Theory and history of Geometry and Representation for the project. Structural Optimization and Topology

Rachele Angela Bernardello; Paolo Borin Andrea Giordano; Federico Panarotto

Dipartimento di Ingegneria Civile, Edile e Ambientale, Università di Padova

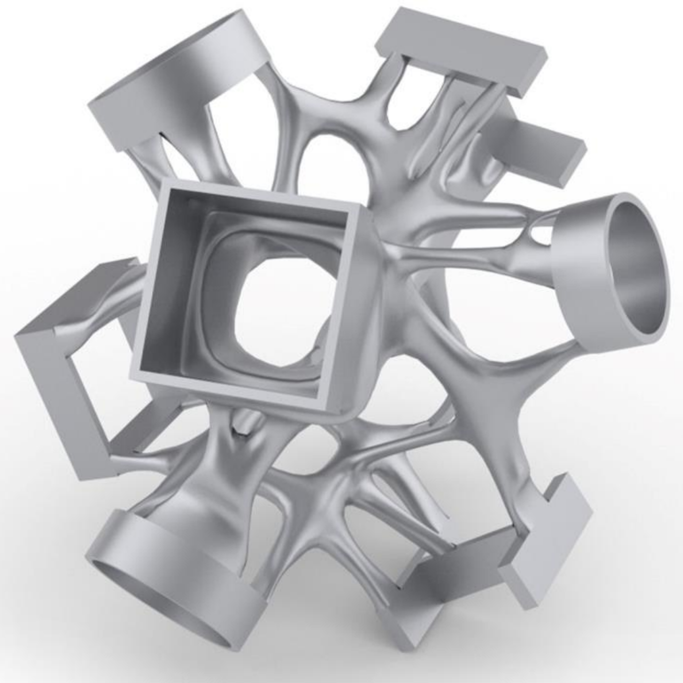

Figure 1. Topology Optimization of a structural steel node.

Abstract

The proposed research intends, in one hand, to place Geometry and Representation in the historical-anthropological climate of their genesis; on the other hand, to read their evolution/impacts for new approaches to the genesis of architectural/engineering artefacts, both from a configurative and structural point of view. However, if we consider the typical primitive/intuitive approaches of Representation, up to its rigorous elaborations based on coherent knowledge of Optics and Geometry, it is possible to recognize the strong links between artistic experience, mathematical contribution and scientific elaboration. It is therefore possible to offer a broad overview of the "state of art" relating to critical sector studies, conducted both in Italy and abroad, to underline:

- how awareness in multiple areas of Geometry is expressed in the mode and in the process of implementation of architecture and engineering, from the ideation to its realization;

- how Representation is placed as a medium between theory and built.

In this sense we intend to focus on Topology and its genesis, for the repercussions in terms of Structural Optimization, for the promising results that these procedures ensure in terms of reducing the use of material and design iterations, without neglecting architectural/engineering quality, also in configurative terms.

Keywords

Topology; History of Geometry/Representation; Structural Optimization; VPL; Configuration.

1. Introduction

The proposed research intends, in one hand, to place Geometry and Representation in the historical-anthropological climate of their genesis; on the other hand, to read their evolution/impacts for new approaches to the genesis of architectural/engineering artefacts, both from a configurative and structural point of view. However, if we consider the typical primitive/intuitive approaches of Representation, up to its rigorous elaborations based on coherent knowledge of Optics and Geometry, it is possible to recognize the strong links between artistic experience, mathematical contribution and scientific elaboration (De Rosa, et al, 2000-2002). It is therefore possible to offer a broad overview of the "state of art" relating to critical sector studies, conducted both in Italy and abroad, to underline:

- how awareness in multiple areas of Geometry is expressed in the mode and in the process of implementation of architecture and engineering, from the ideation to its realization;
- how Representation is placed as a medium between theory and built.

This last statement validates the idea that an architectural/engineering structure - conceived in the mind of the architect / engineer - finds its configuration, even before its constructive reality, in the mediation of Representation, which prefigures and establishes not only its spatial, metric or functional values, but which incorporates a whole series of cultural, theoretical and historical values that belong to it: architecture, in order to be factual, must be represented. It is in this sense that we intend to historically analyse the different forms of Representation by connecting them critically with the design creation. Necessarily, in this complex process, Geometry is therefore an indispensable "tool" for the architect/engineer, allowing him not only a correct and effective representation of particularly complex structures and highly complex spatial configurations, but above all by stimulating every design operation in a creative sense, thus reviving the ancient combination of art-science that has produced so many notable testimonies of human genius (Kemp, 1990). For this reason, the contribution of knowledge of the historical evolution of Geometry also allows us to reach an inventive and design awareness. In particular, the comprehension of Topology - even in historical terms - has fundamental repercussions on what is now called "structural optimization" (Fig. 1). In fact, with this action we mean the design criterion that allows the optimal exploitation of the physical-mechanical characteristics of construction elements through reduced material use and highest performance level. A concept therefore of general validity and inherent in the founding principles on which Science and Art of building have historically been consolidated (Gulli, 2012).

Precisely the **deep connection between Science and Art** is fundamental, renovating/highlighting in this research the **linkage between Representation and Geometry** to obtain a correct scientific congruence between configuration and structure in architecture and engineering.

2. Theory and History of Geometry to "make scientific" the Representation Systems

It is important to underline, therefore, that Gaspard Monge marks the start to scientifize the Representation methods encoding Descriptive Geometry (Taton, 1963); immediately after, during the nineteenth century, a further step towards the maximum generalization of perspective, within the Projective Geometry, will lead to the method of Central Projections, by Henry Jean Victor Poncelet: the perspective is thus freed of all the limitations operated up to then, such as that of the optical cone and visual likelihood, then configured as a mathematical tool capable of establishing a one-to-one correspondence between entities and figures of three-dimensional space and the related plane images, where the properties of the space are translated into equivalent properties in the plane (Poncelet, 1822).

We are therefore witnessing - thanks to Monge and Poncelet - the mathematization of graphic techniques, clearly explaining two very close types of geometric transformations: the central projection and the cylindrical projection. Tracing the main lines of the mathematization of graphic techniques means, therefore, to look for the origins of perspective theory and Descriptive Geometry, precisely through the systematization of two categories of graphic procedures, on the one hand those of designers, painters, sculptors, engravers, on the other hand those of architects, builders, stone cutters and carpenters.

Even today to represent architecture, whether built or in progress, these methods are adopted, perhaps loosing that character that distinguished them, that is, to give a scientific foundation to the architect/engineer ability to figure and therefore recreate reality, a reality filtered by his personality and his culture.

If, on the one hand, it is possible to recognize that from the nineteenth century to today the birth of new non-Euclidean geometries, such as elliptic and hyperbolic ones, or like Topology, has contributed to renewing the architectural and engineering project - and the related representations -, on the other hand, the representation itself has been transformed into a routine, a repetitive habit.

The advent of a *deus ex machina*, the computer, has led to some innovations, despite the fact that orthogonal projections, axonometry, perspective are used once again - and automatically - in graphics programs; in fact we are witnessing the transition, in the general context of an elaboration of the scientific foundations of Representation, from the mathematization of graphic techniques to their computerization: computer/softwares thus systematically and globally organize all the necessary data, transforming those techniques into **information immediately/quickly available.**

3. Topology

Topology is an abstraction of specific geometric concepts such as 'continuity' and 'proximity'. In fact, the word 'topology' derives from the Greek "τοποσ", a place, and "λογοσ", a discourse (Firby, 2001). It was introduced by Listing (1848), a student of Gauss, in the title of the first book devoted to the subject. Another adopted appellation was Analysis situs (Poincaré 1895).

The Poincaré's research provided the first systematic treatment of Topology and revolutionized the subject by using algebraic structures to distinguish between non-homeomorphic topological spaces, founding the field of Algebraic Topology (Dieudonné 2009; James 1999). Another popular term used to convey the more intuitive aspects related to Topology is the geometry of the 'rubber-sheet'. Rozvany (1997) defines Topology as "the model of connectivity or spatial sequence of members or elements in a structure".

Topology, unlike Euclidean Geometry, studies invariants under continuous/invertible transformations. For example, we can shape and stretch a game ball into a filled cube by such transformations, but not into a toroidal shape. When we talk about Topology, we are interested in how spaces and shapes are joined and connected, not how they appear (Zomorodian 2005). Topology is a study of the intrinsic/qualitative properties of geometric shapes, not normally affected by changes in size or shape (Kolarevic 2003). To understand its meaning, consider modeling surfaces as if they were flexible spatial models in a three-dimensional (or higher-dimensional) space. We consider two surfaces to be equal and say they are homeomorphic if one of the spatial patterns can be continuously distorted to resemble the other. By continuous distortion we mean bending, stretching, crushing without 'tearing' or 'gluing' the stitches together (Firby 2001). According to these principles, therefore, the surfaces of a bull and a cup of tea are homeomorphic. Similarly a circle and an ellipse, for example, or a square and a rectangle, can be considered topologically equivalent, since both the circle and the square could be respectively deformed into an ellipse or a rectangle. This quality of homeomorphism is particularly interesting, since the focus is on the relational structure of an object and not on its geometry (Kolarevic 2003). Topology is a much treated and discussed topic especially in relation to traditional geometry and topological space both from an algebraic and spatial point of view (Carter 1995; Reid et al. 2005; Singh yet al. 2019).

Topology Optimization is the most generic system of Structural Optimization (Rozvany et al. 1995). It is based on the concept of Topology.

4. Topology Optimization

Symbol of the culture of these years (and perhaps these centuries), the computer is a 'machine' that transformed a material culture into an immaterial one, in which ocular and media perception are "confused". Yet geometric knowledge and representation are still indispensable for the training of an architect or engineer: geometric training *tout court* must have to be involved with space, teaching how to imagine it, preparing for its prefiguration, and then making it achievable with structures. Thanks to digital, together with geometric awareness, we can now achieve greater manipulability and simulation of an architectural/engineering artefact, strengthening the creative potential: manipulability and simulation must not be mimetic of reality, but must contribute to interpreting / creating it, thus being able to affirm that the purpose of the digital machine (as for drawing) therefore lies in its creative potential. Consequently, the purpose of this research is to outline a process to achieve an optimization that, with the study of Topology, allows us to understand, interpret and finally create optimal spatial configurations (Fig. 2, Fig. 3), ensuring:

- advanced formal concepts;
- reduction of design iterations;
- reduction in the use of material;
- decrease in the waste of resources (and consequently in costs).

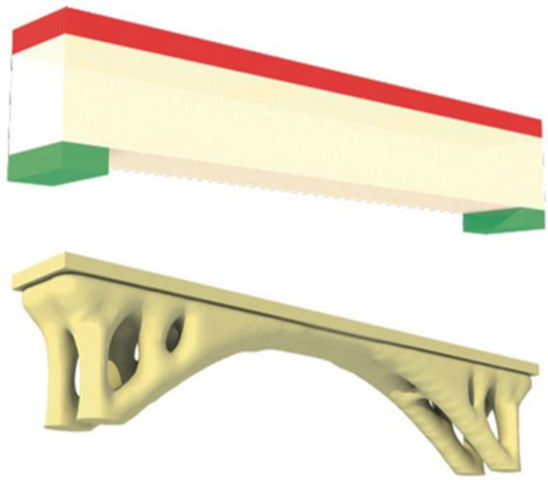

Figure 2. Initial domain of a beam case (volume: 0,72 m^3) and topology optimization result (volume: 0,72 m^3), with 20% of volume target value (image by Gianluca Rolle).

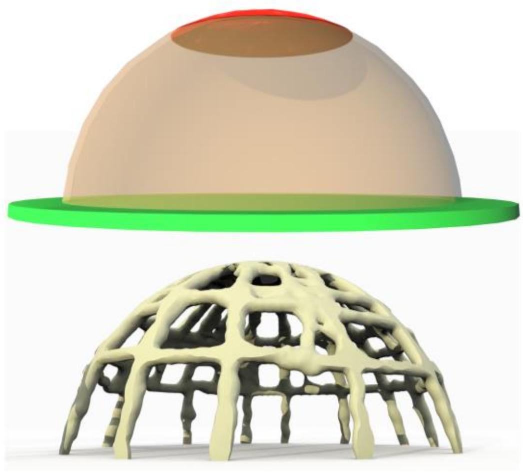

Figure 3. Initial domain of a sferic vault case (volume: 5,38 m^3, thikness: 0,3 m) and topology optimization result (volume: 2,51 m^3), with 30% of volume target value (image by Gianluca Rolle).

Furthermore, the quality - architectural and engineering - should not be overlooked, defined by the development of formal possibilities that such awareness can offer: in fact, the application of structural optimization techniques can be implemented creatively to increase the visual impact of a project and improve the readability of structural actions (Mayencourt et al. 2019). In this research, therefore, strategies strictly linked to Topology will be considered, to obtain structural optimization, such as size optimization (another method of structural optimization) and the theory of minimal surfaces (methodology not included among those of structural optimization but classified as a method for creating shapes).

References

Carter, J., S., 1995. *How Surfaces Intersect in Space*. Vol. 2. WORLD SCIENTIFIC.

De Rosa, A., Sgrosso, A., Giordano, A., 2000-2002. *La geometria nell'Immagine*. Torino: UTET. Voll.3.

Dieudonné, J., 2009. *A History of Algebraic and Differential Topology 1900–1960*. Boston: Birkhäuser Basel.

Firby, P., A., Gardiner. C., F., 2001. Intuitive Ideas. In (edited by Firby P. A. and Gardiner. C. F.) *Surface Topology*, Cambridge: Woodhead Publishing. Pp. 15–31

James, I., M., 1999. *History of Topology*. Amsterdam: Elsevier.

Kemp, M., 1990. *The Science of Art: Optical Themes in Western Art from Brunelleschi to Seurat*. Yale University Press.

Kolarevic, B., 2003. *Architecture in the Digital Age, Design and Manufacturing*. London: Taylor & Francis.

Mayencourt, P., Mueller, C., 2019. *Optimization and shaping of indeterminate frame structure*. In: Structures and Architecture - Bridging the Gap and Crossing Borders: Proceedings of the Fourth International Conference on Structures and Architecture (ICSA 2019), July 24-26, 2019, Lisbon, Portugal. Boca Raton: CRC Press.

Poincaré, H., 1895. *Analysis Situs*. Paris: Journal de l'École Polytechnique 2(1).

Poncelet, J., V., 1822. *Traité des propriétés projectives des figures: useful ouvrage in ceux qui s'occupent des applications de la géométrie descrip tive et d'operations géométriques sur le terrain*. Paris: Bachelier.

Reid, M., Szendroi, B., 2005. *Geometry and Topology*. Cambridge: Cambridge University Press.

Rozvany, G., I., N., ed. 1997. *Topology Optimization in Structural Mechanics*. 1st ed. Vienna: Springer Vienna.

Rozvany, G., I., N., Bendsøe, M. P., Kirsch, U., 1995. *Layout Optimization of Structures*. In: Applied Mechanics Reviews 48(2):41–119.

Singh, M., Song, Y., Wu, J., 2019. *Algebraic Topology and Related Topics*. Singapore: Birkhäuser.

Taton, R., 1963. *History of science*. Basic Books. Vol. 4.

Zomorodian, A. J., 2005. *Topology for Computing*. Cambridge: Cambridge University Press.

Biographical data of the Authors

Rachele Angela Bernardello

Università di Padova, rachele.bernardello@dicea.unipd.it

Short CV: Rachele Bernardello is a PhD student at the University of Padova with a thesis in BIM practices for monitoring and management of existing bridges. Rachele graduated in Building Engineering and Architecture and since 2017 she has worked at LIM.lab - Laboratory of Information Modeling in Padua. Her research interests are in BIM and Information exchange referred both to Infrastructures and Buildings, focus on digital workflows through delivered international standards. Her research also deals with Cultural Heritage Management and Communication based on digital methods.

Paolo Borin

Università di Padova, paolo.borin@dicea.unipd.it

Short CV: Paolo Borin took a PhD at the Università IUAV di Venezia. His thesis explores the science and stereotomy of Guarino Guarini through digital and computational modeling techniques. Borin graduated with a degree in Architectural Engineering from University of Padova (IT), in 2011 with honors. His thesis analyzes how prefabrication and BIM could lead to low cost and hight quality buildings. From 2011 he has been exploring theory and practise of Building Information Modeling and applying it to enhance the building process.

Andrea Giordano

Università di Padova, andrea.giordano@unipd.it

Short CV: Full professor at University of Padova, coordinator of the second level Master "BIM/hBIM" and of LDR (Lab of Drawing and Representation) and LIM (Lab of Information Modeling). His interests are centered on the themes of representation understood as a hermeneutic process of construction and design, with particular attention to the geometric-configurative aspects of architectural spaces. He is a member of the Technical Scientific Committee of the Italian Union of Drawing – UID, and a member of the Steering Committee of the international joint research Visualizing Venice/Visualizing Cities.

Federico Panarotto

Università di Padova, federico.panarotto@unipd.it

Short CV: Federico earned his PhD in Civil Engineering at Civil, Environmental and Architectural-Engineering Department of University of Padua in 2021. Federico's main research focus is on Building Information Modeling (BIM) and on Structural Optimization in the AEC sector. These topics are also dealt with in his teaching activity in Disegno Edile 3 and Laboratorio di disegno automatico (CAD e BIM) courses at the same department. Federico is part of the LIM.Lab research group which mainly focus on BIM with particular reference to geometry and representation topics.