa Díaz 2017) we have done a parametric model that allows to generate the plan ribs using three different methods (figure 1): geometry 1: star construction based on sides midpoints of the plan; geometry 2:: based on outer circle; geometry 3: based on inner circle.

The parameters that you can change are the measure of the sides.. By comparing different models with the plan from mesh models, we are able to evaluate which methods might have been used to set out the plan ribs (Buchanan, 2021).

There are different methods to define ribs curvature. Starting from Villard rules (the curvature is the same for all the ribs) to more complex rules. We have defined the fixed elements and variables to generate a parametric model based on treatises drawings (figure 1). We generated a simplified model without ribs and arches thickness, without web surfaces between ribs and arches. This wireframe model is composed by intrados lines of the ribs: moving the section rib along these lines (extrusion lines) we can modelling the ribs.

The variables data are: ridge configuration (straight or curve), the ribs curvature, that can be generate using different geometric methods (Calvo López 2018), the starting point location of the extrusion lines (Angjeliu 2019), the shape and the spring points of the ribs, that can be located at different levels in relation to the impost plane of the vault. In our model changing the input parameters we can generate different models based on different geometric rules. For example using Villard rule, staring from diagonal arc we define the curvature used for all ribs and we can generate two different models: one for straight ridge hypothesis and the other for curved or inclined ridge. In the first case, straight ridge, the tool are able to automatically determinate the spring points of all elements, ribs, wall arches and transverse arches. In this case the spring points are not on the impost plane and each of them is located in a different position in relation to the horizontal ridge. Overlapping these two models on mesh model we can evaluate if the designer used the Villard rule or not, in this case we can formulate another design hypothesis to match with ideal models generated using other geometric rules.

Figure 3  St Eustache church nave vault. Extrusion lines from point cloud: geometry analysis for parametric tool testing.

Our tool allows us to generate other different models using different set of geometric rules. For examples fixed the start point of extrusion line, the spring point and the end point of the ribs (the keystone level) we can use cord method, two cords methods, three cords method or another geometric set of geometric rules to generate the wireframe models that we can overlap on mesh model to to analyse the design process related to case study. As we know there are not specific rules for wall arches geometry that can be pointed or circle arches with spring point located upper than the impost plan, oval or elliptical arches, for this reason we have to define these elements in relation to case study (figure 1).

4. Realty based models and case study

Another goal of our research is to define an algorithmic for reality based model. We can use two different process: the first is based on extrusion lines from point cloud data (figure 2), in this case the tool allows us to transform the section from mesh model into the best curve that fix the real one (Capone 2021), the second is based on use of some specific measurement (end points and spring points of ribs extrusion line). To define the geometric shape of the extrusion lines we used more than one sections from point cloud. The comparison allow us to define the curve nearest to the real ones (figure 3). Testing the process with some specific case studies the goal is to define new research topics. In our research the aim is to improve the tool in relation to testing results. We are going to describe in detail this process for our case study, St. Eustache church in Paris, in the full paper.

5. Conclusion an future works

Our aim is to define the input parameters that we can use to generate different kinds of star vaults using the same algorithm (figure 4). The inputs are: plan size extrusion lines for wall arches, extrusion lines localization, ribs and arches cross section, the ridge geometry (that can be slightly horizontal, inclined or curved) and the keystones configuration. We are going to test our algorithm for some different case studies to evaluate and to improve the tool. We are going to compare our algorithm with commercial HBIM authoring software Edificius (ACCA), that allows us to model different kinds of vaults using specific parametric objects without any external tools.
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