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***Investigación en Técnicas Numéricas
Basadas en Ecuación Integral para el
Análisis Y Diseño de Dispositivos de
Microondas para Comunicaciones
Espaciales***

*Doctorado en Tecnologías de la Información y las
Comunicaciones*

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Cartagena (2020)

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*INVESTIGACIÓN EN TÉCNICAS NUMÉRICAS BASADAS EN ECUACIÓN INTEGRAL PARA EL
ANÁLISIS Y DISEÑO DE DISPOSITIVOS DE MICROONDAS PARA COMUNICACIONES
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Investigations on Numerical Techniques Based on Integral Equation for the Analysis and Design of Microwave Devices in Space Applications

by

Celia Gómez Molina

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor in Telecommunication Systems Engineering in the Universidad Politécnica de Cartagena, Department of Information Technologies and Communications, Spain.

September 1st, 2020

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“Satisfaction lies in the effort, not in the attainment

Full effort is full victory.”

- Mahatma Gandhi

I dedicate my humble effort to my family and friends, whose affection and support inspire me to get this thesis.

I am highly appreciative of the help that all professors gave me during all these years. This thesis is possible thanks to their work.

Specially, I would like to express my sincere gratitude to my supervisor Alejandro Álvarez Melcón for his commitment, continuous support and immense knowledge.

All this contributes to accomplish this project both at personal and professional levels. This project is specially dedicated to him and his work.

This thesis is also from all of you.

ACKNOWLEDGEMENTS

I would like to thank the people who made this dissertation possible, specially to all the professors that have helped me to achieve the technical knowledge necessary to develop this work.

I wish to express my deepest gratitude to my supervisor and co-supervisor, Prof. Dr. Alejandro Álvarez Melcón and Prof. Dr. Fernando Quesada Pereira, for their invaluable support. I am indebted to them for sharing expertise, and for the encouragement extended to me. Thanks for the long discussions that helped me resolve the technical details for my work. Also, I am very fortunate and grateful to Prof. Dr. Vicente Boria and Prof. Dr. Marco Guglielmi for giving me the opportunity to work together and learn from them. They kept me going on and this work would not have been possible without their input. Finally, I offer special thanks to Prof. Dr. Juan Sebastián Gómez Díaz from the University of California, Davis (EEUU), and Prof. Giuseppe Macchiarella from the Politecnico di Milano (Italy) for the time that they have dedicated during my research stays to discuss and enrich my work.

Last but not least, I would like to mention that this work has been supported by the Spanish Government through the Ministerio de Educación, Cultura y Deporte with Ref. FPU15/02883, EST17/00576 and EST18/00427 and the Ministerio de Economía y Competitividad through the sub-projects 4, 2 and 1 of the coordinated project with Ref. TEC2016-75934-C4-R. Without their support and funding, this project could not have reached its goals.

TABLE OF CONTENTS

DEDICATION	ii
ACKNOWLEDGEMENTS	iii
LIST OF FIGURES	vii
LIST OF TABLES	xxv
LIST OF APPENDICES	xxviii
ABSTRACT	xxix
CHAPTER	
I. Introduction	1
II. Multimode Equivalent Network Technique for Waveguide De- vices	18
2.1 Introduction	18
2.2 MEN Formulation without Asymptotic Admittances	20
2.3 Application of Kummer's Transformation	27
2.3.1 Theoretical Derivations	27
2.3.2 Numerical Results	28
2.4 Efficient Mechanism to Assess Convergence	34
2.4.1 Theoretical Derivations	34
2.4.2 Numerical Results	40
2.5 Conclusions	54
III. Electric Multimode Equivalent Network Technique for Planar Devices	56
3.1 Introduction	56
3.2 Electric MEN for Single-layer Planar Devices	58

3.2.1	Theoretical Derivations	58
3.2.2	Numerical Results	69
3.3	Extension of the Electric MEN for Multilayered Planar Devices	72
3.3.1	Theoretical Derivations	72
3.3.2	Numerical Results	76
3.4	Conclusions	79
IV.	Magnetic Multimode Equivalent Network Technique for Planar Devices	80
4.1	Introduction	80
4.2	Magnetic MEN for Single-layer Planar Devices	83
4.2.1	Theoretical Derivations	83
4.2.2	Numerical Results	97
4.3	Extension of the Magnetic MEN for Multilayered Planar Devices	108
4.3.1	Theoretical Derivations	108
4.3.2	Numerical Results	110
4.4	Conclusions	116
V.	Consideration of Losses in the Magnetic Zero-thickness MEN Formulation	118
5.1	Introduction	118
5.2	Surface Impedance and Admittance of a Low Impedance Plane	120
5.3	Surface Impedance and Admittance of a High Impedance Plane	123
5.4	Lossy Magnetic Zero-thickness MEN	124
5.5	Numerical Results	134
VI.	Magnetic Multimode Equivalent Network Technique for Thick Planar Devices	142
6.1	Introduction	142
6.2	Magnetic MEN for Thick Planar Devices	144
6.2.1	Theoretical Derivations	144
6.2.2	Numerical Results	153
6.3	Conclusions	167
VII.	Waveguide, Hybrid and Microstrip Filter Designs	168
7.1	Introduction	168
7.2	Resonating Aperture Waveguide Filters for X/Ku-band Applications	170
7.2.1	Introduction	170
7.2.2	Study on the Resonating Iris Technology	171
7.2.3	Filter Designs	177

7.2.4	Conclusions	180
7.3	Thick Metallic Bar Filters for Wideband C-band Applications	181
7.3.1	Introduction	181
7.3.2	Thick Bar Technology Characterization	186
7.3.3	Design of Filters Based on Thick Bar Technology . .	191
7.3.4	Prototype Manufacturing and Measurements	202
7.3.5	Conclusions	206
7.4	Stub Loaded Stopband Microstrip Filters for Ka-band Appli- cations	208
7.4.1	Introduction	208
7.4.2	Study on the Undesired Coupling Effect	209
7.4.3	Techniques for Reducing the Undesired Coupling Effect	211
7.4.4	Filters Designs	214
7.4.5	Conclusions	222
7.5	Conclusions	222
VIII. Conclusions		224
8.1	Future Lines of Research	227
8.2	Scientific Outcome	228
8.3	Research Stays	233
APPENDICES		234
A.1	Magnetic and Electric Vector Modal Functions of a Rectangu- lar Waveguide	236
A.2	Electric Vector Modal Functions of a Rectangular Metallic Patch	237
A.3	Coupling Integrals between Rectangular Waveguides for Mag- netic Formulations	240
A.4	Coupling Integrals between a Rectangular Waveguide and a Rectangular Metallic Patch for Electric Formulations	243
A.4.1	Internal Areas	243
A.4.2	Metallic Patch Attached to $x = 0$ Wall	246
A.4.3	Metallic Patch Attached to $x = a$ Wall	247
A.5	Coupling Integrals between a Rectangular Metallization and an Excitation Pulse	248
A.5.1	Metallic Patch Attached to $x = 0$ Wall	248
A.5.2	Metallic Patch Attached to $x = a$ Wall	249
A.6	Coupling Integrals between an Arbitrary Waveguide and an Excitation Pulse	251
B.1	GUI for Microstrip Circuits	256
BIBLIOGRAPHY		279

LIST OF FIGURES

Figure

1.1	Example of a waveguide component and its equivalent network <i>Guglielmi et al.</i> (1999).	7
2.1	Two-dimensional step in waveguide technology.	21
2.2	MEN representation for the waveguide step in Fig. 2.1. (a) Explicitly representation of the localized modes. (b) Representation of the MEN drawing only the accessible modes (most common formalism). . . .	24
2.3	Geometry of two cavities connected through an iris. The height in y-axis is 7.899 mm for all the rectangular waveguide sections. In addition, $a^{(2)} = a^{(4)} = 15.799$ mm, $a^{(3)} = 3.5993$ mm and $a^{(1)} = a^{(5)} = 5.9283$ mm. Frequency sweep from 18 GHz to 25 GHz.	29
2.4	Equivalent network of the device represented in Fig. 2.3.	29
2.5	Magnitude of the scattering parameters of the structure in Fig. 2.3 simulated using the three studied formulations with the parameters in Table 2.1. The three results are coincident.	30
2.6	Scheme in E-plane of the symmetric capacitive transformer under analysis.	31
2.7	Scattering parameters magnitude of the structure in Fig. 2.6 simulated using the three studied formulations with the parameters in Table 2.3. The three results are coincident.	33
2.8	Inductive iris composed of two steps in rectangular waveguide technology. The dimensions are $a_1 = 19.1$ mm, $a_2 = 9.0203$ mm and the height is $b = 8.2$ mm.	41

2.9	Relative convergence error of the element $Z_{N(r),N(r)}^{(r,r)}$ as a function of the accuracy F -factor for the first step in Fig. 2.8.	43
2.10	Single dual-mode inductive bandpass filter under study (dimensions in mm).	45
2.11	Magnitude of the S-parameters for the dual-mode cavity filter in Fig. 2.10 using the parameters adjusted by the proposed method (see Table 2.6) and compared to the convergent performance using MEN formulation and ANSYS HFSS software tool.	46
2.12	Three-pole bandpass filter under study. The dimensions are $a = 19.1$ mm, $a_1 = 9.0203$ mm, $a_2 = 5.1702$ mm and the height is $b = 8.2$ mm. The lengths are $l_1 = 1$ mm, $l_2 = 21.6581$ mm and $l_3 = 23.1495$ mm.	46
2.13	S-parameters magnitude for the three-pole bandpass filter in Fig. 2.12 using the parameters adjusted by the proposed method for $dB_{th} = 10$ dB and $dB_{th} = 20$ dB (see Table 2.7) and compared to the convergent performance using MEN formulation and ANSYS HFSS software tool.	47
2.14	Four-pole dual-mode bandpass filter in the Ku-band. The dimensions are reported in Table 2.8 and the height is $b = 9.525$ mm.	48
2.15	S-parameters magnitude for the four-pole dual-mode bandpass filter in Fig. 2.14 using the parameters adjusted by the proposed method (see Table 2.9) and compared to the convergent performance using MEN formulation and ANSYS HFSS software tool.	49
2.16	Rectangular waveguide bandpass filter under analysis. The dimensions in mm are: $w_1 = 12.937$, $a_1 = 27.572$, $w_2 = 9.487$, $a_2 = 27.137$, $w_3 = 3.928$, $a_3 = 19.05$, $w_4 = 3.915$, $a_4 = 27.173$, $w_5 = 9.471$, $a_5 = 27.602$, $w_6 = 12.922$. The lengths in mm are: $t_1 = 12$, $l_1 = 26.577$, $t_2 = 9$, $l_2 = 29.926$, $t_3 = 1$, $l_3 = 18.946$, $l_4 = 29.919$, $l_5 = 26.585$	51
2.17	S-parameter in-band response for the filter in Fig. 2.16 using the parameters adjusted by the proposed method (see Table 2.11) and compared to the convergent performance using FEST3D and ANSYS HFSS software tool.	52
2.18	S-parameter out-of-band response for the filter depicted in Fig. 2.16 using the parameters adjusted by the proposed method (see Table 2.12) and compared to the convergent performance using FEST3D and ANSYS HFSS software tools.	53

3.1	Two-dimensional zero-thickness printed circuit in planar technology. The gray areas correspond to the printed metallizations and the dark areas to the excitation ports, both contained in the transverse plane. The media on both sides of the discontinuity can be different, for instance the substrate and the air of a microstrip structure.	59
3.2	Multimode Equivalent Network (generalized Y-matrix) representation for the zero-thickness discontinuity in Fig. 3.1.	63
3.3	Schematic of the shielded microstrip printed circuit under study, excited with two coaxial connectors.	67
3.4	Equivalent network representation for the microstrip circuit shown in Fig. 3.3 using the MEN formalism. The metallization at the interface is characterized by its equivalent admittance coupling matrix. Just one accessible mode is considered, which is characterized with transmission lines in the air (l_a) and dielectric (l_d) regions, terminated with a short-circuit representing the top and bottom PEC covers of the cavity box (see Fig. 3.3). The coaxial excitations are represented with transmission lines of characteristic impedance Z_0	67
3.5	Third order dual-bandpass filter under study. The dimensions (in mm) are: $a = 40$, $b = 34$, $L_{in} = L_{out} = 14$, $L_r = 24.54$, $w_1 = 1.8$, $w_2 = 5$, $L_1 = 3$ and $L_2 = 3.14$. The width of all the microstrip lines is 2 mm. The dielectric relative permittivity $\epsilon_r = 2.2$	69
3.6	Comparison of the S-parameters magnitude of the dual-bandpass filter in Fig. 3.5 computed using different full-wave electromagnetic analysis tools (ADS and ANSYS HFSS), and compared to the measured response.	70
3.7	Geometry of the four-pole bandpass filter under study. The dimensions (in mm) are: $a = 25.4$, $b = 25.4$, $L_1 = 2.9$, $L_2 = 2.708$, $L_3 = 2.887$, $w_1 = 0.1$, $w_2 = 0.613$, $w_3 = 0.802$, $h_1 = 3.6$ and $h_2 = 0.4$. The width of all the microstrip lines is 0.355 mm. The dielectric relative permittivity is $\epsilon_r = 9.65$	71
3.8	S-parameters magnitude of the coupled-line bandpass filter in Fig. 3.7 using different electromagnetic analysis tools (ADS and ANSYS HFSS), and compared to the measured response.	71
3.9	Scheme of the geometry under study.	73

3.10	Multimode Equivalent Network corresponding to the multilayer geometry represented in Fig. 3.9, with two levels of metallization. . .	73
3.11	Scheme of the circuit that contains two metallization level and three dielectric substrates.	74
3.12	Multimode Equivalent Network of the multilayer geometry in Fig. 3.11.	75
3.13	Layout of the two metallization levels of the bandpass filter under analysis. Gray areas are the metallizations while dark areas represents the excitation pulses. The dimensions (in mm) are: $L_{in} = L_{out} = 23$, $L_{res1} = L_{res2} = 34.2$, $S_1 = 0.1$, $S_2 = 3.2$ $a = 40$, $b = 20$, $L_{res3} = 33.94$. The width of all lines is 1.49 mm.	77
3.14	S-parameters magnitude of the bandpass filter in Fig. 3.13 obtained using the presented formulation and compared to the results reported in <i>Melcon et al.</i> (1999) and the simulation results obtained using the commercial tool ADS.	78
4.1	Two-dimensional zero-thickness printed circuit in planar technology. The gray areas correspond to the printed metallizations and the dark areas to the excitation and internal ports, both placed on the transverse ($z=0$) plane. The media on both sides of the discontinuity can be different, for instance, the substrate and the air of a microstrip structure. The size of the ports along the x and y -axis is denoted as Δx and Δy , respectively.	84
4.2	Multimode Equivalent Network (generalized Z -matrix) representation for the zero-thickness discontinuity with arbitrarily shaped metallic areas, shown in Fig. 4.1, including one port in the transverse plane.	89
4.3	Schematic of the shielded microstrip printed circuit under study, excited with two coaxial connectors.	93
4.4	Equivalent network representation for the microstrip circuit shown in Fig. 4.3 using the MEN formalism. The metallization at the interface is characterized by its equivalent impedance coupling matrix. Just one accessible mode is considered, which is characterized with transmission lines in the air (l_a) and dielectric (l_d) regions, terminated with the impedance Z_s representing losses in the top and bottom covers of the cavity box (see Fig. 4.3). The coaxial excitations are represented with transmission lines of characteristic impedance Z_0	94

4.5	Lowpass microstrip filter under study. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 67.5 \text{ mm} \times 67.5 \text{ mm}$. With respect to Fig. 4.3, the value of l_a is 9.83 mm. The dielectric relative permittivity is $\epsilon_r = 2.33$ and the substrate thickness is $l_d = 1.57 \text{ mm}$. The filter is centered in the box.	98
4.6	Scattering parameters magnitude of the lowpass filter in Fig. 4.5 computed using the MEN formulation, using a finite elements based electromagnetic analysis tool (ANSYS HFSS), and compared to the measured response.	98
4.7	Hairpin microstrip bandpass filter geometry under study. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 31.2 \text{ mm} \times 30 \text{ mm}$. With respect to Fig. 4.3, the value of l_a is 3.73 mm. The dielectric relative permittivity is $\epsilon_r = 6.15$ and the substrate thickness is $l_d = 1.27 \text{ mm}$. The filter is centered in the box.	100
4.8	In-band S-parameters magnitude of the hairpin bandpass filter shown in Fig. 4.7, using the proposed MEN formulation, and using both the electromagnetic analysis tool ANSYS HFSS and simulation results reported in <i>Hong</i> (2011).	100
4.9	Out-of-band S-parameters magnitude of the hairpin bandpass filter shown in Fig. 4.7, using the proposed MEN formulation and the electromagnetic analysis tool ANSYS HFSS.	101
4.10	Dual-mode microstrip bandpass filter under study. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 25 \text{ mm} \times 20 \text{ mm}$. With respect to Fig. 4.3, the value of l_a is 8.73 mm. The dielectric relative permittivity is $\epsilon_r = 10.8$ and the substrate thickness is $l_d = 1.27 \text{ mm}$. The filter is centered in the box.	102
4.11	In-band S-parameters magnitude of the dual-mode bandpass filter shown in Fig. 4.10, using the proposed lossy MEN formulation, and the electromagnetic analysis tool ANSYS HFSS. Results of simulations (using Sonnet) and measurements from <i>Hong et al.</i> (2007) are also included.	103
4.12	Out-of-band S-parameters magnitude of the dual-mode bandpass filter shown in Fig. 4.10, using the proposed lossy MEN formulation, and the electromagnetic analysis tool ANSYS HFSS. Results of simulations (using Sonnet), and measurements from <i>Hong et al.</i> (2007) are also included.	103

4.13	Wilkinson power divider in microstrip technology. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 30 \text{ mm} \times 16 \text{ mm}$. The value of l_a in Fig. 4.3 is 3 mm. The dielectric relative permittivity is $\varepsilon_r = 4$ and the substrate thickness is $l_d = 1.2 \text{ mm}$. The divider is centered in the box.	104
4.14	S-parameters magnitude of the power divider, shown in Fig. 4.13, using the proposed MEN formulation and the commercial software ANSYS HFSS. The excitation is through Port 1.	105
4.15	S-parameters magnitude of the power divider, shown in Fig. 4.13, using the proposed MEN formulation and the commercial software ANSYS HFSS. The excitation is through Port 2.	105
4.16	S-parameters magnitude of the power divider, shown in Fig. 4.13, when the internal resistor has $R = 10 \Omega$. The simulations are obtained using the MEN formulation and the commercial software ANSYS HFSS. The excitation is through Port 2.	106
4.17	Circuit containing two metallic interfaces, and three dielectric substrates, excited with two coaxial connectors in the lower metallization level.	108
4.18	Multimode Equivalent Network of the multilayer geometry in Fig. 4.17.	109
4.19	First implementation of a microwave resonator with direct input/output coupling, using two metallization interfaces. The dimensions are in mm. Gray areas are the metallizations while dark areas represent the excitation ports. The width of all lines is 1.49 mm. The dimensions of the shielding box are: $a \times b = 40 \text{ mm} \times 20 \text{ mm}$. With respect to Fig. 4.17, the heights of the dielectric layers are: $h_1 = h_2 = h_3 = 5 \text{ mm}$. The relative dielectric permittivities of the substrates are: $\varepsilon_{r1} = \varepsilon_{r3} = 2.33$ and $\varepsilon_{r2} = 1.07$	111
4.20	S-parameters magnitude of the rectangular microwave resonator of Fig. 4.19, obtained using the magnetic MEN formulation, compared with the results obtained using the formulation reported in Chapter III, and to the simulation results obtained with ANSYS HFSS. . . .	112

4.21	Second implementation of a microwave resonator with direct input/output coupling, using two metallizations. In this case, an elliptic resonator is used in the second metallization level. The dimensions are in mm. The width of the rectangular input/output lines is 1.49 mm. The dimensions of the shielding box are: $a \times b = 40 \text{ mm} \times 20 \text{ mm}$. With respect to Fig. 4.17, $h_1 = h_3 = 5 \text{ mm}$ and $h_2 = 6.52 \text{ mm}$. The relative dielectric permittivities of the substrates are: $\varepsilon_{r1} = 2.33$, $\varepsilon_{r2} = 1.07$ and $\varepsilon_{r3} = 4$	112
4.22	S-parameters magnitude of the elliptic microwave resonator of Fig. 4.21, obtained using the presented formulation and compared to the simulation results obtained with the commercial tool ANSYS HFSS. . .	113
4.23	Third implementation of a microwave resonator with direct input/output coupling, using two metallization interfaces. An internal port is used to introduce a direct inductive coupling between input/output ports. The dimensions are in mm. The width of the input/output lines is 1.49 mm. The internal port (hatched area) is used to connect an inductor L . The dimensions of the shielding box are: $a \times b = 40 \text{ mm} \times 20 \text{ mm}$. With respect to Fig. 4.17, $h_1 = h_3 = 5 \text{ mm}$ and $h_2 = 7.5 \text{ mm}$. The relative dielectric permittivities of the substrates are the same as in Fig. 4.21. The value of the inductor is $L = 25 \text{ nH}$. . .	114
4.24	S-parameters magnitude of the elliptic microwave resonator with inductive input/output coupling, as shown in Fig. 4.23, using the proposed MEN formulation and the software tool ANSYS HFSS. . . .	115
5.1	Zero-thickness discontinuity under analysis. The metallizations (gray areas) are made with non perfect conductors. The losses due to the metallization are considered through an equivalent problem: we assume a high impedance surface in the air part (aperture) of the discontinuity.	119
5.2	Low and high impedance surfaces. (a) The low impedance surface is characterized using the finite conductivity σ . Z_c and Y_c are the surface impedance and admittance of this plane, respectively. (b) The high impedance surface is characterized using the resistivity parameter R_m . Z_s and Y_s are the surface impedance and admittance of this plane, respectively.	121
5.3	First option for the implementation of the impedance inverter. This inverter transforms the resistivity of a good conductor ($1/\sigma$) into the resistivity of a high impedance surface (R_m). Z_0 is the characteristic impedance of the inverter and λ is the wavelength of the medium. .	132

5.4	Second option for the impedance inverter. This inverter transforms the surface impedance of a good conductor (Z_c) into the surface impedance of a high impedance plane (Z_s). Z_0 is the characteristic impedance of the inverter and λ is the wavelength.	132
5.5	Dual-mode microstrip bandpass filter under study. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 25 \text{ mm} \times 20 \text{ mm}$. The value of l_a is 8.73 mm and the dielectric relative permittivity is $\varepsilon_r = 10.8$ and the substrate thickness is $l_d = 1.27 \text{ mm}$. The filter is centered in the box.	135
5.6	In-band S-parameters magnitude of the dual-mode bandpass filter shown in Fig. 5.5, using the proposed lossy MEN formulation, and the electromagnetic analysis tool ANSYS HFSS. Results of simulations (using Sonnet) and measurements from <i>Hong et al.</i> (2007) are also included.	135
5.7	Wilkinson power divider in microstrip technology. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 30 \text{ mm} \times 16 \text{ mm}$. The value of l_a is 3 mm. The dielectric relative permittivity is $\varepsilon_r = 4$ and the substrate thickness is $l_d = 1.2 \text{ mm}$. The divider is centered in the box.	137
5.8	S-parameters magnitude of the power divider, shown in Fig. 5.7, using real components in ANSYS HFSS. The excitation is through Port 1.	138
5.9	S-parameters magnitude of the power divider, shown in Fig. 5.7, using real components in ANSYS HFSS. The excitation is through Port 2.	138
5.10	S-parameters magnitude of the power divider, shown in Fig. 5.7, using the proposed MEN formulation and the commercial software ANSYS HFSS, including losses. The excitation is through Port 1.	139
5.11	S-parameters magnitude of the power divider, shown in Fig. 5.7, using the proposed MEN formulation and the commercial software ANSYS HFSS, including losses. The excitation is through Port 2.	140
6.1	Schematic (side view) of the structure under analysis. The metallization has a non negligible thickness (h_2). The structure is excited by two lateral ports (Ports 1 and 2), at the interface $z=0$	145

6.2	Top view of the two discontinuities present in the structure shown in Fig. 6.1. (a) The first discontinuity (at $z = 0$) contains the lateral excitation ports (dark areas) that excite the structure. (b) The second discontinuity (at $z = h_2$) only contains the metallizations (gray areas).	146
6.3	Final equivalent network representation for the structure shown in Fig. 6.1, using the MEN formalism. The discontinuities are characterized by their MEN coupling matrices. The modes are characterized with transmission lines in the different regions. Modes in media 1 and 3 are terminated with a short-circuit to model the top and bottom metal covers (PEC) of the box enclosure (see Fig. 6.1). The coaxial excitations are represented with transmission lines of characteristic impedances Z_{p1} and Z_{p2} for Ports 1 and 2, respectively.	152
6.4	Side (left) and top (right) views of the filters under study. Dark areas (l_p) represent lumped ports that model the direct connection of the coaxial pins to the first and last resonators (Ports 1 and 2). In this case, $\Delta x = l_p$ and $\Delta y = W_{r1}$	154
6.5	Number of accessible modes that are necessary according to the specified threshold (dB_{th}). This test is carried out for an operating frequency $f_0 = 5$ GHz and a thickness h_2 equal to 2 mm.	156
6.6	S-parameters response of the first in-line bandpass filter (with $h_2 = 2$ mm) for different values of the threshold dB_{th}	158
6.7	Magnitude of the S-parameters for the first in-line bandpass filter (with $h_2 = 2$ mm) computed using the <i>thick</i> MEN formulation as compared to the results from ANSYS HFSS. The simulation results using the <i>zero-thickness</i> MEN formulation described in Chapter IV are also included for comparison.	158
6.8	Necessary number of accessible modes according to the thickness of the metallization (h_2 , see Fig. 6.4). This test is done for different power thresholds and for an operating frequency equal to $f_0 = 5$ GHz.	159
6.9	Magnitude of the S-parameters for the second in-line bandpass filter (with $h_2 = 0.5$ mm) computed using the <i>thick</i> MEN formulation and compared to ANSYS HFSS results. The MEN calculations are obtained with a reduced threshold $\text{dB}_{th} = 1$ dB. The simulation results using the <i>zero-thickness</i> MEN formulation described in Chapter IV are also included for comparison.	161

6.10	Magnitude of the S-parameters for the second in-line bandpass filter but with $h_2 = 0.1$ mm, computed using the <i>thick</i> MEN formulation and compared to ANSYS HFSS results. The MEN calculations are obtained with $N = 23$. The simulation results using the <i>zero-thickness</i> MEN formulation described in Chapter IV are also included for comparison.	163
6.11	Magnitude of the S-parameters for the transversal filter computed using the MEN formulation as compared to the ANSYS HFSS results and to measurements from the prototype discussed in Chapter VII. The thickness of the metallizations is $h_2 = 2$ mm. The MEN results are obtained with $\text{dB}_{th} = 4$ dB, leading to $N = 28$ accessible modes. We have used $\tan \delta = 0.0037$ and $\sigma = 3.8 \cdot 10^7$ S/m (aluminum) to model the losses. In the inset, we can see a zoom of the $ S_{21} $ parameter in the passband. In the zoom, the black curve is the ANSYS HFSS result, the green curve is the measured response and the red curve is the MEN simulation.	166
7.1	3D view of the proposed filter technology based on resonating apertures.	172
7.2	3D view of a traditional waveguide filter using half-wavelength cavities.	172
7.3	Magnitude of the Scattering parameters of the bandpass filter in Fig. 7.1 for different sizes of the resonating apertures. The dimensions of the aperture ($\text{ap} = a_{ap} \times b_{ap}$) are in mm.	174
7.4	Magnitude of the Scattering parameters of the filter in Fig. 7.1 for different lengths (l_{ap}) of the resonating apertures.	175
7.5	Magnitude of the Scattering parameters of the bandpass filter in Fig. 7.1 for different lengths of the intermediate waveguide section (WG 3), l_{int}	176
7.6	Magnitude of the Scattering parameters of the bandpass filter in Fig. 7.1 for different lengths of the intermediate waveguide section (WG 3), l_{int}	177
7.7	Magnitude of the Scattering parameters of the three filters obtained with ANSYS HFSS. The dimensions are reported in Table 7.1. . .	178
7.8	Magnitude of the Scattering parameters of the designed filter using Teflon, obtained with the MEN code, with FEST3D and ANSYS HFSS.	179

7.9	3D view of the proposed filter technology based on bulky bars buried in a dielectric medium. The sketch shows an example of a filter using three bar resonators enclosed in a house cavity. I/O couplings are implemented with the step impedance discontinuity produced from a direct connection of the coaxial connectors with the first/last bar resonators.	184
7.10	Unloaded quality factor Q_U of a single bulky bar resonator, as a function of the thickness l_{th} . The geometry under test is shown in the inset. The dielectric is Teflon with a relative permittivity of $\varepsilon_r = 2.1$ and a loss tangent $\tan \delta = 0.001$. The bar is made using copper with conductivity $\sigma_c = 5.8 \cdot 10^7$ S/m, and the box is made of aluminum with conductivity $\sigma_b = 3.8 \cdot 10^7$ S/m. Other dimensions are $l_d = 10$ mm, $l_a = 10$ mm, $l_r = 15$ mm, $w_r = 2$ mm (bar resonator width in the y -axis), $a = 30$ mm and $b = 20$ mm (box dimension in the y -axis). The test frequency is $f_c = 5.56$ GHz.	187
7.11	Side (left panel) and top (right panel) view of two coupled thick bar resonators, showing relevant dimensions. Feeding ports are shown with dark areas l_p	188
7.12	Normalized coupling coefficient as a function of the separation between resonators y_{12} , for various bar thicknesses l_{th} . Coupling coefficient is normalized with the second frequency plan ($f_{c2} = 3.26$ GHz, $BW_2 = 1.15$ GHz, $FBW_2 = 35.28\%$). Other dimensions are: $a = 30$ mm, $b = 28$ mm, $y_{r1} = y_{r2} = 10$ mm, $w_{r1} = w_{r2} = 2$ mm, $x_{12} = 0$ mm, $l_d = 10$ mm, $l_a = 10$ mm. The relative permittivity of the substrate is $\varepsilon_r = 2.1$	189
7.13	External quality factor Q_{ext} as a function of the input port reference impedance Z_p , for various values of the bar thickness l_{th} . The test frequency is $f_c = 3.5$ GHz. Other dimensions are: $l_d = 10$ mm, $l_a = 10$ mm, $y_{r1} = 10$ mm, $w_{r1} = 2$ mm, $a = 38$ mm, $b = 32$ mm. The dark area l_p denotes a lumped port that models the direct connection of the coaxial pin to the resonator.	191
7.14	Top (right panel) and side (left panel) view of the three filters designed using the new proposed technology (one following the in-line topology and two more with a transversal topology). Relevant dimensions are also shown. Thick bars are identified with numbers. For the transversal filters the bars are numbered according to resonators shown in the coupling topology of Fig. 7.17. Dark areas (l_p) represent lumped ports that model the direct connection of the coaxial pins to the first/last resonators.	192

7.15	3D view of the coaxial transition employed to adjust the I/O couplings.	194
7.16	Scattering parameters of the in-line filter obtained with ANSYS HFSS (design) and compared to the coupling matrix response (matrix). The bandwidth is $BW_1 = 800$ MHz ($FBW_1 = 18.78\%$). The dimensions are reported in Table 7.2.	196
7.17	Coupling topology that represents the filter structure shown in Fig. 7.14, with an additional cavity resonance. Bar resonators correspond to resonators 1, 2, and 3 in this topology, while the cavity resonance is represented by resonator 4. Input and output ports are denoted as source (S) and load (L), respectively. Couplings are shown with arrows.	197
7.18	Scattering parameters of the second designed filter obtained with ANSYS HFSS (design) and comparison to the coupling matrix response (matrix). The bandwidth is $BW_2 = 1.15$ GHz ($FBW_2 = 35.28\%$). The dimensions of this second design are reported in the second column of Table 7.3.	200
7.19	Scattering parameters of the third designed filter obtained with ANSYS HFSS (design) and comparison to the coupling matrix response (matrix). The bandwidth is $BW_3 = 1.57$ GHz ($FBW_3 = 47.01\%$). The dimensions of this third design are reported in the third column of Table 7.3.	201
7.20	Comparison for the out-of-band responses of the two last designed transversal filters from ANSYS HFSS simulations. Dimensions of both filters are collected in Table 7.3.	201
7.21	Manufactured prototype. (a) Components of the filter before assembling, and details of the slots in the Teflon substrate used for alignment. The inset inside shows the step discontinuity needed in the radius of the outer conductor for the I/O coupling adjustment. (b) Top view of the structure after assembling. Inset shows details of the assembling process between the coax center pin and the copper bar.	202
7.22	Comparison between the S-parameters simulated using the full-wave commercial software ANSYS HFSS and measured data from the manufactured prototype shown in Fig. 7.21.	203
7.23	Comparison between the S-parameters simulated using the full-wave commercial software ANSYS HFSS and measured data from the manufactured prototype shown in Fig. 7.21 (out-of-band performance). .	204

7.24	Comparison between the S-parameters simulated using the MEN code, the full-wave commercial software ANSYS HFSS and measured data from the manufactured prototype shown in Fig. 7.21.	206
7.25	Schematic of the microstrip test under study: two stubs placed orthogonally (not connected). (a) Two short-circuited lines. (b) Two open-circuited lines.	210
7.26	Microstrip structure. The substrate has a relative permittivity of $\epsilon_r = 2.94$. The thickness of the copper layer is $t = 35$ μm . H corresponds to the thickness of the substrate.	210
7.27	Layout of the stopband filter under study. The microstrip substrate is shown in Fig. 7.26. The substrate has a thickness of $H = 0.25$ mm. (a) Stubs oriented in the same direction. (b) Stubs that alternate their orientations.	212
7.28	Simulated S-parameters for the two filters shown in Fig. 7.27. . . .	212
7.29	Layout of the stopband under study. The microstrip substrate is shown in Fig. 7.26. The substrate has a thickness of $H = 0.25$ mm. (a) Aligned stubs. (b) Stubs in a bent configuration.	213
7.30	Simulated S-parameters for the two filters shown in Fig. 7.29. . . .	213
7.31	Layout of the first implemented filter with straight main line and all stubs in the same orientation.	215
7.32	Layout of the second implemented filter with straight main line and stubs alternating the orientation. The dimensions are in mm.	215
7.33	Photographs of the prototypes based on straight main lines from input to output. (a) Filter given in Fig. 7.31. (b) Filter given in Fig. 7.32.	215
7.34	Measured S-parameters of the two filters implemented using a straight configuration. The layouts are given in Fig. 7.31 (aligned stubs) and Fig. 7.32 (alternated stubs).	216
7.35	Layout of the third implemented filter using bent configuration. . .	217
7.36	Layout of the fourth implemented filter combining bent configuration and alternated stubs. The dimensions are in mm.	217

7.37	Photographs of the prototypes based on bent configurations. (a) Filter given in Fig. 7.35. (b) Filter given in Fig. 7.36.	217
7.38	Measured S-parameters of the two bent filters given in Fig. 7.35 (aligned stubs) and Fig. 7.36 (alternated stubs).	218
7.39	Measured S-parameters of the two alternated stubs filters given in Fig. 7.32 (straight main line) and Fig. 7.36 (bent configuration). . .	218
7.40	LPKF Protolaser S machine fabricating the filters.	219
7.41	Measurement of the filters using the vector network analyzer (R&S ZVA67).	220
7.42	3680 series Universal Test Fixture (UTF) used as the input/output excitations.	220
7.43	S-parameters of the alternated stubs filter given in Fig. 7.32, using the MEN formulation. They are compared to AWR results (using Analyst and AXIEM solvers) and to ADS.	221
A.1	Rectangular waveguide cross-section. The dimensions are: $a \times b$. . .	236
A.2	Rectangular metallic patch located in the cross-section of a rectangular waveguide. The dimensions of the obstacle are: $a_{obs} \times b_{obs}$, and the dimensions of the rectangular waveguide are: $a \times b$	238
A.3	Rectangular metallic patch attached to the $x = 0$ wall of a rectangular waveguide. The dimensions of the obstacle are: $a_{obs} \times b_{obs}$, and the dimensions of the rectangular waveguide are: $a \times b$	239
A.4	Rectangular metallic patch attached to the $x = a$ wall of a rectangular waveguide. The dimensions of the obstacle are: $a_{obs} \times b_{obs}$, and the dimensions of the rectangular waveguide are: $a \times b$	239
A.5	Junction between two rectangular waveguides. The dimensions of the biggest waveguide are: $a \times b$, while the dimensions of the smallest waveguide are: $a_{ap} \times b_{ap}$	241
A.6	Two-dimensional zero-thickness printed rectangular patch in planar technology, excited by a transverse port. The gray area corresponds to the printed metallization and the dark area to the excitation port, both placed on the transverse ($z=0$) plane.	249

A.7	Two-dimensional zero-thickness printed rectangular patch in planar technology, excited by a transverse port. The gray area corresponds to the printed metallization and the dark area to the excitation port, both placed on the transverse ($z=0$) plane.	250
A.8	Two-dimensional zero-thickness printed circuit in planar technology. The gray area corresponds to the printed metallization and the dark area to the excitation port, both placed on the transverse ($z=0$) plane.	251
A.9	Hairpin microstrip bandpass filter geometry under study. The dimensions are in mm. The dimensions of the shielding box are: $a \times b = 31.2 \text{ mm} \times 30 \text{ mm}$. The filter is centered in the box.	253
A.10	Coupling integral $C_{i,0}$ between the mode 5 of the arbitrary waveguide and the excitation pulse, according to the distance Δx of integration.	253
A.11	Coupling integral $C_{i,0}$ between the mode 10 of the arbitrary waveguide and the excitation pulse, according to the distance Δx of integration.	254
A.12	Detail of the port integration. x_2 is the coordinate where the metallization starts. Δx is the distance taken in the coupling integral of equation (A.37).	255
B.1	Schematic of the shielded microstrip printed circuit under analysis/design, excited with two coaxial connectors.	257
B.2	GUI for the analysis and design of shielded microstrip components through the MEN technique. It implements the MEN formulation of Chapter IV for the analysis of planar components excited by two lateral ports, together with an optimizer module. The fields have been completed with the parameters required in the first optimization step.	257
B.3	Second order coupled line filter under analysis/design. It is implemented in shielded microstrip technology. The structure is excited by two lateral ports (dark areas).	258
B.4	Example of the microstrip circuit format file, based on the definition of the segments that compose the circuit.	261
B.5	Layout of the geometry under analysis, plotted by the GUI. The two excitation ports are also drawn.	262

B.6	S-parameters of the component under analysis obtained using the MEN technique for planar devices.	262
B.7	Map of coordinates given by the optimizer. The dimensions are in mm. It helps the user to visualize the optimization variables. In this case, the geometry is for the first optimization step, where only one resonator is considered.	263
B.8	Optimizer window where the user can define the variables and the dependent variables. For the first step of design, the necessary variables are specified.	264
B.9	Window to create the mask. The mask is defined by imposing limits to the values of the magnitude of the S_{11} and S_{21} parameters in different frequency bands. Finally, different weights can be assigned to the S_{11} and S_{21} parameters in the total error computation.	266
B.10	Window to load the mask. The mask is defined in the specified file. The frequencies of the passband and the weight given to the passband in the total error computation can be also defined in this window. Finally, different weights can be assigned to the magnitude of the S_{11} and S_{21} parameters, in the total error computation.	266
B.11	Format of the file that contains the goal mask. The frequencies have to be introduced in GHz and the goal S_{11} and S_{21} in linear units.	267
B.12	Layout of the geometry under design, plotted by the GUI. The initial values of the variables are also given.	268
B.13	S-parameters of the component given in Fig. B.12, obtained by the MEN technique. The MEN result is compared to the partial matrix response (Goal).	268
B.14	Convergence of the optimization algorithm according to the number of iterations. The picture shows the value of the objective function that, in this tool, is defined as the error between the MEN S-parameters and the goal mask from the coupling matrix. This is the convergence rate for the first optimization step.	269
B.15	Layout of the geometry after the first optimization step, plotted by the GUI. The final values of the variables are also given.	269
B.16	S-parameters of the component given in Fig. B.15, obtained by the MEN technique. The MEN result is compared to the partial matrix response (Goal).	270

B.17	GUI for the analysis and design of shielded microstrip components through the MEN technique. It implements the MEN formulation of Chapter IV for the analysis of planar components excited by two lateral ports, together with an optimizer module. The fields have been completed with the parameters required in the second and third optimization steps.	271
B.18	Map of coordinates given by the optimizer. The dimensions are in mm. It helps the user to visualize the optimization variables. In this case, the geometry is for the second and third optimization steps, where the two resonators are considered.	271
B.19	Optimizer window to define the variables and the dependent variables. For the second step of design, the necessary variables are specified.	272
B.20	Window to load the mask. The mask is defined in the specified file. The frequencies of the passband and the weight given to the passband in the total error computation can be also defined in this window. Finally, different weights can be assigned to the S_{11} and S_{21} parameters.	273
B.21	Layout of the geometry under design, plotted by the GUI. The initial value of the variable is also given.	273
B.22	S-parameters of the component given in Fig. B.21, obtained by the MEN technique. The MEN result is compared to the partial matrix response (Goal).	274
B.23	Convergence of the optimization algorithm according to the number of iterations. The picture shows the value of the objective function that, in this tool, is defined as the error between the MEN S-parameters and the goal mask from the coupling matrix. This is the convergence rate for the second optimization step.	274
B.24	Layout of the geometry after the second optimization step, plotted by the GUI. The final value of the variable is also given.	275
B.25	S-parameters of the component given in Fig. B.24, obtained by the MEN technique. The MEN result is compared to the partial matrix response (Goal).	275
B.26	Optimizer window to define the variables and the dependent variables. For the third step of design, the necessary variables are specified.	276

B.27	Window to load the mask. The mask is defined in the specified file. The frequencies of the passband and the weight given to the passband in the total error computation can be also defined in this window. Finally, different weights can be assigned to the S_{11} and S_{21} parameters.	276
B.28	Layout of the geometry under design, plotted by the GUI. The initial value of the variable is also given.	277
B.29	Convergence of the optimization algorithm according to the number of iterations. The picture shows the value of the objective function that, in this tool, is defined as the error between the MEN S-parameters and the goal mask from the coupling matrix. This is the convergence rate for the third optimization step.	277
B.30	Layout of the geometry after the third optimization step, plotted by the GUI. The final value of the variable is also given.	278
B.31	S-parameters of the component given in Fig. B.30, obtained by the MEN technique. The MEN result is compared to the partial matrix response (Goal). This is the final response of the filter after the optimization process.	278

LIST OF TABLES

Table

2.1	Specific parameters for MEN formulations in Fig. 2.3.	29
2.2	Dimensions of the E-plane waveguide transformer in Fig. 2.6.	32
2.3	Specific parameters for MEN formulations in Fig. 2.6.	32
2.4	Number of accessible modes in each section adjusted by the new method according to the length L and the threshold dB_{th}	42
2.5	Parameters adjusted by the method using $\text{dB}_{th} = 10$ dB and $\text{dB}_{th} = 20$ dB for different lengths L	43
2.6	Convergence parameters adjusted automatically by the method using $\text{dB}_{th} = 10$ dB.	45
2.7	Convergence parameters adjusted automatically by the method using $\text{dB}_{th} = 20$ dB, for the different steps in the filter depicted in Fig. 2.12.	47
2.8	Dimensions of the four-pole dual-mode filter in Fig. 2.14	48
2.9	Convergence parameters adjusted automatically by the method using $\text{dB}_{th} = 20$ dB.	49
2.10	Total computational time for different numerical parameters in the MEN implementation using 250 frequency points.	50
2.11	Convergence parameters adjusted automatically by the method using $\text{dB}_{th} = 20$ dB. In-band analysis.	52
2.12	Convergence parameters adjusted automatically by the method using $\text{dB}_{th} = 20$ dB. Out-of-band analysis.	53

4.1	Numerical MEN parameters required by all the microwave components analyzed in this work.	115
4.2	Computational time required by all the microwave components analyzed in this work.	116
5.1	Value of the parameters associated with losses in the structure of Fig. 5.5.	136
5.2	Tuning of the parameter k defined as equation (5.40).	136
5.3	Tuning of the parameter k defined as equation (5.45).	136
5.4	Value of k according to the first approach [equation (5.40)] and the second approach [equation (5.45)], for the two examples studied in this chapter.	140
6.1	Physical dimensions of the two in-line filters studied in this subsection, as defined in Fig. 6.4.	156
6.2	Number of accessible modes and computation time according to the selected threshold dB_{th}	157
6.3	Physical dimensions of the transversal filter, according to Fig. 6.4.	164
7.1	Physical dimensions of the three proposed filters, according to the sketch shown in Fig. 7.1.	178
7.2	Physical dimensions of the designed third order in-line filter, according to the sketches shown in Fig. 7.14 and Fig. 7.15, with foam as a dielectric material ($\varepsilon_r = 1.07$).	195
7.3	Physical dimensions of the two transversal filters, according to the sketches shown in Fig. 7.14 and Fig. 7.15, with Teflon as dielectric material ($\varepsilon_r = 2.1$).	200
7.4	Comparison in terms of Q_U , FBW , IL and electrical size between the manufactured prototype and other resonant filters reported in the literature.	205
7.5	Coupling effect (measured through the $ S_{21} $) for different values of the dielectric thickness (H) and the different operation frequencies (f).	210

7.6	Ka-Band filter specifications implemented in stub loaded microstrip technology.	214
A.1	TE and TM modes of the rectangular waveguide shown in Fig. A.1.	237
A.2	TE and TM modes of the internal rectangular obstacle located in the waveguide cross-section, as shown in Fig. A.2.	238
A.3	TE and TM modes of a rectangular obstacle attached to the $x = 0$ wall of a rectangular waveguide, as shown in Fig. A.3.	239
A.4	TE and TM modes of a rectangular obstacle attached to the $x = a$ wall of a rectangular waveguide, as shown in Fig. A.4.	240

LIST OF APPENDICES

Appendix

A. Vector Modal Functions and the Solution of the Coupling Integrals . . 235

B. Optimization MEN tool 256

ABSTRACT

Investigations on Numerical Techniques Based on Integral Equation for the Analysis and Design of Microwave Devices in Space Applications

by

Celia Gómez Molina

Among all the numerical methods described in the technical literature and implemented in the electromagnetic (EM) full-wave simulators, this Ph. D. dissertation is focused on the Multimode Equivalent Network (MEN) technique. The main objective of this doctoral thesis is to extend the state of the art in the MEN context. This technique is based on the individual characterization of the discontinuities present in the structure. For each discontinuity, it starts by imposing the boundary conditions to obtain the relevant integral equations (IEs) that model the problem. After solving the IEs, the discontinuity is then characterized through an equivalent network, where the interactions between the modes on both sides of the discontinuity are rigorously account for by an impedance or admittance coupling matrix. Finally, the individual MENs are conveniently combined to perform the analysis of the complete device.

This technique was originally developed for the analysis of waveguide components, where the excitations are the guided modes on both sides of the discontinuities. This formulation is revisited in this document as starting point. Several mechanisms to

increase the computational efficiency of this technique for waveguide analyses are also proposed in this doctoral thesis. Then, in order to extend the functionality of this technique and adapt it to other technologies that are nowadays commonly used in certain applications (such as microstrip or stripline), we develop in this doctoral thesis the MEN formulation for single-layer and multilayer planar components. We start formulating the MEN representation for zero-thickness discontinuities composed of rectangular obstacles, containing lateral ports in the transverse plane. This development is based on an electric MEN approach. The introduced lateral excitations are employed to model the coaxial connectors that normally excite the planar shielded structures. The excitation of the components through these ports is the most important difference with respect to the MEN formulations for waveguide problems. It allows to analyze planar devices where the excitation of the structures is lateral, instead of through guided modes (as in waveguide components). Then, we extend this analysis to zero-thickness discontinuities that contain lateral ports and are formed by arbitrarily shaped metallizations. This is possible thanks to the combination of the magnetic MEN technique with the Boundary Integral-Resonant Mode Expansion (BI-RME) method. This extension allows the analysis of a large variety of microwave planar components using the MEN approach. In this development, internal ports are also considered for inter-connection purposes. Based on this MEN formulation, we detail how to consider ohmic losses in the discontinuity when the magnetic approach is used. Finally, we extend the MEN technique to the analysis of multilayer boxed planar microwave circuits that are built using a thick metallization. This formulation is particularly necessary when the thickness of the metallization becomes electrically large.

All the proposed MEN extensions have been programmed in MATLAB. To validate the formulations, we select certain microwave components as examples and we analyze them running our MATLAB MEN codes. Then, we compare the sim-

ulations to different commercial software results, measurements from prototypes (if they are available) and results from the technical literature. In addition, several microwave filters have been designed in this document. They are implemented in different technologies (rectangular waveguide, thick metallic bars and microstrip) for different applications. These filters have been also simulated using the MEN formulations proposed in this doctoral thesis. All these numerical results show good agreement with respect to other full-wave EM tools and measurements, thereby fully validating the MEN extensions proposed in this thesis. Finally, an optimization module has been added to the MEN codes and allows the design of microstrip components using the MEN formulation.

Keywords

Bandpass filters, convergence, coupled line filters, dual-mode filters, hybrid waveguide microstrip technology, integral equations, Method of Moments (MoM), microstrip, microwave filters, MMICs, multilayer devices, multimode equivalent networks, numerical methods, planar junctions, resonator filters, stopband filters, Taylor series, thick junctions, transmission zeros, transversal filters, waveguide filters, waveguide junctions, wideband filters, Wilkinson divider.

CHAPTER I

Introduction

The development of rapid, low cost designs and manufacturing procedures is a constant concern for modern microwave industry. In this respect, numerical methods *Itoh* (1989); *Jin* (2010) for solving electromagnetic problems are attracting more and more attention in recent years, due to their ability of saving development time and experimental work in real communication projects. The goal of any numerical method is to model a continuous problem in terms of a discrete one with acceptable accuracy *Freni* (2017). Substantial research effort has been devoted to the development of fast and accurate computational electromagnetics (CEM) tools for the analysis and design of the microwave communication components required by the industry. Furthermore, a continuous effort is also dedicated to the reduction of the computational burden needed by these tools.

As a result, a very large variety of numerical methods for the analysis and design of microwave devices has been reported in the technical literature and are still under investigation to improve their computational performance. An extensive review of the most commonly used numerical methods is described in *Itoh* (1989); *Sorrentino* (1988). The numerical techniques can be classified in different ways according, for example, to the solution domain and the field propagator. The first classification described in *Freni* (2017) to solve forward (direct) problems is the following:

- Analytical
- Asymptotic methods (also known as High Frequency methods): as Geometrical Optics (GO), Physical Optics (PO) or Geometrical Theory of Diffraction (GTD) *Algar et al.* (2016).
- Integral Equation (IE) methods: as Method of Moments (MoM) *Harrington* (1993), Boundary Element Method (BEM), Mode Matching (MM) *Schorer and Bornemann* (2015) or Fast Multiple Method (FMM). For these methods, a domain decomposition is always necessary. This can be done by using Synthetic Function eXpansion (SFX) or Characteristic Basis Function (CBF), either entire domain or sub-domain basis functions.
- Differential Equation (DE) methods: as Finite Difference Method (FDM) *Salazar-Palma* (1998), Finite Difference Time Domain (FDTD) *Peterson et al.* (1998), Finite Difference Frequency Domain (FDFD) or Finite Element Method (FEM) *Salazar-Palma* (1998). These techniques require always a mesh truncation based on Radiation Boundary Condition (RBC) or Absorbing Boundary Condition (ABC).
- Hybrid methods *Arndt* (2015): as IE/DE hybrid (for instance, FEM/BEM), Numerical/Analytical Hybrid or others such as IE/MM *Catina et al.* (2005).

Based on the previous classification, microwave problems for space applications can be solved using differential equations, as FEM, integral equations, as MM, or a combination between differential and integral equations, as FEM/BEM. The differential equation models are based on the differential form of Maxwell's equations. As described in *Balanis* (2012), "The differential form of Maxwell's equations ... is used to describe and relate the field vectors, current densities, and charge densities at any point in space at any time." They are able to deal with inhomogeneous, non-linear or time varying media. In relation to the computational effort, some of these

techniques such as FEM and FDTD can perform the analysis of a wide variety of different types of arbitrary microwave circuits structures, at the expense, most of the time, of consuming high computational resources. On the other hand, the integral equation models are based on the integral form of Maxwell's equations. As explained in *Balanis* (2012), "The integral form of Maxwell's equations describes the relations of the field vectors, charge densities, and current densities over an extended region of space. They have limited applications, and they are usually utilized only to solve electromagnetic boundary-value problems that possess complete symmetry (such as rectangular, cylindrical, spherical, etc., symmetries)." These methods normally require the computation of the Green's function. The Green's function represents the impulse response of a system (similar to the transfer function of a system) *Balanis* (2012). If the appropriate Green's function is known, then methods based on integral equations can be more suitable. The main difficulty of computing the Green's function is related to the singularities that normally appear when the observation and source points coincide. From a computational point of view, integral equation techniques can normally analyze a more limited variety of geometries, but with important reduction in computational time for particular problems.

The other possible classification of these techniques, according to *Freni* (2017), is based on the domain where the problem is solved. In this respect, we can solve the problem in the space-time domain or in a transformed one either space-frequency domain or spectral-frequency domain *Chen and Ney* (2007). For the time-domain techniques, the time t is one of the variables and the solution is obtained for just one source but many frequencies, as for example FDTD. However, other methods formulate the problem in the frequency-domain like FEM or MM. In this case, the time harmonic variation $e^{j\omega t}$ is assumed and the solution is obtained for many sources but single frequency. To solve the problem in the frequency-domain can be more suitable for lossy and resonant problems, periodic structures, among others. On

the other hand, in case of non-linear problems, time varying problems or broadband problems with single excitation is better to use a time-domain technique.

According to these categories, when a microwave engineer has to solve an electromagnetic problem, it is necessary to select the solution domain (time or frequency domain) and the field propagator (integral equation model or differential equation model).

However, with the development of the microwave industry and the computational hardware advances, microwave engineers normally use any of the available commercial software tools for the design of components. These tools, such as ANSYS High-Frequency Structure Simulator (*HFSS*) (2020), CST Studio Suite (*CST*) (2020), Advanced Design System (*ADS*) (2020), Microwave Office (*AWR*) (2020) or FEST3D (*FEST3D*) (2020), for instance, implement some of the aforementioned techniques. The simulation results using these tools usually show good agreement with measurements, and also exhibit good computational efficiency. However, as already discussed with the numerical techniques, some of these tools are more suitable than others, depending on the structure under analysis. For example, to analyze 3D arbitrary structures a good choice would be a more generic software full-wave tool like ANSYS HFSS or CST. Other software tools seem to be more efficient if the problem can be decomposed into defined canonical waveguide structures, as FEST3D. Finally, other CEM tools, as ADS or AWR, are recommended if the structure under analysis is a multilayer planar device. On this basis, depending on the structure under analysis, the engineer should determine which numerical technique (and software tool) is more suitable to perform an accurate and efficient analysis. For example, in *Algar et al.* (2016), the authors studied the performance of different techniques applied to the problem that they would like to solve: the analysis of the electromagnetic scattering by wind turbines. As Prof. Itoh affirmed in *Itoh* (1985): “When a specific structure is analyzed, one has to make a choice of the method best suited for the structure. Ob-

vously, the choice is not unique. Therefore, the user must make a critical assessment for each candidate method.”

In this doctoral thesis, we focus on studying waveguide and planar devices for space communication applications. In this respect, we are dealing with linear problems consisted on standard structures with homogeneous media. According to the previous classification of numerical methods, for lossy and resonant problems, the best option is a frequency-domain technique. In this case, both differential and integral equation techniques can be suitable. However, since the media are homogeneous and the particular Green’s functions of these problems are known, we choose to work with integral equations methods *Melcon et al.* (1999). Among all the possible integral equation techniques, the Multimode Equivalent Network (MEN) formulation *Guglielmi et al.* (1999) has proved to be an accurate and efficient technique for analyzing two-dimensional discontinuities between waveguides. This technique is based on IEs and is appropriate when the microwave structure is known a priori, since it particularizes the formulation to the circuit under analysis. This method begins with the individual characterization of the different discontinuities that compose the structure under study. For each discontinuity, the formulation starts by imposing the corresponding boundary conditions for the tangential electromagnetic fields to obtain the relevant integral equations. A circuit interpretation of the solutions of the integral equations leads to an equivalent network, where all modal interactions are rigorously accounted for through a generalized impedance or admittance coupling matrix.

If the technique starts by imposing the boundary conditions on the tangential components of magnetic field in the aperture part of the discontinuity, then we call magnetic MEN formulation, and the equivalent network is written in terms of an impedance coupling matrix. However, if we start by imposing the boundary conditions on the tangential components of electric field in the metallization part of the discontinuity, then we call electric MEN formulation and the resulting equivalent net-

work is an admittance coupling matrix. Once all the discontinuities present in the structure are characterized through their equivalent networks, the MENs are properly combined to perform the analysis of the complete device.

The advantage of this formulation compared to other integral equation techniques *Melcon et al.* (1999) is that each discontinuity is analyzed independently from the others. In the direct EFIE implementations *Melcon et al.* (1999); *Rautio and Harrington* (1987), all the unknowns of the problem are included in a single integral equation. This can lead to matrices of very large sizes when several discontinuities are analyzed. The MEN technique, on the other hand, formulates a distinct integral equation to characterize each discontinuity with a different multimode coupling matrix. All discontinuities are then coupled together by cascading the different multimode impedance coupling matrices. The result is that, instead of one large global problem, this technique solves a given number of smaller problems that are then properly combined at network level.

This formulation was originally developed for the analysis of zero-thickness obstacles and thick junctions in waveguide problems, as reported for instance in *Alvarez and Guglielmi* (1995); *Guglielmi et al.* (1999). An example of the equivalent network of a microwave filter based on thick inductive irises in a rectangular waveguide is shown in Fig. 1.1.

State of the art

Before this multimode formalism, single-mode equivalent networks were used in *Marcuvitz* (1986) to analyze microwave structures. This technique was very useful for the analysis of certain type of communication devices. However, if the analyzed component contains discontinuities that are placed close to each other, then a single-mode representation is not accurate for the analysis, even when just one mode is propagating inside the waveguide. The reason is that, for close discontinuities, lower order

Figure 1.1: Example of a waveguide component and its equivalent network *Guglielmi et al. (1999)*.

evanescent modes can reach the following discontinuity with sufficient energy. For a proper characterization of the device, these evanescent modes have to be considered in the equivalent network.

Due to this reason, in 1970, Prof. Palocz and Prof. Oliner started the multimode formalism in *Palocz and Oliner (1970)* for planar gratings. As they said in this first contribution: “This procedure [multimode equivalent network] generalizes the well-known single-mode method by taking multimode propagation into account.” After this publication, this technique was described by Prof. Dr. Guglielmi and his collaborators for several types of waveguide discontinuities. First, the most classical types of discontinuity were analyzed. For example, in *Guglielmi and Newport (1990)* the authors described the MEN analysis for inductive coupling windows. In *Guglielmi and Gheri (1994)*, the authors extended this formulation to capacitive steps. Its usefulness and efficiency in the analysis of two-dimensional discontinuities are shown, for instance, in *Gerini and Guglielmi (1998)*; *Melcon and Guglielmi (1994)*; *Smith et al. (1993)*. Based on this previous work, other types of components like inductive and capacitive multiple posts devices *Guglielmi and Gheri (1995)*, Y-junction of rectangular waveguides *Gimeno and Guglielmi (1997)*, leaky wave antennas *Guglielmi*

and Oliner (1987) and H and E-plane uniform bends in rectangular waveguide *Gi-meno and Guglielmi* (1995) were modeled using the multimode formalism. The type of discontinuities analyzed in all these works are composed of rectangular waveguides, where the entire domain basis functions, used in the Method of Moments, are the waveguide modes, and they are known analytically. Around 1984, Prof. Conciauro and his group developed the Boundary Integral-Resonant Mode Expansion (BI-RME) method *Conciauro et al.* (1984). This method was originally developed for the numerical calculation of the propagating modes and their cutoff frequencies in strongly perturbed rectangular or circular metallic waveguides *Arcioni et al.* (2014). In this context, once the rectangular discontinuities were rigorously analyzed using the MEN technique, some authors in *Cogollos et al.* (2003) used the BI-RME method to extend the MEN technique to the analysis of arbitrary junctions. Thanks to this combination, waveguide components such as cross-shaped iris with rounded corners and dual-mode filters with elliptical waveguide resonators were analyzed by means of the MEN technique. It is important to mention that, in these waveguide components, the discontinuities under analysis are considered electrically thick. This is the reason why the equivalent network that models the discontinuity is in series configuration. However, in the waveguide context, some electrically thin discontinuities (approximated as zero-thickness discontinuities) were also analyzed. For example, in *Guglielmi and Oliner* (1989), the authors derived the electric and magnetic MEN formulations applied to the analysis of zero-thickness metal-strip gratings seen as obstacles or apertures, respectively. As observed in *Guglielmi and Oliner* (1989), when the discontinuity under analysis is considered zero-thickness, the equivalent network remains in a parallel configuration. This formulation allows the analysis of useful zero-thickness waveguide discontinuities, and Frequency Selective Surfaces (FSSs), as treated in *Monni et al.* (2007).

At this point, it is important to note that in all these original formulations, the

structures under study are excited by the guided modes in the input and output waveguide sections. Therefore, in the work done from the first publication in 1970 until the recent publications in 2017, the MEN technique has been employed for the efficient and accurate analysis of all the described waveguide problems.

Objectives of this thesis

The main objective of this thesis is to extend the state of the art in the Multimode Equivalent Network context. For this purpose, we first focus on the traditional MEN formulations applied to waveguide problems. Since this formulation is already particularized to different waveguide structures in the technical literature, the first goal, in the context of waveguide devices, is to improve the performance of these traditional MEN formulations. In the original formulation described for instance in *Guglielmi et al.* (1999); *Melcon and Guglielmi* (1994), some static admittances were used in the equivalent network, together with a Taylor expansion *Guglielmi et al.* (1999), to obtain a frequency independent kernel for the relevant integral equation. In this document, we present a new and improved formulation for the MEN representation of arbitrary waveguide junctions. We reformulate the MEN without using static admittances. Then, the Kummer's transformation *Fructos et al.* (2008) is used to separate the kernel into dynamic and static parts, by introducing higher order extraction terms. The main difference with respect to the old formulation is that the approximation of the kernel is more accurate and the numerical computations are more efficient. After that, we focus on the numerical convergence of this technique when it is applied for analyzing waveguide components. In the implementation of any numerical technique, there are always a set of parameters that must be properly adjusted in order to obtain, at the same time, computational efficiency and numerical accuracy of the results. In this context, therefore, we propose a more intuitive and efficient strategy for choosing these parameters in the MEN formalism. Following this approach, setting only one

global numerical variable is sufficient to adjust automatically the specific convergence parameters of each discontinuity to provide a specific level of numerical accuracy of the results. As a consequence, the computational efficiency is significantly increased. In addition, the user experience is also improved, since this approach eliminates all lengthy convergence tests previously needed to assure enough numerical accuracy.

After the work done to improve the performance of this analysis method for waveguide devices, we focus on the second goal of this doctoral thesis. Since planar implementations are nowadays more often used for certain applications, the second goal of this doctoral thesis is to extend the MEN formulation to the analysis of shielded planar devices.

As previously mentioned, some zero-thickness approaches based on both magnetic field and electric field MENs have also been introduced in the past *Guglielmi and Oliner* (1989). A common feature of these previous implementations is that the excitations of the structure are the modes propagating in the input/output (I/O) waveguide sections used as ports. Thus, the original formulations do not allow the analysis of planar devices, since they do not include the external (or even internal) ports in the transverse plane, where the metallic circuit is placed, that are needed for the excitation of printed microstrip devices. These external transverse ports would represent the real I/O coaxial connectors used to excite planar structures. On the other hand, the internal ports can be used to model connections to external components or lumped elements.

To accomplish this goal, we first derive the electric MEN formulation for the analysis of zero-thickness planar discontinuities that include arbitrary rectangular printed metallizations together with internal and external ports in the transverse plane. This allows the efficient analysis of single-layer and multilayer planar circuits composed of rectangular printed metallizations by using an electric MEN formulation. Following the same principle, when a multilayer circuit is analyzed, this technique

begins with the individual MEN characterization of the different planar circuits (or *discontinuities*) that compose the structure. Then, the various MENs are properly cascaded to obtain the final network that characterizes the complete device. This electric MEN formulation is limited to areas with a rectangular shape, where entire domain basis functions are known analytically.

In order to increase the functionality of the MEN technique for planar devices, we then propose an extension for the analysis of zero-thickness discontinuities formed by metallic areas of arbitrary shape and include both internal and external ports in the transverse plane. For this analysis, the BI-RME method *Cogollos et al.* (2003); *Conciauro et al.* (1984) is used to numerically obtain the basis functions needed to solve the integral equations with complex (arbitrarily shaped) planar geometries. This formulation allows the efficient characterization of single-layer and multilayer planar circuits composed of arbitrary printed metallizations using a magnetic MEN formulation. In the multilayer case, the proposed MEN approach is used to model each metal interface in the multilayer stack with the appropriate equivalent network. As before, the MENs are then conveniently cascaded to perform the analysis of the complete device.

This magnetic planar MEN model is derived including the losses produced by lossy dielectrics placed in the intermediate layers between metallizations and the non-perfect conductor used for shielding the circuit. In most practical cases, the loss parameters have to be tuned, since the models to consider losses are just an approximation. On this basis, we demonstrate that, if some losses (as those related to the metallic circuit) are not considered, their small contribution to the total losses can be very effectively accounted for by the losses in the dielectric(s), and the box. Following this approach, the modification falls on the equivalent transmission lines that represent the guided modes, while the equivalent networks that represent the discontinuities remain unchanged. This strategy allows to efficiently consider losses in this

analysis without modifying the MEN formulation. To go one step further, we also describe in this document the magnetic MEN formulation for lossy zero-thickness discontinuities. For this formulation, the losses related to the metallic circuits are considered from the beginning in the boundary conditions using an equivalent problem. We also demonstrate that the result of this development can be easily applied to the previously derived magnetic planar MEN formulation.

It is important to recall that all mentioned planar formulations are strictly valid only for zero-thickness metallizations (or in practice, if the thickness of the metallizations is electrically very small), since they assume that the metallizations have a negligible thickness. This is really useful when we analyze typically shielded multilayer microstrip circuits of complex geometry. However, when the metallizations have a non negligible thickness, the zero-thickness formulation is not longer valid and an alternative strategy based on thick discontinuities must be followed. This formulation become necessary, for example, when planar circuits are analyzed at high frequencies. In this case, the thickness becomes electrically important and therefore it has to be considered for an accurate analysis. Moreover, a new type of wideband filters *Gomez-Molina et al.* (2020d) are implemented using bulky metallic bar resonators. In this case, the thickness of the resonators is also electrically large even at microwave frequencies. To overcome this limitation, we also describe in this document how to apply the Multimode Equivalent Network technique to the analysis of boxed devices that are excited with lateral ports and are composed of metallizations with non negligible thickness. The MEN analysis is carried out considering the thick metallic circuit as a length section of uniform waveguide. The problem is then divided into two waveguide steps (or junctions). We first obtain the MEN of the two junctions. Then, we cascade them to obtain the MEN of the complete circuit, thereby including rigorously the effect of finite thickness. A key aspect of the proposed MEN formulation is that it includes ports in the transverse plane of the circuit.

Thanks to the extensions presented in this thesis, planar devices made of arbitrarily shaped printed or bulky metallizations can be analyzed using the MEN formulation. In addition to theory, all the described derivations have been implemented in MATLAB (*MATLAB*) (2020), leading to an accurate and computationally efficient MEN tool for these type of devices. These computer codes have been used to validate the formulations proposed in this thesis. In each chapter, after describing the MEN formulation under study, we normally implement, in our tool, some microwave devices that have previously been proposed in the technical literature. Then, we implement the same devices using other commercial software tools (like ANSYS HFSS, ADS or FEST3D). To validate the theory, we compare the MEN results with respect to other software's results and measurements from the manufactured prototypes (if they are available). These comparisons have clearly demonstrated the usefulness and efficiency of the novel MEN extensions in the analysis of practical communication components.

After the validation of the proposed MEN formulations, a graphical user interface (GUI) has been developed to ease the application of the magnetic MEN approach for planar devices. This tool was originally created for the analysis of microstrip components. Furthermore, with the inclusion of an optimizer, this tool can be used not only for analyzing but also for designing new microwave components. This GUI is also described in this document.

Finally, during this research, some microwave filters *Cameron et al.* (2007) have been designed for different applications. In particular, we have designed compact X/Ku-band filters using waveguide technology *Marcuvitz* (1986); *Vallerotonda et al.* (2019) for satellite communication applications. Also, microwave filters for wideband C-band applications have been designed using a novel thick metallic bars technology *Gomez-Molina et al.* (2020d); *Zhu et al.* (2018). Finally, based on microstrip technology *Hong* (2011), stopband filters with high rejection level have been designed

at Ka-band frequencies for mobile communication applications. Some of these designs have been manufactured and measured. The measurements of the prototypes are in good agreement with respect to the full-wave simulations, which validates the proposed designs.

In addition, to show the usefulness of the described MEN extensions, these filters (designed using different technologies and for different operational bands) are analyzed using the theory described in this doctoral thesis. Good agreement between the MEN results and measurements are also obtained for these comparisons, thus fully validating the usefulness of the MEN extensions proposed in this document.

Ph. D. dissertation structure

This document is organized as follows:

- Chapter I:

In this first chapter, we have summarized the most used numerical techniques and carried out an extensive revision of the state of the art. Then, we have defined the main goals of this doctoral thesis and the organization of this document.

- Chapter II:

In this chapter, we review the Multimode Equivalent Formulation applied to the analysis of waveguide components. We reformulate it without using the asymptotic admittances that were traditionally extracted in the network. Then, to improve the performance of this technique, we apply Kummer's transformation to the kernel of the integral equations. Thanks to this new version of the formulation, we propose a mechanism to assess numerical convergence with the minimum computational effort. Some waveguide cavity filters and transformers are analyzed in terms of performance to validate these strategies.

The articles related to this chapter that have been published in technical journals or international conferences are: *Gomez-Molina et al.* (2017) and *Gomez-Molina et al.* (2018b).

- Chapter III:

In this third chapter, we extend for the first time the MEN formulation to the analysis of single-layer and multilayer boxed planar devices composed of printed rectangular metallizations. This analysis is based on the electric MEN analysis of zero-thickness discontinuities that include excitation ports in the transverse plane. Some coupled line microstrip filters are analyzed to validate this theory.

The articles related to this chapter that have been published in technical journals or international conferences are: *Gomez-Molina et al.* (2018a) and *Gomez-Molina et al.* (2019).

- Chapter IV:

In this chapter, we describe the magnetic MEN analysis of zero-thickness discontinuities that include transverse ports and are composed of arbitrarily shaped metallizations. This is possible thanks to the combination between the MEN technique and the BI-RME method. This formulation allows the efficient and accurate characterization of single-layer and multilayer boxed planar devices that are excited laterally through coaxial ports. In this case, we analyze some devices like Wilkinson dividers, hairpin filters or elliptic resonators, as validation.

The article related to this chapter that has been published in a technical journal is: *Gomez-Molina et al.* (2020c).

- Chapter V:

In this fifth chapter, we reformulate the zero-thickness MEN technique considering losses in the discontinuity. This is done by means of an equivalent problem, where the losses are considered to be located in the aperture. Thus, the losses are taken into account from the beginning of the problem, when the boundary conditions are imposed. Some examples from other chapters are resimulated using losses at the interface, showing good agreement with respect to measurements from manufactured prototypes and to other full-wave software tools.

- Chapter VI:

In this chapter, the MEN formulation is particularized to the analysis of boxed devices that are excited through lateral ports and are made of thick metallizations. Since, in some analyzed devices, the thickness of the metallic patches is electrically large, the zero-thickness formulation described in Chapter IV cannot be applied. Several filters that are implemented using thick bar resonators are compared to measurements to validate this particular formulation.

The article related to this chapter that has been published in a technical journal is: *Gomez-Molina et al. (2020b)*.

- Chapter VII:

In this last chapter, we present some filters that have been designed at X, C, Ku and Ka-bands using different technologies (waveguide, bulky metallic bars and microstrip). Some of them have been manufactured, showing good agreement with respect to full-wave simulations. In addition, they have been simulated using the MEN computational code to confirm the usefulness of this numerical method in the analysis of real microwave components.

The articles related to this chapter that have been published in technical journals

or international conferences are: *Gomez-Molina et al.* (2020d) and *Gomez-Molina et al.* (2020a).

*** Appendix A:**

In this appendix, we detail the mathematical expression of the magnetic and the electric vector modal functions of rectangular waveguides and rectangular metallic patches. We also focus on how to solve the integral equations that normally appears in MEN problems using the Method of Moments. If the discontinuity under analysis is defined between two rectangular waveguides, we solve analytically the coupling integrals that are needed to compute the kernel of the integral equations. These theoretical derivations are described in this appendix. On the contrary, if we characterize the discontinuity between a rectangular waveguide and an arbitrary waveguide, we compute numerically the coupling integrals using the BI-RME method. The coupling integrals between arbitrary waveguides or metallic patches and the excitation pulse are also discussed in this appendix.

*** Appendix B:**

In this appendix, we describe the MATLAB tool that has been created for the analysis and design of microstrip devices using the magnetic MEN formulation described in Chapter IV. We explain the parameters that have to be fixed in this tool and also the system of coordinates that the optimizer uses to select the optimization variables. As a validation, a coupled line microstrip filter is designed using this MEN tool.

CHAPTER II

Multimode Equivalent Network Technique for Waveguide Devices

2.1 Introduction

In computational electromagnetic research area, a great interest is devoted to the reduction of the necessary computational burden of the numerical methods that are used for solving electromagnetic problems. The numerical techniques are expected to reach accurate results consuming, at the same time, the minimum computational time in the analysis. As already mentioned, the CEM tools support the microwave engineers in the design process. During the optimization stage, required always in design tasks, the numerical methods implemented in these software tools have to analyze the structure under design several times. This shows the great importance of keeping to the minimum the computational effort required by the tool for just one single analysis.

In addition, the analysis methods that are nowadays used in the microwave industry are implemented numerically. As a consequence, there is always a set of numerical parameters that needs to be properly adjusted in order to reach convergent results for the components under analysis. If the parameters are undersized, the response may not be convergent, that is, not accurate. On the other hand, if we increase the para-

meters more than what it is required, the problem may be oversized. This, in turn, may lead to an unnecessary increment of the computational effort, to ill conditioned matrices, or even to relative convergence problems *Itoh* (1989). It is clear, therefore, that a careful selection of these numerical parameters is needed to find a compromise between accuracy and efficiency. Currently, the correct convergence of the numerical methods that are integrated in commercial software tools continues to be a critical issue. From the user's point of view, this implies the responsibility of selecting the proper parameters, without the complete knowledge (in some cases) of the numerical methods being implemented. This process usually requires repeated simulations of the same structure with increased numerical parameters, until convergence is reached. The whole process can then be very time consuming.

In this chapter, we focus on the enhancement of the numerical efficiency of the Multimode Equivalent Network (MEN) formulation *Guglielmi et al.* (1999) for the analysis of waveguide components. For this purpose, we focus on improving the efficiency of the network formulation and then, on how to properly select the corresponding numerical parameters. In particular, we first reformulate the MEN technique without using asymptotic admittances. Then, we apply Kummer's transformation to compute the heavy part of the kernel independent from the frequency. Finally, we propose a new mechanism to assess convergence for MEN formulation of waveguide discontinuities.

In the original formulation reported for instance in *Guglielmi et al.* (1999) and *Melcon and Guglielmi* (1994), static impedances are used in the equivalent network, together with a Taylor expansion, to obtain a frequency independent kernel for the relevant integral equation. The resulting formulation is indeed efficient, however, it has some limitations and introduces some problems. The main limitation is that the convergence of the Taylor expansion is linked to the number of accessible modes explicitly included in the computations. As a result, the total size of the multi-mode coupling matrix is unnecessarily increased. This affects negatively the overall

efficiency, since a bigger matrix needs to be inverted for each point in frequency.

To overcome these limitations, the MEN is reformulated in Section 2.2 without using static admittances. We then extract the frequency dependence from the kernel by using the Kummer's transformation in Section 2.3. Following this approach, the kernel is separated into static and dynamic parts. The static terms are equivalent to the static terms used in the original formulation. However, the presence of the fast converging dynamic terms assures a kernel computation whose convergence is not dependent on the number of accessible modes explicitly included. In this way, the number of accessible modes can be fixed considering only the presence of nearby discontinuities, thereby keeping the size of the MEN matrix to the strictly required minimum. Two practical examples are discussed to validate this new approach.

A clear advantage of this new formulation is that the influence of the accessible modes is separated from the issue of the convergence of the kernel. As a consequence, all parameters affecting the efficiency of the computations and the accuracy of the results are made independent from each other. To further exploit this advantage, we propose a new strategy to select the numerical parameters that are needed to ensure convergence in the MEN formulation. We first discuss, in Section 2.4, the novel strategy to select the numerical parameters for the MEN formulation and then we discuss a number of practical application examples clearly showing the obtained improvements. These examples fully validate the proposed strategy. Finally, the main conclusions of this chapter are outlined in the Section 2.5.

2.2 MEN Formulation without Asymptotic Admittances

In this section, we derive the general MEN formulation for the two-dimensional thick junction shown in Fig. 2.1, without using the asymptotic admittances originally extracted in *Guglielmi et al. (1999)*. It is important to note that, from now, we use text in bold to indicate vectors and text in bold with bars to indicate matrices.

Figure 2.1: Two-dimensional step in waveguide technology.

To start with the formulation, we impose the continuity of the tangential component of the magnetic field $\mathbf{H}_t^{(\delta)}$ in the aperture at $z = 0$

$$\mathbf{H}_t^{(1)}(s) - \mathbf{H}_t^{(2)}(s) = 0, \quad (2.1)$$

where δ refers to the medium so that $\delta = 1$ or $\delta = 2$ for $z < 0$ or $z > 0$, respectively. In equation (2.1), s indicates a point in the waveguide cross-section at the discontinuity $s = (x, y)$ (see Fig. 2.1).

Now we use the modal expansion formalism described in *Guglielmi et al. (1999)*; *Marcuvitz (1986)*. Using this expansion, the tangential components of the magnetic field are written as an infinite modal summation. Then, equation (2.1) can be rewritten as follows

$$\sum_{m=1}^{+\infty} I_m^{(1)} \mathbf{h}_m^{(1)}(s) - \sum_{m=1}^{+\infty} I_m^{(2)} \mathbf{h}_m^{(2)}(s) = 0, \quad (2.2)$$

where $\mathbf{h}_m^{(\delta)}(s)$ is the magnetic vector modal function of mode m in the homogeneous medium (δ) . These functions are treated in detail in Appendix A. In equation (2.2), $I_m^{(\delta)}$ represents the total modal currents.

The next step is to separate the modes which contribute to the storage of energy (*localized modes*) from the modes that contribute to the energy exchange between cascaded discontinuities (*accessible modes*) *Guglielmi et al. (1999)*. This is done by

splitting the infinite modal series into two terms, thus obtaining

$$\begin{aligned}
& \sum_{n=1}^{N(1)} I_n^{(1)} \mathbf{h}_n^{(1)}(s) - \sum_{n=1}^{N(2)} I_n^{(2)} \mathbf{h}_n^{(2)}(s) \\
&= \sum_{m=N(1)+1}^{+\infty} Y_m^{(1)} V_m^{(1)} \mathbf{h}_m^{(1)}(s) + \sum_{m=N(2)+1}^{+\infty} Y_m^{(2)} V_m^{(2)} \mathbf{h}_m^{(2)}(s),
\end{aligned} \tag{2.3}$$

where $N(\delta)$ is the number of accessible modes in each region. In equation (2.3), $V_m^{(\delta)}$ and $Y_m^{(\delta)}$ represent the total modal voltage and the characteristic modal admittance, respectively. Note that, in equation (2.3), the localized modes have been loaded by their corresponding characteristic modal admittances. For the sake of clarity, the subscript on the left hand side in equation (2.3) has been changed to n . The characteristic modal admittance is defined in the usual form as

$$Y_m^{(\delta)} = \begin{cases} \frac{\sqrt{k^2 - k_{t,m}^{(\delta)}}}{w \mu_0 \mu_r^{(\delta)}} & TE \\ \frac{w \epsilon_0 \epsilon_r^{(\delta)}}{\sqrt{k^2 - k_{t,m}^{(\delta)}}} & TM \end{cases}. \tag{2.4}$$

In equation (2.4), k is the wavenumber of the medium, w is the angular frequency, ϵ_0 and μ_0 are the permittivity and permeability of free space, $\epsilon_r^{(\delta)}$ and $\mu_r^{(\delta)}$ are the permittivity and permeability of the medium (δ) (see Fig. 2.1) and $k_{t,m}^{(\delta)}$ is the cut-off wavenumber of the mode m in medium (δ).

To proceed, we recall the relation between the modal voltage $V_m^{(\delta)}$ and the electric field in the aperture $[\mathbf{z}_0 \times \mathbf{E}]$, and write

$$V_m^{(\delta)} = \int_{ap} [\mathbf{z}_0 \times \mathbf{E}(s')] \cdot \mathbf{h}_m^{(\delta)*}(s') ds', \tag{2.5}$$

where ap is the aperture at the waveguide junction (see Fig. 2.1) and $\mathbf{z}_0$ is the unit vector in the direction of the z -axis. It is important to note that, since s' is an abstract variable that represents the x' and y' coordinates of a plane, the integral in equation

(2.5) is bidimensional (surface integral).

Due to the linearity of the problem, the unknown electric field in the aperture $[\mathbf{z}_0 \times \mathbf{E}]$ must be linearly proportional to the excitations, so that we can write

$$[\mathbf{z}_0 \times \mathbf{E}(s')] = \sum_{n=1}^{N(1)} I_n^{(1)} \mathbf{M}_n^{(1)}(s') - \sum_{n=1}^{N(2)} I_n^{(2)} \mathbf{M}_n^{(2)}(s'), \quad (2.6)$$

where $\mathbf{M}_n^{(\gamma)}$ are unknown partial magnetic-current expansion functions. In this case, $\gamma = 1$ or $\gamma = 2$ for $z < 0$ or $z > 0$, respectively. Substituting now equation (2.6) into equation (2.5), and using the resulting equation in equation (2.3), we can finally obtain the fundamental integral equation of our problem

$$\mathbf{h}_n^{(\gamma)}(s) = \int_{ap} \overline{\mathbf{K}}(s, s') \cdot \mathbf{M}_n^{(\gamma)}(s') ds', \quad (2.7)$$

where $\overline{\mathbf{K}}(s, s')$ is the kernel of the integral equation

$$\overline{\mathbf{K}}(s, s') = \sum_{\delta=1}^2 \sum_{m=N(\delta)+1}^{+\infty} Y_m^{(\delta)} \mathbf{h}_m^{(\delta)}(s) \mathbf{h}_m^{(\delta)*}(s'). \quad (2.8)$$

To complete the rigorous network formulation, equation (2.6) is used in equation (2.5) to write the modal voltages in terms of the modal currents

$$V_m^{(\delta)} = \sum_{n=1}^{N(1)} I_n^{(1)} Z_{m,n}^{(\delta,1)} + \sum_{n=1}^{N(2)} I_n^{(2)} Z_{m,n}^{(\delta,2)}, \quad (2.9)$$

where $Z_{m,n}^{(\delta,\gamma)}$ are the elements of the impedance coupling matrix $\overline{\mathbf{Z}}$

$$Z_{m,n}^{(\delta,\gamma)} = \int_{ap} \mathbf{M}_n^{(\gamma)}(s') \cdot \mathbf{h}_m^{(\delta)*}(s') ds'. \quad (2.10)$$

Then, the resulting impedance coupling matrix $\overline{\mathbf{Z}}$ is of size $(2N \times 2N)$ and is

computed as

$$\bar{\mathbf{Z}} = \begin{bmatrix} Z_{1,1}^{(1,1)} & Z_{1,2}^{(1,1)} & \cdots & Z_{1,N}^{(1,1)} & | & Z_{1,1}^{(1,2)} & Z_{1,2}^{(1,2)} & \cdots & Z_{1,N}^{(1,2)} \\ \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \ddots & \vdots \\ Z_{N,1}^{(1,1)} & Z_{N,2}^{(1,1)} & \cdots & Z_{N,N}^{(1,1)} & | & Z_{N,1}^{(1,2)} & Z_{N,2}^{(1,2)} & \cdots & Z_{N,N}^{(1,2)} \\ - & - & - & - & + & - & - & - & - \\ Z_{1,1}^{(2,1)} & Z_{1,2}^{(2,1)} & \cdots & Z_{1,N}^{(2,1)} & | & Z_{1,1}^{(2,2)} & Z_{1,2}^{(2,2)} & \cdots & Z_{1,N}^{(2,2)} \\ \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \ddots & \vdots \\ Z_{N,1}^{(2,1)} & Z_{N,2}^{(2,1)} & \cdots & Z_{N,N}^{(2,1)} & | & Z_{N,1}^{(2,2)} & Z_{N,2}^{(2,2)} & \cdots & Z_{N,N}^{(2,2)} \end{bmatrix}. \quad (2.11)$$

In terms of network representation, these equations can be interpreted in the forms shown in Fig. 2.2.

Figure 2.2: MEN representation for the waveguide step in Fig. 2.1. (a) Explicitly representation of the localized modes. (b) Representation of the MEN drawing only the accessible modes (most common formalism).

As observed, since we are analyzing a thick discontinuity using a magnetic formulation, the equivalent network is written in terms of impedances and remains in a series configuration. The matrix $\bar{\mathbf{Z}}$ in equation (2.11) rigorously account for the interaction between the modes on both side of the discontinuity. These modes are represented in Fig. 2.2 by their corresponding transmission line model *Pozar* (2011). The localized modes, that are loaded with their corresponding characteristic modal admittances in equation (2.3), can be explicitly depicted in the network as shown in Fig. 2.2(a) or not represented as in Fig. 2.2(b).

The Method of Moments (MoM) and the Galerkin procedure can now be used to solve the integral equation, as reported in *Guglielmi et al.* (1999). In particular, the unknowns of the problem are expanded as follows

$$\mathbf{M}_n^{(\gamma)}(s') = \sum_{k=1}^{N_b} \alpha_{n,k}^{(\gamma)} \mathbf{h}_k(s'), \quad (2.12)$$

where we use as basis ($\mathbf{h}_k$) and test ($\mathbf{h}_i$) functions the magnetic vector modal functions of the small waveguide, in this case, in region (2) [$\mathbf{h}_k = \mathbf{h}_i = \mathbf{h}_m^{(2)}$]. In equation (2.12), N_b represents the number of basis and test functions considered in the MoM-Galerkin procedure. Applying the test procedure, the integral equation (2.7) is finally transformed into the following system of linear equations

$$C_{i,n}^{(\gamma)} = \sum_{k=1}^{N_b} \alpha_{n,k}^{(\gamma)} \sum_{\delta=1}^2 \sum_{m=N+1}^{+\infty} Y_m^{(\delta)} C_{k,m}^{(\delta)} C_{i,m}^{(\delta)}. \quad (2.13)$$

The coefficients $C_{k,m}^{(\delta)}$ and $C_{i,m}^{(\delta)}$ represent the coupling integrals between the localized modes of the waveguide in the medium (δ), and the small waveguide modes used

as basis and test functions, respectively.

$$C_{k,m}^{(\delta)} = \int_{ap} \mathbf{h}_k(s') \cdot \mathbf{h}_m^{(\delta)*}(s') ds' \quad (2.14)$$

$$C_{i,m}^{(\delta)} = \int_{ap} \mathbf{h}_i(s) \cdot \mathbf{h}_m^{(\delta)}(s) ds \quad (2.15)$$

The terms $C_{i,n}^{(\gamma)}$ are the coupling integrals between the accessible modes of the waveguide in the medium (γ) and the small waveguide modes used as test functions.

$$C_{i,n}^{(\gamma)} = \int_{ap} \mathbf{h}_i(s) \cdot \mathbf{h}_n^{(\gamma)}(s) ds \quad (2.16)$$

It is important to mention that, when the junction is located between two waveguides with rectangular shape, the coupling integrals in equations (2.14), (2.15) and (2.16) can be analytically computed as explained in Appendix A. On the contrary, if they are arbitrarily shaped, these coupling integrals can conveniently be computed numerically using the BI-RME method *Cogollos et al.* (2003); *Conciauro et al.* (1984).

Note that the original MEN formulations reported in the technical literature, like for example *Guglielmi et al.* (1999), use the asymptotic admittances in the formulation and that results in a more complex MEN. Furthermore, since the resulting kernel is dependent on frequency, a Taylor expansion is used to transform the kernel into a number of frequency independent terms. To ensure the convergence of the Taylor expansions, however, a sufficiently large number of accessible modes must be included explicitly in the MEN. This, in turn, leads to a bigger size of the MEN impedance matrix. On the contrary, the formulation presented in this section does not use these asymptotic admittances. This can be appreciated in the equivalent network presented in Fig. 2.2, where no asymptotic admittances are placed in parallel in the accessible modes, as they remain in the original formulation form *Guglielmi et al.* (1999). The

kernel in equation (2.8) is until now frequency dependent.

2.3 Application of Kummer's Transformation

To efficiently solve the integral equation in equation (2.7), it is now convenient to split the kernel into static and dynamic parts. To do that, we propose to use the Kummer's transformation that is usually applied to accelerate the convergence of Green's functions *Fructos et al.* (2008).

2.3.1 Theoretical Derivations

Kummer's technique is applied in this case by extracting from the original modal admittance $Y_m^{(\delta)}$, its asymptotic behavior $\hat{Y}_m^{(\delta)}$ when $m \rightarrow \infty$. With this transformation the kernel in equation (2.8) becomes

$$\begin{aligned} \bar{\mathbf{K}}(s, s') &= \bar{\mathbf{K}}_d(s, s') + \bar{\mathbf{K}}_s(s, s') \\ &= \sum_{\delta=1}^2 \sum_{m=N(\delta)+1}^{+\infty} (Y_m^{(\delta)} - \hat{Y}_m^{(\delta)}) \mathbf{h}_m^{(\delta)}(s) \mathbf{h}_m^{(\delta)*}(s') \\ &\quad + \sum_{\delta=1}^2 \sum_{m=N(\delta)+1}^{+\infty} \hat{Y}_m^{(\delta)} \mathbf{h}_m^{(\delta)}(s) \mathbf{h}_m^{(\delta)*}(s'). \end{aligned} \quad (2.17)$$

Now to calculate the asymptotic behavior, we rewrite the modal admittance from equation (2.4) and then, we use Taylor expansions for $\sqrt{(1-x)}$ and $\sqrt{(1-x)}^{-1}$ to approximate it using P terms, thereby obtaining

$$\hat{Y}_m^{(\delta)} \approx \begin{cases} \frac{-j|k_{t,m}^{(\delta)}|}{w\mu_0\mu_r^{(\delta)}} \sum_{p=0}^P B_p^{TE} \left(\frac{k}{k_{t,m}^{(\delta)}} \right)^{2p} & TE \\ \frac{w\epsilon_0\epsilon_r^{(\delta)}}{-j|k_{t,m}^{(\delta)}|} \sum_{p=0}^P B_p^{TM} \left(\frac{k}{k_{t,m}^{(\delta)}} \right)^{2p} & TM \end{cases}, \quad (2.18)$$

where the coefficients B_p are known factors of these Taylor expansions. More information about this type of approximations can be found in *Gomez-Molina et al.*

(2015, 2016). Then, the static part of the kernel $\overline{\mathbf{K}}_s(s, s')$ can be rewritten using the expressions in equation (2.18) as follows

$$\begin{aligned}
\overline{\mathbf{K}}_s(s, s') &= \sum_{\delta=1}^2 \sum_{m=N(\delta)+1}^{+\infty} \hat{Y}_m^{(\delta)} \mathbf{h}_m^{(\delta)}(s) \mathbf{h}_m^{(\delta)*}(s') \\
&= \frac{-j}{w\mu_0} \sum_{p=0}^P B_p^{TE} k^{2p} \sum_{\delta=1}^2 \sum_{\substack{m=N(\delta)+1 \\ mTE}}^{+\infty} \frac{\mathbf{h}_m^{(\delta)}(s) \mathbf{h}_m^{(\delta)*}(s')}{\mu_r^{(\delta)} k_{t,m}^{(\delta)(2p-1)}} \\
&\quad + \frac{w\epsilon_0}{-j} \sum_{p=0}^P B_p^{TM} k^{2p} \sum_{\delta=1}^2 \sum_{\substack{m=N(\delta)+1 \\ mTM}}^{+\infty} \frac{\epsilon_r^{(\delta)} \mathbf{h}_m^{(\delta)}(s) \mathbf{h}_m^{(\delta)*}(s')}{k_{t,m}^{(\delta)(2p+1)}}.
\end{aligned} \tag{2.19}$$

It can be observed that now the summation in m is frequency independent, while the dynamic part of the kernel $\overline{\mathbf{K}}_d(s, s')$ has to be recomputed at each frequency point. This formulation leads to two advantages, namely, a minimum number of accessible modes will be required in the MEN matrix, and the heavy part of the kernel is transformed to be frequency independent. On the other hand, the summation of the dynamic part of the kernel needs to be computed for each frequency. However, this term converges very fast and can be evaluated with little computational effort.

2.3.2 Numerical Results

Example 1: Rectangular waveguide cavity resonators.

The example in Fig. 2.3 is discussed to validate the new formulation using Kummer's transformation. For a computer implementation, we use N_{k_d} terms in the dynamic and N_k in the static part of the kernel in equation (2.17). In the example shown in Fig. 2.3, the lengths of cavities and irises are selected to be electrically long on purpose. In this way, the number of accessible modes (N) can be fixed to the number of modes propagating along the uniform rectangular waveguide sections joining two discontinuities. For the selected frequency range, the number of propagating modes is five for all rectangular waveguide sections.

Figure 2.3: Geometry of two cavities connected through an iris. The height in y-axis is 7.899 mm for all the rectangular waveguide sections. In addition, $a^{(2)} = a^{(4)} = 15.799$ mm, $a^{(3)} = 3.5993$ mm and $a^{(1)} = a^{(5)} = 5.9283$ mm. Frequency sweep from 18 GHz to 25 GHz.

The equivalent network of this device is shown in Fig. 2.4. As observed, after obtaining the equivalent network for each discontinuity present in Fig. 2.3, the MENs are cascaded as shown in 2.4 to analyze the complete device.

Figure 2.4: Equivalent network of the device represented in Fig. 2.3.

In Table 2.1 the parameters that are needed to reach convergence for three different cases, are shown. In all three cases the computations give essentially coincident S-parameters, and are given in Fig. 2.5.

Table 2.1: Specific parameters for MEN formulations in Fig. 2.3.

Original Formulation	Formulation of Section 2.2	Formulation of Section 2.3
N=7 $N_k=2000$	N=5 $N_{k_d}=2000$	N=5 and $N_{k_d}=10$ (P=5) $N_k=2000$

Figure 2.5: Magnitude of the scattering parameters of the structure in Fig. 2.3 simulated using the three studied formulations with the parameters in Table 2.1. The three results are coincident.

The first case corresponds to the original MEN formulation with asymptotic admittances in the network *Guglielmi et al.* (1999); *Melcon and Guglielmi* (1994). By using the Taylor approximation, the kernel is computed independently from frequency, but the number of accessible modes has to be increased from 5 to 7, as shown in the first column of Table 2.1. This is due to the dependence existing between the number of accessible modes and the Taylor approximation of the kernel.

The second case corresponds to the one presented in Section 2.2 without asymptotic admittances. The number of accessible modes can be reduced to the strict minimum, namely to 5 modes (these are the modes propagating along the rectangular waveguide sections). In this example, however, we have not used Kummer's transformation and the kernel in equation (2.8) is still fully frequency dependent.

Finally, the third case corresponds to the new formulation obtained by applying higher order extraction terms based on the Kummer's technique [reported in equations

(2.17)-(2.19)]. If the dynamic part of the kernel is introduced in the formulation, as shown in Section 2.3, then the necessary number of accessible modes is reduced again to 5. The dynamic part needs to be computed for every frequency points. However, the convergence is very fast, and only $N_{k_d}=10$ modes are needed when $P = 5$ Kummer's terms are extracted. Then, the heavy part of the kernel is computed independent from frequency, as in the original formulations.

The advantage of this new formulation is that the computations are basically frequency independent, and yet the size of the coupling matrix is reduced to the required minimum. Using the new approach, the CPU time, in this case, has been reduced 18.34 % in comparison to the original strategy.

Example 2: Capacitive step transformer.

Now, we analyze the transformer shown in Fig. 2.6, using the three MEN strategies discussed previously. This device is a capacitive transformer implemented using WR-75 waveguide technology for the input/output ports. In Table 2.2 we collect the dimensions of this circuit (where $a=19.05\text{mm}$).

Figure 2.6: Scheme in E-plane of the symmetric capacitive transformer under analysis.

This device was used in *Jiménez et al.* (2010) to verify the theoretical model proposed to predict the harmonics generated by the multipactor discharge occurring in realistic passive waveguide components in space conditions. We use this transformer

here as an example to validate the advantages in terms of performance of the new MEN theory.

Table 2.2: Dimensions of the E-plane waveguide transformer in Fig. 2.6.

Waveguide Section	L_i (mm)	b_i (mm)
in	20.00	9.53
1	8.11	5.46
2	8.03	1.44
3	8.24	0.44
4	40.04	0.32

In this case, since the lengths (along z -direction) of all waveguide sections are sufficiently large, the minimum number of accessible modes in each section is equal to the number of propagating modes. This is due to the fact that no evanescent modes reach the following discontinuity with sufficient energy to be accounted for as accessible. The analysis frequency in this example is from $f = 10$ GHz to 31.45 GHz. For this range, the maximum number of modes that are propagating along the waveguide sections is ten.

In Table 2.3 we show the numerical parameters needed by each formulation to reach the convergent response shown in Fig. 2.7.

Table 2.3: Specific parameters for MEN formulations in Fig. 2.6.

Original Formulation	Formulation of Section 2.2	Formulation of Section 2.3
N=14 $N_k=1000$	N=10 $N_{k_d}=1000$	N=10 and $N_{k_d}=4$ $N_k=1000$

As it has been previously explained, the minimum number of accessible modes in this case is 10. However, using the original formulation *Guglielmi et al. (1999)*, we need 14 accessible modes to obtain the convergent response in Fig. 2.7. This is due to the fact that these extra 4 modes do not satisfy the Taylor approximation assumed

Figure 2.7: Scattering parameters magnitude of the structure in Fig. 2.6 simulated using the three studied formulations with the parameters in Table 2.3. The three results are coincident.

in the kernel of the IE in the original formulation. This approximation is applied to compute the kernel in a frequency independent way, but it leads to an increment in the necessary number of accessible modes (because of the modes that cannot be statically approximated). As a result, using the original formulation, the kernel is computed statically, but the number of accessible modes has to be increased from 10 to 14.

On the other hand, using the formulation in Section 2.2, the number of accessible modes can be fixed to its minimum value, that is, 10. In contrast, since we do not use any static approximation, the kernel of the integral equation is computed dynamically. This increases the computational effort for this analysis.

With the objective of achieving a better balance between these two scenarios, we have applied in Section 2.3 an static approximation just for the higher order modes that are computed in the kernel. Then, we compute dynamically in the kernel the

lower order localized modes that do not satisfy this approximation. From Table 2.3, we can see that this strategy allows us to fix again the number of accessible modes to its minimum value ($N=10$). With this idea, the 4 modes that do not fulfill the static approximation applied to the modal admittance, are computed in the dynamic kernel.

It is important to remember that the integral equation is solved for each accessible mode [see equation (2.7)] and the size of the coupling matrix also depends on N [see equation (2.11)]. Thus, the computational time for this analysis is reduced in a 27.11% if we use 10 accessible modes instead of 14.

With these two examples, we can conclude that the formulation proposed in this section allows to compute the heavy part of the kernel independently from frequency, while keeping the size of the impedance coupling matrix to a minimum. This contributes to improve the efficiency of the MEN technique for waveguide components.

2.4 Efficient Mechanism to Assess Convergence

In this section, the numerical parameters, that need to be set when the MEN formulation is used, are first revised according to the formulation described in Section 2.2 and 2.3. Then, we discuss how to efficiently chose the value of these parameters to assess convergence for MEN formulation of waveguide discontinuities.

2.4.1 Theoretical Derivations

First, in equation (2.3), we have separated the accessible from localized modes by splitting the summation into $N(\delta)$ terms. Thus, the first numerical parameter that has to be set is the number of accessible modes $N(\delta)$. This parameter $N(\delta)$ controls the number of modes that are going to be accessible in the resulting equivalent network for the waveguide (δ). These modes are explicitly connected to the following discontinuity (another MEN as in Fig. 2.2 and Fig. 2.4) by length sections of transmission lines.

The rest of the modes in the right hand side of equation (2.3) are taken into account as localized ones in the kernel of the integral equation. They have been loaded by their corresponding characteristic modal admittance.

Secondly, the MoM-Galerkin procedure *Harrington* (1993) was employed to solve the integral equations (2.7). This procedure is explained in detail in Appendix A. It consists in expanding the unknown functions $\mathbf{M}_n^{(\gamma)}(s')$. Assuming that the index r represents the smaller waveguide of the step, we use as basis functions the modes of the smaller waveguide $\mathbf{h}_k^{(r)}$

$$\mathbf{M}_n^{(\gamma)}(s') = \sum_{k=1}^{N_b} \alpha_{n,k}^{(\gamma)} \mathbf{h}_k^{(r)}(s'), \quad (2.20)$$

where N_b determines the number of modes of the smaller waveguide that are used to expand the unknown functions. Thus, the second numerical parameters is N_b that fixes the number of basis and test functions in the MoM-Galerkin procedure.

Finally, we computed the kernel using Kummer's transformation, as explained in Section 2.3. Using this transformation, the kernel can be rigorously split into a dynamic and static parts as in equation (2.17). The numerical parameters N_{k_d} and N_k represent then the number of terms used in the dynamic and static kernel computation, respectively. The number of terms in the static kernel (N_k) controls the upper limit in the summation of the frequency independent part of the Green's function (kernel), which describes the discontinuity at the junction of the two waveguides. In the cases in which some lower order localized modes do not satisfy the asymptotic approximation assumed in the static kernel, additional dynamic terms N_{k_d} need to be computed.

As summary, the numerical parameters that the user has to set when the MEN formulation is applied are: the number of accessible modes N , the number of basis/test functions N_b and the number of terms in the dynamic and static part of the kernel N_{k_d} and N_k . In the regular convergence procedure, the minimum specific set of

parameters that ensures a given degree of accuracy, is identified by increasing manually their value until convergent results are obtained. As it is indicated in (*FEST3D*) (2020), the user should start by simulating the structure with increasing number of accessible modes (using a sufficiently large number of basis functions N_b and terms in the kernel N_k). Once the required number of accessible modes is found, the process must be repeated for the number of basis functions, until the minimum number required for reaching convergence is identified. Finally, to fix the number of terms in the kernel, one has to simulate repeatedly the same structure with a sufficient number of terms and decrease this parameter progressively until the minimum required value is identified. This procedure requires a large number of repeated computations and is clearly very time consuming.

To improve the situation, we propose in this section a new automatic strategy for the identification of the optimum set of computational parameters. It is important to note that the proposed method is possible thanks to the fact that, using the kernel formulation in equation (2.17), the number of accessible modes is not linked anymore to the convergence of the kernel. This is possible because of the fast converging dynamic summation that computes the lower order localized modes that do not satisfy the static kernel approximation. In this way, the number of accessible modes can now be fixed according to the physical structure of the device, independently from the kernel convergence issue. Using equation (2.17) this goal is possible, while the computational burden of the kernel remains frequency independent. In the following subsections, we explain how the specific MEN convergence parameters can be efficiently adjusted automatically.

Number of accessible modes

The first parameter that needs to be adjusted is the number of accessible modes in each waveguide region of the junction (see Fig. 2.1). To establish an appropriate

criterion for this selection, it is important to recall which modes are theoretically needed as accessible in the equivalent network (see Fig. 2.2). Naturally, all the modes which are propagating along the waveguide section must be taken into account as accessible modes, since they are responsible for the active energy exchange between the discontinuities in the structure. Furthermore, it is also known that the evanescent modes are storing energy near the discontinuities. Since in real structures many discontinuities are placed close to each other, some evanescent modes must also be considered as accessible ones, in order to achieve a complete characterization of the involved energy exchange process. In this representation, therefore, more modes than the propagating ones must, in general, be considered as accessible modes for each waveguide region in the equivalent network.

On this basis, the accessible modes in each region must be the propagating modes plus the evanescent ones that reach the next junction with enough energy. An automatic selection procedure must take into account that in waveguide sections with small lengths (such as thin coupling windows) we need to include more evanescent modes, since these modes are interacting with nearby discontinuities. On the other hand, in waveguide sections with large lengths (such as resonant cavities), only the first few propagating modes (with maybe one or two additional evanescent modes) are needed.

To cope with these very different situations, we propose a selection method based on fixing a power attenuation threshold in dB (that we call dB_{th}). The attenuation for each evanescent mode (At_m) along a distance l can be calculated through its attenuation constant α_m , as

$$At_m = \alpha_m \cdot l. \quad (2.21)$$

According to this criterion, an evanescent mode will be considered to be accessible, if the total attenuation when it reaches the next discontinuity is smaller than

the specified threshold. The number of accessible modes will then be given by all propagating modes plus the evanescent modes which satisfy the following condition (in dB)

$$At_m \leq dB_{th}, \quad (2.22)$$

which is imposed at the highest frequency in the analysis loop.

It is easy to see that this process automatically includes more evanescent modes in short waveguide sections, and less evanescent modes in long waveguide sections.

When analyzing a whole device, just a threshold dB_{th} needs to be fixed for the whole structure. The number of accessible modes required in each waveguide section is then adjusted automatically according to condition in equation (2.22).

Number of basis/test functions

As previously mentioned, the basis functions used for the MoM solution of the integral equation (2.7), are the vector modal functions of the smaller waveguide shown in Fig. 2.1. The number of basis functions (N_b) used in this expansion [see equation (2.20)] is equal to the number of test functions, since the Galerkin procedure is normally used. In this case, we need to remember that the last mode we need to represent as excitation of our integral equation is the highest order accessible mode as indicated in equation (2.7). Once the threshold dB_{th} is set, this mode corresponds to the highest order mode that satisfies the condition in equation (2.22). From now on, the cut-off wavenumber of this mode is referred to as kc_{max} . In the proposed procedure, all the modes of the smaller waveguide with a cut-off wavenumber smaller or equal to this value (times an accuracy factor F_b) are automatically selected as basis/test functions. This condition can be written as

$$N_b \rightarrow kc_m \leq F_b \cdot kc_{max}, \quad (2.23)$$

where F_b is considered as a factor of precision. It is clear that if the factor F_b is increased, more modes will satisfy the condition in equation (2.23). This leads to a higher number of basis functions to be used in the MoM solution.

The criterion to identify the correct F_b value is based on the numerical convergence of the value of the last element of the impedance sub-matrix that represents the auto-interaction of the smaller waveguide. According to equation (2.10), this last element is written as $Z_{N(r),N(r)}^{(r,r)}$. We propose to use the relative error in the computation of this element of the impedance matrix to identify the value of the F_b factor, evaluated at the highest frequency in the analysis loop. The computational effort of this evaluation is really small in comparison to what is needed to adjust manually this parameter. Once the dB_{th} value is fixed by the user, the selection of F_b is accomplished automatically, without any user intervention, to ensure convergence.

Number of terms in the kernel

According to equation (2.17), the N_k -parameter refers to the number of terms that are summed in the static kernel of the integral equation. In other words, it represents the number of terms in the frequency independent summation of the Green's functions. As for the basis functions, the kernel convergence depends on the geometry of the step and also on the excitation of the integral equation. In this respect, using the same cut-off wavenumber of the last accessible mode kc_{max} , the number of modes in the kernel (N_k) will be taken as those modes of the bigger waveguide with their cut-off wavenumbers kc_n lower than the selected kc_{max} times an accuracy F_k factor.

This condition can be written as

$$N_k \rightarrow kc_n \leq F_k \cdot kc_{max}, \quad (2.24)$$

where F_k is, as before, the precision factor. The adjustment of this factor is going to be carried out according to the convergence study on the element $Z_{N(r),N(r)}^{(r,r)}$, which is

accomplished automatically.

A convenient option is to select the same factor $F_b = F_k = F$ to fix both the precision associated to the number of basis functions and to the number of terms in the kernel. To simplify the convergence procedure, the same F value is indeed used in the results presented in Subsection 2.4.2. Therefore, if the F -factor is increased, more basis functions and terms in the kernel will be used in the numerical solution of the problem.

When analyzing a complex device, the parameter F , that adjusts the specific parameters N_b and N_k , is automatically computed for each discontinuity, to assure convergence, and therefore the user is released of that responsibility. This results in a significant reduction of the computational effort together with an improved user simulation experience.

The complete picture is finalized with the calculation of the dynamic part of the kernel shown in equation (2.17). This parameter is related to the Taylor expansion of the static approximation, and converges with few terms (N_{k_d}). On this basis, we have taken a fix number $N_{k_d} = 10$ for all the practical examples considered in Subsection 2.4.2.

It is important to remark that the static part of the kernel only contains the higher order modes which satisfy the static approximation. Thanks to the dynamic part, the localized modes that do not satisfy the static approximation are no longer considered as accessible modes (as it is done in the original formulation), but are considered in the dynamic kernel as localized.

2.4.2 Numerical Results

The proposed convergence method is applied now in a MEN implementation to analyze some inductive rectangular waveguide structures. First of all, an iris between two rectangular waveguides will be analyzed using the new mechanism to show how

the convergence parameters are adjusted according to different lengths. Then, some practical inductive bandpass filters are also studied using the new strategy, to show the performance improvement achieved in the analysis of real waveguide structures.

Example 1: Study on simple rectangular waveguide iris.

A simple iris between two waveguides shown in Fig. 2.8 is first analyzed, using the Multimode Equivalent Network formulation together with the convergence method proposed in Subsection 2.4.1. To show the behavior of the described technique according to the length of the iris, the structure in Fig. 2.8 is studied for different values of L .

Figure 2.8: Inductive iris composed of two steps in rectangular waveguide technology. The dimensions are $a_1 = 19.1$ mm, $a_2 = 9.0203$ mm and the height is $b = 8.2$ mm.

At the input and output waveguides, only the propagating modes need to be considered as accessible modes. However, in the iris, the propagating plus some evanescent modes will need to be considered as accessible.

With the new technique, the number of accessible modes is not fixed directly. Instead, we have to select a global threshold dB_{th} . With this power attenuation threshold, the number of accessible modes needed to satisfy the specified attenuation condition level in each region will automatically be fixed.

In this context, in Table 2.4 the number of accessible modes (N) that the proposed method fixes for each medium and for different values of dB_{th} is reported.

As it is noticed, for $\text{dB}_{th} = 0$ dB, since no attenuation is specified, only the propagating modes are accessible in all three regions. Moreover, when the attenuation

Table 2.4: Number of accessible modes in each section adjusted by the new method according to the length L and the threshold dB_{th} .

dB_{th}	$L = 0.5 \text{ mm}$	$L = 1 \text{ mm}$	$L = 5 \text{ mm}$
0 dB	3,1,3	3,1,3	3,1,3
10 dB	3,6,3	3,3,3	3,1,3
20 dB	3,13,3	3,6,3	3,2,3
30 dB	3,19,3	3,10,3	3,2,3

threshold is increased, more evanescent modes are considered to be accessible in the iris. Also, the number of accessible modes increases for thinner irises, as it is required to represent the stronger energy exchange between nearby discontinuities.

Once this parameter has been chosen by the user, the factor F is automatically adjusted, and consequently also the number of basis functions and the number of terms in the static kernel. For the problems treated in this chapter, we have observed that relative errors in the order of 10^{-3} during the calculation of the F -factors lead to good accuracy. Thus, for each discontinuity, the factor F is selected as the one which gives a relative convergence error in the order of 10^{-3} for the element $Z_{N(r),N(r)}^{(r,r)}$ of the generalized impedance matrix.

In Fig. 2.9 we show this relative error as a function of increasing the accuracy (F) parameter for the first step of Fig. 2.8. In this test, we have fixed $\text{dB}_{th} = 10 \text{ dB}$ and $L = 0.5 \text{ mm}$. As it is shown, for this step a factor $F = 3$ is needed to obtain an error in the order of 10^{-3} .

In Table 2.5, the automatically selected parameters F , N_b and N_k are presented for each considered iris length. The absolute convergence errors (ϵ), achieved with respect to the convergent response, are also included. The convergent response is obtained by selecting arbitrary large numerical parameters (in the examples treated in this paper 20 accessible modes, 200 basis functions and 2000 terms in the kernel). Note that, this time, the absolute error is given instead of the relative error. This is

Figure 2.9: Relative convergence error of the element $Z_{N(r),N(r)}^{(r,r)}$ as a function of the accuracy F -factor for the first step in Fig. 2.8.

because the response contains poles and zeros, which make inaccurate the calculation of the relative errors at the corresponding frequencies.

In this case, we also consider two different threshold levels dB_{th} , 10 dB and 20 dB, to see how the factor F is adjusted. For completeness, the number of accessible modes (N) in the three regions is also given.

Table 2.5: Parameters adjusted by the method using $\text{dB}_{th} = 10$ dB and $\text{dB}_{th} = 20$ dB for different lengths L .

	$L = 0.5$ mm	$L = 1$ mm	$L = 5$ mm
10 dB	$N = (3, 6, 3)$	$N = (3, 3, 3)$	$N = (3, 1, 3)$
	$F = 3$	$F = 4$	$F = 9$
	$N_b = 17$	$N_b = 12$	$N_b = 12$
	$N_k = 38$	$N_k = 25$	$N_k = 27$
	$\epsilon = 3.23 \cdot 10^{-4}$	$\epsilon = 6.65 \cdot 10^{-4}$	$\epsilon = 3.10 \cdot 10^{-4}$
20 dB	$N = (3, 13, 3)$	$N = (3, 6, 3)$	$N = (3, 2, 3)$
	$F = 2$	$F = 3$	$F = 6$
	$N_b = 26$	$N_b = 17$	$N_b = 12$
	$N_k = 55$	$N_k = 38$	$N_k = 25$
	$\epsilon = 4.89 \cdot 10^{-4}$	$\epsilon = 3.14 \cdot 10^{-4}$	$\epsilon = 3.23 \cdot 10^{-4}$

It can be seen that, for irises that require a smaller number of accessible modes, a bigger F -value is selected by the algorithm, and this effectively preserves the accuracy of the computations.

As it is noticed, the proposed selection mechanism simplifies the current manual convergence study, since just a physical parameter, the threshold dB_{th} , is required to be fixed according to the desired accuracy level. Then, the factor of precision F is automatically adjusted, and finally, the number of basis functions and terms in the kernel is automatically selected according to the conditions given in equations (2.23) and (2.24), to provide a convergent response.

Another important fact is that until now, for the sake of simplification, MEN users used to select the same parameters for all the discontinuities within a structure (global parameters). This can oversize the less restrictive discontinuities until the others reach convergence. With this new convergence mechanism, the user can still select global parameters, that is, the same threshold dB_{th} for all the discontinuities. However, with the new algorithm this implies to use different specific parameters (number of accessible modes, number of basis functions and terms in the kernel) in each discontinuity according to their specific geometries. As a consequence, all discontinuities have different numerical parameters better suited to their specific needs, still using intuitive and global parameters for the user.

In fact, regular waveguide filters are composed of short and large length sections, mixed together. Thus, the advantage of being able to use a global parameter for the whole structure (while the actual numerical parameters are adjusted differently for each discontinuity) is significant in the following examples.

Example 2: Dual-mode inductive cavity bandpass filter.

The second example is proposed in *Guglielmi et al.* (2001), and consists of a single dual-mode inductive cavity as the one shown in Fig. 2.10.

Figure 2.10: Single dual-mode inductive bandpass filter under study (dimensions in mm).

For a threshold $\text{dB}_{th} = 10$ dB, the factors F and the specific parameters N , N_b and N_k , automatically selected for each discontinuity, are shown in Table 2.6.

Table 2.6: Convergence parameters adjusted automatically by the method using $\text{dB}_{th} = 10$ dB.

	Step 1	Step 2	Step 3	Step 4
N	(1, 3)	(3, 3)	(3, 3)	(3, 1)
F	5	4	4	5
N_b/N_k	15/25	12/44	12/44	15/25

The S-parameters, simulated using the specific parameters in Table 2.6, are compared in Fig. 2.11 to the convergent response for 20 accessible modes, 200 basis functions and 2000 terms in the kernel and to ANSYS HFSS simulation results.

The absolute convergence error is of $4.47 \cdot 10^{-4}$. As we can see, a good agreement between the convergent response and the S-parameters obtained with the proposed method is achieved. In addition, a great reduction in required simulation parameters is noticed, which leads to a reduction in the computational effort. Instead of using 20 accessible modes, 200 basis/test functions and 2000 terms in the kernel for all the discontinuities, the proposed mechanism allows us to use the parameters in Table 2.6 and still have the same response (see Fig. 2.11).

Using an Intel (R) Core i7-6500U CPU @ 2.50 GHz with 12 GB of RAM, the computational time for 150 frequency points required to achieve the ANSYS HFSS results in Fig. 2.11 is 5 minutes and 47 seconds, while for the proposed MEN imple-

Figure 2.11: Magnitude of the S-parameters for the dual-mode cavity filter in Fig. 2.10 using the parameters adjusted by the proposed method (see Table 2.6) and compared to the convergent performance using MEN formulation and ANSYS HFSS software tool.

mentation it is of 2.65 seconds. It is important to remember that our software tool is specifically designed to analyze this type of waveguide structures in contrast to ANSYS HFSS, which is a more generic analysis tool.

Example 3: Three-pole in-line filter.

The third example consists of a three-pole in-line inductive rectangular waveguide bandpass filter (see Fig. 2.12). For a global threshold $dB_{th} = 20$ dB, the selected parameters of each discontinuity are shown in Table 2.7.

Figure 2.12: Three-pole bandpass filter under study. The dimensions are $a = 19.1$ mm, $a_1 = 9.0203$ mm, $a_2 = 5.1702$ mm and the height is $b = 8.2$ mm. The lengths are $l_1 = 1$ mm, $l_2 = 21.6581$ mm and $l_3 = 23.1495$ mm.

Table 2.7: Convergence parameters adjusted automatically by the method using $dB_{th} = 20$ dB, for the different steps in the filter depicted in Fig. 2.12.

	Steps 1 / 8	Steps 2 / 7	Steps 3 / 6	Steps 4 / 5
N	(3, 6)	(6, 1)	(1, 3)	(3, 1)
F	3	3	4	4
N_b/N_k	17/38	17/38	12/44	12/44

It can be observed that the proposed technique increases the value of (F) when the number of accessible modes decreases. This tends to keep a constant numerical accuracy along all the discontinuities of the structure.

The resulting S-parameters, compared to the convergent response for 20 accessible modes, 200 basis functions and 2000 terms in the kernel and to results obtained with the commercial software tool ANSYS HFSS, are shown in Fig. 2.13. In this case, we also include the performance for a lower threshold, $dB_{th} = 10$ dB, to show how the accuracy level can be controlled through this parameter.

Figure 2.13: S-parameters magnitude for the three-pole bandpass filter in Fig. 2.12 using the parameters adjusted by the proposed method for $dB_{th} = 10$ dB and $dB_{th} = 20$ dB (see Table 2.7) and compared to the convergent performance using MEN formulation and ANSYS HFSS software tool.

In this example, for $dB_{th} = 20$ dB, the absolute convergence error is of $6.33 \cdot 10^{-4}$, indicating good numerical accuracy. Moreover, a resulting reduction of the parameters in the numerical method is effectively achieved. The computational time for 150 frequency points required to achieve the ANSYS HFSS results in Fig. 2.13 is more than 35 minutes, while for the proposed MEN implementation it is of 3.65 seconds.

Example 4: Four-pole dual-mode filter.

The fourth example is proposed in *Guglielmi et al.* (2001) and consists of a four-pole dual-mode inductive bandpass filter shown in Fig. 2.14. The dimensions are reported in Table 2.8.

Figure 2.14: Four-pole dual-mode bandpass filter in the Ku-band. The dimensions are reported in Table 2.8 and the height is $b = 9.525$ mm.

Table 2.8: Dimensions of the four-pole dual-mode filter in Fig. 2.14

$a = 19.05$ mm		
$a_1 = 10.931$ mm	$l_1 = 3$ mm	$o_1 = 1$ mm
$a_2 = 30.077$ mm	$l_2 = 28.306$ mm	$o_2 = 1$ mm
$a_3 = 10.782$ mm	$l_3 = 12.481$ mm	$o_3 = 0.87$ mm
$a_4 = 29.567$ mm	$l_4 = 28.852$ mm	$o_4 = 0.87$ mm
$a_5 = 10.596$ mm	$l_5 = 3$ mm	

For a threshold $dB_{th} = 20$ dB, the factors F and the adjusted parameters for each discontinuity, are shown in Table 2.9.

Table 2.9: Convergence parameters adjusted automatically by the method using $dB_{th} = 20$ dB.

	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6
N	(1, 2)	(2, 2)	(2, 1)	(1, 2)	(2, 2)	(2, 1)
F	5	5	10	10	5	5
N_b/N_k	10/17	10/27	10/27	10/27	10/27	10/17

The resulting S-parameters, compared to the convergent response for 20 accessible modes, 200 basis functions and 2000 terms in the kernel and to ANSYS HFSS simulation results, are shown in Fig. 2.15.

Figure 2.15: S-parameters magnitude for the four-pole dual-mode bandpass filter in Fig. 2.14 using the parameters adjusted by the proposed method (see Table 2.9) and compared to the convergent performance using MEN formulation and ANSYS HFSS software tool.

The absolute convergence error in this case is of $8.08 \cdot 10^{-4}$, showing again the effectiveness of the proposed technique. The computational time required for 250 frequency points to achieve the ANSYS HFSS results reported in Fig. 2.13 is 31 minutes and 19 seconds, while the time for the proposed MEN implementation is of 2.02 seconds.

This example is appropriate for an in depth study on the importance of adjusting different F -factors for each discontinuity. As we can see in Table 2.9, if the factor F is selected to be the same, it should be the biggest to assure convergence in all the discontinuities. In this case, it would be $F = 10$ to assure convergence in steps 3 and 4. This would imply that N_b and N_k in the rest of the discontinuities would also be increased. In other words, the parameters for some discontinuities would be increased at the expense of reaching convergence in the most limiting discontinuities. Since this increment is unnecessary, this would result in an inefficient oversizing of the convergence parameters. This problem is avoided with the strategy proposed in Subsection 2.4.1, since the algorithm automatically adjusts different F -factors for each discontinuity.

In Table 2.10 the computational times required by different numerical parameters in the MEN implementation for analyzing this geometry using 250 frequency points are reported. Contrary to the proposed mechanism, in the cases reported in Table 2.10, the same parameters have been used for all the discontinuities.

Table 2.10: Total computational time for different numerical parameters in the MEN implementation using 250 frequency points.

N	N_b	N_k	Time (s)
2	20	55	2.21
5	30	300	2.75
10	50	500	6.81
15	100	1000	28.65

Table 2.10 shows that increasing the numerical MEN parameters affects negatively the computational effort. As comparison, the analysis of the same filter using the parameters fixed by the proposed mechanism (without considering the time for execute the routine that find these optimal parameters) is only 1.32 seconds. Thus, Table 2.10 shows that an important reduction in computational time can be obtained, if

the numerical parameters of the MEN are kept to the strict minimum required, as calculated by the new proposed strategy.

Example 5: Five-pole in-line bandpass filter.

The last example is the analysis of the inductive bandpass filter shown in Fig. 2.16. This device was used in *Perez Escudero et al.* (2015) to prove the technique proposed to enhance the free spurious range in waveguide inductive filters.

Figure 2.16: Rectangular waveguide bandpass filter under analysis. The dimensions in mm are: $w_1 = 12.937$, $a_1 = 27.572$, $w_2 = 9.487$, $a_2 = 27.137$, $w_3 = 3.928$, $a_3 = 19.05$, $w_4 = 3.915$, $a_4 = 27.173$, $w_5 = 9.471$, $a_5 = 27.602$, $w_6 = 12.922$. The lengths in mm are: $t_1 = 12$, $l_1 = 26.577$, $t_2 = 9$, $l_2 = 29.926$, $t_3 = 1$, $l_3 = 18.946$, $l_4 = 29.919$, $l_5 = 26.585$.

The first analysis is the in-band response of the filter. In Table 2.11 we show, for $dB_{th} = 20$ dB, the factors F and the specific numerical parameters N , N_b and N_k , that the proposed mechanism have selected for each discontinuity. In Fig. 2.17, the in-band MEN response using the parameters in Table 2.11 are compared to the response obtained using the commercial software tools ANSYS HFSS and FEST3D (selecting 20 accessible modes, 200 basis and test functions and 2000 terms in the kernel of the IE).

In addition, in Fig. 2.18, we show the out-of-band response of the filter, simulated using the specific parameters in Table 2.12. In this plot, the MEN results are also compared to the response obtained using ANSYS HFSS and FEST3D.

As observed, in both cases (in-band and out-of-band analysis), the S-parameters obtained applying the automatic mechanism are similar to the convergent response from ANSYS HFSS and FEST3D.

Table 2.11: Convergence parameters adjusted automatically by the method using $dB_{th} = 20$ dB. In-band analysis.

Step	N	F	(N_b / N_k)
1 y 12	(1 / 1)	12	(11 / 17)
2 y 11	(1 / 2)	10	(10 / 21)
3 y 10	(2 / 1)	10	(10 / 29)
4 y 9	(1 / 2)	8	(8 / 22)
5 y 8	(2 / 2)	5	(9 / 69)
6 y 7	(2 / 1)	5	(9 / 48)

Figure 2.17: S-parameter in-band response for the filter in Fig. 2.16 using the parameters adjusted by the proposed method (see Table 2.11) and compared to the convergent performance using FEST3D and ANSYS HFSS software tool.

If we now compared the parameters given in Tables 2.11 and 2.12, we can see that the proposed technique is able to select different values of F for each discontinuity, so that the necessary number of basis and kernel terms in each discontinuity is selected to reach accurate results. We can also appreciate that this mechanism is able to correctly select the number of accessible modes and factor F according to the frequency sweep. In general, when the operational frequency increases, more modes should be considered as accessible. Consequently, more basis and test functions and more terms in the kernel of the IE would be needed for a proper characterization of the device.

Table 2.12: Convergence parameters adjusted automatically by the method using $dB_{th} = 20$ dB. Out-of-band analysis.

N	F	(N_b / N_k)
(3 / 2)	8	(16 / 24)
(2 / 4)	9	(18 / 38)
(4 / 1)	12	(16 / 48)
(1 / 4)	9	(12 / 36)
(4 / 2)	5	(9 / 69)
(2 / 3)	5	(9 / 48)

Figure 2.18: S-parameter out-of-band response for the filter depicted in Fig. 2.16 using the parameters adjusted by the proposed method (see Table 2.12) and compared to the convergent performance using FEST3D and ANSYS HFSS software tools.

As a conclusion, instead of performing these two analyses (in-band and out-of-band) using the same numerical parameters common for all the discontinuities, now the algorithm adjusts these parameters differently for each discontinuity present in the structure and for each frequency analysis that has to be performed.

As these examples show, the new mechanism for fixing the convergence parameters in MEN implementations gives accurate responses along with a more efficient computation (thanks to the achieved reduction in the number of the involved numerical parameters).

2.5 Conclusions

In this chapter, a new MEN formulation for the analysis of two-dimensional waveguide junctions is derived. The difference between the classical MEN technique and the new formulation lies in the treatment of the frequency dependent kernel. More precisely, in the classical MEN formulation, asymptotic admittances and a Taylor expansion are used, resulting in a dependence between the number of accessible modes required in the equivalent network and the convergence of the Taylor series. In the new formulation proposed in this chapter, the number of accessible modes is not linked to the convergence of the kernel anymore. This is achieved by reformulating the MEN technique without using asymptotic admittances and then introducing in a novel way a higher order Kummer's transformation. The result is that the number of accessible modes in the coupling matrix can now be kept to a minimum, still having the important computational burden of the kernel independent from frequency. In addition to theory, the numerical convergence of the new formulation is discussed with the analysis of a rectangular waveguide structure and a capacitive step transformer, clearly showing the advantages introduced with respect to classical MEN formulation.

Thanks to this new way to formulate the MEN technique, we could propose a simple and elegant strategy to choose efficiently the numerical parameters needed to reach convergence in the MEN formulation. With the proposed mechanism only the parameter dB_{th} needs to be fixed by the software user. The chosen dB_{th} value will then affect the accuracy of the obtained results. The other computational parameter needed, namely F , is automatically adjusted according to the convergence of the last element of the multimode impedance matrix. This leads to the correct adjustment of all specific computational parameters (namely, N , N_b and N_k), different for each discontinuity, by just fixing the same attenuation threshold for the whole structure.

This convergence mechanism leads to two advantages. The first advantage is related to the fact that, for each discontinuity, the actual numerical parameters are

different according to their specific convergence needs, thus the oversizing of discontinuities is avoided. The second advantage is on user level. The proposed procedure implies a more intuitive way of selecting just a parameter that controls the accuracy of the calculations. This also leads to a huge saving in computational effort, as compared to the convergence study currently required in commercially available MEN implementations. In addition to theory, numerical results achieved using the proposed mechanism have also been provided, indicating that the new procedure leads to more efficient MEN calculations, while producing an excellent numerical accuracy.

Scientific Outcome:

- Related to Section 2.2 and 2.3: *Gomez-Molina et al.* (2017).
- Related to Section 2.4: *Gomez-Molina et al.* (2018b).

CHAPTER III

Electric Multimode Equivalent Network Technique for Planar Devices

3.1 Introduction

The Multimode Equivalent Network formulation was originally developed for the analysis of waveguide problems, as reported for instance in *Guglielmi et al.* (1999). The analysis of two-dimensional waveguide discontinuities using this formulation has been revisited in Chapter II to enhance its efficiency. In Chapter II the discontinuities under analysis are considered thick, since they are normally junctions between two waveguide sections. Nevertheless, some zero-thickness approaches based on both magnetic field and electric field integral equations have also been introduced in the past, as in *Guglielmi and Oliner* (1989). However, in these original formulations, the excitation of the problem are still the guided modes on both sides of the discontinuity under study. Therefore, the work done so far does not allow the analysis of typical planar printed microwave circuits, where external (or even internal) ports are contained in the transverse plane.

In this context, therefore, the first objective of this chapter is to extend the electric MEN formulation for zero-thickness discontinuities to more practical cases that include excitation ports in the transverse plane, where the metallization is located.

The theoretical derivation is described in Subsection 3.2.1. Since we use an electric formulation, the unknowns of the problem are the electric current densities induced on the metallic areas, instead of magnetic current densities defined in the aperture, as happened in the magnetic MEN formulation studied in Chapter II. This formulation allows for the characterization of single-layer boxed planar devices that contain arbitrary rectangular elements in the metallization interface, where the vector modal functions are known analytically *Melcon et al.* (1999). As a verification, two practical single-layer microstrip filters are efficiently analyzed in Subsection 3.2.2. It is important to note that, using this first formulation, boxed microstrip circuits that contain only one printed metallization interface can be analyzed. However, other more complex filter topologies may require multilayer implementations *Melcon et al.* (1999).

Therefore, the second aim of this chapter is to extend the electric MEN formulation studied in Section 3.2 to the analysis of multilayer boxed microstrip circuits that are based on arbitrary rectangular elements and include ports in the transverse plane. For this purpose, each metal interface is characterized using the zero-thickness MEN technique reported in Section 3.2. Then, the individual MENs are cascaded to obtain the final network that models the complete device. This procedure is fully described in Subsection 3.3.1. As a validation, we describe in Subsection 3.3.2 the use of the proposed formulation to analyze a multilayer microwave filter that implements a trisection topology.

The numerical results presented in this chapter fully validate the theoretical formulations, and clearly highlight the accuracy and computational efficiency of the proposed analysis tool. Finally, in Section 3.4, the main conclusions of this chapter are outlined.

It is important to remark that it is the first time that the MEN technique is applied to the analysis of printed planar devices. Other numerical techniques reported

in the technical literature, for instance the Electric Field Integral Equation (EFIE) techniques reported in *Bozzi et al. (2001)*; *Melcon et al. (1999)*, can be also used to analyze this kind of planar devices. However, these formulations do not allow to obtain an equivalent modal network as the MEN technique does. In this case, we solve the problem in the frame of the MEN representation, so that the analysis of planar circuits can be seamlessly integrated with the analysis of complex waveguide structures and antennas. In addition, in the direct EFIE implementations discussed in *Bozzi et al. (2001)*; *Melcon et al. (1999)*, all the unknowns of the problem are included in one global integral equation. This can lead to matrices of very large sizes when several layers are analyzed. The MEN technique that we discuss, on the other hand, formulates one separate integral equation to characterize each discontinuity with a different multimode coupling matrix. All discontinuities are then coupled together by cascading the different multimode admittance coupling matrices. The result is that, instead of one large global problem, we solve a number of smaller problems that are then properly combined at network level.

3.2 Electric MEN for Single-layer Planar Devices

3.2.1 Theoretical Derivations

In this section, we derive the rigorous MEN formulation for two-dimensional zero-thickness obstacles based on arbitrary rectangular printed metallizations (see Fig. 3.1). For the first time, lateral ports in the transverse plane are also included in the formulation. These ports model the coaxial input/output connectors used to excite real microwave components *Edwards and Steer (2013)*; *Sánchez-Soriano et al. (2018)*.

For the sake of simplicity, only one excitation pulse and one metallization (S_1) layer are considered for the presentation of the theory. The extension to more ports

Figure 3.1: Two-dimensional zero-thickness printed circuit in planar technology. The gray areas correspond to the printed metallizations and the dark areas to the excitation ports, both contained in the transverse plane. The media on both sides of the discontinuity can be different, for instance the substrate and the air of a microstrip structure.

and multiple metallization layers is, however, straightforward.

For the exciting electric field $\mathbf{E}_{exc}$ in the lateral port, we use a pulse excitation model *Rautio and Harrington* (1987) (symbol $\prod$ from now on). It has been shown that this model can correctly represent the coaxial connectors normally used as excitation in real shielded planar structures. A delta-gap model *Melcon et al.* (1999) has also been used in the past for this purpose. However, the difference between pulse and a delta-gap model, in the case of the excitations in Fig. 3.1, is that the pulse model includes also the effect of the width (Δx) along the x -axis in the excitation. Based on the pulse excitation model *Rautio and Harrington* (1987), a voltage V_0 constant along the width of the pulse is assumed to be applied at the excitation port. Thus, the following expression is chosen for the exciting electric field $\mathbf{E}_{exc}$

$$\mathbf{E}_{exc}(s) = V_0 \frac{1}{\Delta x} \prod \left(\frac{x - x_0}{\Delta x} \right) \prod \left(\frac{y - y_0}{\Delta y} \right) \mathbf{x}_0, \quad (3.1)$$

where (x_0, y_0) is the coordinate of the center of the pulse and Δx and Δy are the widths of the pulse along x and y direction, respectively (see Fig. 3.1). In equation (3.1), $\mathbf{x}_0$ is the unit vector in the direction of the x -axis. It is important to note that, in equation (3.1), s indicates a point in the waveguide cross-section and it is defined as $s = (x, y)$. Then, we will use indistinctly, in this document, s and (x, y) to refer the

points in the waveguide cross-section. Also, in order to compact the pulse notation, from now on, we will use the following expression

$$\mathbf{E}_{exc}(s) = V_0 \frac{1}{\Delta x} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{x}_0. \quad (3.2)$$

The formulation starts by imposing the condition that the total tangential electric field components must vanish on the obstacle S_1 at $z = 0$, including the excitation contribution

$$\mathbf{E}_t(s) + \mathbf{E}_{exc}(s) = 0. \quad (3.3)$$

We now recognize that, since the metallic circuit is zero-thickness, and the waveguide in region (1) is identical to the one in region (2), this condition can be written, using the well known modal expansion formalism *Guglielmi et al. (1999)*, as follows

$$\sum_{m=1}^{+\infty} V_m \mathbf{e}_m(s) + V_0 \frac{1}{\Delta x} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{x}_0 = 0, \quad (3.4)$$

where $\mathbf{e}_m(s)$ refers to the electric vector modal function *Melcon et al. (1999)* of mode m in the region (1) or (2), and V_m represents the total modal voltage at the interface $z = 0$. It is important to note that m refers to all TE and TM modes in the rectangular waveguide of cross-section $(a \times b)$ in Fig. 3.1.

As mentioned, the ‘zero-thickness condition’ has been assumed, so that the electric vector modal functions in region (1) ($\mathbf{e}_m^{(1)}$) are considered equal to the ones in region (2) ($\mathbf{e}_m^{(2)}$). Then, $\mathbf{e}_m^{(1)}(s) = \mathbf{e}_m^{(2)}(s) = \mathbf{e}_m(s)$. The mathematical expressions of these functions are known analytically since regions (1) and (2) are, in this case, rectangular waveguides. These functions are reported in Appendix A of this document. Also, for this case, the voltages satisfy at $z = 0$ the condition that $V_m^{(1)} = V_m^{(2)} = V_m$.

To proceed, we separate the accessible from localized modes by splitting the infi-

nite series as

$$\begin{aligned} \sum_{n=1}^N V_n \mathbf{e}_n(s) + V_0 \frac{1}{\Delta x} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{x}_0 \\ = \sum_{m=N+1}^{+\infty} Z_m^T \bar{I}_m \mathbf{e}_m(s), \end{aligned} \quad (3.5)$$

where N is the number of modes that will be accessible in the equivalent network. In equation (3.5), $\bar{I}_m$ represents the total modal current at the interface, and is defined as $\bar{I}_m = I_m^{(1)} - I_m^{(2)}$. Also, Z_m^T is the total characteristic modal impedance defined as the parallel configuration between the modal impedances in both media $Z_m^T = Z_m^{(1)} || Z_m^{(2)}$. Note that, for the sake of clarity, the subscript on the left hand side in equation (3.5) has been changed to n .

The next step in the formulation is to recall the relation between the modal current $\bar{I}_m$ and the magnetic field in the metallization $[-\mathbf{z}_0 \times (\mathbf{H}_t^{(1)} - \mathbf{H}_t^{(2)})]$, and write

$$\bar{I}_m = \int_{obs} [-\mathbf{z}_0 \times (\mathbf{H}_t^{(1)} - \mathbf{H}_t^{(2)})(s')] \cdot \mathbf{e}_m^*(s') ds', \quad (3.6)$$

where the integral is extended to the area of the obstacle (*obs*) at the junction, in this case, S_1 (see Fig. 3.1). Due to the linearity of the problem, the unknown magnetic fields in the obstacle $[-\mathbf{z}_0 \times (\mathbf{H}_t^{(1)} - \mathbf{H}_t^{(2)})]$ can be expanded into a sum of partial electric current densities $\mathbf{N}_n(s')$ and $\mathbf{N}_0(s')$ as

$$[-\mathbf{z}_0 \times (\mathbf{H}_t^{(1)} - \mathbf{H}_t^{(2)})(s')] = \sum_{n=1}^N V_n \mathbf{N}_n(s') + V_0 \mathbf{N}_0(s'), \quad (3.7)$$

where $\mathbf{N}_n$ and $\mathbf{N}_0$ are the unknown functions of our problem. Using the equation (3.7) into equation (3.6), and the resulting equation in equation (3.5), we can finally

obtain the fundamental integral equations of the problem

$$\mathbf{e}_n(s) = \int_{S_1} \bar{\mathbf{K}}(s, s') \cdot \mathbf{N}_n(s') ds' \quad (3.8)$$

$$\frac{1}{\Delta x} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{x}_0 = \int_{S_1} \bar{\mathbf{K}}(s, s') \cdot \mathbf{N}_0(s') ds', \quad (3.9)$$

where $\bar{\mathbf{K}}(s, s')$ is the kernel of the integral equations that is expressed through an infinite series, as

$$\bar{\mathbf{K}}(s, s') = \sum_{m=N+1}^{+\infty} Z_m^T \mathbf{e}_m(s) \mathbf{e}_m^*(s'). \quad (3.10)$$

In order to increase the computational efficiency of this theory, the Kummer's transformation explained in Section 2.3 of Chapter II has been applied in this case to extract the first asymptotic term in the kernel of the integral equations [shown in equation (3.10)].

To complete the rigorous network formulation, we can write the modal currents in terms of the modal voltages as follows

$$I_0 = Y_{0,0} V_0 + \sum_{n=1}^N Y_{0,n} V_n \quad (3.11)$$

$$\bar{I}_m = Y_{m,0} V_0 + \sum_{n=1}^N Y_{m,n} V_n \quad (3.12)$$

and then, identify the elements of the generalized admittance coupling matrix as

$$Y_{0,0} = \int_{S_1} \mathbf{N}_0(s') \cdot \left[\frac{1}{\Delta x} \prod \left(\frac{x' - x_0}{\Delta x} \right) \left(\frac{y' - y_0}{\Delta y} \right) \mathbf{x}_0 \right] ds' \quad (3.13)$$

$$Y_{0,n} = \int_{S_1} \mathbf{N}_n(s') \cdot \left[\frac{1}{\Delta x} \prod \left(\frac{x' - x_0}{\Delta x} \right) \left(\frac{y' - y_0}{\Delta y} \right) \mathbf{x}_0 \right] ds' \quad (3.14)$$

$$Y_{m,0} = \int_{S_1} \mathbf{N}_0(s') \cdot \mathbf{e}_m^*(s') ds' \quad (3.15)$$

$$Y_{m,n} = \int_{S_1} \mathbf{N}_n(s') \cdot \mathbf{e}_m^*(s') ds', \quad (3.16)$$

which leads to the following admittance coupling matrix.

$$\bar{\mathbf{Y}} = \begin{bmatrix} Y_{0,0} & Y_{0,1} & Y_{0,2} & \cdots & Y_{0,N} \\ Y_{1,0} & Y_{1,1} & Y_{1,2} & \cdots & Y_{1,N} \\ \vdots & & \ddots & & \vdots \\ Y_{M,0} & Y_{M,1} & Y_{M,2} & \cdots & Y_{M,N} \end{bmatrix} \quad (3.17)$$

The dimension of this matrix is $(N + 1) \times (M + 1)$, since only one port is considered.

Note that these equations can be interpreted in terms of the generalized network representation shown in Fig. 3.2.

Figure 3.2: Multimode Equivalent Network (generalized Y-matrix) representation for the zero-thickness discontinuity in Fig. 3.1.

From Fig. 3.2, we can see that, in this case, the network is written in terms of an admittance coupling matrix [given in equation (3.17)] because the formulation is derived using the electric field boundary conditions. In addition, the network

is in parallel configuration due to the fact that the analyzed discontinuity is zero-thickness. For comparison, the equivalent network derived in Chapter II for thick junctions using a magnetic approach remains in series and is written in terms of an impedance coupling matrix. In addition, we can see that the thick matrix size is twice the size of matrix in equation (3.17).

As observed in equation (3.9), the consideration of the transverse port as excitation of the structure leads to a new integral equation whose excitation is the pulse introduced in equation (3.4). This is an important difference with respect to the traditional discontinuity MEN characterization *Guglielmi et al.* (1999), where the excitations were only the modes on both sides of the discontinuity. This new excitation of the problem also implies new elements ($Y_{0,0}$, $Y_{0,n}$ and $Y_{m,0}$) in the coupling matrix that take into account the interaction between the modes on both sides of the discontinuity and the new lateral port. Moreover, a new input/output port appears in the equivalent network of Fig. 3.2 (at the discontinuity plane) to model the new transverse excitation.

An important issue to consider is how to expand the unknown functions $\mathbf{N}_n(s')$ and $\mathbf{N}_0(s')$ to solve the integral equations (3.8) and (3.9). In this respect, the Method of Moments (MoM) and the Galerkin procedure *Harrington* (1993) are employed. In particular, the electric vector modal functions of the metallization *Melcon et al.* (1999) are used here as basis ($\mathbf{e}_k$) and test ($\mathbf{e}_i$) functions. Their expressions are also given in Appendix A. Following this approach, the unknowns $\mathbf{N}_n(s')$ and $\mathbf{N}_0(s')$ can finally be written as

$$\mathbf{N}_0(s') = \sum_{k=1}^{N_b} \alpha_{0,k} \mathbf{e}_k(s') \quad (3.18)$$

$$\mathbf{N}_n(s') = \sum_{k=1}^{N_b} \alpha_{n,k} \mathbf{e}_k(s'), \quad (3.19)$$

where N_b determines the number of modes that are used to expand the unknown functions in the MoM-Galerkin procedure.

Applying now the test procedure, the integral equations (3.8) and (3.9) lead to the following system of linear equations

$$C_{i,n} = \sum_{k=1}^{N_b} \alpha_{n,k} \sum_{m=N+1}^{+\infty} Z_m^T C_{k,m} C_{i,m} \quad (3.20)$$

$$C_{i,0} = \sum_{k=1}^{N_b} \alpha_{0,k} \sum_{m=N+1}^{+\infty} Z_m^T C_{k,m} C_{i,m}, \quad (3.21)$$

where $C_{k,m}$ and $C_{i,m}$ are the coupling integrals between the localized modes in region (1) and (2), and the obstacle modes used as basis and test functions, respectively.

$$C_{k,m} = \int_{S_1} \mathbf{e}_k(s') \cdot \mathbf{e}_m^*(s') ds' \quad (3.22)$$

$$C_{i,m} = \int_{S_1} \mathbf{e}_i(s) \cdot \mathbf{e}_m(s) ds \quad (3.23)$$

The terms $C_{i,n}$ refer to the coupling integrals between the accessible modes in region (1) and (2) and the (obstacle) modes used as test functions.

$$C_{i,n} = \int_{S_1} \mathbf{e}_i(s) \cdot \mathbf{e}_n(s) ds \quad (3.24)$$

The term $C_{i,0}$, on the other hand, refers to the coupling integral between the test functions and the excitation pulse.

$$C_{i,0} = \int_{S_1} \mathbf{e}_i(s) \cdot \left[\frac{1}{\Delta x} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{x}_0 \right] ds \quad (3.25)$$

All the coupling integrals in equations (3.22), (3.23), (3.24) and (3.25) can be computed analytically for any arbitrary rectangular printed metallization. These integrals are solved in Appendix A. Once the coupling coefficients are computed and the integral equations (3.20) and (3.21) are solved, the elements of the generalized

admittance coupling matrix $\overline{\mathbf{Y}}$ can be calculated as

$$Y_{0,0} = \sum_{k=1}^{N_b} \alpha_{0,k} C_{k,0} \quad ; \quad Y_{0,n} = \sum_{k=1}^{N_b} \alpha_{n,k} C_{k,0} \quad (3.26)$$

$$Y_{m,0} = \sum_{k=1}^{N_b} \alpha_{0,k} C_{k,m} \quad ; \quad Y_{m,n} = \sum_{k=1}^{N_b} \alpha_{n,k} C_{k,m}. \quad (3.27)$$

Note that, with this formulation, we have numerically imposed the boundary conditions that must satisfy the electric field tangential components at the metallization using equation (3.3). The boundary conditions that must satisfy the magnetic field tangential components are imposed in equation (3.7), when the discontinuity in the tangential magnetic field components is set equal to the unknown electric current densities.

We should emphasize that the presented formulation is valid for isotropic piecewise homogeneous substrates. The consideration of anisotropic substrates would require to change the kernel of the integral equations. In addition, the formulation is accurate if the thickness of the metallization is electrically very small, so it can be neglected (zero-thickness approximation).

Application to lossless microstrip circuits

The importance of the presented formulation lies in the fact that single-layer boxed printed circuits including transverse excitation ports, as the one shown in Fig. 3.3, can now be efficiently analyzed using the common frame of the MEN formulation. For this structure, region (2) in Fig. 3.1 is air, while region (1) represents the dielectric substrate. The equivalent network of the device in Fig. 3.3 is shown in Fig. 3.4.

For this analysis, the metallic circuit is characterized through its equivalent network as we have described. Then, the modes in the air region (height l_a) and in the substrate (height l_d) are represented with transmission lines on both sides of the

Figure 3.3: Schematic of the shielded microstrip printed circuit under study, excited with two coaxial connectors.

Figure 3.4: Equivalent network representation for the microstrip circuit shown in Fig. 3.3 using the MEN formalism. The metallization at the interface is characterized by its equivalent admittance coupling matrix. Just one accessible mode is considered, which is characterized with transmission lines in the air (l_a) and dielectric (l_d) regions, terminated with a short-circuit representing the top and bottom PEC covers of the cavity box (see Fig. 3.3). The coaxial excitations are represented with transmission lines of characteristic impedance Z_0 .

discontinuity (see Fig. 3.2). These lines have the propagation constant and the characteristic impedance equal to those of the modes in that medium. As can be seen in Fig. 3.4, they are short-circuited to model the top/bottom covers of the structure. Recall that, for this structure, the input/output coaxial transitions, shown in Fig. 3.3, are represented by the pulse excitations and the transmission line ports of characteristic impedance Z_0 shown in the equivalent network of Fig. 3.4.

An additional very important point that must be noted is that, in this structure, as shown in Fig. 3.4, we only need to consider one accessible mode, independently from the number of modes propagating along z direction. This is because the structure contains only one metallic interface, and therefore the connection to other MENs is not required. This approach is rigorous, if both localized and accessible modes are loaded with the parallel total admittance Y_{tl} shown in Fig. 3.4. As a result, the mesh equations for the equivalent network of Fig. 3.4 are written in matrix form as

$$\begin{bmatrix} I_{p1} \\ I_{p2} \\ \bar{I}_1 \end{bmatrix} = \begin{bmatrix} Y_{p1,p1} & Y_{p1,p2} & Y_{p1,1} \\ Y_{p2,p1} & Y_{p2,p2} & Y_{p2,1} \\ Y_{1,p1} & Y_{1,p2} & Y_{1,1} \end{bmatrix} \cdot \begin{bmatrix} V_{p1} \\ V_{p2} \\ V_1 \end{bmatrix}, \quad (3.28)$$

where only the interactions between the transversal ports $P1$ and $P2$ and the first accessible mode are considered.

To obtain the S-parameters from the equivalent network in Fig. 3.4, one of the pulse ports is loaded with the port impedance (usually $Z_0 = 50 \Omega$), and the other one is taken as the excitation. Since the equivalent admittance coupling matrix $\bar{\mathbf{Y}}$ relates the currents and the voltages in the network ports, as shown in equation (3.28), we can solve the system to obtain the input impedance in the excitation port ($Z_{in} = V_{p1}/I_{p1}$), and the transimpedance between the input and output ports, $P1$ and $P2$ ($Z_{tran} = V_{p2}/I_{p1}$). With this information, the S-parameters can be calculated as

$$S_{11} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \quad (3.29)$$

$$S_{21} = \frac{2Z_{tran}}{Z_{in} + Z_0}. \quad (3.30)$$

It is important to note that, since the coupling integrals for the MEN in Fig. 3.4 are computed analytically, this formulation is restricted to devices that are composed of rectangular metallic patches. However, it allows for the first time the analysis of

planar microwave component using the MEN formalism.

3.2.2 Numerical Results

Example 1: Third order dual-bandpass filter.

To validate the theory presented in the previous section, we first analyze the dual-bandpass filter presented in *Martinez-Mendoza et al. (2008)*. The hybrid waveguide-microstrip structure under analysis is shown in Fig. 3.5. In this filter, two of the resonators are implemented using microstrip lines (L_r in Fig. 3.5) while the other one is implemented using the first resonance of the metallic box.

Figure 3.5: Third order dual-bandpass filter under study. The dimensions (in mm) are: $a = 40$, $b = 34$, $L_{in} = L_{out} = 14$, $L_r = 24.54$, $w_1 = 1.8$, $w_2 = 5$, $L_1 = 3$ and $L_2 = 3.14$. The width of all the microstrip lines is 2 mm. The dielectric relative permittivity $\epsilon_r = 2.2$.

Figure 3.6 shows the filter response computed using the proposed method as compared to the results obtained with commercial full-wave software tools, and to measured results.

As can be noticed, the MEN results show good agreement in comparison with other numerical software codes (ADS and ANSYS HFSS), and also with measurements. It should be noticed that the differences between the simulations and the measurements can be due to the circuit losses and manufacturing issues, which are not considered in the electromagnetic simulations.

Figure 3.6: Comparison of the S-parameters magnitude of the dual-bandpass filter in Fig. 3.5 computed using different full-wave electromagnetic analysis tools (ADS and ANSYS HFSS), and compared to the measured response.

Example 2: Four-pole coupled line filter.

The second example, studied to validate the electric MEN for planar circuits, is presented in Fig. 3.7. As observed, this device consists of a shielded bandpass filter, based on coupled line sections, with two excitation ports in the transverse plane. In this case, the resonators of the filter are implemented using typical microstrip lines. The metallic enclosure is added in this structure to avoid any possible radiation. This filter was proposed in *Melcon et al. (1999)*.

Figure 3.8 shows the results obtained using the electric MEN technique. The simulations obtained by using other commercial tools (ADS and ANSYS HFSS) are also included, together with the measured S-parameters.

For this example, 19000 terms have been included in the static kernel, 3000 in the dynamic part of the kernel and 35 basis functions in the MoM/Galerkin procedure. Just one accessible mode is needed, since the structure contains only one discontinuity (one metallization layer). As we can see, the agreement between MEN results and the

Figure 3.7: Geometry of the four-pole bandpass filter under study. The dimensions (in mm) are: $a = 25.4$, $b = 25.4$, $L_1 = 2.9$, $L_2 = 2.708$, $L_3 = 2.887$, $w_1 = 0.1$, $w_2 = 0.613$, $w_3 = 0.802$, $h_1 = 3.6$ and $h_2 = 0.4$. The width of all the microstrip lines is 0.355 mm. The dielectric relative permittivity is $\epsilon_r = 9.65$.

Figure 3.8: S-parameters magnitude of the coupled-line bandpass filter in Fig. 3.7 using different electromagnetic analysis tools (ADS and ANSYS HFSS), and compared to the measured response.

other simulation responses is also good in this example. The differences with respect to measured results are again essentially due to the circuit losses and manufacturing issues that have not been considered in the simulations.

In this case, using an Intel Xeon E5-2620 v3 @ 2.40 GHz with 32 GB of RAM, the computation time using the MEN approach proposed in this work is about 12.1 seconds for 150 frequency points. It is important to mention that, due to the high

sensitivity of both analyzed examples (Example 1 and 2), some convergence problems have been found in ANSYS HFSS. They have been partially solved by using a selectively refined mesh around the critical gaps of the structure, which takes more than 20 minutes to reach the presented results. On the other hand, ADS seems to achieve very similar results than the new MEN technique, but consuming more computational time in its analysis (about 4 minutes and 5 seconds).

This implies that the proposed technique is not only accurate in the analysis of real single-layer planar printed devices, but also very efficient in terms of computational effort. Even though the geometry under study is restricted to rectangular printed metallizations, a wide class of microwave structures of practical interest can still be analyzed, as it has been shown in this section.

3.3 Extension of the Electric MEN for Multilayered Planar Devices

3.3.1 Theoretical Derivations

In this section, we describe the Multimode Equivalent Network formulation based on the electric field boundary condition applied to the analysis of multilayer boxed microstrip circuits. In particular, we analyze circuits composed of several layers of metallization with a combination of rectangular shaped patches, including also a lateral excitation (ports in the transverse plane). A typical structure under study is shown in Fig. 3.9.

As we can see from Fig. 3.9, the geometry being analyzed consists of different layers of stacked dielectrics separated by metallized interfaces. The structure is excited by lateral coaxial connectors that can be placed in any of the different metallization layers. In order to analyze this structure, each metallic layer in Fig. 3.9 is characterized using the MEN formulation for zero-thickness discontinuities that includes ports

Figure 3.9: Scheme of the geometry under study.

in the transverse plane. This formulation is described in Section 3.2.1.

Once the MEN of each discontinuity in the structure has been computed, the separate networks are conveniently cascaded to obtain the final network shown in Fig. 3.10. This network now describes the complete circuit shown in Fig. 3.9.

Figure 3.10: Multimode Equivalent Network corresponding to the multilayer geometry represented in Fig. 3.9, with two levels of metallization.

As we can see, the accessible modes in each dielectric layer are represented again

in terms of transmission lines (where the propagation constant and the characteristic impedance of the line n are those of the corresponding mode n). Furthermore, in Fig. 3.10, the metallic interfaces that are printed between dielectric media are modeled by their corresponding coupling matrices ($\bar{Y}$ and $\bar{Y}'$). In this case, since the structure contains several metallic interfaces, more than one accessible mode are necessary for a proper characterization of the device. These modes are then connected to the following MEN as shown in Fig. 3.10. It is important to recall that the difference with respect to the traditional MEN formulation *Guglielmi and Oliner* (1989) is that now these metallic interfaces can also contain lateral excitations (modeled by the ports in the transverse plane).

Particularization to devices with three dielectric layers

Let us consider a particular case: a circuit composed of three dielectric layers and two metallizations as the one shown in Fig. 3.11.

Figure 3.11: Scheme of the circuit that contains two metallization level and three dielectric substrates.

In this case, two transverse ports $P1$ and $P2$ have been included. These ports represent the real coaxial excitations that are normally used in boxed microstrip technology. Following the previous procedure, the equivalent network that characterizes the component under study is represented in Fig. 3.12.

As we can see in Fig. 3.12, the top and bottom PEC covers of the box are modeled by terminating the transmission lines representing substrates 1 (h_1, ϵ_{r1}) and 3 (h_3, ϵ_{r3}) with short-circuits. As a consequence, the excitation of the problem is not via the

Figure 3.12: Multimode Equivalent Network of the multilayer geometry in Fig. 3.11.

accessible modes, as in waveguide problems, but through the ports $P1$ and $P2$ that represent the coaxial connectors used as excitation in planar circuits (see Fig 3.12).

Note that in our formulation we use TE and TM modes which are transverse with respect to the axis perpendicular to the dielectric layers (z -axis in Fig. 3.11). These modes do not couple at the interface between two different dielectric layers. Therefore, if no metallic obstacle is present at a dielectric interface, then there are no coupling effects at this interface. This situation is easily tackled in the network of Fig. 3.12 by just connecting, to each accessible mode, a new transmission line with the constitutive parameters of the new medium.

To compute the S-parameters from the network in Fig. 3.12, one of the transverse port is used as excitation and the other one is loaded with the reference impedance

of the port (for us $Z_0 = 50\Omega$). The total matrix ($\bar{\mathbf{Y}}^T$) that models the complete problem can then be written in terms of the following current-voltage relation

$$\begin{bmatrix} I_{p1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{p1,p1}^T & Y_{p1,p2}^T & Y_{p1,1}^T & \cdots & Y_{p1,N}^T \\ Y_{p2,p1}^T & Y_{p2,p2}^T & Y_{p2,1}^T & \cdots & Y_{p2,N}^T \\ Y_{1,p1}^T & Y_{1,p2}^T & Y_{1,1}^T & \cdots & Y_{1,N}^T \\ \vdots & & \ddots & & \vdots \\ Y_{N,p1}^T & Y_{N,p2}^T & Y_{N,1}^T & \cdots & Y_{N,N}^T \end{bmatrix} \cdot \begin{bmatrix} V_{p1} \\ V_{p2} \\ V_1 \\ \vdots \\ V'_N \end{bmatrix}. \quad (3.31)$$

This system relates the external currents of the problem in Fig. 3.12 ($I_n^{(1)}$ and $I_n'^{(2)}$) and the port currents (I_{p1} and I_{p2}) with the external voltages (V_n and V_n') and the port voltages (V_{p1} and V_{p2}). Therefore, the input impedance in port 1 ($Z_{in} = V_{p1}/I_{p1}$) and the transimpedance between port 2 and 1 ($Z_{tran} = V_{p2}/I_{p1}$) can be extracted by solving the system in equation (3.31). The S-parameters are then computed in terms of Z_{in} and Z_{tran} as reported in equations (3.29) and (3.30).

3.3.2 Numerical Results

Example 1: Trisection bandpass filter.

To validate the multilayered MEN theory, the formulation described in the previous subsection is used to perform the analysis of a microwave filter composed of two metallization levels and three dielectric layers (as shown in Fig. 3.11). The metallization layout of both interfaces is shown in Fig. 3.13. This bandpass filter, that was presented for the first time in *Melcon et al.* (1999), implements a trisection topology *Cameron et al.* (2007). As we can see in Fig. 3.13, in this example both excitation ports are placed in the same metallization layer, and on the same side of the box. However, any other arrangement can also be easily analyzed.

The relative dielectric permittivities of the substrates are: $\varepsilon_{r1} = \varepsilon_{r3} = 2.33$ and

Figure 3.13: Layout of the two metallization levels of the bandpass filter under analysis. Gray areas are the metallizations while dark areas represents the excitation pulses. The dimensions (in mm) are: $L_{in} = L_{out} = 23$, $L_{res1} = L_{res2} = 34.2$, $S_1 = 0.1$, $S_2 = 3.2$ $a = 40$, $b = 20$, $L_{res3} = 33.94$. The width of all lines is 1.49 mm.

$\varepsilon_{r2} = 1.07$. The heights of the dielectric layers are: $h_1 = h_3 = 0.51$ mm and $h_2 = 3.30$ mm.

In Fig. 3.14, we show the S-parameters computed using the formulation proposed in Subsection 3.3.1. In this figure, we compare the simulated results to the integral equation analysis and measurements given in *Melcon et al. (1999)*. The structure has also been analyzed using the commercial software tool ADS.

As we can see, the simulated S-parameters are in good agreement with the measurements of the prototype fabricated [see *Melcon et al. (1999)*]. The small differences that can be observed is probably due to manufacturing tolerances, and to the losses in the materials that have been used to manufacture the filter that have not been included in the simulations. The results are also in good agreement with the simulations given in *Melcon et al. (1999)* and to the results obtained using the commercial software ADS. In our code, 33 modes have been used as accessible modes in the equivalent network of Fig. 3.12. After a convergence study, the numerical parameters set for an accurate characterization of each discontinuity are: 125 basis/test functions, 6500 terms in the static kernel and 900 in the dynamic kernel for the first metallization level. For the second metallization we have used 70 basis/test functions, 2000

Figure 3.14: S-parameters magnitude of the bandpass filter in Fig. 3.13 obtained using the presented formulation and compared to the results reported in *Melcon et al.* (1999) and the simulation results obtained using the commercial tool ADS.

terms in the static kernel and 600 terms in the dynamic kernel. The computational time for the analysis of the complete structure is approximately 59 seconds (considering 100 points in the frequency sweep). Using the commercial software ADS, the computational time needed to perform the same analysis is about 6.5 minutes. To analyze the filter with ANSYS HFSS, a very dense mesh needs to be used in the narrow coupling gaps so that, as a result, the analysis takes more than 20 minutes. Despite using this dense mesh, the results are still not convergent in ANSYS HFSS. This is the reason why the ANSYS HFSS results are not reported in Fig. 3.14. We can, therefore, conclude that the analysis of the filter under exam using the proposed MEN formulation is both accurate and computationally very efficient.

3.4 Conclusions

In this chapter, we extend for the first time the electric MEN formulation to the analysis of planar devices. We first describe the electric MEN formulation for the analysis of planar discontinuities based on arbitrary printed rectangular metallizations that contain excitation ports in the transverse plane. The characterization of this basic zero-thickness discontinuity is the basis for the analysis of a very large variety of single-layer boxed microwave components. In fact, the usefulness and efficiency of this novel formulation have been confirmed with two practical microwave filters examples.

Based on this analysis, we have extended the electric MEN formulation to multilayer filter implementations. In order to analyze the complete structure, we first obtain the MEN of each metallization and then, we conveniently cascade the resulting networks. The intermediate dielectric layers are characterized using the corresponding modal transmission lines. In addition to theory, the formulation has been validated through the analysis of a trisection filter designed in boxed multilayer planar technology.

In all the examples analyzed in this chapter, the EM simulation results have shown good agreement with respect to other full-wave software tools and with measurements, thereby fully validating the accuracy and efficiency of the proposed theory.

Scientific Outcome

- Related to Section 3.2: *Gomez-Molina et al.* (2018a).
- Related to Section 3.3: *Gomez-Molina et al.* (2019).

CHAPTER IV

Magnetic Multimode Equivalent Network Technique for Planar Devices

4.1 Introduction

To understand the goal of this chapter, it is important to remember that the original magnetic MEN formulations *Guglielmi et al.* (1999) were developed for the analysis of waveguide components, where the excitations of the problem are the guided modes. Thus, these formulations do not include the ports in the transverse plane that are needed for the excitation of printed microstrip and multilayer circuits. The first MEN formulation applied to planar devices is described in Chapter III, where we derived the electric MEN formulation for the analysis of multilayer planar circuits composed of rectangular printed metallizations. However, this formulation is still limited to areas with a rectangular shape, where entire domain basis functions are known analytically. This is observed in the numerical results of Chapter III, where only coupled line filters were analyzed.

In this context, the goal of this chapter is to extend the method proposed in Chapter III to the case of multilayer circuits that are composed of printed metallic patches with arbitrary shape (like the one shown in Fig. 4.1). For this purpose, the BI-RME method *Cogollos et al.* (2003); *Conciauro et al.* (1984) is used to obtain

the basis functions needed to solve the integral equations with complex (arbitrarily shaped) planar geometries. Note that the BI-RME method has been previously used in MEN formulations applied to the analysis of waveguide discontinuities *Cogollos et al.* (2003). However, this is the first time that a MEN formulation is combined with BI-RME for the analysis of arbitrarily shaped printed metallizations in a multilayered environment. Another important difference with respect to the formulation presented in Chapter III is that, in this chapter, we use a magnetic field formulation to derive the fundamental integral equations. Following this approach, the unknowns of the problem are now magnetic current densities defined in the aperture, instead of the electric current densities induced on the metallic areas. This formulation, therefore, complements the one presented in Chapter III, and allows the analysis of complex multilayered printed circuits by combining electric and magnetic field formulations. The idea of combining electric and magnetic formulations was also proposed in *Guglielmi and Oliner* (1989), in the context of the analysis of zero-thickness discontinuities inside waveguides. With the extension proposed in this chapter, the same concept can now be applied to the analysis of microstrip and multilayered printed circuits. It is also important to note that the ports modeled in the formulation described in this chapter allows to use the MEN technique with coaxial input/output ports, and to connect external lumped-element components to the printed circuits being analyzed.

It is important to mention that the analysis of arbitrary planar structures has already been discussed in the past by several authors *Bozzi et al.* (2001); *Melcon et al.* (1999); *Rautio and Harrington* (1987). In *Melcon et al.* (1999), the authors analyze boxed microwave circuits composed of rectangular printed metallizations, using an electric field integral equation (EFIE). In *Bozzi et al.* (2001), the BI-RME method was efficiently used for the analysis of planar structures with complex geometries. In *Rautio and Harrington* (1987), on the other hand, arbitrary geometries are treated

using subsectional rooftop basis functions, instead of entire domain basis functions as in *Bozzi et al. (2001)*; *Melcon et al. (1999)*. Finally, the analysis of open microstrip structures based on EFIE formulations using closed-form Green's Functions, where rooftop or piecewise-sinusoidal (PS) basis functions are used, is treated in great detail in other related works *Grzegorzcyk and Mosig (2003)*; *Kinayman and Aksun (1997)*; *Marin et al. (1989)*; *Park et al. (1995)*; *Sheng-Gen Pan and Wolff (1994)*. The main difference between the formulation that we discuss in this chapter, and the EFIE techniques discussed in *Bozzi et al. (2001)*; *Melcon et al. (1999)*; *Rautio and Harrington (1987)*, is that we solve the problem in the frame of the MEN representation, so that the analysis of planar circuits can be seamlessly integrated with the analysis of complex waveguide structures and antennas. It is important to note that in the direct EFIE implementations discussed in *Bozzi et al. (2001)*; *Melcon et al. (1999)*; *Rautio and Harrington (1987)*, all the unknowns of the problem are included in one global integral equation. However, it is important to remember that the MEN technique formulates one separate integral equation to characterize each discontinuity with a different multimode coupling matrix. All discontinuities are then coupled together by cascading the different multimode impedance coupling matrices. The result is that, instead of one large global problem, we solve a number of smaller problems that are then properly combined at network level.

We should emphasize that the structures addressed in this chapter are classic multilayered shielded planar circuits. However, the work in Chapter III together with this formulation, can be considered as the first applications of the MEN technique to the analysis of this kind of structures.

To accomplish the goal of this chapter, we first describe the theoretical details of the MEN formulation applied to the analysis of zero-thickness discontinuities that contain ports in the transverse plane in Subsection 4.2.1. Then, we apply this formulation to single-layer planar devices. In particular, we put special emphasis on

how ports in the transverse plane (either external or internal) can be integrated with a magnetic field formulation. To validate the theory, numerical results for several printed microstrip structures (including internal ports) are then discussed in Subsection 4.2.2. Based on this formulation, a Graphical User Interface (GUI) has been created in MATLAB (*MATLAB*) (2020) for the design of microstrip components. This tool combines the single-layer magnetic MEN explained in this chapter with an optimizer algorithm that carries out the variables optimization. This GUI is explained in detail in Appendix B.

Once the formulation for single-layer devices is validated, we extend the formulation to multilayer circuits in Subsection 4.3.1. As previously, in this case, the technique begins with the individual MEN characterization of the different planar circuits (or *discontinuities*) that compose the structure under study. The various MENs are then properly combined to perform the analysis of the complete device. Some numerical examples that contain more than one metallic interface are analyzed in Subsection 4.3.2 to validate this theory. Finally, the main conclusions are outlined in Section 4.4.

4.2 Magnetic MEN for Single-layer Planar Devices

4.2.1 Theoretical Derivations

In this section, we formulate the rigorous MEN representation of a two-dimensional zero-thickness obstacle consisting of arbitrarily shaped printed metallizations (see Fig. 4.1). The theoretical derivations also include internal and external lumped ports in the transverse plane, as shown in Fig. 4.1 (dark areas). It should be emphasized that these ports model the coaxial input/output connectors used to excite real microwave components *Edwards and Steer* (2013); *Sánchez-Soriano et al.* (2018).

For the sake of simplicity, however, just one lumped excitation port is considered

Figure 4.1: Two-dimensional zero-thickness printed circuit in planar technology. The gray areas correspond to the printed metallizations and the dark areas to the excitation and internal ports, both placed on the transverse ($z=0$) plane. The media on both sides of the discontinuity can be different, for instance, the substrate and the air of a microstrip structure. The size of the ports along the x and y -axis is denoted as Δx and Δy , respectively.

in this formulation. The extension to more than one port (either external, or internal for inter-connection purposes) is straightforward.

The formulation starts by imposing the continuity of the tangential components of the magnetic field $\mathbf{H}_t^{(\delta)}$ in the aperture at $z = 0$, including the excitation contribution

$$\mathbf{H}_t^{(1)}(s) - \mathbf{H}_t^{(2)}(s) = \mathbf{z}_0 \times \mathbf{J}_{exc}(s), \quad (4.1)$$

where $\delta = 1$ or $\delta = 2$ for $z < 0$ or $z > 0$, respectively, and $s = (x, y)$ indicates a point in the junction cross-section of Fig. 4.1. The main difference with respect to the traditional MEN technique for waveguide problems is that in equation (4.1) an excitation port has been accounted for by including the surface electric current density $\mathbf{J}_{exc}$. For this excitation current density, we use a pulse excitation model *Rautio and Harrington* (1987). It has been shown that this model can correctly represent the coaxial connectors normally used as excitation in real shielded planar structures. A delta-gap model *Eleftheriades and Mosig* (1996) has also been used in the past for this purpose. However, the difference between pulse and a delta-gap model is that the pulse model includes also the effect of the width (Δy) along the y -axis in the excitation (see Fig. 4.1). Using the pulse model, a current I_0 , constant along the

width of a pulse function, is assumed to be applied at the exciting port. Therefore, the resulting expression for the surface electric current density $\mathbf{J}_{exc}$ is

$$\begin{aligned}\mathbf{J}_{exc}(s) &= -I_0 \frac{1}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \prod \left(\frac{y - y_0}{\Delta y} \right) \mathbf{x}_0 \\ &= -I_0 \frac{1}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{x}_0,\end{aligned}\tag{4.2}$$

where (x_0, y_0) is the coordinate of the center of the pulse, and Δx and Δy are the widths of the pulse in the x -axis and y -axis, respectively (see Fig. 4.1). Note that since in this example the input line is directed along the x -axis, the current exciting the circuit only has x -component. However, the scattered field in the aperture is modeled with all vector components. Therefore, the induced currents in the metallizations will be correctly modeled with all needed vector components.

The amplitude of the pulse is normalized with the $1/\Delta y$ factor to ensure that the power flow P_{exc} defined as

$$P_{exc} = \frac{1}{2} \int_{S_{exc}} (\mathbf{E}_{exc} \times \mathbf{H}_{exc}^*) \cdot \mathbf{z}_0 dS\tag{4.3}$$

is equal to that associated to the equivalent transmission line P_0 defined as

$$P_0 = \frac{1}{2} V_0 I_0^*\tag{4.4}$$

where V_0 is the voltage across the port, and I_0 is the port current. Moreover, the integral in equation (4.3) extends to the pulse area denoted as $(S_{exc} = \Delta x \Delta y)$.

By using this excitation model, and the well known modal expansion formalism *Guglielmi et al.* (1999), equation (4.1) can be written as

$$\sum_{m=1}^{+\infty} I_m^{(1)} \mathbf{h}_m^{(1)}(s) - \sum_{m=1}^{+\infty} I_m^{(2)} \mathbf{h}_m^{(2)}(s) = \frac{-I_0}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0,\tag{4.5}$$

where $\mathbf{h}_m^{(\delta)}(s)$ represents the magnetic vector modal function of mode m in the medium (δ) , and $I_m^{(\delta)}$ refers to the modal currents. It is important to note that m refers to all TE and TM modes in the rectangular waveguide of cross-section $(a \times b)$ of Fig. 4.1.

To continue, we now separate the modes which contribute to the storage of energy (*localized modes*) from the modes that contribute to the energy exchange between cascaded discontinuities (*accessible modes*) Gomez-Molina *et al.* (2018b). This is done by splitting the infinite modal series into two terms, thus obtaining

$$\begin{aligned} & \sum_{n=1}^{N(1)} I_n^{(1)} \mathbf{h}_n^{(1)}(s) - \sum_{n=1}^{N(2)} I_n^{(2)} \mathbf{h}_n^{(2)}(s) + \frac{I_0}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0 \\ &= \sum_{m=N(1)+1}^{+\infty} \frac{V_m^{(1)}}{Z_m^{(1)}} \mathbf{h}_m^{(1)}(s) + \sum_{m=N(2)+1}^{+\infty} \frac{V_m^{(2)}}{Z_m^{(2)}} \mathbf{h}_m^{(2)}(s), \end{aligned} \quad (4.6)$$

where $N(\delta)$ is the number of accessible modes in each region.

In equation (4.6), $V_m^{(\delta)}$ represents the modal voltage and $Z_m^{(\delta)}$ the characteristic modal impedance in medium (δ) . Note that the localized modes have been loaded with their corresponding characteristic modal impedance. For the sake of clarity, the subscript on the left hand side in equation (4.6) has been changed from m to n .

We now recognize that, since the metallic circuit is zero-thickness, and the waveguide cross-section in region (1) is identical to the one in region (2), the magnetic vector modal functions in region (1) ($\mathbf{h}_m^{(1)}$) are identical to the ones in region (2) ($\mathbf{h}_m^{(2)}$). Then, knowing that $\mathbf{h}_m^{(1)}(s) = \mathbf{h}_m^{(2)}(s) = \mathbf{h}_m(s)$ and $V_m^{(1)} = V_m^{(2)} = V_m$, equation (4.6) is transformed into

$$\sum_{n=1}^N \bar{I}_n \mathbf{h}_n(s) + I_0 \frac{1}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0 = \sum_{m=N+1}^{+\infty} V_m Y_m^T \mathbf{h}_m(s), \quad (4.7)$$

where $\bar{I}_n$ represents the total modal current at the interface, defined as $\bar{I}_n = I_n^{(1)} - I_n^{(2)}$. Also, Y_m^T is the total characteristic modal admittance defined as $Y_m^T = Y_m^{(1)} + Y_m^{(2)}$, where $Y_m^{(\delta)}$ represents the characteristic modal admittance of the mode m in the

medium (δ). Note that the number of accessible modes in both regions is assumed to be equal, $N(1) = N(2) = N$, although a different number of accessible modes can also be easily considered.

To continue the formulation, we now recall the following relation between the modal voltage V_m and the electric field in the aperture $\mathbf{E}(s')$

$$V_m = \int_{ap} [\mathbf{z}_0 \times \mathbf{E}(s')] \cdot \mathbf{h}_m^*(s') ds', \quad (4.8)$$

where the integral extends over the entire aperture (ap) at the junction (see Fig. 4.1) and $ds' = dx' dy'$.

Due to the linearity of the problem, the unknown transverse electric field in the aperture must be linearly proportional to the excitations, so that it can be expanded into a sum of partial surface magnetic current densities $\mathbf{M}_n(s')$ and $\mathbf{M}_0(s')$ as follows

$$[\mathbf{z}_0 \times \mathbf{E}(s')] = \sum_{n=1}^N \bar{I}_n \mathbf{M}_n(s') + I_0 \mathbf{M}_0(s'), \quad (4.9)$$

where $\mathbf{M}_n$ and $\mathbf{M}_0$ are the unknown functions of the problem. Substituting now equation (4.9) into equation (4.8), and by using the resulting equation in equation (4.7), we can finally obtain the fundamental integral equations of our problem

$$\mathbf{h}_n(s) = \int_{ap} \bar{\mathbf{K}}(s, s') \cdot \mathbf{M}_n(s') ds' \quad (4.10)$$

$$\frac{1}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0 = \int_{ap} \bar{\mathbf{K}}(s, s') \cdot \mathbf{M}_0(s') ds', \quad (4.11)$$

where $\bar{\mathbf{K}}(s, s')$ is the kernel of the integral equations

$$\bar{\mathbf{K}}(s, s') = \sum_{m=N+1}^{+\infty} Y_m^T \mathbf{h}_m(s) \mathbf{h}_m^*(s'). \quad (4.12)$$

In order to increase the efficiency of the formulation we have just outlined, we also use the Kummer's transformation described in Chapter II for the acceleration of the kernel of the integral equations [shown in equation (4.12)]. Using this strategy, the kernel is divided into static and dynamic parts. As a result, the dynamic part, that needs to be recomputed per each frequency point, exhibits now a faster convergence rate. For more detail about Kummer's transformation, see Chapter II.

To complete the rigorous network formulation, equation (4.9) is used in equation (4.8) to write the modal voltages in terms of the modal currents

$$V_0 = Z_{0,0} I_0 + \sum_{n=1}^N Z_{0,n} \bar{I}_n \quad (4.13)$$

$$V_m = Z_{m,0} I_0 + \sum_{n=1}^N Z_{m,n} \bar{I}_n, \quad (4.14)$$

where $Z_{0,0}$, $Z_{0,n}$, $Z_{m,0}$ and $Z_{m,n}$ are the elements of the generalized impedance coupling matrix $\bar{\mathbf{Z}}$ that are given by

$$Z_{0,0} = \int_{ap} \mathbf{M}_0(s') \cdot \left(\frac{1}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0 \right) ds' \quad (4.15)$$

$$Z_{0,n} = \int_{ap} \mathbf{M}_n(s') \cdot \left(\frac{1}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0 \right) ds' \quad (4.16)$$

$$Z_{m,0} = \int_{ap} \mathbf{M}_0(s') \cdot \mathbf{h}_m^*(s') ds' \quad (4.17)$$

$$Z_{m,n} = \int_{ap} \mathbf{M}_n(s') \cdot \mathbf{h}_m^*(s') ds'. \quad (4.18)$$

The resulting multimode impedance coupling matrix is of size $(N + 1) \times (N + 1)$. We

can therefore write

$$\bar{\mathbf{Z}} = \begin{bmatrix} Z_{0,0} & Z_{0,1} & Z_{0,2} & \cdots & Z_{0,N} \\ Z_{1,0} & Z_{1,1} & Z_{1,2} & \cdots & Z_{1,N} \\ \vdots & & \ddots & & \vdots \\ Z_{N,0} & Z_{N,1} & Z_{N,2} & \cdots & Z_{N,N} \end{bmatrix}. \quad (4.19)$$

In network terms, we finally obtain the representation shown in Fig. 4.2.

Figure 4.2: Multimode Equivalent Network (generalized Z -matrix) representation for the zero-thickness discontinuity with arbitrarily shaped metallic areas, shown in Fig. 4.1, including one port in the transverse plane.

Since the discontinuity is zero-thickness and we are using a magnetic approach, the resulting network remains in parallel configuration (as observed in Fig. 4.2), and it is represented by an impedance coupling matrix.

As observed in equation (4.11), the consideration of the transverse port as excitation of the structure leads to a new integral equation, whose excitation is the pulse introduced in equation (4.1). This is an important difference with respect to the traditional thick discontinuity magnetic characterization *Guglielmi et al.* (1999), normally used in the analysis of waveguide components (described in Chapter II).

This new excitation of the problem also implies new elements ($Z_{0,0}$, $Z_{0,n}$ and $Z_{m,0}$) in the coupling matrix that take into account the interaction between the modes on both sides of the discontinuity and the new lateral port. Again, we verify that, for thick discontinuities, the size of the coupling matrix is twice the zero-thickness matrix size. Finally, we can note that a new input/output port appears in the equivalent network of Fig. 4.2 (at the discontinuity plane) to model the new transverse excitation.

In order to solve the integral equations (4.10) and (4.11), we now expand the unknown partial magnetic current functions $\mathbf{M}_n(s')$ and $\mathbf{M}_0(s')$ using the Method of Moments (MoM) *Guglielmi et al.* (1999). In particular, we use the magnetic vector modal functions in the aperture at the plane of the junction both as basis ($\mathbf{h}_k^B$) and test ($\mathbf{h}_i^B$) functions, applying the Galerkin method *Guglielmi et al.* (1999). Following this approach, the unknowns $\mathbf{M}_n(s')$ and $\mathbf{M}_0(s')$ can finally be written as

$$\mathbf{M}_0(s') = \sum_{k=1}^{N_b} \alpha_{0,k} \mathbf{h}_k^B(s') \quad (4.20)$$

$$\mathbf{M}_n(s') = \sum_{k=1}^{N_b} \alpha_{n,k} \mathbf{h}_k^B(s'), \quad (4.21)$$

where N_b is the numerical parameter that determines the number of basis and test functions used in the MoM procedure, and $\alpha_{0,k}$ and $\alpha_{n,k}$ are the unknown coefficients of the expansions.

Applying the test procedure, the integral equations (4.10) and (4.11) are finally transformed into the following system of linear equations

$$C_{i,n} = \sum_{k=1}^{N_b} \alpha_{n,k} \sum_{m=N+1}^{+\infty} Y_m^T C_{k,m} C_{i,m} \quad (4.22a)$$

$$C_{i,0} = \sum_{k=1}^{N_b} \alpha_{0,k} \sum_{m=N+1}^{+\infty} Y_m^T C_{k,m} C_{i,m}. \quad (4.22b)$$

The coefficients $C_{k,m}$ and $C_{i,m}$ represent the coupling integrals between the localized modes of the rectangular waveguide, and the aperture modes used as basis and test functions, respectively.

$$C_{k,m} = \int_{ap} \mathbf{h}_k^B(s') \cdot \mathbf{h}_m^*(s') ds' \quad (4.23)$$

$$C_{i,m} = \int_{ap} \mathbf{h}_i^B(s) \cdot \mathbf{h}_m(s) ds \quad (4.24)$$

The terms $C_{i,n}$ are the coupling integrals between the accessible modes of the rectangular waveguide and the aperture modes used as test functions.

$$C_{i,n} = \int_{ap} \mathbf{h}_i^B(s) \cdot \mathbf{h}_n(s) ds \quad (4.25)$$

The term $C_{i,0}$, on the other hand, refers to the coupling integral between the test functions and the excitation pulse.

$$C_{i,0} = \int_{ap} \mathbf{h}_i^B(s) \cdot \left(\frac{1}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0 \right) ds \quad (4.26)$$

It is important to mention that, since the metallizations at the junction are arbitrarily shaped, the coupling integrals in equations (4.23), (4.24) and (4.25), that are related to the modes in the aperture, can conveniently be computed numerically using the BI-RME method *Cogollos et al.* (2003); *Conciauro et al.* (1984). The coupling integrals $C_{i,0}$ can then be computed analytically using the numerical expansion in terms of rectangular waveguide modes of the basis functions $[\mathbf{h}_i^B(s)]$, which are also provided by the BI-RME method. To illustrate the procedure, let $\mathbf{h}_m(s)$ be the rectangular waveguide modes used to expand the arbitrary aperture modes $\mathbf{h}_i^B(s)$.

Using the BI-RME method we obtain the following expression

$$\mathbf{h}_i^B(s) = \sum_{m=1}^{N_k} C_{i,m} \mathbf{h}_m(s), \quad (4.27)$$

where $C_{i,m}$ are the expansion coefficients calculated by BI-RME [the same ones as those in equation (4.24)]. According to equation (4.27), the coupling integral in equation (4.26) can be computed as

$$C_{i,0} = \sum_{m=1}^{N_k} C_{i,m} \int_{ap} \mathbf{h}_m(s) \cdot \left(\frac{1}{\Delta y} \prod \left(\frac{x-x_0}{\Delta x} \right) \left(\frac{y-y_0}{\Delta y} \right) \mathbf{y}_0 \right) ds, \quad (4.28)$$

where the remaining integral is now analytical. The solution of this integral is given in Appendix A.

According to this notation, the elements of the generalized impedance coupling matrix $\bar{\mathbf{Z}}$ can be rewritten in a more compact form as

$$Z_{0,0} = \sum_{k=1}^{N_b} \alpha_{0,k} C_{k,0} \quad ; \quad Z_{0,n} = \sum_{k=1}^{N_b} \alpha_{n,k} C_{k,0} \quad (4.29a)$$

$$Z_{m,0} = \sum_{k=1}^{N_b} \alpha_{0,k} C_{k,m} \quad ; \quad Z_{m,n} = \sum_{k=1}^{N_b} \alpha_{n,k} C_{k,m}. \quad (4.29b)$$

Note that, with this formulation, we have numerically imposed the boundary conditions that must satisfy the magnetic field tangential components at the aperture using equation (4.1). The continuity of the electric field tangential components is implicitly imposed when the modal voltages on both sides of the discontinuity are assumed to be equal $V_m^{(1)} = V_m^{(2)} = V_m$ (see Fig. 4.2).

It is also important to note that the circuit in Fig. 4.1 is only an example of a possible microwave circuit. The formulation presented in this chapter is not restricted to any particular shape. In fact, the shape can be arbitrary thanks to the use of the BI-RME method *Cogollos et al.* (2003); *Conciauro et al.* (1984). In addition, we

should emphasize again that the presented formulation is valid for isotropic piecewise homogeneous substrates. The consideration of anisotropic substrates would require to change the kernel of the integral equations. Finally, we should mention that this formulation is accurate if the thickness of the metallization is electrically very small. For large thicknesses, the formulation presented may not be accurate, and an alternative formulation based on thick discontinuities, as the one presented in Chapter VI, may be more suitable. A deep comparison between these two formulations is addressed in Chapter VI.

Application to lossy microstrip circuits

The proposed MEN formulation is now particularized to the analysis of typical shielded printed microstrip circuits. In this case, one single metallization interface will be considered between two different media, together with two lumped ports, which represent the coaxial input/output connectors. For this structure, region (2) in Fig. 4.1 is air, while region (1) represents the dielectric substrate. A schematic of the structure under analysis is shown in Fig. 4.3. The inclusion of losses in the structure under analysis is also discussed.

Figure 4.3: Schematic of the shielded microstrip printed circuit under study, excited with two coaxial connectors.

Once we have characterized the metallization at the interface using the proposed MEN technique, the modes in the air region (height l_a) and in the substrate (height l_d) are represented with transmission lines on both sides of the discontinuity (see Fig. 4.2). As a result, the equivalent circuit of Fig. 4.3 becomes the one shown in

Fig. 4.4.

Figure 4.4: Equivalent network representation for the microstrip circuit shown in Fig. 4.3 using the MEN formalism. The metallization at the interface is characterized by its equivalent impedance coupling matrix. Just one accessible mode is considered, which is characterized with transmission lines in the air (l_a) and dielectric (l_d) regions, terminated with the impedance Z_s representing losses in the top and bottom covers of the cavity box (see Fig. 4.3). The coaxial excitations are represented with transmission lines of characteristic impedance Z_0 .

As we can see in Fig. 4.4, the transmission lines that represent the accessible modes are terminated by a surface impedance Z_s that models the losses in the top/bottom covers of the shielding cavity *Rautio and Demir* (2003). Thus, Z_s can be zero for a Perfect Electric Conductor (PEC) box, or it can be computed as

$$Z_s = (1 + j) \sqrt{\frac{w\mu}{2\sigma_{box}}}, \quad (4.30)$$

if the box is considered to be lossy. In this expression, σ_{box} is the finite conductivity of the metallic shielding box. Moreover, w is the angular frequency and μ is the permeability.

In addition, losses in the substrate [region (1)] can be taken into account by using

a complex propagation constant in the transmission line model of each mode

$$\gamma_m^{(1)} = \sqrt{k_{cm}^2 - k_0^2 \varepsilon' (1 - j \tan \delta)} = \alpha_{dm}^{(1)} + j\beta_m^{(1)}, \quad (4.31)$$

where k_{cm} is the cut-off wavenumber of the mode m and k_0 is the free space wavenumber. In equation (4.31), $\varepsilon' = \varepsilon_0 \varepsilon_r$ and the loss tangent ($\tan \delta$) models the losses in the dielectric substrate.

Losses in the lateral metallic walls of the box can also be considered by adding an attenuation term α_{cm} , due to the finite conductivity of the lateral walls of the box, to the complex propagation constants of the transmission lines $\gamma_m^{(1)} = \alpha_{cm}^{(1)} + \alpha_{dm}^{(1)} + j\beta_m^{(1)}$. This attenuation term can be calculated as described in *Marcuvitz* (1986) for each mode of the rectangular waveguide.

The last contribution to the losses in the structure of Fig. 4.3 is due to the printed metallizations. In *Taroncher et al.* (2005), a formulation was developed to account for these losses in the impedance matrix of the discontinuity. However, in *Taroncher et al.* (2005) the authors show that the lossy magnetic formulation does not offer accurate results, while increasing the computational time. Furthermore, in most practical microstrip circuits, for an accurate prediction of the losses, the fine tuning of the various loss constants it is almost always required. Therefore, the exact computation of the losses in the metallization is not included in this section. In this chapter, the small contribution to the total losses due to the metallic circuit(s) are very effectively accounted for by the losses in the dielectric(s), and the box. A deep study on the consideration of these losses is explained in Chapter V. In Chapter V, we formulate an equivalent problem to consider these losses at the aperture, where the boundary conditions are imposed when the magnetic formulation is employed.

An additional very important point that must be noted is that, in this structure, as shown in Fig. 4.4, we only need to consider one accessible mode. This is because the

structure contains only one metallic interface, and therefore the connection to other MENs is not required. This approach is rigorous, if both localized and accessible modes are loaded with the parallel total impedance Z_{tl} shown in Fig. 4.4. As a result, the mesh equations for the equivalent network of Fig. 4.4 are written in matrix form as

$$\begin{bmatrix} V_{p1} \\ V_{p2} \\ V_1 \end{bmatrix} = \begin{bmatrix} Z_{p1,p1} & Z_{p1,p2} & Z_{p1,1} \\ Z_{p2,p1} & Z_{p2,p2} & Z_{p2,1} \\ Z_{1,p1} & Z_{1,p2} & Z_{1,1} \end{bmatrix} \cdot \begin{bmatrix} I_{p1} \\ I_{p2} \\ \bar{I}_1 \end{bmatrix}, \quad (4.32)$$

where only the interactions between the transversal ports $P1$ and $P2$ and the first accessible mode are considered. Recall that, for this structure, the input/output coaxial transitions, shown in Fig. 4.3, are represented by the pulse excitations and the transmission line ports of characteristic impedance Z_0 shown in the equivalent network of Fig. 4.4.

To obtain the S-parameters from the equivalent network in Fig. 4.4, one of the pulse ports is loaded with the port impedance (usually $Z_0 = 50 \Omega$), and the other one is taken as the excitation. Since the equivalent impedance coupling matrix $\bar{\mathbf{Z}}$ relates the voltages and the currents in the network ports, as shown in equation (4.32), we can solve the system to obtain the input admittance in the excitation port ($Y_{in} = I_{p1}/V_{p1}$), and the transadmittance between the input and output ports, $P1$ and $P2$ ($Y_{tran} = I_{p2}/V_{p1}$). With this information, the S-parameters can be calculated as

$$S_{11} = \frac{1 - Z_0 Y_{in}}{1 + Z_0 Y_{in}} \quad (4.33)$$

$$S_{21} = Y_{tran} \frac{(-2Z_0)}{1 + Z_0 Y_{in}}. \quad (4.34)$$

An advantage with respect to the electric MEN formulation presented in Chapter III is that, this formulation allows the analysis of a larger variety of microwave components. This is because the shape of the metallization can be arbitrary (not restricted

to any particular shape as happened in Chapter III). On the contrary, the presented formulation will require to compute the modes in the aperture numerically, using BI-RME, which increases the total computational time with respect to the formulation in Chapter III (that all the modes are known analytically).

Finally, it is important to note that this particular formulation (without considering losses) has been used to program the GUI explained in Appendix B, for the efficient analysis and design of microstrip shielded components. For the design process, an optimizer, that runs this MEN formulation several times, is included.

4.2.2 Numerical Results

To validate the theoretical derivations presented in Subsection 4.2.1, several examples of practical microwave printed circuits are analyzed in this section using the MEN formulation. The results are compared to the simulations obtained using other commercial software tools such as ANSYS HFSS, other numerical techniques and measurements from the prototypes.

Example 1: Lowpass microstrip filter.

The first example consists of a stepped-impedance lowpass microstrip filter, first presented in *Eleftherlades et al.* (1995). The structure of the filter under analysis is shown in Fig. 4.5. The dark areas correspond to the lumped ports.

In Fig. 4.6 we show the filter response calculated with the MEN formulation proposed in this chapter. In the same figure, the results obtained with a commercial software tool (ANSYS HFSS) and the measured results are also reported.

As we can see, the MEN results are in good agreement with the response provided by ANSYS HFSS, and also with measurements. The peak around 3 GHz is observed both in ANSYS HFSS and our MEN code results, which indicates that it is not a numerical error but most probably a box resonance. To obtain this result, we

Figure 4.5: Lowpass microstrip filter under study. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 67.5 \text{ mm} \times 67.5 \text{ mm}$. With respect to Fig. 4.3, the value of l_a is 9.83 mm. The dielectric relative permittivity is $\epsilon_r = 2.33$ and the substrate thickness is $l_d = 1.57 \text{ mm}$. The filter is centered in the box.

Figure 4.6: Scattering parameters magnitude of the lowpass filter in Fig. 4.5 computed using the MEN formulation, using a finite elements based electromagnetic analysis tool (ANSYS HFSS), and compared to the measured response.

have used 4000 terms in the kernel [3900 in the static part and 100 in the dynamic part of the kernel *Gomez-Molina et al. (2017)*]. Furthermore, 600 basis and test

functions (computed using the BI-RME method) have been used in the MoM-Galerkin procedure. Losses have not been included in this MEN simulation.

From the computational point of view, we first execute the BI-RME module to obtain the relevant numerical coupling integrals and then run our MEN code. It is important to note that the static part of the kernel and the coupling integrals are not re-computed for each frequency point during the frequency sweep, since they are frequency independent. For the results presented above, using an Intel (R) Core^(TM) i7-6500U CPU @ 2.50 GHz with 12 GB of RAM, the computational time required to obtain the coupling integrals is less than 3 minutes. The MEN code takes about 11.85 seconds for all the frequency sweep (100 points). Even though ANSYS HFSS implements other numerical techniques to solve the same problem, the computational time for achieving the results presented in this section is also given for completeness. In this case, ANSYS HFSS takes 9 minutes and 3 seconds to obtain the response reported in Fig. 4.6.

Example 2: Hairpin bandpass filter in microstrip technology.

The next example is presented in Fig. 4.7. This is a hairpin microstrip bandpass filter with two excitation ports in the transverse plane, originally proposed in *Hong* (2011).

Figure 4.8 shows the in-band response obtained using the MEN formulation. In this figure, the MEN results are compared to those obtained using the commercial software ANSYS HFSS and the simulated results from *Hong* (2011). No losses are considered in any of the simulation results. For this example, we used 5000 terms in the kernel [4500 in the static part and 500 in the dynamic part of the kernel *Gomez-Molina et al.* (2017)] and 310 basis and test functions in the MoM-Galerkin procedure. The computational time required for the MEN simulation is about 6.08 seconds (200 frequency points). It is important to mention that to obtain accurate results with

Figure 4.7: Hairpin microstrip bandpass filter geometry under study. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 31.2 \text{ mm} \times 30 \text{ mm}$. With respect to Fig. 4.3, the value of l_a is 3.73 mm. The dielectric relative permittivity is $\epsilon_r = 6.15$ and the substrate thickness is $l_d = 1.27 \text{ mm}$. The filter is centered in the box.

Figure 4.8: In-band S-parameters magnitude of the hairpin bandpass filter shown in Fig. 4.7, using the proposed MEN formulation, and using both the electromagnetic analysis tool ANSYS HFSS and simulation results reported in *Hong (2011)*.

ANSYS HFSS, we need to use, in this case, a selectively refined mesh around the critical gaps. This increases to more than 30 minutes the computational time required by ANSYS HFSS to obtain the results presented in Fig. 4.8. Additionally, Fig. 4.9

shows the out-of-band response of this filter. As we can see, in both the in-band

Figure 4.9: Out-of-band S-parameters magnitude of the hairpin bandpass filter shown in Fig. 4.7, using the proposed MEN formulation and the electromagnetic analysis tool ANSYS HFSS.

and out-of-band simulations, the MEN results show good agreement with respect to ANSYS HFSS and the simulation data reported in *Hong* (2011).

Example 3: Dual-mode bandpass filter.

The third example is the analysis of a dual-mode microstrip filter, first presented in *Hong et al.* (2007). The geometry of the microstrip circuit with the excitation port locations (dark areas) is shown in Fig. 4.10. In Fig. 4.11 we present the passband response obtained using the proposed MEN formulation. In this case, losses are included in the MEN analysis as explained in Subsection 4.2.1. Details of the insertion losses in the passband are shown in the inset of Fig. 4.11, giving $IL = 0.79$ dB. In particular, the following values have been used for modeling the losses: $\tan \delta = 0.0047$ and $\sigma_{box} = 3.8 \cdot 10^7$ S/m (aluminum). The response obtained is compared to that provided by ANSYS HFSS and also to the simulations and measurements reported in

Figure 4.10: Dual-mode microstrip bandpass filter under study. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 25 \text{ mm} \times 20 \text{ mm}$. With respect to Fig. 4.3, the value of l_a is 8.73 mm. The dielectric relative permittivity is $\epsilon_r = 10.8$ and the substrate thickness is $l_d = 1.27 \text{ mm}$. The filter is centered in the box.

Hong et al. (2007). The simulations reported in *Hong et al.* (2007) have been obtained using the commercially available full-wave electromagnetic simulator Sonnet (*Sonnet*) (2020). As it is reported in *Hong et al.* (2007), all the simulations are normalized in the frequency axis using $f_0 = 1.07 \text{ GHz}$ (design center frequency) and the measurements are normalized using $f_0 = 0.9975 \text{ GHz}$ (measured center frequency).

To obtain the in-band response, 4000 terms have been used in the kernel [3700 in the static part and 300 in the dynamic part of the kernel *Gomez-Molina et al.* (2017)]. In the MoM-Galerkin method, 450 basis and test functions (computed using the BI-RME method) have been used. The computational time required to obtain the coupling integrals is less than 6 minutes. The MEN code takes about 11.41 seconds for the complete frequency sweep (120 points). To achieve the results presented in Fig. 4.11, ANSYS HFSS takes 23 minutes and 6 seconds. Finally, the out-of-band response is shown in Fig. 4.12, where the results obtained using the MEN technique are also compared to the ANSYS HFSS response and the data from *Hong et al.* (2007). As we can see, the results are also in good agreement with respect to the theoretical and experimental data retrieved from *Hong et al.* (2007).

Figure 4.11: In-band S-parameters magnitude of the dual-mode bandpass filter shown in Fig. 4.10, using the proposed lossy MEN formulation, and the electromagnetic analysis tool ANSYS HFSS. Results of simulations (using Sonnet) and measurements from *Hong et al. (2007)* are also included.

Figure 4.12: Out-of-band S-parameters magnitude of the dual-mode bandpass filter shown in Fig. 4.10, using the proposed lossy MEN formulation, and the electromagnetic analysis tool ANSYS HFSS. Results of simulations (using Sonnet), and measurements from *Hong et al. (2007)* are also included.

Example 4: Wilkinson power divider.

The fourth example is the Wilkinson power divider shown in Fig. 4.13. This circuit has been chosen to validate the formulation for devices containing internal ports. In this circuit, the resistor R is modeled as an internal port (hatched area) with reference impedance equal to $Z_p = R = 2 Z_0 = 100 \Omega$. In addition, three external pulses (dark areas) are included to model the three external ports (Port 1, Port 2 and Port 3) of the power divider.

Figure 4.13: Wilkinson power divider in microstrip technology. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 30 \text{ mm} \times 16 \text{ mm}$. The value of l_a in Fig. 4.3 is 3 mm. The dielectric relative permittivity is $\epsilon_r = 4$ and the substrate thickness is $l_d = 1.2 \text{ mm}$. The divider is centered in the box.

In Fig. 4.14, we show the response of the structure obtained using the MEN formulation and we compare it to the ANSYS HFSS results when the divider is excited through Port 1. As we can see, the obtained S-parameters are in good agreement with the ANSYS HFSS simulation. At the design frequency, half of the input power goes to Port 2 ($|S_{21}| \approx -3 \text{ dB}$) while the other half goes to Port 3 ($|S_{31}| \approx -3 \text{ dB}$). Port 1 exhibits good matching ($|S_{11}| < -20 \text{ dB}$). In this case the resistor is not influencing the response. To further validate the internal port performance, in Fig. 4.15 we show the response obtained using the MEN formulation and we compare it to the

Figure 4.14: S-parameters magnitude of the power divider, shown in Fig. 4.13, using the proposed MEN formulation and the commercial software ANSYS HFSS. The excitation is through Port 1.

ANSYS HFSS results, when the structure is excited through Port 2. As we can see,

Figure 4.15: S-parameters magnitude of the power divider, shown in Fig. 4.13, using the proposed MEN formulation and the commercial software ANSYS HFSS. The excitation is through Port 2.

good agreement between our results and the ANSYS HFSS results is obtained also

in this case. From Fig. 4.15 we can verify the good performance of the internal port (used to connect the resistor), since half of the power goes from Port 2 to Port 1 ($|S_{12}| \approx -3$ dB) and Port 3 remains isolated ($|S_{32}| < -37$ dB). Additionally, the resistor dissipates half of the power, since Port 2 remains with good matching levels ($|S_{22}| < -20$ dB).

In the last test, we show how the internal port behaves when the value of the resistor is changed from the ideal nominal value of $R = 100\ \Omega$, to a much lower value $R = 10\ \Omega$. As we can see from Fig. 4.16, in this situation, Port 3 is not isolated

Figure 4.16: S-parameters magnitude of the power divider, shown in Fig. 4.13, when the internal resistor has $R = 10\ \Omega$. The simulations are obtained using the MEN formulation and the commercial software ANSYS HFSS. The excitation is through Port 2.

anymore, and Port 2 exhibits poor matching. Again, the MEN results are in good agreement with the commercial software ANSYS HFSS. In particular, our formulation correctly predicts a drop in the matching level of Port 2 and in the isolation to Port 3, giving $|S_{22}| = |S_{32}| = -7.61$ dB at the design frequency. These same values are also predicted by the commercial tool ANSYS HFSS.

To obtain the responses reported in Fig. 4.14, Fig. 4.15 and Fig. 4.16, 4000 terms

have been used in the kernel [3900 in the static part and 100 in the dynamic part of the kernel *Gomez-Molina et al.* (2017)]. In the MoM-Galerkin procedure, 350 basis and test functions (computed using the BI-RME method) have been used. The computational time required to obtain the coupling integrals is less than 4 minutes. The MEN code takes about 4.54 seconds to achieve the results in Fig. 4.14 and Fig. 4.15 (for 100 frequency points) and the same computational time for the results in Fig. 4.16. The coupling integrals do not have to be recomputed for the second case ($R = 10 \Omega$) since the metallization has not changed as compared to the first case ($R = 100 \Omega$). ANSYS HFSS takes about 10 minutes to obtain the results in Fig. 4.14 and Fig. 4.15 and the same time to obtain the results in Fig. 4.16.

It is important to mention that, for the four examples discussed so far, only one accessible mode is required to solve the problem, since all structures contain only one discontinuity in the transverse plane (only one metallic circuit, including the two input and output ports). Besides this advantage, we should remark that the analysis of structures with very thin dielectric layers and several metallization interfaces in close proximity, will require more computational effort. In the case of the MEN technique discussed in this chapter, this situation will require the use of more accessible modes in the equivalent network, to correctly characterize the energy exchange process between nearby discontinuities. The use of more accessible modes will increase the size of the final matrix, and consequently the time required to solve large systems of linear equations will be also higher. This increase in computational time will be evident in the examples discussed in Subsection 4.3.2.

4.3 Extension of the Magnetic MEN for Multilayered Planar Devices

4.3.1 Theoretical Derivations

In this subsection, we illustrate the use of the MEN formulation described in subsection 4.2.1 to analyze boxed planar circuits that are composed of two metallic interfaces and three dielectric substrates, as the one shown in Fig. 4.17. Following the same strategy, the extension to an arbitrary number of dielectric and circuit layers is straightforward.

Figure 4.17: Circuit containing two metallic interfaces, and three dielectric substrates, excited with two coaxial connectors in the lower metallization level.

As in Subsection 3.3.1 of Chapter III, the idea is to analyze separately all the planar discontinuities (metallic interfaces) of the circuit under study, using the formulation that we described. Once the different individual equivalent networks are computed, they can be simply cascaded as shown in Fig. 4.18.

In this network, the transmission lines represent the accessible modes in each dielectric layer. The impedance coupling matrices, connected in parallel, represent the different printed metallic circuits shown in Fig. 4.17. As we can see, two coaxial connectors are used as input and output ports. The connectors are modeled by two transverse ports $P1$ and $P2$ in the first equivalent network shown in Fig. 4.18 (lower planar circuit). Again, as we can see, the top and bottom covers of the box are modeled by terminating the transmission lines representing substrates 1 (h_1, ϵ_{r1})

Figure 4.18: Multimode Equivalent Network of the multilayer geometry in Fig. 4.17.

and 3 (h_3, ϵ_{r3}) with a surface impedance Z_s . As in Subsection 4.2.1, this impedance models the losses in the top and bottom covers of the box, and is computed using equation (4.30).

As discussed in Chapter III, if no metallic obstacle is present at a dielectric interface, then there are no coupling effects between modes at this interface. This situation is easily tackled in the network of Fig. 4.18 by just connecting, to each accessible mode, a new transmission line with the constitutive parameters of the new medium.

Once we have characterized the complete circuit with the network in Fig. 4.18, the S-parameters are computed by using one of the transverse port as excitation, and loading the other one with the reference impedance of the port ($Z_0 = 50 \Omega$). The total matrix ($\bar{\mathbf{Z}}^T$) that models the complete problem can then be written in terms of

the following voltage-current relation

$$\begin{bmatrix} V_{p1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{p1,p1}^T & Z_{p1,p2}^T & Z_{p1,1}^T & \cdots & Z_{p1,N}^T \\ Z_{p2,p1}^T & Z_{p2,p2}^T & Z_{p2,1}^T & \cdots & Z_{p2,N}^T \\ Z_{1,p1}^T & Z_{1,p2}^T & Z_{1,1}^T & \cdots & Z_{1,N}^T \\ \vdots & & \ddots & & \vdots \\ Z_{N,p1}^T & Z_{N,p2}^T & Z_{N,1}^T & \cdots & Z_{N,N}^T \end{bmatrix} \begin{bmatrix} I_{p1} \\ I_{p2} \\ I_1^{(1)} \\ \vdots \\ I_N^{(2)} \end{bmatrix}. \quad (4.35)$$

This system of linear equations relates the external voltages (V_n and V_n') and the port voltages (V_{p1} and V_{p2}) in Fig. 4.18 to the external currents ($I_n^{(1)}$ and $I_n^{(2)}$) and the port currents (I_{p1} and I_{p2}). In equation (4.35), the external voltages have been replaced by their corresponding voltage-current relations. The input admittance ($Y_{in} = I_{p1}/V_{p1}$) and the transadmittance between ports ($Y_{tran} = I_{p2}/V_{p1}$) can then be extracted by solving the linear system in equation (4.35). The S-parameters are then calculated in terms of Y_{in} and Y_{tran} using again equations (4.33) and (4.34).

4.3.2 Numerical Results

The example discussed in this section consists of a microwave resonator with direct input and output coupling, implemented through a multilayer stack as shown in Fig. 4.17. The circuit is composed of three dielectric layers and two metallic interfaces (the microstrip circuits). In this section, we discuss three different implementations of this filter.

Example 1: First implementation using a rectangular resonator.

The first implementation is based on rectangular metallizations. The layout of the two metallic interfaces is shown in Fig. 4.19. The first (lower) level contains the input and output microstrip lines. The second level contains a rectangular microstrip

resonator. This multilayer implementation allows for a direct capacitive coupling between input and output ports ($P1$ - $P2$ coupling), so that one transmission zero (TZ) appears above the passband. Again, the structure is completely shielded, and the excitations are the two lateral coaxial connectors (see Fig. 4.17) that are modeled with pulse ports (dark areas in Fig. 4.19).

Figure 4.19: First implementation of a microwave resonator with direct input/output coupling, using two metallization interfaces. The dimensions are in mm. Gray areas are the metallizations while dark areas represent the excitation ports. The width of all lines is 1.49 mm. The dimensions of the shielding box are: $a \times b = 40 \text{ mm} \times 20 \text{ mm}$. With respect to Fig. 4.17, the heights of the dielectric layers are: $h_1 = h_2 = h_3 = 5 \text{ mm}$. The relative dielectric permittivities of the substrates are: $\epsilon_{r1} = \epsilon_{r3} = 2.33$ and $\epsilon_{r2} = 1.07$.

The performance of the filter, obtained using the proposed MEN formulation, is presented in Fig. 4.20. In this figure, we compare the results with the full-wave simulation using the electric MEN formulation reported in Chapter III, and the results from ANSYS HFSS. As we can see, the simulated S-parameters are in good agreement with the results obtained using the commercial software ANSYS HFSS. Also, the response is similar to the one obtained using the electric multilayer MEN formulation, although with a slight frequency shift.

Example 2: Second implementation using an elliptical resonator.

Another possible implementation of this filter is using an elliptical resonator, as shown in Fig. 4.21. In this case, since the resonator is not rectangular, the analysis

Figure 4.20: S-parameters magnitude of the rectangular microwave resonator of Fig. 4.19, obtained using the magnetic MEN formulation, compared with the results obtained using the formulation reported in Chapter III, and to the simulation results obtained with ANSYS HFSS.

Figure 4.21: Second implementation of a microwave resonator with direct input/output coupling, using two metallizations. In this case, an elliptic resonator is used in the second metallization level. The dimensions are in mm. The width of the rectangular input/output lines is 1.49 mm. The dimensions of the shielding box are: $a \times b = 40 \text{ mm} \times 20 \text{ mm}$. With respect to Fig. 4.17, $h_1 = h_3 = 5 \text{ mm}$ and $h_2 = 6.52 \text{ mm}$. The relative dielectric permittivities of the substrates are: $\epsilon_{r1} = 2.33$, $\epsilon_{r2} = 1.07$ and $\epsilon_{r3} = 4$.

cannot be performed using the electric MEN technique described in Chapter III or the direct EFIE formulation proposed in *Melcon et al. (1999)*. However, this analysis

can still be carried out using the MEN formulation proposed in this chapter, together with the BI-RME method.

The MEN results are compared to ANSYS HFSS simulations in Fig. 4.22. As

Figure 4.22: S-parameters magnitude of the elliptic microwave resonator of Fig. 4.21, obtained using the presented formulation and compared to the simulation results obtained with the commercial tool ANSYS HFSS.

we can see, good agreement between the MEN simulation and ANSYS HFSS results is obtained also in this case. The resonator in Fig. 4.21 has been tuned so that its response is similar to the rectangular resonator of Fig. 4.19.

Example 3: Third implementation for shifting the TZ.

The two previous implementations have a TZ above the passband due to the direct capacitive coupling between input and output ports ($P1-P2$). If the TZ is required to be below the passband, the sign of the direct coupling $P1-P2$ should be changed. This can be implemented through an inductive direct coupling between the ports. The third design shows a practical implementation of this concept (see Fig. 4.23).

Here, we propose to use an inductor L (connected to an internal port) between the input and the output lines to change the sign of the direct $P1$ - $P2$ coupling.

Figure 4.23: Third implementation of a microwave resonator with direct input/output coupling, using two metallization interfaces. An internal port is used to introduce a direct inductive coupling between input/output ports. The dimensions are in mm. The width of the input/output lines is 1.49 mm. The internal port (hatched area) is used to connect an inductor L . The dimensions of the shielding box are: $a \times b = 40 \text{ mm} \times 20 \text{ mm}$. With respect to Fig. 4.17, $h_1 = h_3 = 5 \text{ mm}$ and $h_2 = 7.5 \text{ mm}$. The relative dielectric permittivities of the substrates are the same as in Fig. 4.21. The value of the inductor is $L = 25 \text{ nH}$.

Fig. 4.24 shows the response obtained using the proposed MEN formulation. In this figure, the MEN results are compared to those obtained using the commercial software ANSYS HFSS. As can be seen, the MEN simulation is in good agreement with respect to the analysis performed using the commercial software ANSYS HFSS. As expected, with the inductance, the TZ is shifted below the passband. The dimensions of the filter, given in Fig. 4.23, have been tuned for the same center frequency as for the microwave resonators in Fig. 4.19 and Fig. 4.21. This example further validates the treatment of internal ports presented in this chapter.

For these simulations, we have used 30 accessible modes for the equivalent network of Fig. 4.18. To characterize the two metallic interfaces, we have included 400 basis and test functions in the MoM-Galerkin procedure, 3700 terms in the static kernel, and 300 in the dynamic kernel. To analyze the complete structure, the computational time required by the MEN code for each design is about 1 min and 40 seconds (for 100 points in the frequency sweep). The computational time required to obtain the

Figure 4.24: S-parameters magnitude of the elliptic microwave resonator with inductive input/output coupling, as shown in Fig. 4.23, using the proposed MEN formulation and the software tool ANSYS HFSS.

coupling integrals is less than 3 minutes. For each simulation of this last example, ANSYS HFSS takes about 13 minutes using the same frequency sweep.

Finally, in Table 4.1 and Table 4.2, we summarize the numerical simulation data for all the examples analyzed in this chapter (in Subsection 4.2.2 and in this subsection). These data demonstrate that the MEN approach described in this chapter for analysis of practical single-layer and multilayer shielded printed circuits is both

Table 4.1: Numerical MEN parameters required by all the microwave components analyzed in this work.

Device	N	N_b	Terms in dynamic / static kernel
Lowpass filter	1	600	100/3900
Hairpin filter	1	310	500/4500
Dual-mode filter	1	450	300/3700
Wilkinson divider	1	350	100/3900
Multilayer filters	30	400	300/3700

Table 4.2: Computational time required by all the microwave components analyzed in this work.

Device	Num. of frequency points	BI-RME time	MEN time
Lowpass filter	100	3 min	11.85 s
Hairpin filter	200	4 min	6.08 s
Dual-mode filter	120	6 min	11.41 s
Wilkinson divider	100	4 min	4.54 s
Multilayer filters	100	3 min + 3 min	1 min 40 s

efficient and accurate.

In conclusion, with the results presented in this subsection and in Subsection 4.2.2, the theory, proposed for the rigorous analysis of zero-thickness printed circuits composed of metallizations with arbitrary shape that can contain internal and external ports, has been fully validated. Structures that contain more than one metallic level can also be efficiently analyzed. The numerical results also confirm that, although other numerical methods are much more powerful from the point of view of the structures they can analyze, if a modal analysis formulation can be obtained, the computational time required to perform the electromagnetic analysis is reduced.

4.4 Conclusions

In this chapter, the applicability of the rigorous MEN technique has been extended to the analysis of planar circuits (or *discontinuities*) that include arbitrarily shaped metallizations together with internal and external ports in the transverse plane. Until Chapter III, the only type of discontinuities that can be represented through its equivalent network are those composed of rectangular patches. This limited the kind of planar components that this technique could analyze. Now, using the novel formulation reported in this chapter, a very large variety of practical single-layer and multilayer boxed microwave printed circuits can be efficiently analyzed in the common frame of the MEN technique. This is possible thanks to the combination of

the magnetic MEN technique with the BI-RME method, that allows the analysis of discontinuities with arbitrary shape. The inclusion of losses in the structure under analysis has also been discussed in this chapter.

In addition to theory, the analysis of a number of practical planar structures has been performed, showing good agreement with respect to other electromagnetic analysis tools, and with measured results. Special attention has been given to the validation of internal ports in some examples. Then, the examples discussed in this chapter fully validate the magnetic MEN for planar circuits, and clearly confirm the usefulness and efficiency of the novel procedure in the analysis of real shielded planar microstrip circuits.

Acknowledgment

This work has been possible thanks to Prof. Dr. Stephan Marini and Prof. Dr. Miguel Ángel Sánchez Soriano from Universidad de Alicante, who provided us with the BI-RME module.

Scientific Outcome

- Related to Section 4.2 and 4.3: *Gomez-Molina et al.* (2020c).

CHAPTER V

Consideration of Losses in the Magnetic Zero-thickness MEN Formulation

5.1 Introduction

In this chapter, we focus our attention on the losses in a metallization interface, as the one shown in Fig. 5.1, when the magnetic zero-thickness MEN formulation is employed. This formulation is described in Chapter IV to analyze microstrip and multilayered planar microwave components. However, in this previous approach, we only consider losses in the substrate(s) and in the box that shields the microwave circuit. For an accurate prediction of losses, in Chapter IV, losses due to the printed metallizations were effectively accounted for by the losses in the dielectric(s), and the box. This is only an approximation that worked very well as observed in the numerical results of Chapter IV.

Nevertheless, a more theoretical model for this contribution of losses can be derived by reformulating the MEN theory for zero-thickness planar discontinuities. In this context, it is important to remember that the magnetic MEN formulation starts by imposing the boundary conditions in the aperture part (air) of the discontinuity. However, the physical losses are in the metallic part of the discontinuity. This means that the consideration of the losses due to the printed metallizations is straightfor-

Figure 5.1: Zero-thickness discontinuity under analysis. The metallizations (gray areas) are made with non perfect conductors. The losses due to the metallization are considered through an equivalent problem: we assume a high impedance surface in the air part (aperture) of the discontinuity.

ward in the electric approach *Melcon et al. (1999)*; *Taroncher et al. (2005)*, where the boundary conditions are imposed in the metallic patches. However, the inclusion of these losses in magnetic field integral equation methods is a more complicate problem. The first approach to consider the losses in the magnetic MEN formulation was proposed in *Taroncher et al. (2005)*. In *Taroncher et al. (2005)*, the authors combine the electric and magnetic boundary conditions to account for the losses in the metallic patches. Nevertheless, they show that the proposed lossy magnetic formulation does not offer accurate results, while increasing the computational time. This is, indeed, due to the addition of new integral equations, what leads to an over-dimensioned problem and ill-conditioned MoM matrix. Instead of using a perturbation theory over the real unknowns, they try to represent the perturbation as such. This leads to stability problems which increase the computational effort of this technique.

In this context, the objective of this chapter is to review this problem to propose an alternative way to consider these losses. In particular, the formulation presented in this chapter does not include additional boundary conditions imposed on the metallic patches, as was done in *Taroncher et al. (2005)*. The consideration of these losses is based on an equivalent problem: we assume that there is an equivalent losses term in the aperture part of the discontinuity. Using the electric formulation, losses are considered by imposing the boundary conditions of a low impedance surface (good

conductor). Thus, in this chapter, we formulate a dual problem in the magnetic domain where we consider a high impedance surface at the discontinuity (in the air part) to take into account the losses. We will demonstrate that this approach is equivalent to introduce a perturbation on the lossless problem unknowns.

To accomplish this goal, we first demonstrate the surface impedance/admittance of a low and high impedance plane (see Fig. 5.2) in Section 5.2 and 5.3, respectively. Then, we review in Section 5.4 the magnetic zero-thickness MEN formulation considering losses in the aperture, where the boundary conditions are imposed. Also, we discuss the parameter R_m that models how lossy is the aperture. Finally, in Section 5.5, we show the performance of this formulation through two examples that were already treated in Chapter IV. The first one is validated through measurements, while the second one is compared to a full-wave commercial software result. The adjusted R_m values are also discussed for these two examples. Finally, the main conclusions are outlined.

5.2 Surface Impedance and Admittance of a Low Impedance Plane

In this section, we review the procedure to obtain the surface impedance/admittance of a thin sheet with low impedance, as the one shown in Fig. 5.2(a). In this case, we start with the Ampere's law for time-harmonic fields *Balanis* (2012)

$$\nabla \times \mathbf{H} = jw\mathbf{D} + \mathbf{J}, \quad (5.1)$$

where w is the angular frequency, $\mathbf{D}$ is the electric displacement field and $\mathbf{J}$ represents the total electric current density on the metal. Then, we write the magnetic field $\mathbf{H}$

Figure 5.2: Low and high impedance surfaces. (a) The low impedance surface is characterized using the finite conductivity σ . Z_c and Y_c are the surface impedance and admittance of this plane, respectively. (b) The high impedance surface is characterized using the resistivity parameter R_m . Z_s and Y_s are the surface impedance and admittance of this plane, respectively.

in terms of the electric field $\mathbf{E}$

$$\nabla \times \mathbf{H} = jw\varepsilon'\mathbf{E} + \sigma\mathbf{E} \quad (5.2)$$

$$\nabla \times \mathbf{H} = jw \left(\varepsilon' + \frac{\sigma}{jw} \right) \mathbf{E}, \quad (5.3)$$

where ε' is the electric permittivity of the medium and σ the finite conductivity.

From equation (5.3) we can define ε as

$$\varepsilon = \varepsilon' + \frac{\sigma}{jw} = \varepsilon' \left(1 + \frac{\sigma}{jw\varepsilon'} \right). \quad (5.4)$$

Considering now just the imaginary part of ε , which is the one associated with the losses due to the finite conductivity of the metal, we can approximate ε as

$$\varepsilon \approx \varepsilon' \left(\frac{\sigma}{jw\varepsilon'} \right) = \frac{\sigma}{jw}. \quad (5.5)$$

Then the surface impedance Z_c that models the low impedance surface in Fig. 5.2(a) can be computed as

$$Z_c = \eta = \sqrt{\frac{\mu}{\varepsilon}} \approx \sqrt{\frac{\mu}{\frac{\sigma}{jw}}} = (1 + j) \sqrt{\frac{w\mu}{2\sigma}}, \quad (5.6)$$

where η is the intrinsic impedance of the medium and μ is the magnetic permeability. An alternative strategy to obtain this expression is using the procedure described in *Pozar* (2011). Following this approach, we first approximate the propagation constant γ as

$$\gamma = jw\sqrt{\mu\varepsilon} \approx jw\sqrt{\mu\frac{\sigma}{jw}}, \quad (5.7)$$

and then we obtain the surface impedance as follows

$$Z_c = \frac{jw\mu}{\gamma} \approx \frac{jw\mu}{jw\sqrt{\mu\frac{\sigma}{jw}}} = \sqrt{j}\sqrt{\frac{w\mu}{\sigma}} = (1 + j) \sqrt{\frac{w\mu}{2\sigma}}. \quad (5.8)$$

On this basis, the surface admittance Y_c is written as

$$Y_c = \frac{1}{Z_c} \approx (1 - j) \sqrt{\frac{\sigma}{2w\mu}}. \quad (5.9)$$

As observed in equation (5.8) and equation (5.9), if $\sigma \rightarrow \infty$, which corresponds to a Perfect Electric Conductor (PEC), then $Z_c = 0$ and $Y_c \rightarrow \infty$. In this case, no losses would be considered in the electric formulation, since we would impose that the total tangential components of the electric field vanish ($Z_c = 0$) on the metallic patch.

5.3 Surface Impedance and Admittance of a High Impedance Plane

In this section, we obtain the surface impedance/admittance of a thin sheet with high impedance, as the one shown in Fig. 5.2(b). This is the dual case of the procedure in Section 5.2. In this case, we start with the Faraday's law for time-harmonic fields *Balanis* (2012)

$$\nabla \times \mathbf{E} = -jw\mathbf{B} - \mathbf{M}, \quad (5.10)$$

where $\mathbf{B}$ is the magnetic flux density and $\mathbf{M}$ represents the total magnetic current density in the aperture. Now, we write the electric field $\mathbf{E}$ in terms of the magnetic field $\mathbf{H}$

$$\nabla \times \mathbf{E} = -jw\mu'\mathbf{H} - R_m\mathbf{H} \quad (5.11)$$

$$\nabla \times \mathbf{E} = -jw \left(\mu' + \frac{R_m}{jw} \right) \mathbf{H}, \quad (5.12)$$

where R_m is the resistivity of the high impedance surface in Fig. 5.2(b).

From equation (5.12) we can define μ as

$$\mu = \mu' + \frac{R_m}{jw} = \mu' \left(1 + \frac{R_m}{jw\mu'} \right). \quad (5.13)$$

Considering the imaginary part of μ , which is the one associated with the losses, we can approximate μ as

$$\mu \approx \mu' \left(\frac{R_m}{jw\mu'} \right) = \frac{R_m}{jw}. \quad (5.14)$$

Then, the surface impedance Z_s that models the high impedance surface in Fig. 5.2(b) can be computed as follows

$$Z_s = \eta = \sqrt{\frac{\mu}{\varepsilon}} \approx \sqrt{\frac{\frac{R_m}{jw}}{\varepsilon}} = (1 - j) \sqrt{\frac{R_m}{2w\varepsilon}}. \quad (5.15)$$

An alternative approach to demonstrate this result is by following the procedure described in *Pozar* (2011). On this basis, we first approximate the propagation constant as

$$\gamma = jw\sqrt{\mu\varepsilon} \approx jw\sqrt{\frac{R_m}{jw}}\varepsilon, \quad (5.16)$$

and then we compute the surface impedance Z_s as

$$Z_s = \frac{\gamma}{jw\varepsilon} \approx \frac{jw\sqrt{\frac{R_m}{jw}}\varepsilon}{jw\varepsilon} = (1 - j)\sqrt{\frac{R_m}{2w\varepsilon}}. \quad (5.17)$$

Then, the surface admittance Y_s of a high impedance surface is approximated as

$$Y_s = \frac{1}{Z_s} \approx (1 + j)\sqrt{\frac{w\varepsilon}{2R_m}}. \quad (5.18)$$

From equations (5.17) and (5.18), we can observe that if $R_m \rightarrow \infty$, then $Z_s \rightarrow \infty$ and $Y_s = 0$. In this case, we would not consider any losses in the magnetic formulation, since we would impose the continuity ($Y_s = 0$) of the tangential components of the magnetic field on the aperture.

5.4 Lossy Magnetic Zero-thickness MEN

In this section, we describe how to introduce losses in the aperture when the magnetic zero-thickness MEN formulation is applied to the analysis of the discontinuity shown in Fig. 5.1. This formulation is a revisited version of the one described in Chapter IV, but instead of consider a perfect dielectric interface at the junction (in the air part at $z = 0$), we consider to have a high impedance surface (as the one treated in Section 5.3).

In the formulation presented in this chapter, no lateral excitation ports are considered for simplicity. However, the combination of this formulation with the lateral port theory described in Chapter IV is straightforward.

The formulation starts by imposing the boundary conditions on the tangential components of the magnetic field $\mathbf{H}_t^{(\delta)}$ in the aperture at $z = 0$, including the magnetic current density $\mathbf{M}_s$ that models the losses in the aperture

$$\mathbf{H}_t^{(1)}(s) - \mathbf{H}_t^{(2)}(s) = -Y_s \mathbf{M}_s(s), \quad (5.19)$$

where $\delta = 1$ or $\delta = 2$ for $z < 0$ or $z > 0$, respectively, and $s = (x, y)$ indicates a point in the junction cross-section of Fig. 5.1. In equation (5.19), Y_s represents the surface admittance of the high impedance surface assumed in the aperture (at $z = 0$).

The magnetic current density $\mathbf{M}_s$ can be written in terms of the unknown electric field in the aperture $[\mathbf{z}_0 \times \mathbf{E}(s)]$ *Balanis* (2012) as follows

$$\mathbf{M}_s(s) = -[\mathbf{z}_0 \times \mathbf{E}(s)]. \quad (5.20)$$

By using the well known modal expansion formalism *Guglielmi et al.* (1999), equation (5.19) can be written as

$$\sum_{m=1}^{+\infty} I_m^{(1)} \mathbf{h}_m^{(1)}(s) - \sum_{m=1}^{+\infty} I_m^{(2)} \mathbf{h}_m^{(2)}(s) = Y_s [\mathbf{z}_0 \times \mathbf{E}(s)], \quad (5.21)$$

where $\mathbf{h}_m^{(\delta)}(s)$ represents the magnetic vector modal function of mode m in the medium (δ) , and $I_m^{(\delta)}$ refers to the modal currents. It is important to note that m refers to all TE and TM modes in the rectangular waveguide cross-section in Fig. 5.1.

To continue, we now separate the accessible from the localized modes *Guglielmi et al.* (1999), thus obtaining

$$\begin{aligned} & \sum_{n=1}^{N(1)} I_n^{(1)} \mathbf{h}_n^{(1)}(s) - \sum_{n=1}^{N(2)} I_n^{(2)} \mathbf{h}_n^{(2)}(s) \\ &= Y_s [\mathbf{z}_0 \times \mathbf{E}(s)] + \sum_{m=N(1)+1}^{+\infty} \frac{V_m^{(1)}}{Z_m^{(1)}} \mathbf{h}_m^{(1)}(s) + \sum_{m=N(2)+1}^{+\infty} \frac{V_m^{(2)}}{Z_m^{(2)}} \mathbf{h}_m^{(2)}(s), \end{aligned} \quad (5.22)$$

where $N(\delta)$ is the number of accessible modes in each region.

In equation (5.22), $V_m^{(\delta)}$ represents the modal voltage and $Z_m^{(\delta)}$ the characteristic modal impedance in medium (δ) . Note that, for the sake of clarity, the subscript on the left hand side in equation (5.22) has been changed from m to n .

We now recognize that, since the metallic circuit is zero-thickness, and the waveguide cross-section in region (1) is identical to the one in region (2), the magnetic vector modal functions in region (1) ($\mathbf{h}_m^{(1)}$) are identical to the ones in region (2) ($\mathbf{h}_m^{(2)}$). Then, knowing that $\mathbf{h}_m^{(1)}(s) = \mathbf{h}_m^{(2)}(s) = \mathbf{h}_m(s)$ and $V_m^{(1)} = V_m^{(2)} = V_m$, equation (5.22) is transformed into

$$\sum_{n=1}^N \bar{I}_n \mathbf{h}_n(s) = Y_s [\mathbf{z}_0 \times \mathbf{E}(s)] + \sum_{m=N+1}^{+\infty} V_m Y_m^T \mathbf{h}_m(s), \quad (5.23)$$

where $\bar{I}_n$ represents the total modal current at the interface, defined as $\bar{I}_n = I_n^{(1)} - I_n^{(2)}$. Also, Y_m^T is the total characteristic modal admittance defined as $Y_m^T = Y_m^{(1)} + Y_m^{(2)}$, where $Y_m^{(\delta)}$ represents the characteristic modal admittance of the mode m in the medium (δ) . Note that the number of accessible modes in both regions is assumed to be equal, $N(1) = N(2) = N$.

To continue the formulation, we now recall the following relation between the modal voltage V_m and the electric field in the aperture $\mathbf{E}(s')$

$$V_m = \int_{ap} [\mathbf{z}_0 \times \mathbf{E}(s')] \cdot \mathbf{h}_m^*(s') ds', \quad (5.24)$$

where the integral extends over the entire aperture (ap) at the junction (see Fig. 5.1) and $ds' = dx' dy'$.

Due to the linearity of the problem, the unknown transverse electric field in the aperture must be linearly proportional to the excitations, so that it can be expanded

into a sum of partial surface magnetic current densities $\mathbf{M}_n(s')$ as follows

$$[\mathbf{z}_0 \times \mathbf{E}(s')] = \sum_{n=1}^N \bar{I}_n \mathbf{M}_n(s'), \quad (5.25)$$

where $\mathbf{M}_n$ are the unknown functions of our problem. In the same way, we can write

$$[\mathbf{z}_0 \times \mathbf{E}(s)] = \sum_{n=1}^N \bar{I}_n \mathbf{M}_n(s). \quad (5.26)$$

Substituting now equation (5.25) into equation (5.24), and by using the resulting expression and equation (5.26) in equation (5.23), we obtain the following expression

$$\sum_{n=1}^N \bar{I}_n \mathbf{h}_n(s) = Y_s \sum_{n=1}^N \bar{I}_n \mathbf{M}_n(s) + \sum_{n=1}^N \bar{I}_n \int_{ap} \sum_{m=N+1}^{+\infty} Y_m^T \mathbf{h}_m(s) \mathbf{h}_m^*(s') \cdot \mathbf{M}_n(s') ds', \quad (5.27)$$

where we can identify the fundamental integral equation of our problem

$$\mathbf{h}_n(s) = Y_s \mathbf{M}_n(s) + \int_{ap} \sum_{m=N+1}^{+\infty} Y_m^T \mathbf{h}_m(s) \mathbf{h}_m^*(s') \cdot \mathbf{M}_n(s') ds'. \quad (5.28)$$

As observed, the magnetic current density $\mathbf{M}_s$ that models the losses has been written in terms of the unknowns of the problem. As a result, this procedure does not lead to an increment of the number of unknowns or integral equations, as happened in *Taroncher et al.* (2005). Also, the excitations of the integral equations are still the magnetic vector modal functions of the guided modes in the rectangular waveguides ($\mathbf{h}_n$). What we are doing in equation (5.28) is perturbing the unknowns that we already had in the lossless case.

In order to solve the integral equation (5.28), we now expand the unknown partial magnetic current functions $\mathbf{M}_n$ using the Method of Moments (MoM) *Guglielmi et al.*

(1999). In particular, we use the magnetic vector modal functions in the aperture at the plane of the junction both as basis ($\mathbf{h}_k^B$) and test ($\mathbf{h}_i^B$) functions, applying the Galerkin method *Guglielmi et al.* (1999). These functions are numerically computed using the BI-RME method *Bozzi et al.* (2001). Following this approach, the unknown $\mathbf{M}_n(s')$ can finally be written as

$$\mathbf{M}_n(s') = \sum_{k=1}^{N_b} \alpha_{n,k} \mathbf{h}_k^B(s'), \quad (5.29)$$

where N_b is the numerical parameter that determines the number of basis and test functions used in the MoM procedure, and $\alpha_{n,k}$ are the unknown coefficients of the expansions. In the same way, we can write

$$\mathbf{M}_n(s) = \sum_{k=1}^{N_b} \alpha_{n,k} \mathbf{h}_k^B(s). \quad (5.30)$$

Using this expansion, equation (5.28) can be written as

$$\mathbf{h}_n(s) = Y_s \sum_{k=1}^{N_b} \alpha_{n,k} \mathbf{h}_k^B(s) + \sum_{k=1}^{N_b} \alpha_{n,k} \sum_{m=N+1}^{+\infty} Y_m^T \int_{ap} \mathbf{h}_k^B(s') \mathbf{h}_m(s) \mathbf{h}_m^*(s') ds'. \quad (5.31)$$

Applying the test procedure, the integral equation (5.31) is transformed into

$$\begin{aligned} \int_{ap} \mathbf{h}_i^B(s) \cdot \mathbf{h}_n(s) ds &= \sum_{k=1}^{N_b} \alpha_{n,k} Y_s \int_{ap} \mathbf{h}_k^B(s) \cdot \mathbf{h}_i^B(s) ds \\ &+ \sum_{k=1}^{N_b} \alpha_{n,k} \sum_{m=N+1}^{+\infty} Y_m^T \int_{ap} \mathbf{h}_k^B(s') \cdot \mathbf{h}_m^*(s') ds' \int_{ap} \mathbf{h}_i^B(s) \cdot \mathbf{h}_m(s) ds. \end{aligned} \quad (5.32)$$

Since the vector modal functions are orthonormal, the coupling integral between

the modes of the arbitrary waveguide is a Kronecker delta.

$$\int_{ap} \mathbf{h}_k^B(s) \cdot \mathbf{h}_i^B(s) ds = \delta_{i,k} \quad (5.33)$$

Then, equation (5.32) can be written as the following system of linear equations

$$C_{i,n} = \sum_{k=1}^{N_b} \alpha_{n,k} \left[Y_s \delta_{i,k} + \sum_{m=N+1}^{+\infty} Y_m^T C_{k,m} C_{i,m} \right]. \quad (5.34)$$

The coefficients $C_{k,m}$ and $C_{i,m}$ represent the coupling integrals between the localized modes of the rectangular waveguide, and the aperture modes used as basis and test functions, respectively.

$$C_{k,m} = \int_{ap} \mathbf{h}_k^B(s') \cdot \mathbf{h}_m^*(s') ds' \quad (5.35)$$

$$C_{i,m} = \int_{ap} \mathbf{h}_i^B(s) \cdot \mathbf{h}_m(s) ds \quad (5.36)$$

The terms $C_{i,n}$ are the coupling integrals between the accessible modes of the rectangular waveguide and the aperture modes used as test functions.

$$C_{i,n} = \int_{ap} \mathbf{h}_i^B(s) \cdot \mathbf{h}_n(s) ds \quad (5.37)$$

If we observe the system of equations (5.34) and we compare it to equation (4.22a), we realize that the addition of losses modifies the diagonal of the MoM matrix $\bar{\mathbf{A}}_{lossy}$

with respect to the lossless case $\overline{\mathbf{A}}_{lossless}$

$$C_{i,n} = \sum_{k=1}^{N_b} \alpha_{n,k} \underbrace{\left[Y_s \delta_{i,k} + \underbrace{\sum_{m=N+1}^{+\infty} Y_m^T C_{k,m} C_{i,m}}_{\overline{\mathbf{A}}_{lossless}(s,s')} \right]}_{\overline{\mathbf{A}}_{lossy}(s,s')} \quad (5.38)$$

where

$$\overline{\mathbf{A}}_{lossy}(s, s') = \overline{\mathbf{A}}_{lossless}(s, s') + \begin{bmatrix} Y_s & 0 & \cdots & 0 & 0 \\ 0 & Y_s & \cdots & 0 & 0 \\ \vdots & 0 & \ddots & 0 & 0 \\ \vdots & 0 & \cdots & Y_s & 0 \\ 0 & \cdots & \cdots & 0 & Y_s \end{bmatrix}. \quad (5.39)$$

On this basis, this formulation remains dual to the lossy EFIE formulation presented in *Melcon et al.* (1999), where the term Z_c (surface impedance of a low impedance plane) is summed in the diagonal of the MoM matrix. In the magnetic case, the term Y_s (surface admittance of a high impedance plane) is summed in the diagonal of the MoM matrix.

The described procedure can be easily extended to the formulation presented in Chapter IV. Thus, the summation of the term Y_s in the diagonal of the MoM matrix in equations (4.22a) and (4.22b) is enough to consider the losses in the magnetic MEN formulation for planar devices with lateral excitation ports.

Contrary to the formulation described in *Taroncher et al.* (2005), with the proposed strategy, the size of the MoM matrix is not increased, since the number of unknowns and integral equations is maintained. The difference with respect to the lossless formulation is a perturbation in the diagonal of this matrix.

Finally, it is important to note that the admittance Y_s can be computed using equation (5.18), where the parameter R_m , that models how lossy is the surface, is set

as explained next.

How to select the value of R_m

As mentioned before, in real microwave circuits, the losses due to the printed metallizations are in the metallic part of the discontinuity (at $z = 0$, see Fig. 5.1). However, the boundary conditions in the magnetic MEN are imposed in the aperture (air), where there are no real losses. If we continue imposing the boundary conditions only in the aperture for the magnetic MEN formulation, one alternative is to approximate the real losses in the non perfect metallizations using a high impedance surface in the air part, as was done in this chapter. This surface is characterized by the resistivity parameter R_m as demonstrated in Section 5.3. In this subsection, we focus the attention on how to select the value of this parameter.

For this purpose, it is important to remember that, in the case of a conductive plane, the parameter σ models how good is the conductor. Intuitively, when the conductor is perfect ($\sigma \rightarrow \infty$), we will have no losses in the structure and $Z_c = 0$. In the magnetic MEN formulation this implies the continuity of the tangential components of the magnetic field on the aperture. To cope with this situation, in the proposed lossy MEN formulation, $\mathbf{M}_s$ has to be null and also Y_s has. Therefore, the parameter R_m has to tend to infinite, $R_m \rightarrow \infty$. If $R_m \rightarrow \infty$, then $Y_s = 0$ and the continuity condition is imposed in equation (5.19). If the magnetic field is continuous, no losses are considered. This is also verified in equation (5.39). If $Y_s = 0$, we can see that $\overline{\mathbf{A}}_{lossy} = \overline{\mathbf{A}}_{lossless}$.

This argument implies that σ and R_m has to be, somehow, directly related. In fact, this relation can be explained through an impedance inverter that transforms a short circuit ($Z_c = 0$) into an open circuit condition ($Y_s = 0$). Fig. 5.3 shows this transformation.

Figure 5.3: First option for the implementation of the impedance inverter. This inverter transforms the resistivity of a good conductor ($1/\sigma$) into the resistivity of a high impedance surface (R_m). Z_0 is the characteristic impedance of the inverter and λ is the wavelength of the medium.

On this basis, the parameter R_m is then computed as

$$R_m = \frac{Z_0^2}{1/\sigma} = k^2 \sigma. \quad (5.40)$$

Since we do not know the characteristic impedance of the inverter, k (Ω) is a parameter that has to be tuned.

The second alternative to implement the impedance inverter is using the surface impedances of the low and high impedance planes, Z_c and Z_s , respectively. This is shown in Fig. 5.4.

Figure 5.4: Second option for the impedance inverter. This inverter transforms the surface impedance of a good conductor (Z_c) into the surface impedance of a high impedance plane (Z_s). Z_0 is the characteristic impedance of the inverter and λ is the wavelength.

In this case, the relation between impedances is

$$Z_s = \frac{Z_0^2}{Z_c} = k^2 Y_c. \quad (5.41)$$

Using equations (5.9) and (5.17), this condition is expressed as

$$(1 - j) \sqrt{\frac{R_m}{2w\varepsilon}} = k^2 (1 - j) \sqrt{\frac{\sigma}{2w\mu}} \quad (5.42)$$

$$\sqrt{\frac{R_m}{\varepsilon}} = k^2 \sqrt{\frac{\sigma}{\mu}} \quad (5.43)$$

$$\sqrt{\frac{R_m}{\sigma}} = k^2 \sqrt{\frac{\varepsilon}{\mu}}. \quad (5.44)$$

Thus, using this second strategy, R_m is computed as follows

$$R_m = k^4 \left(\frac{\varepsilon \sigma}{\mu} \right), \quad (5.45)$$

where again k (Ω) is the parameter that has to be adjusted.

If we compare equations (5.40) and (5.45), we can observe that the difference, between these two strategies to set R_m , is that equation (5.45) is normalized by a factor proportional to the intrinsic impedance of the air ($\eta = \sqrt{\frac{\mu}{\varepsilon}}$) with respect to the expression in equation (5.40).

In this procedure, a parameter (k) has to be tuned. This does not implies an extra effort for the consideration of losses since, in the approach followed in Chapter IV, the losses due to the dielectric were also tuned to finally match the real losses. The advantage of the new proposed procedure compared to the previous one is that now, the losses parameters of dielectric(s), box and metallizations, can be set to their real values, and we just need to tune k for an exact computation of losses. The

new procedure considers losses at the discontinuity in a more rigorous way (using an equivalent problem) than in Chapter IV, where they were not considered at all and simply accounted for by other loss terms.

5.5 Numerical Results

In this section, the mechanism proposed in this chapter to include the losses due to the printed metallization is validated through two numerical examples. In the first one, we show how to tune the value of the parameter k according to equations (5.40) and (5.45). In this case, the insertion losses (IL) are compared to measurements. In the second example, we analyze a Wilkinson power divider considering losses in our code and in ANSYS HFSS. A final comparison between the values of k obtained in both examples and using the two proposed strategies is reported.

Example 1: Dual-mode open loop bandpass filter.

The first example is the dual-mode microstrip filter shown in Fig. 5.5. This filter was treated in Subsection 4.2.2 considering losses just in the box and in the dielectric. However, in this previous simulation, no losses were considered in the metallization. The small contribution to the total losses due to the metallic circuit were accounted for by the losses in the dielectric and the box. As observed in the numerical results of Chapter IV, this approximation proved to be very effective. Now, we revisit this example to validate the new strategy that allows to consider also the losses in the discontinuity, when the MEN formulation is used.

In Fig. 5.6 we present the passband response obtained using the MEN formulation in Chapter IV, where the details of the insertion losses are shown in the inset. In particular, from measurements we got $IL = 0.7$ dB at $f/f_0 = 1$ GHz.

In Table 5.1, we present in the first row the real parameters of the materials that we assume were used to manufacture the prototype: the loss tangent of the

Figure 5.5: Dual-mode microstrip bandpass filter under study. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 25 \text{ mm} \times 20 \text{ mm}$. The value of l_a is 8.73 mm and the dielectric relative permittivity is $\epsilon_r = 10.8$ and the substrate thickness is $l_d = 1.27 \text{ mm}$. The filter is centered in the box.

Figure 5.6: In-band S-parameters magnitude of the dual-mode bandpass filter shown in Fig. 5.5, using the proposed lossy MEN formulation, and the electromagnetic analysis tool ANSYS HFSS. Results of simulations (using Sonnet) and measurements from *Hong et al.* (2007) are also included.

dielectric ($\tan \delta$), the electric conductivity of the metallizations (σ_{met}) and the electric conductivity of the box (σ_{box}).

If we use these real values in the MEN code without considering losses in the metallization, then the insertion losses are $IL = 0.36 \text{ dB}$, as reported in the second row of Table 5.1. To obtain the real IL value ($IL = 0.7 \text{ dB}$), we had to increase the

value of the loss tangent until $\tan \delta = 0.0047$, as reported in the third row of Table 5.1.

Table 5.1: Value of the parameters associated with losses in the structure of Fig. 5.5.

	$\tan \delta$	σ_{met} (S/m)	σ_{box} (S/m)	IL at $f/f_0 = 1$ GHz
Real prototype	0.002	$5.8 \cdot 10^7$	$3.8 \cdot 10^7$	0.7 dB
MEN simulation	0.002	∞	$3.8 \cdot 10^7$	0.36 dB
MEN simulation	0.0047	∞	$3.8 \cdot 10^7$	0.7 dB

Using the mechanism proposed in Section 5.4, we are able to fix the parameters ($\tan \delta$, σ_{met} and σ_{box}) to the real ones and tune the value of k to adjust the simulated losses to the real measured losses (in this case $IL = 0.7$ dB at $f/f_0 = 1$ GHz). On this basis, we present the IL obtained for different values of k adjusted using equation (5.40) in Table 5.2 and adjusted using equation (5.45) in Table 5.3.

Table 5.2: Tuning of the parameter k defined as equation (5.40).

$\tan \delta$	σ_{met} (S/m)	k	σ_{box} (S/m)	IL at $f = 1$ GHz
0.002	$5.8 \cdot 10^7$	1	$3.8 \cdot 10^7$	5.85 dB
0.002	$5.8 \cdot 10^7$	3	$3.8 \cdot 10^7$	2.48 dB
0.002	$5.8 \cdot 10^7$	8	$3.8 \cdot 10^7$	1.22 dB
0.002	$5.8 \cdot 10^7$	12	$3.8 \cdot 10^7$	0.94 dB
0.002	$5.8 \cdot 10^7$	20	$3.8 \cdot 10^7$	0.7 dB

Table 5.3: Tuning of the parameter k defined as equation (5.45).

$\tan \delta$	σ_{met} (S/m)	k	σ_{box} (S/m)	IL at $f = 1$ GHz
0.002	$5.8 \cdot 10^7$	20	$3.8 \cdot 10^7$	5.8 dB
0.002	$5.8 \cdot 10^7$	40	$3.8 \cdot 10^7$	1.91 dB
0.002	$5.8 \cdot 10^7$	60	$3.8 \cdot 10^7$	1.08 dB
0.002	$5.8 \cdot 10^7$	87	$3.8 \cdot 10^7$	0.7 dB

The result using the new lossy MEN formulation is the same as the response presented in Fig. 5.6. However, as shown in Table 5.2 and 5.3, the new mechanism allows us to use the real values of the parameters: $\tan \delta = 0.002$, $\sigma_{met} = 5.8 \cdot 10^7$ S/m and $\sigma_{box} = 3.8 \cdot 10^7$ S/m, and then tune k to retrieve the real IL . In particular, for this example, if we define k as equation (5.40) then $k = 20$. On the contrary, if we use equation (5.45), then $k = 87$.

Example 2: Wilkinson power divider.

The second example is the Wilkison power divider shown in Fig. 5.7. This circuit was studied in Subsection 4.2.2 as well. This component was simulated using the MEN technique and ANSYS HFSS, but no real measurements are available. In Chapter IV, no losses were included in none of the simulations (neither using the MEN code nor ANSYS HFSS). In this subsection, we simulate again this microwave component using the loss theory proposed in this chapter. We also resimulate the ANSYS HFSS example to consider the losses.

Figure 5.7: Wilkinson power divider in microstrip technology. The dimensions are in mm. The dimensions of the shielding box section are: $a \times b = 30$ mm $\times$ 16 mm. The value of l_a is 3 mm. The dielectric relative permittivity is $\epsilon_r = 4$ and the substrate thickness is $l_d = 1.2$ mm. The divider is centered in the box.

Let suppose that the loss tangent of the employed substrate is $\tan \delta = 0.002$. The metallization is made of copper with $\sigma_{met} = 5.8 \cdot 10^7$ S/m, while the external housing is made of aluminum with $\sigma_{box} = 3.8 \cdot 10^7$ S/m. In Fig. 5.8 and 5.9, we show

the S-parameters simulated using ANSYS HFSS and the previous parameters. As observed, the insertion losses are $IL = 3.11$ dB at $f = 2.54$ GHz.

Figure 5.8: S-parameters magnitude of the power divider, shown in Fig. 5.7, using real components in ANSYS HFSS. The excitation is through Port 1.

Figure 5.9: S-parameters magnitude of the power divider, shown in Fig. 5.7, using real components in ANSYS HFSS. The excitation is through Port 2.

Using the proposed strategy, in the MEN code, we can set the value of these

parameters to their real values [$\tan \delta = 0.002$, $\sigma_{met} = 5.8 \cdot 10^7$ (S/m) and $\sigma_{box} = 3.8 \cdot 10^7$ (S/m)] and tune the value of k . To obtain $IL = 3.11$ dB at $f = 2.54$ GHz, using the first approach k is equal to $k = 15$, while using the second approach this parameter is $k = 71$. In Fig. 5.10 and Fig. 5.11, we show the response of the structure obtained using the proposed lossy MEN formulation and ANSYS HFSS including losses as well, when the divider is excited through Port 1 and 2, respectively.

Figure 5.10: S-parameters magnitude of the power divider, shown in Fig. 5.7, using the proposed MEN formulation and the commercial software ANSYS HFSS, including losses. The excitation is through Port 1.

As we can see, the S-parameters obtained are in good agreement with the ANSYS HFSS simulation, considering losses. The IL matches very well in both software tools. This confirms that, using the lossy MEN formulation proposed in this chapter, we are able to simulate the required IL by tuning the value of k .

Finally, we compare in Table 5.4, the values of k used in both examples to model the metallization losses. At this point, it is important to mention that the first example is a microwave bandpass filter at 1.06 GHz that is compared to real measurements, while the second example is a Wilkinson divider working at 2.54 GHz that is compared

Figure 5.11: S-parameters magnitude of the power divider, shown in Fig. 5.7, using the proposed MEN formulation and the commercial software ANSYS HFSS, including losses. The excitation is through Port 2.

to ANSYS HFSS simulation results.

Table 5.4: Value of k according to the first approach [equation (5.40)] and the second approach [equation (5.45)], for the two examples studied in this chapter.

	k (first approach)	k (second approach)
Example 1	20	87
Example 2	15	71

As we can observe in Table 5.4, the value of k is similar in both examples for the two different approaches. Also, it can be noticed that the difference between both approaches is the intrinsic impedance of the air. This agrees the relation between equation (5.40) and equation (5.45).

Conclusions

In this chapter, a new strategy to consider the losses in microstrip components due to the printed metallization is proposed. In this case, the metallic losses in the microstrip circuit are represented using an equivalent problem, where a high impedance surface is considered in the aperture (air) part of the discontinuity. In this way, the boundary conditions applied when the magnetic zero-thickness formulation is derived, account for a magnetic surface current density that models the lossy high impedance aperture. As demonstrated, this procedure results in a perturbation on the lossless problem unknowns. This is equivalent to the way to consider metallic losses in EFIE formulations. By following this approach, the problem is not over-dimensioned, so the stability and convergence of the method are assured.

In most practical microstrip circuits, for an accurate prediction of the losses, the fine tuning of the various loss constants it is almost always required. For example, in real cases, the effective conductivity of the metallization is different from the ideal conductivity of the material used for manufacturing. If these losses are not explicitly considered, they have to be included by tuning the loss constants of other materials. At the end, a tuning process is required. An advantage of the proposed theory is that the parameters of the materials used for manufacturing can be fixed at their specified values. Then, it is the characteristic impedance of the inverter that models the transformation from a low impedance surface to a high impedance surface, the one that has to be tuned.

The good agreement between the lossy MEN results and the measurements from a prototype and numerical simulations using a commercial software validates the theory proposed in this chapter.

CHAPTER VI

Magnetic Multimode Equivalent Network Technique for Thick Planar Devices

6.1 Introduction

In previous chapters, we extended the Multimode Equivalent Network formulation to the analysis of multilayer boxed planar devices, that include lateral excitation ports that could model the excitation of planar circuits through real coaxial connectors *Edwards and Steer* (2013); *Marini et al.* (2018). The electric MEN formulation described in Chapter III allows to analyze multilayer planar components that are based on arbitrary rectangular zero-thickness elements. In Chapter IV, we extended this analysis to multilayer planar devices that consist of arbitrarily shaped zero-thickness metallizations. This was possible thanks to the combination of the MEN formulation and the BI-RME method *Bozzi et al.* (2001).

In many cases of practical interest, however, the thickness of the metallization cannot be neglected, and then the zero-thickness formulations described in Chapters III and IV are no longer accurate. If the thickness is non negligible, but is electrically small, alternative integral equation techniques, as the one described in *Stevanovic et al.* (2006), can be used to transform the thin aperture or metallization into an equivalent zero-thickness object. However, if the metallization is electrically thick, a

more accurate formulation is required. This is indeed the case, for instance, when planar circuits are analyzed at very high frequencies. In this situation, the thickness becomes electrically important and, therefore, it must be explicitly included in the computations in order to obtain accurate results. Furthermore, a new class of thick bar resonators has been recently proposed to implement wideband filters *Gomez-Molina et al.* (2020d). The thicker metallic resonators exhibit better unloaded quality factors than regular printed microstrip resonators. Moreover, higher coupling levels (between resonators and with the input/output ports) than the ones obtained with thinner (standard) planar microstrip technology can be achieved. These filters are explained in detail in Chapter VII. Since the thickness of the resonators in these filters is electrically large, even at microwave frequencies, the zero-thickness formulations described in Chapters III and IV are not suitable anymore.

In this context, therefore, the objective of this chapter is to extend the state of the art of the analysis of planar microwave circuits by discussing a MEN formulation that can analyze rigorously planar microwave circuits based on thick metallizations, including also coaxial input and output ports, and several dielectric layers, as shown in Fig. 6.1. To cope with this analysis, we consider the thick metallic circuit as a length of uniform waveguide. The problem is then divided into two waveguide steps (or junctions). We first obtain the equivalent network of the two junctions. Then, we cascade them to obtain the MEN of the complete circuit, thereby including rigorously the effect of finite thickness.

A key aspect of the proposed MEN formulation is that it includes ports in the transverse plane of the circuit. This is an important difference with respect to the thick formulation presented in Chapter II, for waveguide problems. In Chapter II, the excitation of the structure was the propagating modes in the input/output waveguide sections. In this chapter, the components under analysis are completely shielded (in z direction), and the excitation is through these lateral ports (Port 1 and 2 in Fig. 6.1).

The effect of losses, due to metal and dielectric materials of the multilayer circuits, will be also discussed in this chapter. It is also important to mention that this problem is solved using a magnetic field approach. The combination of the formulation presented in this chapter, together with the BI-RME method *Arcioni et al.* (2014), allows the analysis of devices that consist of arbitrarily shaped thick metallizations.

To accomplish the goal of this chapter, in Subsection 6.2.1, we first describe the analysis of thick discontinuities including ports in the transverse plane. We then explain how to cascade the equivalent networks to build the final model that describes the complete structure. Then, in Subsection 6.2.2, we study the numerical convergence of the formulation for two in-line microwave filters implemented with thick coupled lines. In Subsection 6.2.2, we also compare the simulation results for a transversal four-pole filter with measurements obtained from a manufactured prototype, showing very good agreement, and thereby fully validating the new MEN formulation. Finally, a summary of the obtained results is provided in the conclusions.

6.2 Magnetic MEN for Thick Planar Devices

6.2.1 Theoretical Derivations

The goal of this section is to extend the MEN technique to the analysis of devices containing transverse ports, and a combination of several thick metallizations and dielectric layers, as shown in Fig. 6.1. Since the metallizations have a non negligible thickness (h_2 in Fig. 6.1), the zero-thickness MEN formulations described in Chapters III and IV cannot be used to analyze such structures. As we can see in Fig. 6.1, the structure can be decomposed into two discontinuities (one at $z = 0$ and the other one at $z = h_2$).

A top view of the two discontinuities is shown in Fig. 6.2. The first disconti-

Figure 6.1: Schematic (side view) of the structure under analysis. The metallization has a non negligible thickness (h_2). The structure is excited by two lateral ports (Ports 1 and 2), at the interface $z=0$.

nuity [see Fig. 6.2(a)] is analyzed using the MEN formulation with transverse ports described in this subsection. It should be emphasized that the transverse ports can be either external or internal, as discussed in Chapter IV. In Fig. 6.2(a), we show two external ports to model the coaxial input/output connectors used to excite real microwave components *Edwards and Steer* (2013); *Marini et al.* (2018). However, the treatment of internal ports, that can be used to connect the metallization to external lumped-element components, is identical. The second discontinuity present in the structure of Fig. 6.1 [see Fig. 6.2(b)] is analyzed using the traditional MEN for waveguide junctions (reviewed in Chapter II), since, in this discontinuity, no lateral ports are considered.

It is important to note that this is only an example of a possible microwave planar circuit with thick metallizations. The formulation presented in this chapter is not restricted to any particular shape. As previously mentioned, the shape can be arbitrary thanks to the use of the BI-RME method *Cogollos et al.* (2003). Once both MENs are obtained, they are cascaded, as explained in this subsection, to compute the final network that characterizes the complete structure in Fig. 6.1.

Figure 6.2: Top view of the two discontinuities present in the structure shown in Fig. 6.1. (a) The first discontinuity (at $z = 0$) contains the lateral excitation ports (dark areas) that excite the structure. (b) The second discontinuity (at $z = h_2$) only contains the metallizations (gray areas).

MEN for discontinuities of thick junctions including transverse ports

First, we derive the general MEN formulation for two-dimensional discontinuities of thick junctions that include transverse ports [as shown in Fig. 6.2(a)]. The formulation starts by applying the boundary conditions on the tangential components of the magnetic field $\mathbf{H}_t^{(\delta)}$ in the aperture at $z = 0$, where $\delta = 1$ or $\delta = 2$ for $z < 0$ or $z > 0$, respectively. In this condition, we include the lateral excitation through the surface electric current density $\mathbf{J}_{exc}$ as follows,

$$\mathbf{H}_t^{(1)}(s) - \mathbf{H}_t^{(2)}(s) = \mathbf{z}_0 \times \mathbf{J}_{exc}(s). \quad (6.1)$$

In equation (6.1), $s = (x, y)$ indicates a point in the waveguide cross-section of the discontinuity, and $\mathbf{z}_0$ is the unit vector in the direction of the z -axis (see Fig. 6.1).

For simplicity, only one transverse port is considered explicitly in this development, but the extension to more than one port (either external or internal) is straightforward. To continue, we use the well known modal expansion formalism in *Guglielmi et al. (1999)* to expand the magnetic field, and the pulse model in *Rautio and Harrington (1987)* for the surface electric current density representing the excitation [see

Fig. 6.2(a)]. More information about the surface current density model can be found in Chapter IV. We also separate the accessible modes from the localized ones, as indicated in *Guglielmi et al.* (1999), to obtain

$$\begin{aligned} \sum_{n=1}^{N(1)} I_n^{(1)} \mathbf{h}_n^{(1)}(s) - \sum_{n=1}^{N(2)} I_n^{(2)} \mathbf{h}_n^{(2)}(s) + \frac{I_0}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0 = \\ \sum_{m=N(1)+1}^{+\infty} Y_m^{(1)} V_m^{(1)} \mathbf{h}_m^{(1)}(s) + \sum_{m=N(2)+1}^{+\infty} Y_m^{(2)} V_m^{(2)} \mathbf{h}_m^{(2)}(s), \end{aligned} \quad (6.2)$$

where $\mathbf{h}_m^{(\delta)}(s)$ is the magnetic vector modal function of mode m in the medium (δ) and $I_m^{(\delta)}$ represents the total modal current. In equation (6.2), $V_m^{(\delta)}$ and $Y_m^{(\delta)}$ represent the total modal voltage and the characteristic modal admittance, respectively. Also, $N(\delta)$ is the number of accessible modes considered in each region. Regarding the pulse, (x_0, y_0) denotes the position of the center of the pulse, and Δx and Δy are the widths of the pulse in the x -axis and y -axis, respectively [see Fig. 6.2(a)].

It is now important to note a difference between the zero-thickness formulation presented in Chapter IV, and the formulation discussed in this chapter. The difference is that the modal voltages on both sides of the discontinuity $V_m^{(1)}$ and $V_m^{(2)}$ are not equal in this case. In addition, the magnetic vector modal functions in medium 1, $\mathbf{h}_m^{(1)}(s)$, are those corresponding to the rectangular waveguide while, in medium 2, the functions $\mathbf{h}_m^{(2)}(s)$ are the modes of the aperture (with arbitrary shape) that are obtained using the BI-RME method. However, in the zero-thickness formulation presented in Chapter IV, we assumed that the modal voltages and the vector modal functions were equal in both media (since the metallization was considered zero-thickness).

To continue the formulation, we now use the relation between the modal voltage

$V_m^{(\delta)}$ and the unknown electric field in the aperture $\mathbf{E}(s')$,

$$V_m^{(\delta)} = \int_{ap} [\mathbf{z}_0 \times \mathbf{E}(s')] \cdot \mathbf{h}_m^{(\delta)*}(s') ds', \quad (6.3)$$

where ap is the aperture at the junction (see Fig. 6.2). Due to the linearity of the problem, the electric field must be proportional to the excitation, so that we can write

$$[\mathbf{z}_0 \times \mathbf{E}(s')] = \sum_{n=1}^{N(1)} I_n^{(1)} \mathbf{M}_n^{(1)}(s') - \sum_{n=1}^{N(2)} I_n^{(2)} \mathbf{M}_n^{(2)}(s') + I_0 \mathbf{M}_0(s'), \quad (6.4)$$

where $\mathbf{M}_n^{(\gamma)}$ and $\mathbf{M}_0$ are the unknowns of our problem and $\gamma = 1$ or $\gamma = 2$ for $z < 0$ or $z > 0$, respectively. Substituting now equation (6.4) into equation (6.3), and using the resulting equation in equation (6.2), we obtain the system of integral equations that represents the problem under study

$$\mathbf{h}_n^{(\gamma)}(s) = \int_{ap} \overline{\mathbf{K}}(s, s') \cdot \mathbf{M}_n^{(\gamma)}(s') ds' \quad (6.5)$$

$$\frac{1}{\Delta y} \Pi \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0 = \int_{ap} \overline{\mathbf{K}}(s, s') \cdot \mathbf{M}_0(s') ds', \quad (6.6)$$

where $\overline{\mathbf{K}}(s, s')$ is the kernel of the two previous integral equations

$$\overline{\mathbf{K}}(s, s') = \sum_{\delta=1}^2 \sum_{m=N(\delta)+1}^{+\infty} Y_m^{(\delta)} \mathbf{h}_m^{(\delta)}(s) \mathbf{h}_m^{(\delta)*}(s'). \quad (6.7)$$

To increase the computational efficiency, the kernel is evaluated using the Kummer's technique described in Chapter II.

Finally, we write the modal voltages in terms of the modal currents as

$$V_0 = Z_{0,0} I_0 + \sum_{n=1}^{N(1)} I_n^{(1)} Z_{0,n}^{(1)} + \sum_{n=1}^{N(2)} I_n^{(2)} Z_{0,n}^{(2)} \quad (6.8)$$

$$V_m^{(\delta)} = Z_{m,0}^{(\delta)} I_0 + \sum_{n=1}^{N(1)} I_n^{(1)} Z_{m,n}^{(\delta,1)} + \sum_{n=1}^{N(2)} I_n^{(2)} Z_{m,n}^{(\delta,2)}, \quad (6.9)$$

where $Z_{0,0}$, $Z_{0,n}^{(\gamma)}$, $Z_{m,0}^{(\delta)}$ and $Z_{m,n}^{(\delta,\gamma)}$ are the elements of the multimode impedance coupling matrix for the discontinuity under study. Then, the resulting multimode impedance coupling matrix is of size $(2N + 1) \times (2N + 1)$. We can therefore write

$$\bar{\mathbf{Z}} = \begin{bmatrix} Z_{0,0} & | & Z_{0,1}^{(1)} & Z_{0,2}^{(1)} & \cdots & Z_{0,N}^{(1)} & | & Z_{0,1}^{(2)} & Z_{0,2}^{(2)} & \cdots & Z_{0,N}^{(2)} \\ - & + & - & - & - & - & - & - & - & - & - \\ Z_{1,0}^{(1)} & | & Z_{1,1}^{(1,1)} & Z_{1,2}^{(1,1)} & \cdots & Z_{1,N}^{(1,1)} & | & Z_{1,1}^{(1,2)} & Z_{1,2}^{(1,2)} & \cdots & Z_{1,N}^{(1,2)} \\ \vdots & & & & \ddots & & & & & \ddots & \vdots \\ Z_{N,0}^{(1)} & | & Z_{N,1}^{(1,1)} & Z_{N,2}^{(1,1)} & \cdots & Z_{N,N}^{(1,1)} & | & Z_{N,1}^{(1,2)} & Z_{N,2}^{(1,2)} & \cdots & Z_{N,N}^{(1,2)} \\ - & | & - & - & - & - & + & - & - & - & - \\ Z_{1,0}^{(2)} & | & Z_{1,1}^{(2,1)} & Z_{1,2}^{(2,1)} & \cdots & Z_{1,N}^{(2,1)} & | & Z_{1,1}^{(2,2)} & Z_{1,2}^{(2,2)} & \cdots & Z_{1,N}^{(2,2)} \\ \vdots & & & & \ddots & & & & & \ddots & \vdots \\ Z_{N,0}^{(2)} & | & Z_{N,1}^{(2,1)} & Z_{N,2}^{(2,1)} & \cdots & Z_{N,N}^{(2,1)} & | & Z_{N,1}^{(2,2)} & Z_{N,2}^{(2,2)} & \cdots & Z_{N,N}^{(2,2)} \end{bmatrix}. \quad (6.10)$$

As shown in equation (6.6), the inclusion of the transverse port as excitation of the structure leads to a new integral equation, where the excitation is the pulse function introduced in equation (6.1). This is an important difference with respect to the traditional waveguide junctions normally used in the analysis of waveguide components (described in Chapter II). Note that, the presence of the new excitation term results into the new elements ($Z_{0,0}$, $Z_{0,n}^{(\gamma)}$ and $Z_{m,0}^{(\delta)}$) in the coupling matrix, that take into account the interactions between the modes on both sides of the discontinuity

and the new lateral excitation port. These terms do not appear in the waveguide coupling matrix given in equation (2.11). In comparison to the zero-thickness coupling matrix given in equation (4.19), we can see that the size of the matrix for thick junctions is twice the size of the zero-thickness coupling matrix.

Finally, the Method of Moments and the Galerkin procedure can be used to solve the system of integral equations [equations (6.5) and (6.6)], as was done in previous chapters. However, in this case, we use the magnetic vector modal functions of modes in medium 2 (see Fig. 6.1) as basis and test functions [$\mathbf{h}_k^B = \mathbf{h}_i^B = \mathbf{h}_m^{(2)}$]. The required modes are computed very efficiently using the BI-RME method *Bozzi et al.* (2001); *Cogollos et al.* (2003). In particular, the unknowns of the problem are expanded as follows

$$\mathbf{M}_0(s') = \sum_{k=1}^{N_b} \alpha_{0,k} \mathbf{h}_k^B(s') \quad (6.11)$$

$$\mathbf{M}_n^{(\gamma)}(s') = \sum_{k=1}^{N_b} \alpha_{n,k}^{(\gamma)} \mathbf{h}_k^B(s'), \quad (6.12)$$

Applying the test procedure, the integral equations (6.5) and (6.6) are finally transformed into the following system of linear equations

$$C_{i,n}^{(\gamma)} = \sum_{k=1}^{N_b} \alpha_{n,k}^{(\gamma)} \sum_{\delta=1}^2 \sum_{m=N+1}^{+\infty} Y_m^{(\delta)} C_{k,m}^{(\delta)} C_{i,m}^{(\delta)} \quad (6.13)$$

$$C_{i,0} = \sum_{k=1}^{N_b} \alpha_{0,k} \sum_{\delta=1}^2 \sum_{m=N+1}^{+\infty} Y_m^{(\delta)} C_{k,m}^{(\delta)} C_{i,m}^{(\delta)}. \quad (6.14)$$

The coefficients $C_{k,m}^{(\delta)}$ and $C_{i,m}^{(\delta)}$ represent the coupling integrals between the localized modes of the waveguide in the medium (δ), and the arbitrary waveguide modes

used as basis and test functions, respectively.

$$C_{k,m}^{(\delta)} = \int_{ap} \mathbf{h}_k^B(s') \cdot \mathbf{h}_m^{(\delta)*}(s') ds' \quad (6.15)$$

$$C_{i,m}^{(\delta)} = \int_{ap} \mathbf{h}_i^B(s) \cdot \mathbf{h}_m^{(\delta)}(s) ds \quad (6.16)$$

The terms $C_{i,n}^{(\gamma)}$ are the coupling integrals between the accessible modes of the waveguide in the medium (γ) and the arbitrary waveguide modes used as test functions.

$$C_{i,n}^{(\gamma)} = \int_{ap} \mathbf{h}_i^B(s) \cdot \mathbf{h}_n^{(\gamma)}(s) ds \quad (6.17)$$

The term $C_{i,0}$, on the other hand, refers to the coupling integral between the test functions and the excitation pulse.

$$C_{i,0} = \int_{ap} \mathbf{h}_i^B(s) \cdot \left(\frac{1}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0 \right) ds \quad (6.18)$$

When $\delta = 1$ or $\gamma = 1$, the coupling integrals in equations (6.15), (6.16) and (6.17) are computed numerically using the BI-RME method. On the contrary, when $\delta = 2$ or $\gamma = 2$, these integrals become a Kronecker delta function $\delta_{k,m}$, $\delta_{i,m}$ and $\delta_{i,n}$, for equations (6.15), (6.16) and (6.17), respectively. This is because the integrals are defined between the arbitrary waveguide modes $[\mathbf{h}_k^B = \mathbf{h}_i^B = \mathbf{h}_m^{(2)}]$ and, since they are orthonormal functions, the resulting integral is 1 when the index is the same ($k = m$ or $i = m$) and zero when they are not. More details about the computation of the coupling integrals required to solve this problem are reported in Appendix A.

Final MEN assembly

Once we have computed the equivalent network for both discontinuities in Fig. 6.2, we can cascade them, as shown in Fig. 6.3.

Figure 6.3: Final equivalent network representation for the structure shown in Fig. 6.1, using the MEN formalism. The discontinuities are characterized by their MEN coupling matrices. The modes are characterized with transmission lines in the different regions. Modes in media 1 and 3 are terminated with a short-circuit to model the top and bottom metal covers (PEC) of the box enclosure (see Fig. 6.1). The coaxial excitations are represented with transmission lines of characteristic impedances Z_{p1} and Z_{p2} for Ports 1 and 2, respectively.

The first discontinuity contains, in addition, two ports that represent the lateral excitations of the structure [Ports 1 and 2 in Fig. 6.1, represented with dark areas in Fig. 6.2(a)]. The modes in the different media are represented by transmission lines (where the propagation constant and the characteristic impedance are those corresponding to the mode in that medium). The lengths of the transmissions lines correspond to the heights (h_1, h_2 and h_3) in Fig. 6.1. As we can see in Fig. 6.3, the modes that represent the first and the last layer (h_1 and h_3) are short-circuited to represent the top and bottom PEC covers of the external metal housing.

Since the discontinuities are thick junctions, and we are using a magnetic approach, the resulting network remains in series configuration, and it is modeled by the concatenation of two impedance coupling matrices.

To analyze the complete structure using this final network, we write the external voltages ($V_n^{(1)}$ and $V_n'^{(2)}$) and the port voltages (V_{p1} and V_{p2}) in terms of the external current ($I_n^{(1)}$ and $I_n'^{(2)}$) and the port currents (I_{p1} and I_{p2}), obtaining the following system of linear equations

$$\begin{bmatrix} V_{p1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{p1,p1}^T & Z_{p1,p2}^T & Z_{p1,1}^T & \cdots & Z_{p1,N}^T \\ Z_{p2,p1}^T & Z_{p2,p2}^T & Z_{p2,1}^T & \cdots & Z_{p2,N}^T \\ Z_{1,p1}^T & Z_{1,p2}^T & Z_{1,1}^T & \cdots & Z_{1,N}^T \\ \vdots & & \ddots & & \vdots \\ Z_{N,p1}^T & Z_{N,p2}^T & Z_{N,1}^T & \cdots & Z_{N,N}^T \end{bmatrix} \begin{bmatrix} I_{p1} \\ I_{p2} \\ I_1^{(1)} \\ \vdots \\ I_{N(2)}'^{(2)} \end{bmatrix}, \quad (6.19)$$

where we have loaded the modes with the corresponding total impedance, and Port 2 is loaded with the appropriate reference impedance (in our case Z_{p2}). We then excite through Port 1, and solve the linear system to obtain the input admittance ($Y_{in} = I_{p1}/V_{p1}$) and the transadmittance between ports ($Y_{tran} = I_{p2}/V_{p1}$). The S-parameters are then calculated in terms of Y_{in} and Y_{tran} as shown next

$$S_{11} = \frac{1 - Z_{p1}Y_{in}}{1 + Z_{p1}Y_{in}} \quad (6.20)$$

$$S_{21} = Y_{tran} \frac{(-2Z_{p2})}{1 + Z_{p1}Y_{in}} \sqrt{\frac{Z_{p1}}{Z_{p2}}}. \quad (6.21)$$

6.2.2 Numerical Results

In this subsection, we analyze three filters that are based on the basic geometry shown in Fig. 6.1. These filters are described in detail in Chapter VII. A general schematic of the structure under analysis is shown in Fig. 6.4.

Figure 6.4: Side (left) and top (right) views of the filters under study. Dark areas (l_p) represent lumped ports that model the direct connection of the coaxial pins to the first and last resonators (Ports 1 and 2). In this case, $\Delta x = l_p$ and $\Delta y = W_{r1}$.

The filters use three thick metallic bars as resonators. The first and the third bars are coupled by proximity to the second bar. Due to the non negligible thickness of the resonators (h_2), a strong capacitive coupling is induced between bars, obtaining higher inter-resonator couplings in comparison to standard microstrip filters. In Chapter VII, it is also demonstrated that the unloaded quality factor of these thick resonators is substantially higher as compared to regular printed microstrip resonators. Moreover, the I/O couplings are implemented through the direct connection of the lateral ports (that model the coaxial connectors) to the first and last thick bars. Following this approach, the traditional I/O feeding lines are not required. Finally, in Chapter VII, we also demonstrate that an additional resonant mode can be excited in the shielding cavity, if it is partially filled with dielectric *Martinez-Mendoza et al. (2007)*, *Martinez-Mendoza et al. (2008)*. If this happens, this structure allows for the implementation of transversal topologies. On the contrary, if this mode is not resonating, the structure is implementing a classical in-line topology.

The numerical behavior of the MEN technique for standard waveguide problems has been studied in Chapter II. For this new problem, the computational novelty is the study of the number of accessible modes that is required as a function of the thickness h_2 (medium 2, see Fig. 6.3) for an accurate characterization of the whole

circuit. This was not necessary in Chapter IV because the thickness was neglected. However, in this problem, the number of accessible modes determines the size of the equivalent network (as shown in Fig. 6.3). It is important to remember that the higher order modes (*localized modes*) that have been loaded with their corresponding modal admittances, are taken into account rigorously in equation (6.7) by using the Kummer's transformation.

To show the behavior of this MEN formulation, we first perform two numerical studies for two in-line filters. In both of them, we use the theory described in Chapter II that allows to select the right number of accessible modes ($N(1) = N(2) = N(3) = N$) according to a power attenuation threshold in dB (called dB_{th}). Using this strategy, the number of accessible modes in each region will be composed of all propagating modes plus the evanescent modes that undergo a total attenuation level, when reaching the next discontinuity, that is smaller than the specified threshold dB_{th} .

Finally, in this subsection, we analyze a transversal filter in order to validate through measurements the MEN theory that we propose in this chapter.

Example 1: First in-line bandpass filter.

In the first example, we study the numerical convergence of this formulation according to the parameter dB_{th} , for a fixed value of the metallization thickness.

For the first two filters, a foam substrate with $\varepsilon_r = 1.05$ is used to avoid the excitation of box modes. The resonators of the filters are thus formed by the three thick bars. The result is two third order in-line bandpass filters, whose dimensions are collected in Table 6.1.

For this test, we use the first in-line filter shown in Table 6.1. In this case, the metallizations that compose the filter have a thickness h_2 equal to 2 mm. Based on the mechanism proposed in Chapter II, we show in Fig. 6.5 the required number of

Table 6.1: Physical dimensions of the two in-line filters studied in this subsection, as defined in Fig. 6.4.

Parameters of the filter	Dimensions first filter (in-line)	Dimensions second filter (in-line)
$h_1 = h_3$	10 mm	10 mm
h_2	2 mm	0.5 mm
$l_{r1} = l_{r3}$	16.6 mm	15.25 mm
l_{r2}	30.65 mm	29.3 mm
$w_{r1} = w_{r2} = w_{r3}$	2 mm	2 mm
$y_{r1} = y_{r3}$	16.8 mm	16.8 mm
$y_{12} = y_{23}$	4.5 mm	4.5 mm
$x_{12} = x_{23}$	11.325 mm	10.7 mm
a	42.2 mm	39.4 mm
b	48.6 mm	48.6 mm
ε_r	1.05	1.05
$Z_{p1} = Z_{p2}$	45 Ω	40 Ω

accessible modes as a function of the specified threshold dB_{th} . As shown in Fig. 6.5,

Figure 6.5: Number of accessible modes that are necessary according to the specified threshold (dB_{th}). This test is carried out for an operating frequency $f_0 = 5$ GHz and a thickness h_2 equal to 2 mm.

when the parameter dB_{th} is larger, more evanescent modes need to be considered as

accessible ones. This will increase the size of the total matrix in equation (6.19), thus increasing the computational effort. This is confirmed in Table 6.2, that specifies the number of accessible modes required to reach different levels of threshold, and the related computational time for each case. To check how the value of the threshold affects the numerical accuracy of the technique, we plot in Fig. 6.6 the filter response for different values of dB_{th} .

Table 6.2: Number of accessible modes and computation time according to the selected threshold dB_{th} .

Attenuation Threshold dB_{th} (in dB)	Number of accessible modes (N)	Time (in seconds)
1	8	6.84
1.5	8	6.84
2	10	7.88
2.5	14	10.15
3	15	10.26
3.5	20	13.86
4	23	14.52
4.5	28	17.09
5	33	20.57
5.5	36	22.38
6	42	26.64

As we can see, the results are stable with values of the threshold $\text{dB}_{th} > 4$ dB. Selecting $\text{dB}_{th} = 4$ dB requires $N = 23$ accessible modes. The S-parameters, simulated using the MEN formulation with 23 accessible modes, are compared to the results obtained with ANSYS HFSS in Fig. 6.7. As we can see, the two software tools provide similar results. This validates, at the same time, both the theory and the threshold value we have selected.

In Fig. 6.7, we also show the simulation results for this filter using the zero-thickness MEN formulation discussed in Chapter IV. As we can see, the zero-thickness results are clearly different from the responses obtained using both ANSYS HFSS and the proposed thick MEN formulation. This is because the thickness of the metalliza-

Figure 6.6: S-parameters response of the first in-line bandpass filter (with $h_2 = 2$ mm) for different values of the threshold dB_{th} .

Figure 6.7: Magnitude of the S-parameters for the first in-line bandpass filter (with $h_2 = 2$ mm) computed using the *thick* MEN formulation as compared to the results from ANSYS HFSS. The simulation results using the *zero-thickness* MEN formulation described in Chapter IV are also included for comparison.

tions ($h_2 = 2$ mm) is electrically large and cannot be neglected. This result confirms that for this filter, zero-thickness approximations are not accurate. Therefore, it also demonstrates the importance of the formulation proposed in this chapter to analyze this kind of microwave components.

Example 2: Second in-line filter.

The second test is related to the number of accessible modes required according to the thickness of the metallizations (h_2 , medium 2 in Fig. 6.1) for a fixed value of the power threshold dB_{th} . In Fig. 6.8, we show the required number of accessible modes according to this thickness (h_2), and for different values of the power threshold.

Figure 6.8: Necessary number of accessible modes according to the thickness of the metallization (h_2 , see Fig. 6.4). This test is done for different power thresholds and for an operating frequency equal to $f_0 = 5$ GHz.

As we can see in Fig. 6.8, more accessible modes are required when the metallization is thinner. This is what happens in waveguide problems, as was concluded in Chapter II. For example, for thin coupling windows (nearby discontinuities) more accessible modes need to be considered, since more evanescent modes arrive with

enough energy to the following discontinuity. According to the test performed in Fig. 6.8, a filter with metallization thickness $h_2 = 0.5$ mm would require 260 accessible modes in the equivalent network to maintain the same power attenuation threshold as in the previous example (i.e $\text{dB}_{th} = 4$ dB). This indicates that the proposed technique becomes less efficient for thinner metallizations. However, when the thickness decreases, the condition on the power threshold can be relaxed. To show that this is indeed the case, we have designed a second in-line bandpass filter, but with a thinner metallization of $h_2 = 0.5$ mm. After optimization, the dimensions of this filter are shown in Table 6.1 (second filter). A similar numerical study for this new filter leads to a required power threshold of only $\text{dB}_{th} = 1$ dB to obtain good convergence. As shown in Fig. 6.8, this threshold value requires only $N = 23$ accessible modes. In Fig. 6.9, we show the S-parameters of this filter simulated using the thick MEN formulation with these numerical parameters ($\text{dB}_{th} = 1$ dB). The results are also compared with simulations obtained using ANSYS HFSS.

From Fig. 6.9 we can see that the MEN results, obtained using 23 accessible modes, are in good agreement with respect to ANSYS HFSS. The number of accessible modes required in this case (for $h_2 = 0.5$ mm) is the same as for the first in-line bandpass filter (with $h_2 = 2$ mm). This indicates that the efficiency of the technique does not substantially degrade if the thickness of the metallization decreases. As demonstrated, in this situation, more accessible modes are needed to maintain a certain level of the power threshold parameter ($\text{dB}_{th} = 4$ dB). However, this increment is compensated by the fact that the power threshold can be lowered to obtain good numerical convergence ($\text{dB}_{th} = 1$ dB for $h_2 = 0.5$ mm). This behavior is the opposite of what occurs in thick junctions of waveguide problems. To explain this fact, it is important to remember that, in waveguide problems, the excitations are the modes in the input/output waveguide sections. Thus, the energy propagation is along z direction. However, in planar components, even though we decompose the structure

Figure 6.9: Magnitude of the S-parameters for the second in-line bandpass filter (with $h_2 = 0.5$ mm) computed using the *thick* MEN formulation and compared to ANSYS HFSS results. The MEN calculations are obtained with a reduced threshold $\text{dB}_{th} = 1$ dB. The simulation results using the *zero-thickness* MEN formulation described in Chapter IV are also included for comparison.

into discontinuities along z direction, the energy is not propagating along z . The structure is excited by the lateral ports that are contained in the (XY) plane. On this basis, the evanescent modes in each discontinuity, for this problem, represent the reactive fields of the discontinuity, instead of the propagating field along z direction. This explains why the number of evanescent modes does not have to increase when the metallization becomes thinner. Intuitively, the accessible modes in this problem are related to the dispersion of the electromagnetic fields at the wedges of the metallization, and along the z direction. This is different to the situation encountered in waveguide problems treated in Chapter II, where the accessible modes are related to the main propagation of the energy along the z direction. When the main propagation of the energy is along the z direction, we demonstrated that more accessible modes were needed when the discontinuities are closer to each other. However, in this case,

the dispersion of the electromagnetic fields seems to be well represented along the z direction using a small number of accessible modes, even when the metallization is thin.

For these two previous examples, we have used 180 basis and test functions in the MoM-Galerkin procedure, 40 dynamic terms and 5000 static terms to compute the kernel of the integral equations. As previously mentioned, the shape of the metallization can be arbitrary, since the modes in medium 2 are computed by using the BI-RME method. It is important to remember that, from the computational point of view, we first run the BI-RME module to obtain the modes of the arbitrary waveguide. We then execute the MEN code to perform the analysis of the complete structure. The computational time required by the BI-RME code is 4 minutes and 46 seconds, whereas the MEN code for 100 frequency points takes only 14.30 seconds.

In Fig. 6.9 we have included the response obtained using the zero-thickness MEN formulation described in Chapter IV. The zero-thickness MEN code takes 8.5 seconds for the same full-wave analysis. This significant reduction in time is due to the fact that when the zero-thickness MEN formulation is applied to single-layer microwave circuits, just one accessible mode is required in the equivalent network. However, as we can see in Fig. 6.9, the response is not very accurate as compared to the one obtained using ANSYS HFSS or the thick MEN formulation proposed in this chapter. If we compare the results in Fig. 6.7 for $h_2 = 2$ mm and in Fig. 6.9 for $h_2 = 0.5$ mm, we can see that, as the thickness decreases, the response obtained with the zero-thickness MEN formulation is more similar to the one of the thick MEN formulation. This is confirmed in Fig. 6.10, where we show the responses of the same filter (second filter in Table 6.1) but with a smaller thickness ($h_2 = 0.1$ mm). In this case, the analysis is performed again using both the thick formulation and the zero-thickness formulation described in Chapter IV.

As we can see in Fig. 6.10, the resonant frequencies of the resonators and the

Figure 6.10: Magnitude of the S-parameters for the second in-line bandpass filter but with $h_2 = 0.1$ mm, computed using the *thick* MEN formulation and compared to ANSYS HFSS results. The MEN calculations are obtained with $N = 23$. The simulation results using the *zero-thickness* MEN formulation described in Chapter IV are also included for comparison.

inter-resonator couplings have changed with the thickness, and the filter response is not equalized anymore. However, this example shows that for thin metallizations, the response obtained using the zero-thickness formulation is similar to the thick response. When the thickness is decreased, more basis and test functions are needed to obtain accurate results, since the currents in the metallization become more singular at the edges. This, together with the required number of accessible modes ($N = 23$) results in a significant increase of the computational time for the thick formulation. On the other hand, the zero-thickness formulation just needs one accessible mode in the equivalent network, which leads to a reduction of 90% in the computational time required to achieve the same result.

Example 3: Transversal filter.

In this last example, we validate through measurements the MEN theory that we formulate in this chapter. For this purpose, we analyze another filter implemented using the same scheme as the one shown in Fig. 6.4. However, in this case, the use of Teflon ($\varepsilon_r = 2.1$) as dielectric in media 1 and 2 allows the box modes to resonate. By using the first resonance of the box, transversal coupling topologies can be implemented as described in Chapter VII. In order to implement a wideband response with high unloaded quality factors, the thickness of the metallic bars in this filter is $h_2 = 2$ mm. After optimization, the dimensions of the design filter are reported in Table 6.3.

Table 6.3: Physical dimensions of the transversal filter, according to Fig. 6.4.

Parameters of the filter	Dimensions third filter (transversal)
$h_1 = h_3$	10 mm
h_2	2 mm
$l_{r1} = l_{r3}$	16.6 mm
l_{r2}	30.2 mm
$w_{r1} = w_{r2} = w_{r3}$	2 mm
$y_{r1} = y_{r3}$	16.8 mm
$y_{12} = y_{23}$	4.5 mm
$x_{12} = x_{23}$	11.1 mm
a	42.2 mm
b	48.6 mm
ε_r	2.1
$Z_{p1} = Z_{p2}$	45 Ω

The S-parameters simulated using the MEN code and ANSYS HFSS are compared to measurements in Fig. 6.11. In the MEN simulation, losses due to the dielectric in media 1 and 2 have been considered as explained in Chapter IV. In addition, losses due to the finite conductivity of the metallic box are also taken into account. For the rectangular waveguide modes in media 1 and 3 (see Fig. 6.4), these losses are included

using an attenuation term calculated as in *Marcuvitz* (1986). To consider the losses due to the metallic box walls in medium 2, the theory explained in *Marini et al.* (2014) is used to compute the complex propagation constant for the arbitrary waveguide modes. This is an important difference with respect to the formulation presented in Chapter IV, where these losses were not considered because this waveguide was zero-thickness. Finally, the modes in media 1 and 3 of Fig. 6.3 are terminated with a surface impedance instead of a short-circuit to model the losses in the top and bottom covers of the box enclosure (as was already done in Chapter IV). According to these considerations, we have used in the MEN results of Fig. 6.11, a loss tangent for the Teflon of $\tan \delta = 0.0037$ and a conductivity for the box equal to $\sigma = 3.8 \cdot 10^7$ S/m (aluminum). A zoom of the insertion losses (IL) in the passband is shown in the inset of Fig. 6.11.

As can be seen, good agreement between simulations and measurements is observed also in this case. The insertion losses in the MEN simulation agree well with the IL reported in Chapter VII ($IL = 0.16$ dB at the center frequency $f_c = 3.26$ GHz). The small differences between the simulations (MEN and ANSYS HFSS) and the measured response could be due to manufacturing issues. For these simulations, we have selected $\text{dB}_{th} = 4$ dB which leads to 28 accessible modes in the network of Fig. 6.3. In the MoM-Galerkin procedure, 200 basis and test functions have been used to expand the unknowns of the problem. Finally, 40 dynamic terms and 5000 static terms have been used in the kernel of the integral equations. The computational time required by the BI-RME code is 6 minutes. The MEN code for 100 frequency points takes only 20.1 seconds. For completeness, ANSYS HFSS takes more than 10 minutes to perform the same full-wave analysis.

In this example, we have also studied the effect of the thickness of the metallization on the numerical convergence of the algorithm. We have found that also for this structure the condition on the power threshold can be relaxed when the thickness h_2

Figure 6.11: Magnitude of the S-parameters for the transversal filter computed using the MEN formulation as compared to the ANSYS HFSS results and to measurements from the prototype discussed in Chapter VII. The thickness of the metallizations is $h_2 = 2$ mm. The MEN results are obtained with $\text{dB}_{th} = 4$ dB, leading to $N = 28$ accessible modes. We have used $\tan \delta = 0.0037$ and $\sigma = 3.8 \cdot 10^7$ S/m (aluminum) to model the losses. In the inset, we can see a zoom of the $|S_{21}|$ parameter in the passband. In the zoom, the black curve is the ANSYS HFSS result, the green curve is the measured response and the red curve is the MEN simulation.

is reduced. Therefore, the number of accessible modes that need to be considered in the equivalent network does not increase substantially for thinner metallizations. In particular, the convergence study shows that if the bar resonators have a reduced thickness of $h_2 = 1$ mm, then the needed dB_{th} value is equal to 2 dB, leading again to 28 accessible modes.

From this numerical example we can conclude that the MEN technique for this kind of structures is both accurate and efficient. The three examples presented in this subsection fully validate the MEN formulation applied to the analysis of microwave circuits composed of thick metallizations, including loss effects and the excitation through lateral coaxial connectors represented by transverse ports. It is important to

note that the formulation that we propose remains rigorous for large thicknesses, as no approximations have been introduced in the formulation of the integral equations. As a consequence, the MEN remains valid even if the thickness of the metallization is increased.

6.3 Conclusions

In this chapter, we have extended the Multimode Equivalent Network formulation to the analysis of shielded microwave devices consisting of several thick metallizations and dielectric layers, including the excitation through lateral ports representing coaxial connectors, internal ports and loss effects.

To rigorously account for the thick metallizations, the structure is decomposed into two discontinuities which are characterized through their corresponding MENs. One of them contains the transverse ports that excite the structure. The two MENs are then cascaded to obtain the final network that models the complete structure.

The numerical convergence of the technique with respect to the thickness of the metallizations has also been confirmed. From the numerical tests, we can conclude that, when this thickness is electrically very small, the zero-thickness MEN formulation described in Chapter IV can be directly used to analyze the structure. On the contrary, when the metallizations have a non negligible thickness, the MEN technique proposed in this chapter should be used for obtaining results with optimal accuracy.

Finally, the discussed practical filters show good agreement with respect to results from another full-wave software tool and measurements, thereby fully validating the MEN extension proposed in this chapter.

Scientific Outcome

- Related to Section 6.2: *Gomez-Molina et al. (2020b)*.

CHAPTER VII

Waveguide, Hybrid and Microstrip Filter Designs

7.1 Introduction

Filters *Cameron et al.* (2007) are important building elements in all RF/microwave communication systems. They are used to select a frequency band in the spectrum. Depending on the application, these filters have to work in a certain operating frequency, with a certain bandwidth and specific characteristics in their response. Most of the communication applications require always compact solutions for filters, with good performance in terms of insertion losses (IL), return losses (RL), out-of-band performance, among other requirements *Cameron et al.* (2007). There are several technologies to implement these filters. As before, the technology employed normally depends on the application. For instance, if the level of power that these filters have to handle and the requirements in terms of performance are high, they are normally built in waveguide technology *Marcuvitz* (1986). On the contrary, if the easy-manufacturing and low-cost are the most restrictive issues, then planar technologies such as microstrip *Hong* (2011) or stripline *Macchiarella et al.* (2018) are more suitable. In addition, there are several hybrid technologies such as Substrate Integrated Waveguide (SIW) *Bozzi et al.* (2011); *Lee et al.* (2019) that offer a good trade-off between performance and cost.

In this context, we present in this chapter three different families of filters that

have been designed for some specific applications. The first one is a family of filters for satellite communication applications at X/Ku-band. Due to the requirements in terms of power, these filters are implemented using waveguide technology. Special attention is devoted to reduce the footprint for these filter solutions. The second family of filters is for wideband C-band applications. In this case, we look for a trade-off technology, that offers good performance in terms of unloaded quality factor (Q_U) and compactness. To cope with these requirements, we propose a bulky metal bar to act as a resonator. This new technology allows to implement high inter-resonator couplings and input/output (I/O) couplings, needed to synthesize wideband responses. The last family of filters is implemented using microstrip technology, since the requirements are low-cost and easy-integration with other components. These filters are designed for 5G communication applications at Ka-band.

Moreover, all the designed filters are also simulated using the MEN formulations described in this document. Thus, the waveguide filters are simulated using the thick MEN technique for waveguide components explained in Chapter II. We analyze the bulky bars filters using the thick MEN formulation with transverse ports described in Chapter VI. Finally, the magnetic zero-thickness MEN formulation for planar devices reported in Chapter IV is used to simulate the microstrip filters.

In all the cases, full-wave simulations obtained with commercial software tools and measurements from the manufactured prototypes are in good agreement with respect to the MEN results. This validates the design procedures presented in this chapter and the usefulness of the MEN extensions proposed in this doctoral thesis for the analysis of real microwave components.

7.2 Resonating Aperture Waveguide Filters for X/Ku-band Applications

The aim of this section is to design compact bandpass filter solutions at X/Ku-band frequencies for satellite communication applications.

7.2.1 Introduction

Microwave filters and multiplexers *Kudsia et al.* (1992) are important components in the communication chain of a satellite. The requirements for these devices vary according to the stage where they are placed. For instance, in the low power module, these components can be implemented in planar or hybrid technologies such as microstrip *Hong* (2011) or SIW *Bozzi et al.* (2011). However, in the output module, the high power that these components have to handle, forces the use of more robust technologies such as waveguide *Vallerotonda et al.* (2019). Also, waveguide filters normally offer a better performance in terms of unloaded quality factors than planar or hybrid technologies. In contrast, the footprint and manufacturing cost of waveguide filters increase considerably with respect to planar technologies. This fact becomes critical in space communication applications, where the footprint and low-weight are highly restricted for the payload components. Due to this, a great interest in the research activity has been devoted to waveguide compact solutions *Guglielmi et al.* (2001); *Nocella et al.* (2016).

Following this concern, in this section, we propose a rectangular waveguide configuration that allows compact solutions for filters. These filters use iris apertures as resonators. Then, a propagating waveguide, bigger than the apertures, is used to control the coupling between the resonating apertures. We will demonstrate that the proposed iris resonating solution allows to design very compact filters in rectangular waveguide technology.

Among all the frequency bands in the microwave spectrum, the X-band is most often used for radar applications, including Earth observation missions and meteorological satellites for monitoring weather conditions. On the other hand, Ku-band is mostly used for satellite communication as well, for instance, for the direct broadcast satellite services. Due to the interest of the satellite industry in these bands, the filters described in this section are designed to operate at X/Ku-band frequencies.

To cope with these objectives, this section is organized as follows. First, we study the proposed configuration for these filters in Subsection 7.2.2. We analyze the influence on the filter response of the size and length of the resonating apertures. Also, we show how to impact the coupling between apertures using the length of the propagating intermediate waveguide section. Based on these studies, in Subsection 7.2.3, we design three filters with different bandwidths centered at the same frequency and with the same insertion loss (IL) level. These designs show the high flexibility of the proposed technology to synthesize the filter requirements. Finally, we propose a last design using rectangular waveguide sections filled with Teflon. As expected, the filter response is then shifted to lower frequencies. This last filter is also simulated using the MEN formulation described in Chapter II. The good agreement between the MEN results and the simulations obtained with other software tools confirms the usefulness of the equivalent network technique in the analysis of these waveguide filters for space applications.

7.2.2 Study on the Resonating Iris Technology

In this subsection, we perform three different studies to characterize the resonating iris technology proposed to implement the required filters. The structure used for these tests is the one shown in Fig. 7.1.

As observed, it consists of five rectangular waveguide sections (from WG 1 to WG 5). Two of them are used as input/output ports (WG 1 and 5). On the other hand,

Figure 7.1: 3D view of the proposed filter technology based on resonating apertures.

WG 2 and 4, that are normally used as coupling windows, are now the two resonating irises that implement the resonators of the filter. Finally, WG 3 is a propagating waveguide that is used to control the coupling level between the two resonators. As observed, this filter is more compact than traditional rectangular waveguide filters (as the one shown in Fig. 7.2).

Figure 7.2: 3D view of a traditional waveguide filter using half-wavelength cavities.

To implement the same filter using traditional waveguide technology, we would need seven waveguide sections. Two for exciting the structure (WG 6 and 12 in Fig. 7.2). Two for implementing the input/output couplings (WG 7 and 11). The two half-wavelength resonant cavities (WG 8 and 10) and the inter-resonator coupling window (WG 9). In this section, we will demonstrate that, in the proposed technology, the intermediate waveguide (WG 3) does not need to be half-wavelength long since it is not acting as a resonator. On the contrary, the fundamental mode is propagating

along this section, so the it makes the coupling between the resonators (WG 2 and 4) possible. In addition, we will demonstrate that the resonating apertures (WG 2 and 4) can be zero-thickness, which allows to implement very compact resonators.

To cope with these objectives, in this subsection, we characterize the structure proposed in Fig. 7.1. For this, we first study the influence of the size of the apertures (WG 2 and 4) on the response of this structure. Then, we test how the length of the apertures (WG 2 and 4) and the length of the coupling waveguide section (WG 3) affect the filter response. In all these tests, we use WR-90 rectangular waveguide sections to implement WG 1, 3 and 5 (22.86 mm $\times$ 10.16 mm). Since the filters are designed for X/Ku-band applications, the fundamental mode is propagating along these sections.

Size of the iris

To study the influence on the filter of the size of the iris, we use the structure in Fig. 7.1. The length of the intermediate waveguide section (WG 3) is $l_{int} = 5$ mm. The resonating apertures are zero-thickness. We perform an analysis of the filter for different dimensions of WG 2 and 4 ($a_{ap} \times b_{ap}$). The simulated results using ANSYS HFSS are reported in Fig. 7.3.

As observed in Fig. 7.3, we have a second order response where the apertures (WG 2 and 4) are resonating. We can also observe that, if we increase the x -dimension (width) of the aperture a_{ap} , the aperture becomes more inductive and then the resonant frequencies of the apertures decrease. On the contrary, if we increase the y -dimension (height) of the aperture b_{ap} , we decrease the capacitance so then the resonant frequencies increase. This aperture is then behaving as an LC resonator *Cameron et al.* (2007), where the resonant frequency is

$$w_o = \frac{1}{\sqrt{LC}}. \quad (7.1)$$

Figure 7.3: Magnitude of the Scattering parameters of the bandpass filter in Fig. 7.1 for different sizes of the resonating apertures. The dimensions of the aperture ($ap = a_{ap} \times b_{ap}$) are in mm.

By changing the size of the apertures (WG 2 and 4) we can control the resonant frequencies of the filter in Fig. 7.1. Also, we can appreciate in Fig. 7.3 that, by changing the size of the apertures, the couplings have also changed. The bigger the aperture is, the more energy is coupled between WG 1, 3 and 5 and the resonators.

Length of the iris

In this test, we show the influence on the filter response of the length of the iris (WG 2 and 4) along z -axis, l_{ap} . In the previous test, zero-thickness apertures were considered. However, from a manufacturing point of view, they have to be thick waveguide sections. To see how the filter response changes according to thickness of the apertures l_{ap} , we implement a filter using the structure in Fig. 7.1 with an aperture of $a_{ap} \times b_{ap} = 15 \text{ mm} \times 4 \text{ mm}$. Again, the length of the intermediate waveguide section (WG 3) is $l_{int} = 5 \text{ mm}$. Then, we simulate the S-parameters for different values of l_{ap} . These simulations are compared in Fig. 7.4.

Figure 7.4: Magnitude of the Scattering parameters of the filter in Fig. 7.1 for different lengths (l_{ap}) of the resonating apertures.

As observed, the filter is also working for thick apertures. We can notice that, when the thickness of the apertures is increased, the second reflection zero (RL) shifts to lower frequencies and then the two poles come closer. However, the return loss level is maintained. This means that, when the thickness is increased, the couplings are decreased and then lower bandwidths can be synthesized. As a result, we can conclude that the thickness of the resonating apertures allows to control the couplings in the filter. With very thin apertures, we will be able to synthesize wideband filters while thick apertures will yield more narrowband filters. The fact that we can control the coupling acting on the resonator implies that we are changing the slope parameter of the resonator *Cameron et al. (2007)*.

Length of the coupling waveguide

In this last test, we investigate the influence on the filter response of the length along z -axis of the intermediate rectangular waveguide section (WG 3, in Fig. 7.1). For that, we implement a filter using the structure depicted in Fig. 7.1 with zero-

thickness apertures of $a_{ap} \times b_{ap} = 15 \text{ mm} \times 4 \text{ mm}$. We perform the analysis of the filter for different lengths of the intermediate waveguide section (WG 3), l_{int} . For this test, it is important to remember that WG 3 follows the rectangular waveguide standard WR-90. Since we are above cut-off, the fundamental mode is propagating along this section. This is an important difference between the proposed configuration and the traditional waveguide filters, where the coupling windows are normally below cut-off. In this case, the fundamental mode is coupling the two apertures. This propagative waveguide section neither acts as the traditional impedance inverters normally used as coupling windows, since it does not need to be quarter-wavelength long. The coupling mechanism occurs due to the discontinuity between the aperture and the WR-90 waveguide. This applies also to the input/output coupling (between WG 1/3 and WG 2/4).

In Fig. 7.5, we show the filter response for different lengths of WG 3. As observed, the coupling between resonators can be controlled using this length. Also, it can be

Figure 7.5: Magnitude of the Scattering parameters of the bandpass filter in Fig. 7.1 for different lengths of the intermediate waveguide section (WG 3), l_{int} .

appreciated that, when this section becomes larger, the coupling decreases. However,

this behavior is periodic (since the mode is propagating). This means that, if we continue increasing the length of this section, the coupling starts to increase again. This is shown in Fig. 7.6.

Figure 7.6: Magnitude of the Scattering parameters of the bandpass filter in Fig. 7.1 for different lengths of the intermediate waveguide section (WG 3), l_{int} .

From these tests, we can conclude that:

- Using the size of the apertures, we can select the resonant frequencies.
- Using the length of the apertures, we can synthesize wideband or narrowband filters.
- Using the length of the intermediate waveguide, the coupling between resonances can be fine-tuned.

7.2.3 Filter Designs

In this subsection, we implement three different filter responses based on the studies performed in the previous subsection. These filters have the same center frequency $f_c = 12.64$ GHz and the same return loss level $RL = 20$ dB. However,

to show the flexibility of the proposed technology, they have different bandwidths $BW_1 = 3.84$ GHz, $BW_2 = 7.11$ GHz and $BW_3 = 1.77$ GHz. In these designs, WG 1, 3 and 5 in Fig. 7.1 are built in WR-90 ($22.86 \text{ mm} \times 10.16 \text{ mm}$). The apertures are considered zero-thickness. We modify the size of the apertures (WG 2 and 4) and the distance between apertures (length of WG 3) to manually adjust the filters. The final dimensions and data for the three filters are collected in Table 7.1.

Table 7.1: Physical dimensions of the three proposed filters, according to the sketch shown in Fig. 7.1.

	First Design	Second Design	Third Design
Aperture size (mm)	15×4	18×5.8	13.2×2
WG 3 length (mm)	5	4.5	7
Bandwidth	$BW_1 = 3.84$ GHz	$BW_2 = 7.11$ GHz	$BW_3 = 1.77$ GHz

The S-parameter response of the three designs are compared in Fig. 7.7. As ob-

Figure 7.7: Magnitude of the Scattering parameters of the three filters obtained with ANSYS HFSS. The dimensions are reported in Table 7.1.

served, the filters are centered at the same frequency and with the same RL . However, the three designs have different bandwidths. As previously mentioned, for tuning the

filter, we have acted on the size of the apertures and on the length of the coupling waveguide. From the previous tests, we know that, this two parameters are enough to control the resonant frequencies and the couplings of the filter. This is proved with the designs in Fig. 7.7. The fractional bandwidths of these designs are $FBW_1 = 30.38\%$, $FBW_2 = 56.25\%$ and $FBW_3 = 14\%$. The second design confirms the possibility of implementing wideband filters using the proposed resonating iris technology. This high bandwidths are difficult to synthesize in traditional waveguide technology [see, for instance, *Perez Soler et al. (2007)*] due to the difficulty of implementing high couplings using below cut-off coupling windows.

Finally, to see the effect of considering dielectrics, we fill all the waveguide sections of the first design (second column of Table 7.1) with Teflon ($\epsilon_r = 2.1$). The S-parameters response obtained using the MEN formulation of Chapter II is compared in Fig. 7.8 with the results from FEST3D and ANSYS HFSS.

Figure 7.8: Magnitude of the Scattering parameters of the designed filter using Teflon, obtained with the MEN code, with FEST3D and ANSYS HFSS.

The first thing that can be noticed is that the filter shifts to lower frequencies. Specifically, the new design in X-band has a center frequency of $f_c = 7.616$ GHz. Also,

we see that MEN results are in good agreement with respect to some commercial tools like FEST3D or ANSYS HFSS, which fully validates the MEN formulation described in Chapter II.

The designs presented in this subsection show the flexibility of this technology to synthesize compact rectangular waveguide filters with different bandwidths for X/Ku-band space communication applications.

7.2.4 Conclusions

In this section, a resonating iris technology using WR-90 rectangular waveguides is proposed to implement compact filters. The motivation of the work presented in this section is to synthesize filter responses with different bandwidths for X/Ku satellite applications. For this, we first performed three different studies related to the effect of the size and length of the resonating apertures and the length of the intermediate waveguide that influences the inter-resonator coupling. Based on these tests, we propose three different designs using this technology. The filters are centered at the same frequency and have the same return loss level. Finally, a lower frequency design is also implemented using rectangular waveguide sections filled with Teflon. This filter has been analyzed using the MEN technique described in Chapter II, showing good agreement with respect to ANSYS HFSS and FEST3D results.

The reported designs validate the proposed structure and show its high flexibility to synthesize different solutions according to the requirements. An important point of these solutions is the reduced footprint as compared to the same filters implemented in traditional rectangular waveguide technology. This is specially interesting for space communication applications, where the size is always an important constraint. In addition, another important characteristic of these filters in comparison to traditional waveguide filters is the possibility of implementing wideband solutions thanks to the coupling mechanism employed in these designs.

Acknowledgment

This work has been developed in collaboration with the students Antonio M. Huéscar and Eva Bernal, from the Universidad Politécnica de Cartagena.

7.3 Thick Metallic Bar Filters for Wideband C-band Applications

The aim of this section is to design compact wideband bandpass filter solutions at C-band frequencies with good performance in terms of unloaded quality factors.

7.3.1 Introduction

Wideband filters are required in modern communication systems, including wireless communications, 5G, satellites, or in artificial intelligence platforms. Most of these applications require compact solutions for filters, with good performance in terms of insertion losses, return losses and unloaded quality factors.

In applications where the footprint and volume are critical, planar technologies, such as microstrip *Cameron et al.* (2007); *Hong* (2011), stripline *Jokela* (1980) or coplanar *Thomson and Hong* (2007); *Williams and Schwarz* (1983) are usually employed. Using these technologies, the level of losses is normally sacrificed in order to produce structures with reduced dimensions and low weight.

Among the planar solutions, microstrip technology *Hong* (2011) is one of the most employed in practice, due to the achieved compactness, ease of integration and low-cost fabrication. Unfortunately, the most important limitations faced by microstrip resonator filters *Cameron et al.* (2007) for these applications are normally small bandwidths and low Q_U . Some previous works have tried to implement broadband filters using microstrip line resonators. For instance, in *Athukorala and Budimir* (2012), the authors use concentric open-loop resonators to reduce the size of the filter and achieve

a fractional bandwidth $FBW \approx 10\%$. It is challenging to enhance the bandwidth of these microwave filters beyond these limits, mainly because very strong couplings are required between resonators and in the input/output (I/O) ports. However, the coupling levels achievable with microstrip line resonators are low, which limit in practice the implementation of wideband responses. The second limiting factor of printed microstrip technology is the relatively low Q_U . This factor is typically less than 150 for this technology. The low Q_U is mainly due to the field distribution of the fundamental microstrip mode (quasi-TEM), and the very small thickness of printed resonators.

Alternative hybrid technologies are the Substrate Integrated Waveguide (SIW) *Bozzi et al.* (2011); *Liu et al.* (2016), Empty SIW (ESIW) *Belenguer et al.* (2014), Quarter Mode SIW (QMSIW) *Moscato et al.* (2016) or Air Filled SIW (ASIW) *Silvestri et al.* (2019). With all these types of substrate integrate technology, the Q_U is increased with respect to microstrip technology while the reduced dimensions are still maintained. Thus, this technology represents a good trade-off between volume and Q_U . For instance, in *Liu et al.* (2016), the achievable Q_U is around 640 for a $FBW = 16.5\%$. The main disadvantage of this technology is the transition to other planar technologies. In SIW filters, loss performance is also sacrificed at the expense of compactness.

In applications where higher Q_U and robustness are strictly required, other non-planar alternatives, such as dielectric resonators *Zhu et al.* (2018), *Pelliccia et al.* (2013) or waveguide technology *Vallerotonda et al.* (2019); *Vanin et al.* (2004) can be used. However, the compactness and low-cost manufacturing features of planar technologies are compromised.

In *Zhu et al.* (2018), the authors proposed the use of high dielectric constant strip resonators deposited on a low permittivity substrate. Using these dielectric resonators, filters with $Q_U \approx 880$ were demonstrated. As observed, the achieved Q_U using this technology is higher as compared to the traditional microstrip or SIW

technologies [see *Zhu et al. (2018)*]]. However, in that work, a standard tapped feeding line was employed to implement the I/O couplings. Due to this arrangement, rather low I/O coupling values could be implemented *Zhu et al. (2018)*, resulting into a fractional bandwidth of only $FBW = 3.3\%$. Higher Q_U can be achieved using dielectric resonators in waveguide technology as in *Pelliccia et al. (2013)*. In *Pelliccia et al. (2013)*, the fractional bandwidth is also $FBW \approx 3\%$ but with a higher quality factor of $Q_U \approx 1750$. However, the footprint is increased in comparison to the solution in *Zhu et al. (2018)*.

Instead, using directly waveguide resonators the achievable fractional bandwidths can be increased as compared to the previous technologies (typically less than 20%), as it was for instance reported in *Vanin et al. (2004)*. However, much higher Q_U factors are possible in waveguide technology. For instance, in *Vallerotonda et al. (2019)* measurements demonstrate an unloaded quality factor around $Q_U = 2700$ and a fractional bandwidth of $FBW = 8.3\%$. Even though compact solutions have been proposed *Vallerotonda et al. (2019)*, the sizes of waveguide filters are normally higher than with other implementations using planar or hybrid technologies.

In this section, we present an alternative technology to realize wideband filters with reduced footprints in comparison to waveguide and dielectric resonator technologies, while maintaining large Q_U as compared to planar technologies. A 3D view of the proposed technology is shown in Fig. 7.9.

The proposed technology employs thick half-wavelength metallic bars buried in a dielectric medium. A metallic enclosure is employed to avoid spurious radiation (see Fig. 7.9). In comparison to planar technologies, we demonstrate that the thick bars significantly increase the achievable coupling levels among resonators, while providing larger Q_U . The use of metallic bars with large thickness also produces a strong decrease in the characteristic impedance of the guided TEM mode. Consequently, we demonstrate that I/O couplings can be implemented by introducing a step impedance

Figure 7.9: 3D view of the proposed filter technology based on bulky bars buried in a dielectric medium. The sketch shows an example of a filter using three bar resonators enclosed in a house cavity. I/O couplings are implemented with the step impedance discontinuity produced from a direct connection of the coaxial connectors with the first/last bar resonators.

discontinuity between the coaxial connectors and the first/last resonators. In this way, explicit I/O feeding lines are avoided. The adjustment of this step impedance discontinuity together with the capability to implement very large inter-resonator couplings, enables to implement wideband frequency responses. Since no I/O lines are required, the footprint of the structure is reduced as compared to standard boxed microstrip printed technology and SIW. However, the manufacturing complexity and volume are increased with respect to these strictly planar technologies.

In comparison to non-planar technologies, thick bar resonators exhibit Q_U s that are similar than the ones obtained using dielectric resonators, but lower than waveguide resonators. However, the I/O mechanism proposed in this paper together with the high inter-resonator couplings achievable with the thick bars allow to increase the achievable fractional bandwidths. The manufacturing complexity is similar to these non-planar technologies, while the weight, footprint and volume are significantly re-

duced.

We also show that an additional resonant mode can be excited in the shielding cavity partially filled with dielectric, similar to the idea proposed in *Martinez-Mendoza et al.* (2007), *Martinez-Mendoza et al.* (2008). In those works, the I/O feeding lines were used to excite an additional cavity box resonance. In this new structure, some of the bar resonators, when adequately placed inside the cavity, can conveniently couple to this additional resonant mode. This allows for the implementation of transversal topologies *Perez Soler et al.* (2007), which can be used to produce transmission zeros at finite frequencies in the transmission coefficient of the filter.

To demonstrate these features, three filters are designed using the proposed concept. First, a third order in-line filter is designed using only three thick bar resonators (not using the cavity mode). The filter is designed at C-band, with center frequency $f_{c1} = 4.26$ GHz, and bandwidth $BW_1 = 800$ MHz, leading to fractional bandwidth of $FBW_1 = 18.78\%$. Then, a transversal-based topology of fourth order is implemented with three bar resonators and one box resonator. The transversal filters are also designed at C-band, with center frequencies $f_{c2} = 3.26$ GHz and $f_{c3} = 3.35$ GHz, and bandwidths $BW_2 = 1.15$ GHz and $BW_3 = 1.575$ GHz, leading to fractional bandwidths of $FBW_2 = 35.28\%$ and $FBW_3 = 47.01\%$, respectively.

To validate the new concept, the first transversal filter with lower bandwidth (BW_2) has been manufactured using thick bars made of copper, and Teflon as the dielectric material. The external housing is made from aluminum. Measurements obtained from the manufactured hardware show very good agreement against initial target specifications, and with respect to full-wave simulations obtained with the MEN formulation from Chapter VI and ANSYS HFSS (*HFSS*) (2020). The results demonstrate that the proposed technology represents a good trade-off between high performance (in terms of IL , RL and Q_U) and compactness, allowing the implementation of very wideband responses, and with good spurious free range (SFR)

performance.

7.3.2 Thick Bar Technology Characterization

In this subsection, we first characterize the proposed technology in terms of the resonators Q_U , inter-resonator couplings and input/output couplings. Special attention will be given to the effect of the metallization thickness in these parameters, since this is the distinct feature as related to standard planar technologies.

Quality factor of resonators

The proposed technology is first characterized in terms of achievable Q_U , by using one isolated metallic bulky bar acting as a microwave resonator (see inset of Fig. 7.10). The behavior of the resonator according to its length (l_r) and width (w_{r2} , see Fig. 7.9) is similar to a microstrip resonator. However, in contrast to printed microstrip resonators, this technology uses bars with a non negligible thickness (l_{th} , see Fig. 7.9). In Fig. 7.10 we plot the value of Q_U for different thickness of the resonator l_{th} ($0.1 \text{ mm} < l_{th} < 5 \text{ mm}$). The bar resonator is made of copper with conductivity $\sigma_c = 5.8 \cdot 10^7 \text{ S/m}$, and it is placed above a dielectric medium made of Teflon ($\epsilon_r = 2.1$, $\tan \delta = 0.001$) with thickness $l_d = 10 \text{ mm}$. All the elements are located inside a cavity made of aluminum with conductivity $\sigma_b = 3.8 \cdot 10^7 \text{ S/m}$.

According to Fig. 7.10, results show a strong dependence of Q_U with the resonator thickness l_{th} . Maximum Q_U of 1360 is obtained for the largest thickness $l_{th} = 5 \text{ mm}$. This value drops to $Q_U = 830$ for the thinnest considered metalization of $l_{th} = 0.1 \text{ mm}$. This study, therefore, shows the first benefit of the proposed technology, namely, the possibility of having significantly larger Q_U values as compared to those obtained using printed resonators. This large Q_U can be attributed to the extended metallic surface provided by the bulky bars, through which the induced currents can flow.

Figure 7.10: Unloaded quality factor Q_U of a single bulky bar resonator, as a function of the thickness l_{th} . The geometry under test is shown in the inset. The dielectric is Teflon with a relative permittivity of $\epsilon_r = 2.1$ and a loss tangent $\tan \delta = 0.001$. The bar is made using copper with conductivity $\sigma_c = 5.8 \cdot 10^7$ S/m, and the box is made of aluminum with conductivity $\sigma_b = 3.8 \cdot 10^7$ S/m. Other dimensions are $l_d = 10$ mm, $l_a = 10$ mm, $l_r = 15$ mm, $w_r = 2$ mm (bar resonator width in the y -axis), $a = 30$ mm and $b = 20$ mm (box dimension in the y -axis). The test frequency is $f_c = 5.56$ GHz.

Inter-resonator couplings

The next step is to characterize the inter-resonator coupling when the bulky bar technology is used. For this purpose, let us consider two symmetric coupled resonators as illustrated in Fig. 7.11. The dark areas in this figure l_p represent the feeding ports of the resonators. With this setup, we calculate a normalized inter-resonator coupling coefficient M_{ij} between the two resonators, using the described frequency plan for the bandwidth BW_2 Cameron et al. (2007)

$$M_{ij} = \frac{f_{c2}}{BW_2} k_{ij}, \quad (7.2)$$

Figure 7.11: Side (left panel) and top (right panel) view of two coupled thick bar resonators, showing relevant dimensions. Feeding ports are shown with dark areas l_p .

where k_{ij} is the inter-resonator coupling coefficient extracted from the even/odd frequencies of the two coupled resonators. As usual, the test is performed under very weak I/O couplings.

In Fig. 7.12, the normalized coupling coefficient M_{ij} is shown as a function of the separation between bars y_{12} , and for different thicknesses l_{th} . Similarly to two coupled microstrip printed resonators, the inter-resonator coupling in this structure can be controlled using both y_{12} and the overlapping distance along the x -axis (x_{12} , see Fig. 7.11). This is observed in the plot, since the normalized coupling coefficient clearly increases as y_{12} is decreased. What it is more interesting is the behavior of the inter-resonator coupling with the bar thickness. The plot clearly indicates that the inter-resonator coupling can be significantly increased using thick bars. In particular, if the thickness of the bar resonator is small ($l_{th} = 0.1$ mm), the maximum normalized coupling value is $M_{ij} = 0.78$. This coupling can be increased to $M_{ij} = 1.15$ if l_{th} is increased to 5 mm. This study shows the second benefit of the proposed technology, which is the possibility to obtain larger inter-resonator couplings. They can be attributed to a stronger capacitive effect that is formed between bars, when the thickness is increased.

Figure 7.12: Normalized coupling coefficient as a function of the separation between resonators y_{12} , for various bar thicknesses l_{th} . Coupling coefficient is normalized with the second frequency plan ($f_{c2} = 3.26$ GHz, $BW_2 = 1.15$ GHz, $FBW_2 = 35.28\%$). Other dimensions are: $a = 30$ mm, $b = 28$ mm, $y_{r1} = y_{r2} = 10$ mm, $w_{r1} = w_{r2} = 2$ mm, $x_{12} = 0$ mm, $l_d = 10$ mm, $l_a = 10$ mm. The relative permittivity of the substrate is $\epsilon_r = 2.1$.

Input/output couplings

The implementation of wideband responses requires not only of large inter-resonator couplings, but also a mechanism to implement large I/O couplings. In this context, it is important to mention that another difference between this technology and traditional microstrip implementations is the characteristic impedance of the transmission line (TEM) mode, formed by the thick bar structure. In general, microstrip filters are designed with standard coaxial transitions with $50\ \Omega$ or $75\ \Omega$ transmission line impedances. The microstrip line is usually dimensioned to match the coaxial impedance, in order to avoid unnecessary reflections between the first/last printed lines and the source/load. Since no impedance mismatch is introduced, the first/last metalizations in these structures act as the feeding lines for the printed coupled-line

resonators of the filter *Hong* (2011). However, when the thickness of the metalization increases, the characteristic impedance substantially drops. Consequently, large impedance mismatch can be introduced between the coaxial connectors and the thick bar acting as a transmission line.

With this technology, the idea is to implement the I/O couplings of the filter by directly using this impedance mismatch. Following this idea, the feeding lines traditionally used in regular printed microstrip filters are eliminated. The I/O couplings are simply implemented by the step impedance discontinuity introduced when directly connecting the pin of the coaxial connector to the first/last thick bar resonators (represented with dark areas l_p in Fig. 7.11). On this basis, the third advantage of the proposed technology is reduced footprint in comparison to non-planar technologies but also to regular boxed microstrip filters (that require two additional I/O feeding lines).

To illustrate the concept, a test with a singly loaded resonator is performed to obtain the external quality factor Q_{ext} as a function of the characteristic impedance Z_p of the input coaxial connector, modeled as a lumped port connected to the thick bar resonator. The structure employed for this test is depicted in the inset of Fig. 7.13. The test is performed by computing the maximum value of the group delay for the S_{11} parameter in this singly terminated configuration *Cameron et al.* (2007). Results are presented in Fig. 7.13 as a function of the port reference impedance Z_p for different bar thickness l_{th} . The values obtained for small port impedances are around $Q_{ext} = 70 - 100$, while $Z_p > 10 \Omega$ produce very low external quality factors ($Q_{ext} < 10$). We can also observe that, when Z_p is small, the use of thicker bars leads to smaller Q_{ext} . For instance, the largest Q_{ext} is around 100 when l_{th} is small ($l_{th} = 1 \text{ mm}$). This value decreases to 70 if a thicker bar is used ($l_{th} = 5 \text{ mm}$). However, the effect of the thickness in the Q_{ext} is marginal when $Z_p > 10 \Omega$.

The observed behavior indicates that very large I/O couplings are indeed achiev-

Figure 7.13: External quality factor Q_{ext} as a function of the input port reference impedance Z_p , for various values of the bar thickness l_{th} . The test frequency is $f_c = 3.5$ GHz. Other dimensions are: $l_d = 10$ mm, $l_a = 10$ mm, $y_{r1} = 10$ mm, $w_{r1} = 2$ mm, $a = 38$ mm, $b = 32$ mm. The dark area l_p denotes a lumped port that models the direct connection of the coaxial pin to the resonator.

able by properly adjusting the step impedance discontinuity introduced between the coaxial connector and the first/last resonator, and therefore there is no need to use extra I/O feeding lines. The opportunity to implement very large I/O couplings, finally demonstrates another benefit of the proposed technology, which is the possibility to synthesize very wideband responses.

7.3.3 Design of Filters Based on Thick Bar Technology

In this subsection, an in-line filter and two additional filters implementing a transversal topology are designed using the proposed thick bar resonator technology.

Example 1: In-line filter.

To demonstrate the feasibility of the described technology, the design of a third order filter using the classical in-line topology is presented. The filter is centered at $f_{c1} = 4.26$ GHz, and has a bandwidth of $BW_1 = 800$ MHz ($FBW_1 = 18.78\%$). The structure of the filter is presented in Fig. 7.9. It incorporates three thick bar resonators with the geometry sketched in Fig. 7.14. The three bars represent the resonators of the filter. In Fig. 7.14, the first and the third bars are coupled by proximity to the second bar resonator placed at the center of the box (bar 2). For the practical realization of the structure, the three thick bar resonators are embedded in a dielectric material of height l_d . To avoid the excitation of box modes, a foam substrate with $\epsilon_r = 1.07$ has been used in this design as dielectric. It should be

Figure 7.14: Top (right panel) and side (left panel) view of the three filters designed using the new proposed technology (one following the in-line topology and two more with a transversal topology). Relevant dimensions are also shown. Thick bars are identified with numbers. For the transversal filters the bars are numbered according to resonators shown in the coupling topology of Fig. 7.17. Dark areas (l_p) represent lumped ports that model the direct connection of the coaxial pins to the first/last resonators.

remarked that the previously described mechanism for the I/O ports is used in this design. This is the reason why we cannot see extra I/O feeding lines in the structure shown in Fig. 7.14.

The $(N + 2)$ coupling matrix of the designed in-line filter for a transfer function with return losses $RL = 20$ dB is *Cameron et al.* (2007)

$$M = \begin{bmatrix} 0 & 1.082 & 0 & 0 & 0 \\ 1.082 & 0 & 1.03 & 0 & 0 \\ 0 & 1.03 & 0 & 1.03 & 0 \\ 0 & 0 & 1.03 & 0 & 1.082 \\ 0 & 0 & 0 & 1.082 & 0 \end{bmatrix}. \quad (7.3)$$

First, the I/O coupling ($M_{S1} = M_{3L} = 1.082$) can be adjusted with the external quality factor Q_{ext} , which can be computed from the normalized I/O coupling as *Cameron et al.* (2007)

$$Q_{ext} = \frac{f_{c1}}{M_{S1}^2 BW_1}, \quad (7.4)$$

leading to $Q_{ext} = 4.548$. From this expression it can be observed that wide bandwidths lead to small values of Q_{ext} , thus requiring large I/O couplings. Fortunately, these large couplings can be implemented by adjusting the step impedance discontinuity between the coaxial pin and the first/last bar resonator. In fact, the plot shown in Fig. 7.13 indicates that the required value of Q_{ext} is about the minimum values that can be obtained with the proposed I/O coupling mechanism.

In order to employ a commercial I/O connector, and at the same time permitting the implementation of the required Q_{ext} value, we propose to introduce a step discontinuity in the radius of a standard coaxial connector. Details of the arrangement are shown in Fig. 7.15. The nominal radii of the inner and outer conductors of the connector are R_1 and r_1 respectively, giving a characteristic impedance of 50Ω . Next, a length l_{ju} is reserved in the cavity wall to introduce a step discontinuity in these radii. The internal conductor radius is provided by the commercial connector, and therefore it is easier to leave it untouched ($r_2 = r_1$). In order to lower the characteris-

Figure 7.15: 3D view of the coaxial transition employed to adjust the I/O couplings.

tic impedance of the coax, a step discontinuity is introduced in the radius of the outer conductor R_2 . By selecting $R_2 < R_1$ and $r_2 = r_1$ (see Fig. 7.15), the characteristic impedance of the corresponding section can be substantially lowered.

For the implementation of the required I/O couplings, the commercial connector PE4128 from PASTERNAK was selected for the external section shown in Fig. 7.15, leading to outer and inner conductors radii of $R_1 = 2.1$ mm and $r_1 = 0.65$ mm. As mentioned before, the internal section that implements the step discontinuity takes the same radius for the inner conductor ($r_2 = r_1$), while it changes the radius of the outer conductor to $R_2 = 0.9$ mm. This example shows the high flexibility of the proposed technique to obtain very large I/O couplings, while employing commercial $50\ \Omega$ I/O coaxial connectors.

Once the I/O couplings are implemented, the next challenge is to implement the required normalized inter-resonator couplings ($M_{12} = M_{23} = 1.03$). Equation (7.2) clearly indicates that wide bandwidths again require very large inter-resonator couplings ($k_{12} = k_{23}$). This large couplings can be implemented by increasing the thickness of the bars, as detailed in the previous section. Once a proper thickness value is selected for the bar resonators, the inter-resonator coupling can be fine adjusted by acting on the variables $x_{12} = x_{23}$ and $y_{12} = y_{23}$ of Fig. 7.14. This is easily accomplished by producing charts similar to the one shown in Fig. 7.12.

It is important to point out that the design of filters using this coupling matrix approach is only accurate for narrow band filters. This is because of the intrinsic narrow band approximation implied in deriving the equivalent circuit represented by the coupling matrix (based on lumped elements and ideally constant impedance or admittance inverters). Due to dispersive effects in the elements of the real physical implementation, a final optimization step is needed to recover the desired target specifications. After this optimization step, the final dimensions of the designed filter are collected in Table 7.2. The total size of the filter is 35.99 mm $\times$ 26.2 mm $\times$ 16 mm.

Table 7.2: Physical dimensions of the designed third order in-line filter, according to the sketches shown in Fig. 7.14 and Fig. 7.15, with foam as a dielectric material ($\varepsilon_r = 1.07$).

Parameters of the filter	Dimensions (in mm) In-line filter
l_a	8
l_d	6
l_{th}	2
$l_{r1} = l_{r3}$	16.35
l_{r2}	29.09
$w_{r1} = w_{r2} = w_{r3}$	2
$y_{r1} = y_{r3}$	5.1
$y_{12} = y_{23}$	5.1
$x_{12} = x_{23}$	13.9
R_1	2.1
R_2	0.9
$r_1 = r_2$	0.65
l_{ju}	4
a	35.99
b	26.2

The in-band response of the in-line filter is shown in Fig. 7.16. The full-wave response of the filter is compared to the coupling matrix response, showing that the design process is effective for the presented filter. It can be observed very good agreement between the full-wave response and the circuit model inside the passband. In

Figure 7.16: Scattering parameters of the in-line filter obtained with ANSYS HFSS (design) and compared to the coupling matrix response (matrix). The bandwidth is $BW_1 = 800$ MHz ($FBW_1 = 18.78\%$). The dimensions are reported in Table 7.2.

particular, the right center frequency, bandwidth and return loss levels have been correctly recovered. Out of the passband, the rejection slopes between the full-wave response and the circuit model start to differ from each other. Again, this is due to the large bandwidth of the filter, that introduces, in the physical implementation, important dispersion effects in resonators and coupling elements. However, as observed in Fig. 7.16, the optimization process can effectively compensate for these effects to correctly recover the response inside the bandwidth.

Example 2: Transversal filters.

Now we address the design of two fourth order transversal filters with two different bandwidths. The first design ($f_{c2} = 3.26$ GHz, $BW_2 = 1.15$ GHz) has a prescribed transmission zero at $f_{z2} = 4.15$ GHz, while the second design ($f_{c3} = 3.35$ GHz, $BW_3 = 1.575$ GHz) has a prescribed transmission zero at $f_{z3} = 4.5$ GHz. The positions of the transmission zeros have been adjusted to achieve a ripple level of

approximately 20 dB in the rejection band. Note that both filters have quite large fractional bandwidths ($FBW_2 = 35.28\%$ and $FBW_3 = 47.01\%$).

The 3D structure to be designed can be seen in Fig. 7.9. It will incorporate three thick bar resonators with the geometry sketched again in Fig. 7.14. As before, the three thick bar resonators are embedded in the dielectric material. The main difference with respect to the in-line filter is that now the dielectric material that partially fills the cavity is Teflon, and this allows the excitation of an additional box resonance [as described in *Martinez-Mendoza et al.* (2007, 2008)]. This box resonance will form the fourth resonator of our fourth order filter structure. The first/last bar resonators (bars 1 and 3 in Fig. 7.14) placed close to the left/right side walls of the cavity will couple to this resonance. On the contrary, the thick bar placed at the center (bar 2 in Fig. 7.14), will barely couple to this mode *Martinez-Mendoza et al.* (2007). As a result, the coupling topology implemented with this arrangement is of the type known as transversal *Amari and Rosenberg* (2002), and it is illustrated in Fig. 7.17. This coupling topology can implement one transmission zero at finite frequencies *Amari and Rosenberg* (2002).

Figure 7.17: Coupling topology that represents the filter structure shown in Fig. 7.14, with an additional cavity resonance. Bar resonators correspond to resonators 1, 2, and 3 in this topology, while the cavity resonance is represented by resonator 4. Input and output ports are denoted as source (S) and load (L), respectively. Couplings are shown with arrows.

The $(N + 2)$ coupling matrix of the first transversal filter for a transfer function

with return losses $RL = 20$ dB is *Cameron et al.* (2007)

$$M_1 = \begin{bmatrix} 0 & 1.03 & 0 & 0 & 0 & 0 \\ 1.03 & 0.07 & 0.816 & 0 & -0.402 & 0 \\ 0 & 0.816 & 0.378 & 0.816 & 0 & 0 \\ 0 & 0 & 0.816 & 0.07 & 0.402 & 1.03 \\ 0 & -0.402 & 0 & 0.402 & -0.949 & 0 \\ 0 & 0 & 0 & 1.03 & 0 & 0 \end{bmatrix}. \quad (7.5)$$

The implementation of the input/output couplings as well as the couplings between bars can be accomplished by following similar considerations described for the in-line filter.

From the matrix shown in equation (7.5), we can see that, in this case, one of the four inter-resonator couplings must have negative sign, necessary for implementing single-band responses. This is given by the elements $M_{14} = -M_{34}$, representing the couplings between the first/last (1,3) bars and the cavity resonance. To implement this sign change, bars 1 and 3 are located at opposite side walls of the cavity (left and right side walls, as shown in Fig. 7.14). The sign change is finally produced by a direction inversion in the electric field of the box mode when going from the left to the right side walls of the cavity. More details about the field distribution of this mode are given in *Martinez-Mendoza et al.* (2007). The absolute value of this coupling will be dependent on the distance from these bars to the top and bottom sidewalls (parameter $y_{r1} = y_{r3}$ in Fig. 7.14), as well as on the air and dielectric heights (parameters l_a and l_d in Fig. 7.14). Since the x -component of the electric field is zero at the top/bottom side walls, this coupling increases when y_{r1} and y_{r3} increase.

The second transversal filter requires a stronger coupling, since it has a wider bandwidth (BW_3). This is achieved by increasing $y_{r1} = y_{r3}$. However, this distance also affects the width of the box b , therefore modifying the resonant frequency of the

cavity mode (M_{44} of the coupling matrix). To correct for this shift of the resonant frequency, the overlapping distance between bars $x_{12} = x_{23}$ can be shortened, thus effectively decreasing the length of the box a . Alternatively, the resonant frequency of the cavity mode can also be adjusted by modifying the air and/or dielectric heights, l_a and l_d . We note that the modification of the overlapping distance $x_{12} = x_{23}$ will, in turn, change the couplings between bars $M_{12} = M_{24}$. However, the effect is compensated, once more, with the adjustment of the separation between them $y_{12} = y_{23}$. After all couplings and resonant frequencies are adjusted, a global optimization is applied to the whole structure for fine-tuning. Again, this is mainly needed due to the large bandwidths of the implemented filters, that require for the compensation of the dispersive behavior of the filter elements. After this final optimization step, the final dimensions of the two designed filters are included in Table 7.3. The total size of the two transversal filters are $42.2 \text{ mm} \times 48.6 \text{ mm} \times 22 \text{ mm}$ (for the second design) and $32.76 \text{ mm} \times 51 \text{ mm} \times 18 \text{ mm}$ (for the third design).

Details of the in-band responses for the two designed filters are shown in Fig. 7.18 and Fig. 7.19. Comparisons with respect to the responses of the coupling matrix are also included, showing that the design process was effective for both filters. In the two examples the transmission zero is placed approximately at the same distance from the filter upper cutoff frequency, thus demonstrating that the passband characteristics can be controlled with the proposed structure, independently from the position of the transmission zero. As observed from Fig. 7.18 and Fig. 7.19, the transmission zero appears above the passband. However, by swapping the resonant frequencies of the resonators 2 and 4 (see Fig. 7.17), the transmission zero can be shifted below the passband [this is known in the scientific literature as the zero shifting property *Amari et al.* (2004)].

To conclude, a comparison between the two last filters in terms of spurious free range (*SFR*) has been performed. The out-of-band responses of both filters are

Table 7.3: Physical dimensions of the two transversal filters, according to the sketches shown in Fig. 7.14 and Fig. 7.15, with Teflon as dielectric material ($\epsilon_r = 2.1$).

Parameters of the filter	Dimensions (in mm) Second design ($BW_2 = 1.15$ GHz)	Dimensions (in mm) Third design ($BW_3 = 1.57$ GHz)
l_a	10	8
l_d	10	8
l_{th}	2	2
$l_{r1} = l_{r3}$	16.1	15.98
l_{r2}	30.2	26.6
$w_{r1} = w_{r2} = w_{r3}$	2	2
$y_{r1} = y_{r3}$	16.8	19.3
$y_{12} = y_{23}$	4.5	3.2
$x_{12} = x_{23}$	11.1	13.9
R_1	2.1	2.1
R_2	1.2	1.4
$r_1 = r_2$	0.65	0.65
l_{ju}	4	4
a	42.2	32.76
b	48.6	51

Figure 7.18: Scattering parameters of the second designed filter obtained with ANSYS HFSS (design) and comparison to the coupling matrix response (matrix). The bandwidth is $BW_2 = 1.15$ GHz ($FBW_2 = 35.28\%$). The dimensions of this second design are reported in the second column of Table 7.3.

Figure 7.19: Scattering parameters of the third designed filter obtained with ANSYS HFSS (design) and comparison to the coupling matrix response (matrix). The bandwidth is $BW_3 = 1.57$ GHz ($FBW_3 = 47.01\%$). The dimensions of this third design are reported in the third column of Table 7.3.

compared in Fig. 7.20. As it can be observed, the SFR of the two filters is similar. In

Figure 7.20: Comparison for the out-of-band responses of the two last designed transversal filters from ANSYS HFSS simulations. Dimensions of both filters are collected in Table 7.3.

both cases, the first spurious band appears at $f_{s1} = 6$ GHz, and is due to the excitation of higher order resonances in the shielding cavity. This gives $SFR_2 \approx 2.1$ GHz (54 %)

for the second filter and $SFR_3 \approx 1.8$ GHz (43 %) for the third design.

7.3.4 Prototype Manufacturing and Measurements

The first transversal filter designed in the previous subsection (second design) has been manufactured and measured. A photograph of the prototype is presented in Fig. 7.21. Details of one of the copper thick bars, before assembling, can be observed in Fig. 7.21(a). As shown, the thick bars are buried into slots mechanized in the Teflon substrate. The bars have been manufactured using a traditional milling process. The aluminium enclosure used for shielding is manufactured in two pieces that are

Figure 7.21: Manufactured prototype. (a) Components of the filter before assembling, and details of the slots in the Teflon substrate used for alignment. The inset inside shows the step discontinuity needed in the radius of the outer conductor for the I/O coupling adjustment. (b) Top view of the structure after assembling. Inset shows details of the assembling process between the coax center pin and the copper bar.

aligned using four dowel pins [see Fig. 7.21(a)]. This allows to easily mechanize in the left/right side walls of the cavity the step discontinuities required in the radius of the outer conductor for the I/O coupling adjustment. A detail of this process is shown in the inset of Fig. 7.21(a). Details of the assembling process of the connector pins with the copper bars are shown in the inset of Fig. 7.21(b). The two pieces of the enclosure are finally assembled with the use of 24 pressure screws. Rounded

corners of 2 mm are introduced in the different parts of the structure, as required by the milling process. They have shown to have little effect on the electrical behavior of the filter.

Measurements corresponding to the prototype in Fig. 7.21 are compared to the in-band full-wave response obtained using ANSYS HFSS in Fig. 7.22. As can be

Figure 7.22: Comparison between the S-parameters simulated using the full-wave commercial software ANSYS HFSS and measured data from the manufactured prototype shown in Fig. 7.21.

observed, the in-band measured response shows good agreement with respect to the full-wave simulation, thus fully validating the proposed concept. The small differences could be due to manufacturing issues and losses in real components.

From Fig. 7.22, we can see that the transmission zero is slightly closer to the passband than expected. This raises the stopband ripple from 20 dB to 16 dB. Moreover, the measured bandwidth is $BW = 1.06$ GHz being slightly smaller than the target value. This finally leads to a measured fractional bandwidth of 32%. Note that this large bandwidth is difficult to achieve using planar technologies based on resonant elements and even with dielectric or waveguide resonators.

As observed in Fig. 7.22, the return loss level is around 20 dB. We note that

two poles of the transfer function are complex, maybe due to manufacturing errors and imprecisions during the assembling process. It should be emphasized that the presented results are obtained directly from the manufactured prototype without applying any post-tuning operation. The insertion of tuning elements in this filter would complicate the manufacturing process. Furthermore, the measured insertion losses are $IL = 0.16$ dB. This corresponds to an approximate value for the unloaded quality factor of $Q_U = 1180$.

Now, we check the measured out-of-band response for the manufactured prototype and compare it to full-wave simulations in Fig. 7.23. Good agreement between

Figure 7.23: Comparison between the S-parameters simulated using the full-wave commercial software ANSYS HFSS and measured data from the manufactured prototype shown in Fig. 7.21 (out-of-band performance).

measured results and the ideal response is also achieved in this case. These results experimentally confirm that the first spurious band appears at $f_{s1} = 6$ GHz, confirming a $SFR \approx 2.1$ GHz (54%). This performance is also good as compared to other transversal filters implementations previously reported. See for instance the transversal filter implemented in waveguide technology with $SFR = 21\%$, reported in *Perez Soler et al. (2007)*.

In Table 7.4, we compare the proposed solution to other resonant filters reported in the technical literature, in terms of Q_U , FBW , IL and electrical size (using the guided wavelength for the center frequency of the filter). As observed in Table 7.4,

Table 7.4: Comparison in terms of Q_U , FBW , IL and electrical size between the manufactured prototype and other resonant filters reported in the literature.

Technology	Reference	Q_U	FBW	IL
Microstrip	<i>Athukorala and Budimir</i> (2012)	-	10%	1.2 dB
SIW	<i>Liu et al.</i> (2016)	640	16.5%	1.5 dB
Dielectric bars	<i>Zhu et al.</i> (2018)	880	3.3%	0.6 dB
Diel.-loaded cav.	<i>Pelliccia et al.</i> (2013)	1750	3%	0.4 dB
Waveguide	<i>Vallerotonda et al.</i> (2019)	2700	8.3%	0.2 dB
Thick bars	(second design)	1180	32%	0.16 dB

Technology	Reference	Electrical size
Microstrip	<i>Athukorala and Budimir</i> (2012)	$0.06 \times 0.06 \times 0.0042$
SIW	<i>Liu et al.</i> (2016)	$0.72 \times 0.72 \times 0.036$
Dielectric bars	<i>Zhu et al.</i> (2018)	$0.874 \times 0.4545 \times 0.3497$
Diel.-loaded cav.	<i>Pelliccia et al.</i> (2013)	$0.3438 \times 0.2865 \times 0.1719$
Waveguide	<i>Vallerotonda et al.</i> (2019)	$1.4 \times 1.27 \times 2.14$
Thick bars	(second design)	$0.464 \times 0.5347 \times 0.242$

the obtained Q_U value is greater than values typically reported for microstrip technology ($Q_U < 150$) and SIW technology ($Q_U < 640$) *Athukorala and Budimir* (2012); *Liu et al.* (2016). This is achieved at the expense of a higher volume of the filter. However, the footprint does not increase significantly, since no I/O feeding lines are required. This filter also has a Q_U higher than the filter based on dielectric resonators proposed in *Zhu et al.* (2018), where a $Q_U = 880$ was reported. However, as expected, the achieved Q_U is lower than those typically obtained using dielectric-loaded cavities *Pelliccia et al.* (2013) and waveguide technology *Vallerotonda et al.* (2019). In contrast, comparing to dielectric *Zhu et al.* (2018) and waveguide *Vallerotonda et al.* (2019) resonators, this structure offers a more compact solution with reduced footprint and IL for wideband applications.

Finally, this filter has been simulated using the Multimode Equivalent Network formulation described in Chapter VI. This is possible thanks to the inclusion of transverse ports (that represent the I/O coaxial connectors plus the coax transition used to excite this filter) in the thick MEN formulation. The ANSYS HFSS simulations and the prototype measurements are compared to the MEN response in Fig. 7.24.

Figure 7.24: Comparison between the S-parameters simulated using the MEN code, the full-wave commercial software ANSYS HFSS and measured data from the manufactured prototype shown in Fig. 7.21.

The good agreement between the MEN results, the ANSYS HFSS simulations and measurements validates the usefulness of the MEN technique for the analysis of this kind of devices.

7.3.5 Conclusions

An alternative technology for the implementation of compact wideband filters has been proposed and experimentally validated. The advantages of thick metallic bars as microwave resonators have been verified through various examples. Such advantages include the possibility to implement wide passbands, enabled by the high couplings between thick bars. Also, input/output feeding lines are avoided by exploiting the

strong mismatch between bar resonators and standard coaxial connectors. This leads to more compact structures, and allows for the implementation of very strong I/O couplings, which is also required for implementing wideband responses. Moreover, the proposed bulky resonators exhibit higher unloaded quality factors Q_U than other technologies such as microstrip or SIW. Comparing with non-planar technologies, the weight and volume is significantly reduced. Finally, we have demonstrated that the suitable excitation of an additional box resonance allows for the implementation of transversal coupling topologies. In addition to the theoretical design process, a fourth order filter with fractional bandwidth of $FBW = 32\%$ and $Q_U = 1180$, has been experimentally demonstrated. Measured return losses of 20 dB and $SFR > 50\%$ were obtained. Measurements and simulated results show good agreement, which confirm the performance of the proposed technology. The reported data experimentally demonstrate that this technology offers good trade-off between high performance, compact size and manufacturing complexity, for filters intended for wideband communication systems. Finally, measurements from the manufactured prototype agree the Multimode Equivalent Network simulations, which demonstrates the usefulness of this technique to analyze this type of microwave components.

Acknowledgment

This work has been developed in collaboration with Dr. Alejandro Pons-Abenza and the University of Davis, California under the supervision of Prof. Dr. Juan Sebastián Gómez Díaz.

7.4 Stub Loaded Stopband Microstrip Filters for Ka-band Applications

The aim of this section is to design microstrip bandpass filter solutions at Ka-band frequencies with good performance in terms of isolation between passband and stopband (A).

7.4.1 Introduction

In recent years, filter solutions at Ka-band frequencies (from 26.5 GHz to 40 GHz) are demanded because of the increasing interest in millimeter wave systems and communication applications.

Most of these applications require compact solutions for filters, with good performance in terms of insertion loss (IL) and return loss (RL). The low weight and cost and simplicity in the manufacturing process are also constraints imposed in the filter design for these applications. In addition, other requirements are easy integration with other components in the circuit board and high stopband rejection. This last characteristic is of critical importance when the filters are part of a diplexer component (consisting of two filters, also called RX and TX filters). Filters with strong requirement on the isolation level in the band of the adjacent one (around 45 dB) could be implemented using waveguide technology *Amari et al.* (2006). However, based on the mentioned restrictions, planar technologies are often a more suitable option. Among the different planar solutions, we have selected for this application microstrip technology *Hong* (2011). Even though SIW filters *Lee et al.* (2019) have demonstrated good performance in terms of losses, the size is usually larger than with the microstrip alternative. In addition, stripline technology has been used in the past *Macchiarella et al.* (2018), achieving good isolation levels. However, stripline technology requires a multi-layer structure, more expensive and critical to implement.

Due to these considerations, in this section, we explore microstrip solutions. The well known drawbacks of microstrip technology at these frequencies are the excitation of surface waves and the radiation properties of open microstrip discontinuities *Oliner and Lee* (1986). These phenomena are extensively treated in the technical literature *Aguilera et al.* (2004); *Jackson et al.* (1993); *Mesa et al.* (1999). They increase the losses and produce spurious couplings that degrade the performance of the filter. Considering the limitations associated with this technology, several microstrip filters for Ka-band applications are reported in the technical literature *Zhang et al.* (2009). For example, in *Zhang et al.* (2009), measurements show an out-of-band rejection around -20 dB, $IL \approx 3$ dB and $RL \approx 15$ dB, in the passband.

In this context, the goal in this section is to improve the performance of microstrip Ka-band stub loaded filters *Hong* (2011) that require high selectivity in the stopband. Measurements clearly demonstrate that, thanks to the reduction of undesired coupling phenomena, stub loaded microstrip Ka-band filters with $RL = 16$ dB, $IL = 2$ dB and isolation between passband and stopband of $A = 43.7$ dB can be synthesized.

7.4.2 Study on the Undesired Coupling Effect

First, we characterize the undesired coupling effect in microstrip lines using the electromagnetic (EM) solver AXIEM in AWR (*AWR*) (2020).

The test consists of analyzing two stubs that are not connected but they are placed orthogonally close to each other. The schematic of the test under study is shown in Fig. 7.25. The microstrip substrate used for this work is shown in Fig. 7.26. As observed, no enclosure is considered in the structure.

We perform the analysis considering that the two stubs are short-circuited [see Fig. 7.25(a)] and open-circuited [see Fig. 7.25(b)]. For this purpose, we excite the structure using a lumped port in one of the stubs and we measure the power transferred to the orthogonal stub to calculate the transfer parameter S_{21} , in the configurations

Figure 7.25: Schematic of the microstrip test under study: two stubs placed orthogonally (not connected). (a) Two short-circuited lines. (b) Two open-circuited lines.

Figure 7.26: Microstrip structure. The substrate has a relative permittivity of $\epsilon_r = 2.94$. The thickness of the copper layer is $t = 35 \text{ um}$. H corresponds to the thickness of the substrate.

shown in Fig. 7.25.

Ideally, no coupling between them should be observed, since they are not connected and they are placed orthogonally to each other. However, using the EM solver, the S_{21} parameter is not negligible. In particular, we compare this residual coupling value for different values of H (thickness of the substrate) and for different operation frequencies (f) in Table 7.5.

As observed in the EM simulations, the propagation of surface waves is causing the non negligible coupling between the two adjacent stubs. From Table 7.5, we observe that this effect is reduced when a thinner substrate is used. Moreover, a better

Table 7.5: Coupling effect (measured through the $|S_{21}|$) for different values of the dielectric thickness (H) and the different operation frequencies (f).

f (GHz)	H (mm)	Short stubs	Open stubs
30	0.7	-41.2 dB	-28.9 dB
30	0.25	-61.8 dB	-43.6 dB
3	0.25	-76.24 dB	-69.16 dB

isolation between the stubs (lower $|S_{21}|$ value) is obtained when the stubs are short-circuited. Finally, we notice that the coupling effect increases with the frequency. These conclusions agree with well known surface wave theory *Oliner and Lee* (1986). This coupling effect will limit the isolation level in the stopband that can be achieved when these elements are used to design stub loaded microstrip filters *Hong* (2011).

7.4.3 Techniques for Reducing the Undesired Coupling Effect

Once we have characterized the undesired coupling effect, we propose two solutions to minimize this effect at Ka-band frequencies when stub loaded stopband filters are designed. According to the previous test, the substrate has been chosen to be as smaller as possible. In this case, the frequency is determined by the application. Finally, even short-circuited stubs have shown to be less affected by the undesired coupling effect, we work with open stubs, for simplicity in the manufacturing process.

Strategy I: Alternating the stubs

For this strategy, we focus our attention in a structure with three open-circuited stubs that implements a stopband filter, as shown in Fig. 7.27. Let us consider that the required isolation is 35 dB from $f_{min} = 16$ GHz to $f_{max} = 22.5$ GHz. The filters are implemented using a main transmission line that connects the input and output ports. The extracted transmission zeros (TZs) are implemented using open stub resonators [see Fig. 7.27(a)]. We propose to alternate the orientation of the stubs to reduce the undesired coupling effect, as shown in Fig. 7.27(b).

In Fig. 7.28, we compare the circuit and the EM simulations of the filters in Fig. 7.27. As observed, the ideal response with three TZs is only obtained with the circuit response. The TZ frequencies correspond to the resonant frequencies of the stubs. In the EM simulation of the filter when all the stubs are oriented in the same direction, two of these TZs become complex so that the $|S_{21}|$ level increases

Figure 7.27: Layout of the stopband filter under study. The microstrip substrate is shown in Fig. 7.26. The substrate has a thickness of $H = 0.25$ mm. (a) Stubs oriented in the same direction. (b) Stubs that alternate their orientations.

Figure 7.28: Simulated S-parameters for the two filters shown in Fig. 7.27.

above the rejection requirement in the upper part of the stopband. This is due to the electromagnetic couplings between stubs, which undermine the selectivity of the stopband from 21 GHz to 22.5 GHz. Finally, we can see that the EM response corresponding to the alternated stubs configuration, can recover the required isolation level ($A > 35$ dB), improving the selectivity with respect to the filter with all the stubs in the same direction.

Strategy II: Bent configuration

In this subsection, we explore the possibility of bending the main line of the filter to decrease the coupling effect. The test consists of comparing the performance of a stopband filter composed of two aligned stubs [see Fig. 7.29(a)] with the bent geometry proposed in Fig. 7.29(b).

Figure 7.29: Layout of the stopband under study. The microstrip substrate is shown in Fig. 7.26. The substrate has a thickness of $H = 0.25$ mm. (a) Aligned stubs. (b) Stubs in a bent configuration.

The S-parameters responses of these two filters are reported in Fig. 7.30. The

Figure 7.30: Simulated S-parameters for the two filters shown in Fig. 7.29.

two structures have been optimized at a circuit level to have the same ideal response.

However, from an EM point of view, the solution based on the bent configuration is clearly closer to the ideal circuit response. The achievable isolation level for the bent configuration at the frequency of the stopband is $A \approx 38$ dB. On the other hand, at the same frequency, the configuration with both stubs aligned shows $A \approx 30$ dB. Therefore, the bent solution seems to be less affected by undesired couplings, producing a better isolation in the stopband (around 8 dB of difference).

7.4.4 Filters Designs

Now we design four implementations of a microwave filter in Ka-band using stub loaded microstrip technology. The goal of these designs is to validate the strategies to reduce the undesired coupling effect proposed previously. The specifications of the filter are given in Table 7.6. The four filters have been homemade manufactured using a laser milling machine (LPKF Protlaser S). The prototypes have been measured with a vector network analyzer (R&S ZVA67) to experimentally validate the designs.

Table 7.6: Ka-Band filter specifications implemented in stub loaded microstrip technology.

Passband	27.75 GHz - 30.3 GHz
Stopband	16.4 GHz - 22.5 GHz
Return losses	$RL < 16$ dB
Isolation	$A > 38$ dB
Insertion losses	$IL < 2$ dB

The filter is designed with a stopband response using the synthesis described in *Macchiarella et al. (2006)*. Using *Macchiarella et al. (2006)*, initial dimensions are obtained for the filter to fulfill the specifications shown in Table 7.6. A fine EM optimization is then required to finally obtain the desired filter response.

Based on the test performed in Subsection 7.4.2, we choose a very thin substrate to reduce the undesired EM coupling effect. In particular, we use RT/Duroid 6002 for the substrate in Fig. 7.26 (with $\varepsilon_r = 2.94$, $H = 0.254$ mm, $\tan \delta = 0.0012$ and the

copper thickness is $t = 35 \text{ um}$).

The first and second filters are based on a traditional straight configuration. In the first implementation, the stubs are oriented in the same direction (see Fig. 7.31), while in the second one we alternate the orientation of the stubs (see Fig. 7.32). The dimensions are the same in both cases and are given in Fig. 7.32. The photographs of the prototypes are shown in Fig. 7.33.

Figure 7.31: Layout of the first implemented filter with straight main line and all stubs in the same orientation.

Figure 7.32: Layout of the second implemented filter with straight main line and stubs alternating the orientation. The dimensions are in mm.

Figure 7.33: Photographs of the prototypes based on straight main lines from input to output. (a) Filter given in Fig. 7.31. (b) Filter given in Fig. 7.32.

In Fig. 7.34 we compare the measured S-parameters of these two filters. As observed, both implementations satisfy the RL and IL specifications in the passband.

Figure 7.34: Measured S-parameters of the two filters implemented using a straight configuration. The layouts are given in Fig. 7.31 (aligned stubs) and Fig. 7.32 (alternated stubs).

However, the first design, where the coupling effect is expected to be more significant, does not satisfy the stopband requirements. From the test performed in previous subsection, when the stubs are placed alternated, the coupling effect is reduced, and the selectivity can be improved. This is what happens in the second design [see Fig. 7.33(b)]. Using this solution, we can see in Fig. 7.34 that the isolation level is above 38 dB in all the stopband and the selectivity is improved with respect to the first design.

The third and fourth examples are implemented by applying the bent configuration proposed in Subsection 7.4.3. The third example has all the stubs in the same direction (see Fig. 7.35), and the fourth one alternates the stubs (see Fig. 7.36). The dimensions are the same in both cases and are reported in Fig. 7.36. In Fig. 7.37 we show the photographs of the prototypes given in Fig. 7.35 and Fig. 7.36.

We compare the performance of these two bent filters in Fig. 7.38. From Fig. 7.38,

Figure 7.35: Layout of the third implemented filter using bent configuration.

Figure 7.36: Layout of the fourth implemented filter combining bent configuration and alternated stubs. The dimensions are in mm.

Figure 7.37: Photographs of the prototypes based on bent configurations. (a) Filter given in Fig. 7.35. (b) Filter given in Fig. 7.36.

we can see that also in this case the alternation of the stubs improves the selectivity in the stopband. Due to this selectivity improvement, the fourth filter clearly satisfies the requirements in the stopband while the third one does not.

Finally, we compare the performances of the second [see Fig. 7.33(b)] and the fourth design [see Fig. 7.37(b)] in Fig. 7.39. Both alternate the stubs, but the second example is implemented with a straight scheme while the fourth filter is bent.

As we can see in Fig. 7.39, the requirements in the passband are fulfilled in both implementations. Moreover, with the straight configuration (second design) the isolation level in the stopband is $A > 38$ dB. However, using the proposed bent

Figure 7.38: Measured S-parameters of the two bent filters given in Fig. 7.35 (aligned stubs) and Fig. 7.36 (alternated stubs).

Figure 7.39: Measured S-parameters of the two alternated stubs filters given in Fig. 7.32 (straight main line) and Fig. 7.36 (bent configuration).

scheme together with the alternation of the stubs (four design), the isolation level can be increased to $A > 43.7$ dB.

Therefore, the measurements presented for these designs experimentally validate the two alternatives proposed in Subsection 7.4.2 to reduce the undesired coupling effects. In particular, the measurements of the last prototype verify that Ka-band filters can be implemented in stub loaded microstrip technology, achieving good performance both in the passband ($IL < 2$ dB and $RL > 16$ dB) and in the stopband (isolation $A > 43.7$ dB).

As previously mentioned, the four filters have been homemade manufactured using a laser milling machine (LPKF Protolaser S). A photograph of the laser machine building these filters is shown in Fig. 7.40. After manufacturing the designs, we have measured the prototypes with a vector network analyzer (R&S ZVA67). A photograph of this last step is given in Fig. 7.41. As observed, no input/output coaxial connectors are welded in the filter since the 3680 series Universal Test Fixture (UTF) shown in Fig. 7.42 is used to excite the filters.

Figure 7.40: LPKF Protolaser S machine fabricating the filters.

Finally, the filter in Fig. 7.32 has been simulated using the Multimode Equivalent

Figure 7.41: Measurement of the filters using the vector network analyzer (R&S ZVA67).

Figure 7.42: 3680 series Universal Test Fixture (UTF) used as the input/output excitations.

Network formulation proposed in Chapter IV for the analysis of planar devices. At this point, it is important to remember that the analysis of microstrip components using the MEN technique is possible thanks to the inclusion of transverse ports that represent the input/output excitations. The MEN results are compared in Fig. 7.43 to the simulations obtained using two different solvers in AWR (*AWR*) (2020) (Analyst and AXIEM) and ADS (*ADS*) (2020).

Figure 7.43: S-parameters of the alternated stubs filter given in Fig. 7.32, using the MEN formulation. They are compared to AWR results (using Analyst and AXIEM solvers) and to ADS.

As observed, the MEN results are in good agreement with respect to the other software tools. It is important to note that, in ADS and AWR results, losses have been included, while in the MEN code they are not considered. Another important point is that this filter is implemented in non-encapsulated microstrip technology. This can be directly simulated in ADS and AWR. However, the Multimode Equivalent Network technique always requires an enclosure to solve the problem. In this way, we have included a very thin box in the upper part of the circuit and we have left that part in open-circuited condition. This strategy seems to work very well, according to the results in Fig. 7.43. Using this approach, no resonant box modes appear in the band of the filter and the performance of the filter is not affected by the box employed to solve the MEN.

7.4.5 Conclusions

We have studied the undesired coupling effect that is present when stopband Ka-band filters are designed using microstrip technology. First, we measure this effect using a simple test performed with the EM solver of AWR. Thanks to this characterization, we propose two alternatives to reduce this effect: alternate the direction of the stubs and use a bent configuration. To validate these strategies, we fabricate four different implementations of a Ka-band filter using RT/Duroid 6002. Details of the manufacturing and measurement procedures are given. The measured data of these prototypes validate the two proposed solutions. The test experimentally demonstrates that the selectivity is improved, and that to obtain good isolation level ($A > 43.7$ dB) is possible in stub loaded microstrip designs through the combination of the two solutions proposed in this study. Finally, one the filter solution has been simulated using the MEN formulation for planar devices proposed in this thesis, showing good agreement with respect to other commercial tools.

Acknowledgment

This work has been developed under the supervision of Prof. Dr. Giuseppe Macchiarella from the Politecnico di Milano, Italy. The manufacturing and measurements of these prototypes have been carried out with the help of Prof. Dr. Juan Hinojosa from Universidad Politcnica de Cartagena.

7.5 Conclusions

In this chapter, we have proposed several filter solutions using different technologies, according to the application and frequency band of operation. First, for satellite communication applications at X/Ku-band frequencies, we have designed several compact waveguide filters with different bandwidths using a resonating iris technology.

Then, we have proposed a novel thick bar technology that allows to synthesize wide-band filter responses. This technology has proved to be a good trade-off between good performance and compactness for C-band applications. One of the filters has been manufactured and measured, showing good agreement with respect to full-wave simulations. Finally, different microstrip solutions at Ka-band have been discussed in order to minimize the surface wave problem that normally degrades the achievable isolation in this technology. These filters have been homemade manufactured and measured to validate the proposed solutions.

In conclusion, we have proved in this chapter that, depending on the application and requirements, we have to select the right technology for implementing the filter. We show that, in cases where the requirements in terms of performance are high, filter solutions based on waveguide or thick bar resonators may be the most suitable choices. On the contrary, if low-cost manufacturing and ease of integration with other components are the most critical requirements, then microstrip solutions could be the best option.

Finally, all the described filters have been simulated using the MEN formulations described in this doctoral thesis. MEN results are in good agreement with respect to measurements from prototypes and other software tool results, thus validating the usefulness of this technique in the analysis of these real microwave components.

Scientific Outcome

- Related to Section 7.3: *Gomez-Molina et al.* (2020d).
- Related to Section 7.4: *Gomez-Molina et al.* (2020a).

CHAPTER VIII

Conclusions

The main conclusions of each subject treated in this thesis have been outlined at the end of each chapter. In this last chapter, we summarize the contribution to the numerical methods and filter research topics of the work developed in this document.

Among all the numerical methods reported in the technical literature, we focus in this work on the Multimode Equivalent Network technique. As mentioned before, this technique was originally developed for the analysis of waveguide components. This technology is employed in some applications, for instance, those that require high performance components and high power handling. However, in other communication systems, planar technologies are eventually more often used, since they allow a reduction in footprint, manufacturing cost and weight. Due to this reason, the main objective of this Ph.D. dissertation was to extend the use of this technique to the analysis of multilayer planar components. This MEN extension makes possible to use this efficient method to the analysis of new components that are used nowadays in the microwave industry.

To accomplish this goal, we first reviewed the MEN formulation for waveguide problems in Chapter II. We proposed two strategies to improve the efficiency of this analysis. First, we reformulated this approach without using the asymptotic admittances that were employed in the original technique. Then, we applied the

Kummer's transformation to the kernel of the integral equations. This allows to improve the efficiency of this analysis, since the heavy part of the kernel is computed independently from the frequency. In addition, the dynamic part, that needs to be recomputed for each frequency point, exhibits a much faster convergence rate. Thanks to this new approach, we could propose an automatic mechanism to assess convergence in the MEN technique for waveguide problems. Using this mechanism, the user has to select one global parameter and the specific numerical parameters of each discontinuity present in the structure are automatically fixed to their optimum values. We proved that these two improvements result in a significant reduction in the computation time.

After this waveguide centered review, we move to the analysis of planar structures. We first started using the electric approach of the MEN technique to analyze multilayer shielded planar components composed of different rectangular metallizations, in Chapter III. This is possible thanks to the inclusion of lateral excitation ports in the zero-thickness electric MEN formulation. These ports model the coaxial connectors normally used to excite this type of structures. This formulation was validated through the analysis of some coupled line filters, since it is restricted to rectangular metallization circuits.

To be able to analyze more generic structures composed of arbitrarily shaped metallizations, we included transverse ports to the magnetic zero-thickness MEN formulation in Chapter IV. In this case, we considered both external and internal lumped ports. The internal ports can be used for inter-connection purposes. This formulation allows the analysis of some microstrip components normally used in the microwave industry such as stepped-impedance lowpass filters, hairpin filters, Wilkinson power dividers or multilayer filters implementing transversal topologies. These devices were simulated using this MEN extension and the results were compared to measurements and other EM software results, thus fully validating the magnetic MEN technique

for planar devices proposed in this chapter. This formulation has been implemented in the graphical interface described in Appendix B, for the analysis and design of microstrip components.

For some of the previous analyses, we included losses in the dielectric substrates and in the metallic enclosure. For a more realistic analysis, in Chapter V, we also discussed the consideration of losses in the metallization interface using an equivalent problem. This approach is based on considering, in the aperture of the discontinuity, a high impedance surface. Some of the previous microstrip circuits were resimulated considering these losses. MEN simulations show good agreement with respect to both measurements and other software results, which validates this approach.

In the previously treated formulations, no thickness for the metallization was considered. This is an optimal approach when this thickness is electrically very small and then, it can be neglected. On the contrary, if the thickness becomes electrically large, the formulation described in Chapter VI must be used for accurate results. In this particular formulation, we integrated the excitation ports to the magnetic thick MEN approach. This allows the analysis of devices that contain electrically thick metallizations at microwave frequencies. Also, thin metallizations at very high frequencies, where the thickness becomes electrically important, can be analyzed with this particular formulation.

Finally, after the MEN extensions, some filter solutions have been proposed in Chapter VII. These filters have been designed for different applications and different bands of operation. As mentioned before, according to the filter requirements, the designer has to select the most suitable technology for the implementation. In our case, we have used waveguide technology to propose compact solutions for satellite filters. Also, we have proposed a trade-off technology that offers good performance and compactness for wideband applications. Finally, we studied some microstrip filter configurations that reduce the surface wave propagation that normally degrades

the performance of microstrip filters at high frequencies. Some of the previously mentioned filters have been manufactured and measured to validate the proposed design procedures.

Despite the fact that the filters presented in Chapter VII are implemented for different applications and in different technologies, all of them have been analyzed using the Multimode Equivalent Network formalisms. The examples considered in each chapter together with these last results, fully validate the usefulness and efficiency of the MEN extensions described in this doctoral thesis.

As a final conclusion, in this document, we have presented several extensions to the state of the art in the MEN context. The most important contribution is that the analysis of encapsulated multilayer planar devices is, from now on, possible using the Multimode Equivalent Network technique.

8.1 Future Lines of Research

Regarding the Multimode Equivalent Network technique, the future tasks that could be accomplished would be:

- The analysis of multilayer microwave structures combining both the electric and the magnetic MEN formulations detailed in Chapter III and IV, respectively. The idea would be to analyze some metallic interfaces using the electric MEN technique and the others using the magnetic formalism. This would allow to use the more efficient formalism in each metallic interface, according to its geometry.
- Explore other excitation models. In this doctoral thesis, we have used a pulse excitation with a reference impedance that normally is a resistor of $50\ \Omega$. However, in order to better model the coaxial connector, one possibility is to load these ports using complex impedance (with a capacitive or inductive part that could model the reactive energy in the port).

- About the MEN programs, a future line will be to improve the efficiency of the codes and to make them more generic (even creating a graphical interface). In addition, a future work would be to use the GUI presented in Appendix B to design microstrip filters with a more complex geometry.
- At a personal level, I would like to continue learning about numerical methods, specially those based on integral equations, to be able to get and use the physical and the numerical insight for solving the electromagnetic problems that nowadays concerns society.

Regarding the filter design topic, the future tasks that could be accomplished would be:

- Go into detail about the characterization of the compact waveguide filters reported in Section 7.2, and report the design curves needed to propose a more general design mechanism for this type of filters.
- About the thick metallic bars filters reported in Section 7.3, a possible future work is to implement complex topologies or other filter geometries using this type of resonators. A large number of different bars distributions (not just coupled line bars) could be explored to obtain other filter topologies.
- At a personal level, I would like to continue the microwave component design career mainly focus on Earth Observation missions, renewable energies and environmental radar applications.

8.2 Scientific Outcome

The scientific outcome related to the research activity developed during this doctoral thesis is summarized below:

- National Conferences:

1. C. Gómez Molina, F. Quesada Pereira, A. Álvarez Melcón, V. E. Boria, M. Guglielmi, “Transformación Eficiente del Kernel en la Red Multimodo Equivalente para el Análisis de Dispositivos Pasivos en Guía de Onda”, in *XXXII SIMPOSIUM NACIONAL DE LA UNION CIENTIFICA INTERNACIONAL DE RADIO URSI 2017*, Cartagena, Sept., 2017.
2. C. Gómez Molina, F. Quesada Pereira, A. Álvarez Melcón, V. E. Boria, M. Guglielmi, “Estudio sobre la Convergencia en la Red Multimodal Equivalente para el Análisis de Dispositivos en Guía de Onda”, in *XXXIII SIMPOSIUM NACIONAL DE LA UNION CIENTIFICA INTERNACIONAL DE RADIO URSI 2018*, Granada, Sept., 2018.
3. C. Gómez Molina, F. Quesada Pereira, A. Álvarez Melcón, V. E. Boria, M. Guglielmi, “Formulación Eléctrica de Red Multimodal Equivalente Aplicada al Análisis de Circuitos Impresos Multicapa Encapsulados”, in *XXXIV SIMPOSIUM NACIONAL DE LA UNION CIENTIFICA INTERNACIONAL DE RADIO URSI 2019*, Sevilla, Sept., 2019.
4. C. Gómez Molina, F. Quesada Pereira, S. Marini, M. A. Sánchez Soriano, A. Álvarez Melcón, V. E. Boria, M. Guglielmi, “Formulación de Red Multimodal Equivalente para el Análisis de Dispositivos Microstrip Encapsulados”, in *XXXV SIMPOSIUM NACIONAL DE LA UNION CIENTIFICA INTERNACIONAL DE RADIO URSI 2020*, On-line, Sept., 2020.

- International Conferences:

1. C. Gómez Molina, F. Quesada Pereira, A. Álvarez Melcón, V. E. Boria, and M. Guglielmi, “Efficient Formulation of Multimode Equivalent Networks for 2-D Waveguide Steps through Kummers Transformation,” in

IEEE MTT-S Int. Conf. on Numerical Electromagnetic and Multiphysics Modeling and Optimization for RF, Microwave and Terahertz Applications, Sevilla, May, 2017.

2. C. Gómez Molina, F. Quesada Pereira, A. Álvarez Melcón, S. Marini, Miguel A. Sánchez-Soriano, V. E. Boria, and M. Guglielmi, "Rigorous Multimode Equivalent Network Representation of Multilayer Planar Circuits," in *IEEE MTT-S Int. Conf. on Numerical Electromagnetic and Multiphysics Modeling and Optimization for RF, Microwave and Terahertz Applications*, Reykjavik, Aug., 2018.
3. C. Gómez Molina, F. Quesada Pereira, A. Álvarez Melcón, V. E. Boria, and M. Guglielmi, "Electric Multimode Equivalent Network Technique for Multilayer Shielded Circuits Based on Arbitrary Rectangular Elements," in *International Conference on Electromagnetics in Advanced Applications 2019*, Granada, Sept., 2019.
4. C. Gómez Molina, J. Hinojosa, F. Quesada Pereira, V. E. Boria, M. Guglielmi, A. Alvarez Melcon, and G. Macchiarella, "Alternative Solutions for Reducing the Undesired Coupling Effect in Stub Loaded Microstrip Filters for Ka-band Applications," accepted in *European Microwave Conference*, Utrecht, Jan., 2021.

- Scientific Journals:

1. C. Gómez Molina, F. Quesada Pereira, A. Álvarez Melcón, V. E. Boria, and M. Guglielmi, "An Efficient Technique to Assess the Convergence of the Multimode Equivalent Network for Waveguide Devices," *IEEE Trans. Microw. Theory Techn.*, vol. 66, no. 2, pp. 651 - 658, Feb. 2018.
2. C. Gómez Molina, A. Pons Abenza, J. Do, F. Quesada Pereira, X. Liu, J. S. Gómez Díaz and A. Álvarez Melcón, "Wideband Bandpass Filters

- Using a Novel Thick Metallization Technology,” in *IEEE Access*, vol. 8, no. 1, pp. 34962 - 34972, Feb. 2020. doi: 10.1109/ACCESS.2020.2974552
3. C. Gómez Molina, F. Quesada Pereira, A. Álvarez Melcón, S. Marini, Miguel A. Sánchez-Soriano, V. E. Boria, and M. Guglielmi, “Multimode Equivalent Network for Boxed Multilayer Arbitrary Planar Circuits,” *IEEE Trans. Microw. Theory Techn.*, vol. 68, no. 7, pp. 2501 - 2514, July 2020. doi: 10.1109/TMTT.2020.2984230
 4. C. Gómez Molina, F. Quesada Pereira, S. Marini, A. Álvarez Melcón, V. E. Boria, and M. Guglielmi, “Multimode Equivalent Networks for Shielded Microwave Circuits with Thick Metallizations,” *IEEE Trans. Microw. Theory Techn.*, Under Revision.

The scientific articles arisen from other collaborations made during this Ph. D. period are summarized below:

- National Conferences:

1. F. Quesada Pereira, C. Gómez Molina, A. Álvarez Melcón, V. E. Boria, M. Guglielmi, “Nueva Formulación de Ecuación Integral en el Dominio Espacial para la Caracterización de Discontinuidades en guía rectangular”, in *XXXII SIMPOSIUM NACIONAL DE LA UNION CIENTIFICA INTERNACIONAL DE RADIO URSI 2017*, Cartagena, Sept., 2017.
2. F. Quesada Pereira, C. Gómez Molina, A. Álvarez Melcón, V. E. Boria, M. Guglielmi, “Nueva Técnica de Ecuación Integral para el Análisis de Discontinuidades Capacitivas entre Guías de Onda Rectangulares que Incluyen Postes Dieléctricos y/o Metálicos de Forma Arbitraria”, in *XXXIV SIMPOSIUM NACIONAL DE LA UNION CIENTIFICA INTERNACIONAL DE RADIO URSI 2019*, Sevilla, Sept., 2019.

- International Conferences:

1. M. A. Sánchez Soriano and S. Marini and J. Vague and C. Gómez and F. Quesada and A. Álvarez and V. E. Bori and M. Guglielmi, “Study on Multipactor Breakdown in Coaxial to Microstrip Transitions,” in *IEEE MTT-S Int. Conf. on Numerical Electromagnetic and Multiphysics Modeling and Optimization for RF, Microwave and Terahertz Applications*, Reykjavik, Aug., 2018.
2. F. Quesada Pereira, C. Gómez Molina, A. Álvarez Melcón, V. E. Boria, M. Guglielmi, “Integral Equation Analysis of Multiport H-plane Devices Containing Arbitrarily Shaped Metallic and/or Dielectric Posts by Using Two-Dimensional Cavity and Parallel Plate Greens Functions”, in *2019 IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting - AP-S/URSI 2019*, Atlanta, Georgia, United State of America, July, 2019.
3. F. Quesada Pereira, C. Gómez Molina, A. Álvarez Melcón, V. E. Boria, and M. Guglielmi, “Integral Equation Analysis of Multiport Hplane Devices Containing Arbitrarily Shaped Metallic and/or Dielectric Posts by Using Two-Dimensional Cavity and Parallel Plate Greens Functions”, in *International Conference on Electromagnetics in Advanced Applications 2019* , Granada, Sept., 2019.

- Scientific Journals:

1. F. Quesada Pereira, C. Gómez Molina, A. Álvarez Melcón, V. Boria Esbert, M. Guglielmi, “Novel Spatial Domain Integral Equation Formulation for the Analysis of Rectangular Waveguide Steps close to Arbitrarily Shaped Dielectric and/or Conducting Posts”, *Radio Science*, vol. 53, no. 4, pp. 406 - 419. AGU Journal, March, 2018.

2. F. Quesada Pereira, C. Gómez Molina, A. Álvarez Melcón, V. Boria Esbert, M. Guglielmi, “On the Analysis of Capacitive Rectangular Waveguide Discontinuities Close to Arbitrarily Shaped Conducting and Dielectric Posts”, *International Journal of Electronics and Communications (AEU)*. vol. 113, no. 152976, pp. 1 - 7, Jan., 2020.

8.3 Research Stays

1. From the 1st of April 2018 to the 22nd of June 2018 at the University of California, Davis, USA, under the supervision of Prof. Dr. Juan Sebastián Gómez Díaz.
2. From the 2st of April 2019 to the 28th of June 2019 at the Politecnico di Milano, Italy, under the supervision of Prof. Dr. Giuseppe Macchiarella.

APPENDICES

APPENDIX A

Vector Modal Functions and the Solution of the Coupling Integrals

In this appendix, we collect the magnetic and electric vector modal functions of a rectangular waveguide. Also, we provide the expression for the electric vector modal functions of a rectangular metallic patch located in the cross-section of a rectangular waveguide. These functions are then used to solve analytically the coupling integrals in some of the MEN problems treated in this doctoral thesis. In particular, we solve the coupling integrals between two rectangular waveguides for the magnetic formulation, and between a waveguide and a rectangular metallic obstacle for the electric formulation. Moreover, we also detail the solution for the coupling integrals between the metallic obstacle modes and the excitation pulse, needed for the electric planar MEN formulation. All the previously mentioned coupling integrals are solved analytically since all the geometries that compose the structures under analysis are rectangular, and then the vector modal functions are known analytically. In this respect, it is important to mention that, the coupling integrals between arbitrary waveguides need to be solved numerically by other methods such as the BI-RME method *Cogollos et al.* (2003). Details about how to calculate the coupling integrals

between an arbitrary waveguide and an excitation pulse, needed for the magnetic MEN planar formulation, are also given in this appendix.

A.1 Magnetic and Electric Vector Modal Functions of a Rectangular Waveguide

The magnetic and electric vector modal functions of a rectangular waveguide as the one shown in Fig. A.1 are reported in Table A.1 *Guglielmi et al.* (1999).

Figure A.1: Rectangular waveguide cross-section. The dimensions are: $a \times b$.

In Table A.1, the constant $A_{m,n}$ and the cut-off wavenumber $k_{c,m,n}$ are defined as

$$A_{m,n} = \frac{\sqrt{\epsilon_m \epsilon_n}}{\pi} \sqrt{\frac{ab}{(mb)^2 + (na)^2}} = \sqrt{\frac{\epsilon_m \epsilon_n}{ab}} \frac{1}{k_{c,m,n}} \quad (\text{A.1})$$

$$k_{c,m,n} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}, \quad (\text{A.2})$$

where

$$\epsilon_i = \begin{cases} 1, & \text{if } i = 0 \\ 2, & \text{if } i \neq 0 \end{cases}.$$

The integers m and n give the number of the maximums of the field along the x and y -axis. In rectangular waveguides, the indexes m and n for TE modes are $m, n = 0, 1, 2, \dots$ but must be non zero simultaneously in order not to have a trivial

Table A.1: TE and TM modes of the rectangular waveguide shown in Fig. A.1.

	TE _{<i>m,n</i>}	TM _{<i>m,n</i>}
h_x	$-A_{m,n} \frac{m\pi}{a} \sin\left(\frac{m\pi(x-x_0)}{a}\right) \cos\left(\frac{n\pi(y-y_0)}{b}\right)$	$A_{m,n} \frac{n\pi}{b} \sin\left(\frac{m\pi(x-x_0)}{a}\right) \cos\left(\frac{n\pi(y-y_0)}{b}\right)$
h_y	$-A_{m,n} \frac{n\pi}{b} \cos\left(\frac{m\pi(x-x_0)}{a}\right) \sin\left(\frac{n\pi(y-y_0)}{b}\right)$	$-A_{m,n} \frac{m\pi}{a} \cos\left(\frac{m\pi(x-x_0)}{a}\right) \sin\left(\frac{n\pi(y-y_0)}{b}\right)$
h_z	$A_{m,n} \frac{k_{c,m,n}^2}{k} \cos\left(\frac{m\pi(x-x_0)}{a}\right) \cos\left(\frac{n\pi(y-y_0)}{b}\right)$	0
e_x	$-A_{m,n} \frac{n\pi}{b} \cos\left(\frac{m\pi(x-x_0)}{a}\right) \sin\left(\frac{n\pi(y-y_0)}{b}\right)$	$-A_{m,n} \frac{m\pi}{a} \cos\left(\frac{m\pi(x-x_0)}{a}\right) \sin\left(\frac{n\pi(y-y_0)}{b}\right)$
e_y	$A_{m,n} \frac{m\pi}{a} \sin\left(\frac{m\pi(x-x_0)}{a}\right) \cos\left(\frac{n\pi(y-y_0)}{b}\right)$	$-A_{m,n} \frac{n\pi}{b} \sin\left(\frac{m\pi(x-x_0)}{a}\right) \cos\left(\frac{n\pi(y-y_0)}{b}\right)$
e_z	0	$A_{m,n} \frac{k_{c,m,n}^2}{k} \sin\left(\frac{m\pi(x-x_0)}{a}\right) \sin\left(\frac{n\pi(y-y_0)}{b}\right)$

solution. For TM modes $m, n = 1, 2, 3, \dots$

A.2 Electric Vector Modal Functions of a Rectangular Metallic Patch

The electric vector modal functions of an internal rectangular metallic patch, as the one shown in Fig. A.2, are reported in Table A.2 *Melcon et al.* (1999).

In this case, the indexes m and n for TE modes are $m, n = 1, 2, 3, \dots$ and for TM are $m, n = 0, 1, 2, \dots$ but must be non zero simultaneously in order not to have a trivial solution.

Figure A.2: Rectangular metallic patch located in the cross-section of a rectangular waveguide. The dimensions of the obstacle are: $a_{obs} \times b_{obs}$, and the dimensions of the rectangular waveguide are: $a \times b$.

Table A.2: TE and TM modes of the internal rectangular obstacle located in the waveguide cross-section, as shown in Fig. A.2.

	TE _{<i>m,n</i>}	TM _{<i>m,n</i>}
e_x	$-A_{m,n} \frac{n\pi}{b_{obs}} \sin\left(\frac{m\pi(x-x_1)}{a_{obs}}\right) \cos\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$	$A_{m,n} \frac{m\pi}{a_{obs}} \sin\left(\frac{m\pi(x-x_1)}{a_{obs}}\right) \cos\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$
e_y	$A_{m,n} \frac{m\pi}{a_{obs}} \cos\left(\frac{m\pi(x-x_1)}{a_{obs}}\right) \sin\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$	$A_{m,n} \frac{n\pi}{b_{obs}} \cos\left(\frac{m\pi(x-x_1)}{a_{obs}}\right) \sin\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$

In Table A.2, the constant $A_{m,n}$ is defined as

$$A_{m,n} = \frac{\sqrt{\epsilon_m \epsilon_n}}{\pi} \sqrt{\frac{a_{obs} b_{obs}}{(mb_{obs})^2 + (na_{obs})^2}}. \quad (A.3)$$

The electric vector modal functions of a rectangular metallic patch attached to the $x = 0$ wall of a rectangular waveguide (see Fig. A.3) are reported in Table A.3 *Melcon et al.* (1999).

In this case, the indexes m and n for TE modes are $m = 1, 3, 5, \dots$ (odd) and $n = 1, 2, 3, \dots$. For TM modes the index are $m = 1, 3, 5, \dots$ (odd) and $n = 0, 1, 2, \dots$

Finally, the electric vector modal functions of a rectangular metallic patch at-

Figure A.3: Rectangular metallic patch attached to the $x = 0$ wall of a rectangular waveguide. The dimensions of the obstacle are: $a_{obs} \times b_{obs}$, and the dimensions of the rectangular waveguide are: $a \times b$.

Table A.3: TE and TM modes of a rectangular obstacle attached to the $x = 0$ wall of a rectangular waveguide, as shown in Fig. A.3.

	TE _{m,n}	TM _{m,n}
e_x	$-A_{m,n} \frac{n\pi}{b_{obs}} \cos\left(\frac{m\pi(x-x_1)}{2a_{obs}}\right) \cos\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$	$-A_{m,n} \frac{m\pi}{2a_{obs}} \cos\left(\frac{m\pi(x-x_1)}{2a_{obs}}\right) \cos\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$
e_y	$-A_{m,n} \frac{m\pi}{2a_{obs}} \sin\left(\frac{m\pi(x-x_1)}{2a_{obs}}\right) \sin\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$	$A_{m,n} \frac{n\pi}{b_{obs}} \sin\left(\frac{m\pi(x-x_1)}{2a_{obs}}\right) \sin\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$

tached to the $x = a$ wall of a rectangular waveguide (see Fig. A.4) are reported in Table A.4 *Melcon et al. (1999)*.

Figure A.4: Rectangular metallic patch attached to the $x = a$ wall of a rectangular waveguide. The dimensions of the obstacle are: $a_{obs} \times b_{obs}$, and the dimensions of the rectangular waveguide are: $a \times b$.

Table A.4: TE and TM modes of a rectangular obstacle attached to the $x = a$ wall of a rectangular waveguide, as shown in Fig. A.4.

	TE _{m,n}	TM _{m,n}
e_x	$-A_{m,n} \frac{n\pi}{b_{obs}} \sin\left(\frac{m\pi(x-x_1)}{2a_{obs}}\right) \cos\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$	$+A_{m,n} \frac{m\pi}{2a_{obs}} \sin\left(\frac{m\pi(x-x_1)}{2a_{obs}}\right) \cos\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$
e_y	$A_{m,n} \frac{m\pi}{2a_{obs}} \cos\left(\frac{m\pi(x-x_1)}{2a_{obs}}\right) \sin\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$	$A_{m,n} \frac{n\pi}{b_{obs}} \cos\left(\frac{m\pi(x-x_1)}{2a_{obs}}\right) \sin\left(\frac{n\pi(y-y_1)}{b_{obs}}\right)$

In this case, the indexes m and n for TE modes are $m = 1, 3, 5, \dots(\text{odd})$ and $n = 1, 2, 3, \dots$. For TM modes the index are $m = 1, 3, 5, \dots(\text{odd})$ and $n = 0, 1, 2, \dots$

In Table A.3 and Table A.4, the constant $A_{m,n}$ is defined as

$$A_{m,n} = \frac{\sqrt{\epsilon_m \epsilon_n}}{\pi} \sqrt{\frac{4a_{obs}b_{obs}}{(mb_{obs})^2 + (n2a_{obs})^2}}. \quad (\text{A.4})$$

As observed in this section, the modes of the metallic patch are different if it is not touching any wall, attached to $x = 0$ wall or attached to $x = a$ wall. This is because the boundary conditions applied in these three cases are different.

A.3 Coupling Integrals between Rectangular Waveguides for Magnetic Formulations

Let suppose that we have two rectangular waveguides as shown in Fig. A.5.

The biggest waveguide is characterized by its magnetic vector modal functions $\mathbf{h}_m$ for the modes (m, n) , as defined in Section A.1. On the other hand, the smallest waveguide is represented by its corresponding magnetic vector modal functions $\mathbf{h}_k$ for the modes (p, q) . The expression of these functions are also given in Section A.1.

Figure A.5: Junction between two rectangular waveguides. The dimensions of the biggest waveguide are: $a \times b$, while the dimensions of the smallest waveguide are: $a_{ap} \times b_{ap}$.

The coupling integrals between these two waveguides are defined as

$$C_{k,m} = \int_{ap} \mathbf{h}_k(s) \cdot \mathbf{h}_m(s) ds, \quad (\text{A.5})$$

where the integral extends along the aperture (ap in Fig. A.5). In equation (A.5), s indicates a point in the waveguide cross-section at the discontinuity, $s = (x, y)$.

These integrals are computed, depending on the nature of the modes (TE or TM), as follows

$$C_{k,m}^{TE,TE} = A_{m,n} A_{p,q} \left[\frac{mp\pi^2}{a \cdot a_{ap}} ss_x cc_y + \frac{nq\pi^2}{b \cdot b_{ap}} cc_x ss_y \right] \quad (\text{A.6})$$

$$C_{k,m}^{TE,TM} = -A_{m,n} A_{p,q} \left[\frac{np\pi^2}{b \cdot a_{ap}} ss_x cc_y - \frac{mq\pi^2}{a \cdot b_{ap}} cc_x ss_y \right] \quad (\text{A.7})$$

$$C_{k,m}^{TM,TE} = -A_{m,n} A_{p,q} \left[\frac{mq\pi^2}{a \cdot b_{ap}} ss_x cc_y - \frac{np\pi^2}{b \cdot a_{ap}} cc_x ss_y \right] \quad (\text{A.8})$$

$$C_{k,m}^{TM,TM} = A_{m,n} A_{p,q} \left[\frac{nq\pi^2}{b \cdot b_{ap}} ss_x cc_y + \frac{mp\pi^2}{a \cdot a_{ap}} cc_x ss_y \right], \quad (\text{A.9})$$

where

$$\begin{aligned}
ss_x &= \int_{x_1}^{x_2} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{p\pi}{a_{ap}}(x-x_1)\right) dx = \\
&\left(\frac{p\pi}{a_{ap}} - \frac{m\pi}{a}\right)^{-1} \cos\left[\frac{p\pi}{a_{ap}}\left(\frac{x_2-x_1}{2}\right) - \frac{m\pi}{a}\left(\frac{x_2+x_1}{2}\right)\right] \\
&\quad \sin\left[\frac{p\pi}{a_{ap}}\left(\frac{x_2-x_1}{2}\right) - \frac{m\pi}{a}\left(\frac{x_2-x_1}{2}\right)\right] \\
&- \left(\frac{p\pi}{a_{ap}} + \frac{m\pi}{a}\right)^{-1} \cos\left[\frac{p\pi}{a_{ap}}\left(\frac{x_2-x_1}{2}\right) + \frac{m\pi}{a}\left(\frac{x_2+x_1}{2}\right)\right] \\
&\quad \sin\left[\frac{p\pi}{a_{ap}}\left(\frac{x_2-x_1}{2}\right) + \frac{m\pi}{a}\left(\frac{x_2-x_1}{2}\right)\right]
\end{aligned} \tag{A.10}$$

$$\begin{aligned}
cc_x &= \int_{x_1}^{x_2} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{p\pi}{a_{ap}}(x-x_1)\right) dx = \\
&\left(\frac{p\pi}{a_{ap}} - \frac{m\pi}{a}\right)^{-1} \cos\left[\frac{p\pi}{a_{ap}}\left(\frac{x_2-x_1}{2}\right) - \frac{m\pi}{a}\left(\frac{x_2+x_1}{2}\right)\right] \\
&\quad \sin\left[\frac{p\pi}{a_{ap}}\left(\frac{x_2-x_1}{2}\right) - \frac{m\pi}{a}\left(\frac{x_2-x_1}{2}\right)\right] \\
&+ \left(\frac{p\pi}{a_{ap}} + \frac{m\pi}{a}\right)^{-1} \cos\left[\frac{p\pi}{a_{ap}}\left(\frac{x_2-x_1}{2}\right) + \frac{m\pi}{a}\left(\frac{x_2+x_1}{2}\right)\right] \\
&\quad \sin\left[\frac{p\pi}{a_{ap}}\left(\frac{x_2-x_1}{2}\right) + \frac{m\pi}{a}\left(\frac{x_2-x_1}{2}\right)\right]
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
ss_y &= \int_{y_1}^{y_2} \sin\left(\frac{n\pi}{b}y\right) \sin\left(\frac{q\pi}{b_{ap}}(y-y_1)\right) dy = \\
&\left(\frac{q\pi}{b_{ap}} - \frac{n\pi}{b}\right)^{-1} \cos\left[\frac{q\pi}{b_{ap}}\left(\frac{y_2-y_1}{2}\right) - \frac{n\pi}{b}\left(\frac{y_2+y_1}{2}\right)\right] \\
&\quad \sin\left[\frac{q\pi}{b_{ap}}\left(\frac{y_2-y_1}{2}\right) - \frac{n\pi}{b}\left(\frac{y_2-y_1}{2}\right)\right] \\
&- \left(\frac{q\pi}{b_{ap}} + \frac{n\pi}{b}\right)^{-1} \cos\left[\frac{q\pi}{b_{ap}}\left(\frac{y_2-y_1}{2}\right) + \frac{n\pi}{b}\left(\frac{y_2+y_1}{2}\right)\right] \\
&\quad \sin\left[\frac{q\pi}{b_{ap}}\left(\frac{y_2-y_1}{2}\right) + \frac{n\pi}{b}\left(\frac{y_2-y_1}{2}\right)\right]
\end{aligned} \tag{A.12}$$

$$\begin{aligned}
cc_y = \int_{y_1}^{y_2} \cos\left(\frac{n\pi}{b}y\right) \cos\left(\frac{q\pi}{b_{ap}}(y - y_1)\right) dy = \\
\left(\frac{q\pi}{b_{ap}} - \frac{n\pi}{b}\right)^{-1} \cos\left[\frac{q\pi}{b_{ap}}\left(\frac{y_2 - y_1}{2}\right) - \frac{n\pi}{b}\left(\frac{y_2 + y_1}{2}\right)\right] \\
\sin\left[\frac{q\pi}{b_{ap}}\left(\frac{y_2 - y_1}{2}\right) - \frac{n\pi}{b}\left(\frac{y_2 - y_1}{2}\right)\right] + \left(\frac{q\pi}{b_{ap}} + \frac{n\pi}{b}\right)^{-1} \cos\left[\frac{q\pi}{b_{ap}}\left(\frac{y_2 - y_1}{2}\right) + \frac{n\pi}{b}\left(\frac{y_2 + y_1}{2}\right)\right] \\
\sin\left[\frac{q\pi}{b_{ap}}\left(\frac{y_2 - y_1}{2}\right) + \frac{n\pi}{b}\left(\frac{y_2 - y_1}{2}\right)\right]
\end{aligned} \tag{A.13}$$

The indeterminate forms that can appear in these expressions can be easily solved by particularizing the integral in equation (A.5) to that specific case or by solving the corresponding limit.

A.4 Coupling Integrals between a Rectangular Waveguide and a Rectangular Metallic Patch for Electric Formulations

In this section, we solve analytically the coupling integrals related to rectangular metallic patches located in the cross-section of a rectangular waveguide. The metallic patch can be placed without touching any wall of the waveguide (internal area), can be attached to $x = 0$ wall or to $x = a$ wall. The solution for these cases are given in the following subsections.

A.4.1 Internal Areas

Let suppose that we have a rectangular waveguide that contains in the cross-section a rectangular metallic patch as shown in Fig. A.2.

The waveguide is characterized by its electric vector modal functions $\mathbf{e}_m$, for the

modes (m, n) , defined in Table A.1. On the other hand, the rectangular metallic patch is represented by its corresponding electric vector modal functions $\mathbf{e}_{\mathbf{k}}$ for the modes (p, q) . These functions are defined in Table A.2.

The coupling integrals between these two elements are defined as

$$C_{k,m} = \int_{obs} \mathbf{e}_{\mathbf{k}}(s) \cdot \mathbf{e}_{\mathbf{m}}(s) ds, \quad (\text{A.14})$$

where the integral extends over the obstacle area (*obs*, in Fig. A.2).

They are computed, depending on the nature of the modes (TE or TM), as follows

$$C_{k,m}^{TE,TE} = A_{m,n} A_{p,q} \left[\frac{nq\pi^2}{b \cdot b_{obs}} cs_x sc_y + \frac{mp\pi^2}{a \cdot a_{obs}} sc_x cs_y \right] \quad (\text{A.15})$$

$$C_{k,m}^{TE,TM} = A_{m,n} A_{p,q} \left[\frac{mq\pi^2}{a \cdot b_{obs}} cs_x sc_y - \frac{np\pi^2}{b \cdot a_{obs}} sc_x cs_y \right] \quad (\text{A.16})$$

$$C_{k,m}^{TM,TE} = -A_{m,n} A_{p,q} \left[\frac{np\pi^2}{b \cdot a_{obs}} cs_x sc_y - \frac{mq\pi^2}{a \cdot b_{obs}} sc_x cs_y \right] \quad (\text{A.17})$$

$$C_{k,m}^{TM,TM} = -A_{m,n} A_{p,q} \left[\frac{mp\pi^2}{a \cdot a_{obs}} cs_x sc_y + \frac{nq\pi^2}{b \cdot b_{obs}} sc_x cs_y \right], \quad (\text{A.18})$$

where

$$\begin{aligned}
cS_x &= \int_{x_1}^{x_2} \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{p\pi}{a_{obs}}(x-x_1)\right) dx = \\
&\left(\frac{p\pi}{a_{obs}} + \frac{m\pi}{a}\right)^{-1} \sin\left[\frac{p\pi}{a_{obs}}\left(\frac{x_2-x_1}{2}\right) + \frac{m\pi}{a}\left(\frac{x_2+x_1}{2}\right)\right] \\
&\quad \sin\left[\frac{p\pi}{a_{obs}}\left(\frac{x_2-x_1}{2}\right) + \frac{m\pi}{a}\left(\frac{x_2-x_1}{2}\right)\right] \\
&+ \left(\frac{p\pi}{a_{obs}} - \frac{m\pi}{a}\right)^{-1} \sin\left[\frac{p\pi}{a_{obs}}\left(\frac{x_2-x_1}{2}\right) - \frac{m\pi}{a}\left(\frac{x_2+x_1}{2}\right)\right] \\
&\quad \sin\left[\frac{p\pi}{a_{obs}}\left(\frac{x_2-x_1}{2}\right) - \frac{m\pi}{a}\left(\frac{x_2-x_1}{2}\right)\right]
\end{aligned} \tag{A.19}$$

$$\begin{aligned}
sC_x &= \int_{x_1}^{x_2} \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{p\pi}{a_{obs}}(x-x_1)\right) dx = \\
&\left(\frac{p\pi}{a_{obs}} + \frac{m\pi}{a}\right)^{-1} \sin\left[\frac{p\pi}{a_{obs}}\left(\frac{x_2-x_1}{2}\right) + \frac{m\pi}{a}\left(\frac{x_2+x_1}{2}\right)\right] \\
&\quad \sin\left[\frac{p\pi}{a_{obs}}\left(\frac{x_2-x_1}{2}\right) + \frac{m\pi}{a}\left(\frac{x_2-x_1}{2}\right)\right] \\
&- \left(\frac{p\pi}{a_{obs}} - \frac{m\pi}{a}\right)^{-1} \sin\left[\frac{p\pi}{a_{obs}}\left(\frac{x_2-x_1}{2}\right) - \frac{m\pi}{a}\left(\frac{x_2+x_1}{2}\right)\right] \\
&\quad \sin\left[\frac{p\pi}{a_{obs}}\left(\frac{x_2-x_1}{2}\right) - \frac{m\pi}{a}\left(\frac{x_2-x_1}{2}\right)\right]
\end{aligned} \tag{A.20}$$

$$\begin{aligned}
cS_y &= \int_{y_1}^{y_2} \cos\left(\frac{n\pi}{b}y\right) \sin\left(\frac{q\pi}{b_{obs}}(y-y_1)\right) dy = \\
&\left(\frac{q\pi}{b_{obs}} + \frac{n\pi}{b}\right)^{-1} \sin\left[\frac{q\pi}{b_{obs}}\left(\frac{y_2-y_1}{2}\right) + \frac{n\pi}{b}\left(\frac{y_2+y_1}{2}\right)\right] \\
&\quad \sin\left[\frac{q\pi}{b_{obs}}\left(\frac{y_2-y_1}{2}\right) + \frac{n\pi}{b}\left(\frac{y_2-y_1}{2}\right)\right] \\
&+ \left(\frac{q\pi}{b_{obs}} - \frac{n\pi}{b}\right)^{-1} \sin\left[\frac{q\pi}{b_{obs}}\left(\frac{y_2-y_1}{2}\right) - \frac{n\pi}{b}\left(\frac{y_2+y_1}{2}\right)\right] \\
&\quad \sin\left[\frac{q\pi}{b_{obs}}\left(\frac{y_2-y_1}{2}\right) - \frac{n\pi}{b}\left(\frac{y_2-y_1}{2}\right)\right]
\end{aligned} \tag{A.21}$$

$$\begin{aligned}
sc_y = \int_{y_1}^{y_2} \cos\left(\frac{n\pi}{b}y\right) \sin\left(\frac{q\pi}{b_{obs}}(y - y_1)\right) dy = \\
\left(\frac{q\pi}{b_{obs}} + \frac{n\pi}{b}\right)^{-1} \sin\left[\frac{q\pi}{b_{obs}}\left(\frac{y_2 - y_1}{2}\right) + \frac{n\pi}{b}\left(\frac{y_2 + y_1}{2}\right)\right] \\
\sin\left[\frac{q\pi}{b_{obs}}\left(\frac{y_2 - y_1}{2}\right) + \frac{n\pi}{b}\left(\frac{y_2 - y_1}{2}\right)\right] \cdot \\
- \left(\frac{q\pi}{b_{obs}} - \frac{n\pi}{b}\right)^{-1} \sin\left[\frac{q\pi}{b_{obs}}\left(\frac{y_2 - y_1}{2}\right) - \frac{n\pi}{b}\left(\frac{y_2 + y_1}{2}\right)\right] \\
\sin\left[\frac{q\pi}{b_{obs}}\left(\frac{y_2 - y_1}{2}\right) - \frac{n\pi}{b}\left(\frac{y_2 - y_1}{2}\right)\right]
\end{aligned} \tag{A.22}$$

The indeterminate forms that can appear in these expressions can be easily solved by particularizing the integral in equation (A.14) to that specific case or by solving the corresponding limit.

A.4.2 Metallic Patch Attached to $x = 0$ Wall

Let suppose that we have a rectangular waveguide that contains in the cross-section a rectangular metallic patch attached to $x = 0$ wall, as shown in Fig. A.3.

The waveguide is characterized by its electric vector modal functions $\mathbf{e}_m$, for the modes (m, n) , defined in Table A.1. On the other hand, the rectangular metallic patch is represented by its corresponding electric vector modal functions $\mathbf{e}_k$ for the modes (p, q) . These functions are defined in Table A.3.

The coupling integrals between these two elements are defined as

$$C_{k,m} = \int_{obs} \mathbf{e}_k(s) \cdot \mathbf{e}_m(s) ds, \tag{A.23}$$

and they are computed, depending on the nature of the modes (TE or TM), as follows

$$C_{k,m}^{TE,TE} = A_{m,n}A_{p,q} \left[\frac{nq\pi^2}{b \cdot b_{obs}} cc_x sc_y - \frac{mp\pi^2}{a \cdot 2a_{obs}} ss_x cs_y \right] \quad (\text{A.24})$$

$$C_{k,m}^{TE,TM} = A_{m,n}A_{p,q} \left[\frac{mq\pi^2}{a \cdot b_{obs}} cc_x sc_y + \frac{np\pi^2}{b \cdot 2a_{obs}} ss_x cs_y \right] \quad (\text{A.25})$$

$$C_{k,m}^{TM,TE} = A_{m,n}A_{p,q} \left[\frac{np\pi^2}{b \cdot 2a_{obs}} cc_x sc_y + \frac{mq\pi^2}{a \cdot b_{obs}} ss_x cs_y \right] \quad (\text{A.26})$$

$$C_{k,m}^{TM,TM} = A_{m,n}A_{p,q} \left[\frac{mp\pi^2}{a \cdot 2a_{obs}} cc_x sc_y - \frac{nq\pi^2}{b \cdot b_{obs}} ss_x cs_y \right], \quad (\text{A.27})$$

where the integrals cc_x and ss_x are solved as in equations (A.11) and (A.10), but substituting a_{ap} by $2a_{obs}$. In addition, the solutions of the integrals sc_y and cs_y are given in equations (A.22) and (A.21), respectively.

The indeterminate forms that can appear in these expressions can be easily solved by particularizing the integral in equation (A.23) to that specific case or by solving the corresponding limit.

A.4.3 Metallic Patch Attached to $x = a$ Wall

Let suppose that we have a rectangular waveguide that contains in the cross-section a rectangular metallic patch attached to $x = a$ wall, as shown in Fig. A.4.

The waveguide is characterized by its electric vector modal functions $\mathbf{e}_m$ for the modes (m, n) , as defined in Table A.1. On the other hand, the rectangular metallic patch is represented by its corresponding electric vector modal functions $\mathbf{e}_k$ for the modes (p, q) . These functions are defined in Table A.4.

The coupling integrals between these two waveguides are defined as

$$C_{k,m} = \int_{obs} \mathbf{e}_k(s) \cdot \mathbf{e}_m(s) ds, \quad (\text{A.28})$$

and they are computed, depending on the nature of the modes (TE or TM), as follows

$$C_{k,m}^{TE,TE} = A_{m,n}A_{p,q} \left[\frac{nq\pi^2}{b \cdot b_{obs}} cs_x sc_y + \frac{mp\pi^2}{a \cdot 2a_{obs}} sc_x cs_y \right] \quad (\text{A.29})$$

$$C_{k,m}^{TE,TM} = A_{m,n}A_{p,q} \left[\frac{mq\pi^2}{a \cdot b_{obs}} cs_x sc_y - \frac{np\pi^2}{b \cdot 2a_{obs}} sc_x cs_y \right] \quad (\text{A.30})$$

$$C_{k,m}^{TM,TE} = -A_{m,n}A_{p,q} \left[\frac{np\pi^2}{b \cdot 2a_{obs}} cs_x sc_y - \frac{mq\pi^2}{a \cdot b_{obs}} sc_x cs_y \right] \quad (\text{A.31})$$

$$C_{k,m}^{TM,TM} = -A_{m,n}A_{p,q} \left[\frac{mp\pi^2}{a \cdot 2a_{obs}} cs_x sc_y + \frac{nq\pi^2}{b \cdot b_{obs}} sc_x cs_y \right], \quad (\text{A.32})$$

where the integrals cs_x and sc_x are solved as in equations (A.19) and (A.20), but substituting a_{obs} by $2a_{obs}$. In addition, the solutions of the integrals sc_y and cs_y are given in equations (A.22) and (A.21), respectively.

The indeterminate forms that can appear in these expressions can be easily solved by particularizing the integral in equation (A.28) to that specific case or by solving the corresponding limit.

A.5 Coupling Integrals between a Rectangular Metallization and an Excitation Pulse

A.5.1 Metallic Patch Attached to $x = 0$ Wall

In this case, the coupling integral $C_{i,0}$ models the interaction between the modes of a metallic patch attached to the $x = 0$ wall of a rectangular waveguide and the excitation pulse, both contained in the transverse plane. The electric vector modal functions ($\mathbf{e}_i$) of the metallic patch are given in Table A.3. The schematic of the situation under analysis is shown in Fig. A.6.

excitation pulse, both contained in the transverse plane. The electric vector modal functions ($\mathbf{e}_i$) of the metallic patch are given in Table A.4. The schematic of the situation under analysis is shown in Fig. A.7.

Figure A.7: Two-dimensional zero-thickness printed rectangular patch in planar technology, excited by a transverse port. The gray area corresponds to the printed metallization and the dark area to the excitation port, both placed on the transverse ($z=0$) plane.

The expression of this integral is the following

$$C_{i,0} = \int_{obs} \mathbf{e}_i(s) \cdot \left(\frac{1}{\Delta x} \Pi \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{x}_0 \right) ds. \quad (\text{A.35})$$

After using the expression for $\mathbf{e}_i(s)$ given in Table A.4, the coupling integral $C_{i,0}$ can be computed as

$$\begin{aligned} C_{i,0} &= \frac{1}{\Delta x} \int_{x_2}^{x_3} \int_{y_1}^{y_2} N_m^{TE/TM} \sin \left(\frac{m\pi}{2a_{obs}} (x - x_1) \right) \cos \left(\frac{n\pi}{b_{obs}} (y - y_1) \right) dx dy \\ &= \frac{b_{obs}}{\Delta x} N_m^{TE/TM} \frac{2a_{obs}}{m\pi} \text{sinc}(n) \left[-\cos \left(\frac{m\pi}{2a_{obs}} (x_3 - x_1) \right) + \cos \left(\frac{m\pi}{2a_{obs}} (x_2 - x_1) \right) \right], \end{aligned} \quad (\text{A.36})$$

where the factors $N_m^{TE/TM}$ are defined as

$$\begin{cases} N_m^{TE} = -\frac{n\pi}{b_{obs}} A_{m,n} \\ N_m^{TM} = \frac{m\pi}{2a_{obs}} A_{m,n} \end{cases}.$$

A.6 Coupling Integrals between an Arbitrary Waveguide and an Excitation Pulse

In this case, the coupling integral $C_{i,0}$ models the interaction between the modes of an arbitrary waveguide, characterized by its magnetic vector modal functions ($\mathbf{h}_i^B$), and an excitation pulse, contained in the transverse plane, as shown in Fig. A.8.

Figure A.8: Two-dimensional zero-thickness printed circuit in planar technology. The gray area corresponds to the printed metallization and the dark area to the excitation port, both placed on the transverse ($z=0$) plane.

The expression of this integral is the following

$$C_{i,0} = \int_{ap} \mathbf{h}_i^B(s) \cdot \left(\frac{1}{\Delta y} \prod \left(\frac{x - x_0}{\Delta x} \right) \left(\frac{y - y_0}{\Delta y} \right) \mathbf{y}_0 \right) ds. \quad (\text{A.37})$$

In all the magnetic MEN formulations with transverse ports treated in this doctoral thesis, the modes $\mathbf{h}_i^B(s)$ are computed numerically using the BI-RME method *Arcioni et al.* (2014). However, the integral in equation (A.37) can then be computed analytically using the numerical expansion in terms of rectangular waveguide modes of the functions $[\mathbf{h}_i^B(s)]$. To illustrate the procedure, let $\mathbf{h}_m(s)$ be the rectangular waveguide modes used to expand the arbitrary aperture modes $\mathbf{h}_i^B(s)$. The expression

of the modes $\mathbf{h}_m(s)$ is given in Table A.1. Using the BI-RME method, we obtain the following expression

$$\mathbf{h}_i^B(s) = \sum_{m=1}^{N_k} C_{i,m} \mathbf{h}_m(s), \quad (\text{A.38})$$

where $C_{i,m}$ are the expansion coefficients calculated by BI-RME. Then, the coupling integral $C_{i,0}$ can be computed as

$$\begin{aligned} C_{i,0} &= \sum_{m=1}^{N_k} C_{i,m} \int_{ap} \mathbf{h}_m(s) \cdot \left(\frac{1}{\Delta y} \prod \left(\frac{x-x_0}{\Delta x} \right) \left(\frac{y-y_0}{\Delta y} \right) \mathbf{y}_0 \right) ds \\ &= \sum_{m=1}^{N_k} C_{i,m} \int_{ap} N_m^{TE/TM} \cos \left(\frac{m\pi}{a} x \right) \sin \left(\frac{n\pi}{b} y \right) \frac{1}{\Delta y} \prod \left(\frac{x-x_0}{\Delta x} \right) \left(\frac{y-y_0}{\Delta y} \right) ds \\ &= \sum_{m=1}^{N_k} C_{i,m} N_m^{TE/TM} \frac{1}{\Delta y} \int_{x_1}^{x_2} \cos \left(\frac{m\pi}{a} x \right) dx \int_{y_1}^{y_2} \sin \left(\frac{n\pi}{b} y \right) dy \\ &= \sum_{m=1}^{N_k} C_{i,m} N_m^{TE/TM} \frac{1}{\Delta y} \left[x_2 \operatorname{sinc} \left(\frac{m}{a} x_2 \right) - x_1 \operatorname{sinc} \left(\frac{m}{a} x_1 \right) \right] \\ &\quad \left(\frac{b}{n\pi} \right) \left[-\cos \left(\frac{n\pi}{b} y_2 \right) + \cos \left(\frac{n\pi}{b} y_1 \right) \right], \end{aligned} \quad (\text{A.39})$$

where the factors $N_m^{TE/TM}$ are defined as

$$\begin{cases} N_m^{TE} = -\frac{n\pi}{b} A_{m,n} \\ N_m^{TM} = \frac{m\pi}{a} A_{m,n} \end{cases}.$$

In this case, when $n = 0$, then the coupling integral in equation (A.37) is zero.

From a computational point of view, it is important to mention that the integration in x -dimension must be extended more than the value of x_2 (see Fig. A.8). To prove this, we study the numerical value of the coupling integral $C_{i,0}$ for several modes i in the “Example 2: Hairpin filter in microstrip technology” of Chapter IV. The geometry of this filter is given in Fig. A.9, for completeness.

Figure A.9: Hairpin microstrip bandpass filter geometry under study. The dimensions are in mm. The dimensions of the shielding box are: $a \times b = 31.2 \text{ mm} \times 30 \text{ mm}$. The filter is centered in the box.

In Fig. A.10 and Fig. A.11 we show the value of the coupling integral $C_{i,0}$, according to Δx (the distance in x -dimension used in the computational integration), for mode $i = 5$ and $i = 10$, respectively.

Figure A.10: Coupling integral $C_{i,0}$ between the mode 5 of the arbitrary waveguide and the excitation pulse, according to the distance Δx of integration.

In this example, the value of x_2 according to Fig. A.9 is 1mm. However, as observed in Fig. A.10 and A.11, for Δx equal to 1mm, the coupling integral value is

Figure A.11: Coupling integral $C_{i,0}$ between the mode 10 of the arbitrary waveguide and the excitation pulse, according to the distance Δx of integration.

not stabilized yet. We should integrate in equation (A.39) over more than 1 mm in x -dimension to achieve the right values for these coupling integral coefficients.

The explanation for this issue is the following. We use the magnetic vector modal functions of the arbitrary waveguide of Fig. A.9 (aperture part) to expand the tangential components of the magnetic field. Since the magnetic field is discontinuous when we move from the air region to the metallization (at x_2 in Fig. A.9), the arbitrary modes must be also discontinuous at x_2 . These modes are expanded numerically as a Fourier expansion using the rectangular waveguide modes (sine and cosine functions, spatial harmonics) in equation (A.38). When an abrupt change or discontinuity is represented as a Fourier series, an effect called Gibbs Phenomenon *Gibbs Phenomenon* (2020) appears. The Gibbs phenomenon reflects the difficulty inherent in approximating a discontinuous function by a finite series of continuous sine and cosine waves. Due to this phenomenon, the discontinuity in the magnetic field is not perfectly modeled in the point where the metallization starts (at x_2). This is why, although the coupling integral extends theoretically until x_2 , for a proper com-

putation of the coupling coefficients in equation (A.39), we should extend the integral over more than x_2 . A zoom of the port with the details of integration is shown in Fig. A.12. The previous explanation is experimentally validated in Fig. A.10 and Fig. A.11.

Figure A.12: Detail of the port integration. x_2 is the coordinate where the metallization starts. Δx is the distance taken in the coupling integral of equation (A.37).

APPENDIX B

Optimization MEN tool

In this appendix, we explain a Graphical User Interface (GUI) that has been created for users to analyze and design microstrip shielded circuits with the MEN formulation.

B.1 GUI for Microstrip Circuits

A schematic of the type of structures that this tool can analyze or design is shown in Fig. B.1. This tool implements the magnetic MEN technique explained in Chapter IV for the analysis of single-layer microstrip circuits that contain arbitrary metallizations and two excitation ports. Additionally, an optimizer module has been included, so the design of microwave components can also be carried out using this tool by optimization. A screenshot of the GUI main window is shown in Fig. B.2.

This tool is programmed in MATLAB (*MATLAB*) (2020). It offers the possibility of just analyzing a microstrip component or designing it by optimization. If we use it to analyze the component, the tool execute the MEN formulation of Chapter IV to obtain the circuit response. On the other hand, if we use it for the design of a component, the tool runs an optimizer module which requires the execution of several

Figure B.1: Schematic of the shielded microstrip printed circuit under analysis/design, excited with two coaxial connectors.

Figure B.2: GUI for the analysis and design of shielded microstrip components through the MEN technique. It implements the MEN formulation of Chapter IV for the analysis of planar components excited by two lateral ports, together with an optimizer module. The fields have been completed with the parameters required in the first optimization step.

MEN analyses, until the optimizer converges. The optimizer module implements the Nelder-Mead simplex algorithm *Lagarias et al.* (1998).

To show the functionalities of this tool, in this appendix, we analyze and design a second order coupled line microstrip filter. A schematic of the circuit under analysis/design is shown in Fig. B.3. It is important to note that this is only an example of a possible geometry. Thank to the integration of the BI-RME module in the MEN

code, the shape of the circuit can be arbitrary, as we showed in the examples treated in Chapter IV.

It is also important to mention that, for the design process, we use the segmentation method proposed in *Martinez Martinez et al. (2017)*. This technique divides the design of the structure into different steps (partial optimization steps). As a consequence, the design is done by optimizing several variables in each step, instead of optimizing all the variables in just one global process. For the example treated in this appendix, the design is divided in two steps: first, we optimize an structure with just one resonator and, in the second step, we introduce the second resonator (see Fig. B.3). The goal response for each step is obtained by applying the segmentation technique to the coupling matrix *Cameron et al. (2007)*. For that, the synthesis of in-line filters described in *Cameron et al. (2007)* has been also programmed. Then, we obtain the scaled coupling matrix, needed to represent the partial responses that will be used as goals.

Figure B.3: Second order coupled line filter under analysis/design. It is implemented in shielded microstrip technology. The structure is excited by two lateral ports (dark areas).

To run this tool, we open the ‘OPTIMIZER.m’ file in MATLAB and we execute it. The main window shown in Fig. B.2 will appear. Either if we analyze or design a component, the parameters that the user has to fix, according to Fig. B.2, are:

- **Shielded Box Data:** a graphical description of the following parameters is

given in the two schemes shown in Fig. B.2.

- **a**: x -dimension of the metallic enclosure (see also Fig. B.3).
 - **b**: y -dimension of the metallic enclosure (see also Fig. B.3)
 - ε_r : relative permittivity of the dielectric (see also Fig. B.1).
 - **l(air)**: height in z -dimension of the air part in the metallic enclosure (l_a in Fig. B.1).
 - **l(subs)**: height in z -dimension of the substrate part in the metallic enclosure (l_d in Fig. B.1).
- **Excitations**: a graphical description of the following parameters is given in the first scheme of Fig. B.2.
 - Δx : width in x -dimension of the excitation port integration. For a right choice of this value, see Section A.6 of Appendix A.
 - Δy : width in y -dimension of the excitation port integration. It should be the same as the width of the input/output lines (see Fig. B.3).
 - **dy 1**: height in y -dimension of the first excitation port.
 - **dy 2**: height in y -dimension of the second excitation port.

In this case, the first port is considered to be from the left wall of the box to the input line, and the second one from the output line to the right wall of the box, as shown in Fig. B.3.

- **Convergence Parameters**: These are the numerical parameters that need to be fixed when the MEN formulation is used. These parameters are explained in detail in Subsection 2.4.1. Since the component is single-layer, only one accessible mode is needed in this case. As explained in Section 2.3, to compute the kernel of the integral equations, the Kummer's transformation is applied.

The number of terms in the dynamic part of the kernel is fixed internally to a sufficient value for a right convergence. According to these considerations, the left parameters that the user must fix are:

- **Num. of basis/test:** Number of basis and test functions used in the MoM-Galerkin procedure.
- **Num. of terms in kernel:** Number of terms considered in the static part of the kernel.
- **Frequency Sweep:** Frequency plan for the analysis.
 - **Min. Freq. (GHz):** Minimum frequency of analysis (in GHz).
 - **Max. Freq. (GHz):** Maximum frequency of analysis (in GHz).
 - **Number of freq. points:** Number of frequency points considered in the analysis.
- **Optimizer Parameters:** These parameters are related to the optimizer module implemented in the GUI for the design of components. These parameters need to be filled only in the case we need to optimize the structure (not for the analysis). They are used to control when the optimizer has to stop.
 - **Max. Num. Iterations:** Maximum number of iterations for the optimization algorithm.
 - **Max. Num. Exe:** Maximum number of MEN program executions. For each iteration of the optimization algorithm, the optimizer runs the MEN code several times. This is why we can stop the optimizer by limiting the number of iterations or the number of MEN executions.
 - **Tol. Obj. Function:** Termination tolerance on the function value.
 - **Tol. Variables:** Termination tolerance on the variables.

For more details about the optimizer module, see MATLAB *fminsearch* function description (MATLAB) (2020).

After completing the previous data in the main window of the tool, the user has to define the microstrip circuit on a file called ‘microstrip_original.asc’ located in the same directory of the MEN tool. The rest of the files needed to execute the BI-RME method have to be also located in this directory. The microstrip circuit has to be defined as it is done in FEST3D (FEST3D) (2020) for arbitrary waveguides. That is, the user has to define the coordinates of the segments that compose the structure. An example of this file format is shown in Fig. B.4. We can use FEST3D graphical interface to create the circuit and copy the output file (.out) of FEST3D, since it has the required format for the BI-RME module.

Figure B.4: Example of the microstrip circuit format file, based on the definition of the segments that compose the circuit.

Once the geometry is defined, the user can choose between analyze, by clicking on **Analyze** button, or optimize the structure by clicking on **Optimize** button, in Fig. B.2.

→ If we click on **Analyze**, the program draws the geometry as shown in Fig. B.5, and then execute the BI-RME method and MEN code of Chapter IV to analyze the

structure. As observed in Fig. B.5, the analyzed geometry is the same as the one shown in Fig. B.3. The process can be followed by the user in the MATLAB program screen. When it finishes, the S-parameters are plotted, as shown in Fig. B.6.

Figure B.5: Layout of the geometry under analysis, plotted by the GUI. The two excitation ports are also drawn.

Figure B.6: S-parameters of the component under analysis obtained using the MEN technique for planar devices.

→ If we click on **Optimize**, the optimization module is executed. Two new windows appear. The first one, shown in Fig. B.7, is a map of the vertexes coordinates of the segments that compose the microstrip circuit. As observed, each point (k) is

Figure B.7: Map of coordinates given by the optimizer. The dimensions are in mm. It helps the user to visualize the optimization variables. In this case, the geometry is for the first optimization step, where only one resonator is considered.

defined through a pair the coordinates (x_k, y_k) in x -axis and y -axis, respectively. Since we apply the segmentation technique to design this component, in this first step, only one resonator is drawn in Fig. B.7.

The second window that appears is shown in Fig. B.8. In this window, we have to set the following parameters:

- **Num. of variables:** Number of variables that are going to be optimized.
- **Num. dep variables:** Number of dependent variables. These are the points of the geometry that have to be changed by the optimizer according to the selected variables.
- **dy1:** Coordinate y_k where the first pulse starts.
- **dy2:** Coordinate y_k where the second pulse starts.

Once the mentioned parameters are filled as shown in Fig. B.8, we click on **CREATE** button. According to the previous parameters, new fields to introduce

Figure B.8: Optimizer window where the user can define the variables and the dependent variables. For the first step of design, the necessary variables are specified.

the variables will be created. As observed in Fig. B.8, for this example, we have created three variables and nine dependent variables. According to the coordinate map given in Fig. B.7, we have to introduce the components x_k and y_k that have to be taken as variables. In this case, we want to optimize the length of the resonator, and the input and output couplings (different from each other) in Fig. B.7. For this, the first variable is the coordinate y_{11} , since it allows to adjust the input coupling. To maintain an input rectangular line of 2 mm, the associated dependent variables are then y_{12} , y_9 and y_{10} . The second variable is x_2 , since it modifies the length of the resonator. The associated variables are x_1 , x_3 and x_4 . Finally, to modify the output coupling, we select y_7 as the last variable. Again, to maintain the width and the shape of the output line, y_5 , y_6 and y_8 are the dependent variables. In Fig. B.8, we show how to define the variables and the associated values of the dependent variables. Finally, we observe that we have associated the first pulse to the variable y_9 , and the second

one to y_8 . In this way, when the height (in y -dimension) of the input/output lines changes, the pulses will change their height accordingly to maintain the excitation to the lines.

After including the variables, we can either define a mask for the optimization (**Create Goal Mask**) or load a mask (**Load Goal Mask**). The objective function for the optimizer is then the error (in natural units) between the mask and the computed S-parameters. In each design process step, we can choose any of these options, so we can alternate between load or create the mask, as needed.

→ → If we choose to create the mask, by clicking on **Create Goal Mask**, the window shown in Fig. B.9 will appear. In this window, the user can assign a limit value for magnitude of the S_{11} and S_{21} parameters in certain frequency bands. A graphical schematic is included in Fig. B.9, to help the user in the definition of these limits. As shown in Fig. B.9, the user can define the maximum value (in dB) for the magnitude of the S_{11} parameter between frequency f_3 and f_4 . Then, the user can define the maximum value (in dB) for the magnitude of the S_{21} parameter in two different bands (between f_1 and f_2 , and between f_5 and f_6). All the frequencies are introduced in GHz. Finally, we can set a different weight in the error computation for the S_{11} and S_{21} parameters. This is done in the field **Weigth S11 (vs S21)**. This value must be between 0 (the error is computed only with the S_{21} parameter) and 1 (the error is computed only with the S_{11} parameter).

Once all the fields are completed, we click on **DONE**. Then, we run the optimizer by clicking on **RUN** in the window of Fig. B.8.

→ → On the contrary, if we want to load the mask, we click on **Load Goal Mask** and the window shown in Fig. B.10 will appear.

Here, we have to indicate the name of the file where the mask is defined in **Name File**. The format for this file is shown in Fig. B.11. The frequencies have to be introduced in GHz and the values of the S_{11} and S_{21} parameters in linear units.

Figure B.9: Window to create the mask. The mask is defined by imposing limits to the values of the magnitude of the S_{11} and S_{21} parameters in different frequency bands. Finally, different weights can be assigned to the S_{11} and S_{21} parameters in the total error computation.

Figure B.10: Window to load the mask. The mask is defined in the specified file. The frequencies of the passband and the weight given to the passband in the total error computation can be also defined in this window. Finally, different weights can be assigned to the magnitude of the S_{11} and S_{21} parameters, in the total error computation.

Then, we can select to give more weight in the optimizer function to the in-band frequencies. This is done by specifying the minimum and maximum frequencies in

S_step1 =

2.4881	0.9911	0.1332
2.4939	0.9909	0.1345
2.4997	0.9907	0.1359
2.5055	0.9905	0.1372
2.5114	0.9903	0.1386
2.5172	0.9901	0.1400
2.5231	0.9899	0.1415
2.5290	0.9897	0.1430
2.5349	0.9895	0.1445
2.5409	0.9893	0.1460
2.5468	0.9891	0.1476
2.5528	0.9888	0.1492
2.5588	0.9886	0.1508
2.5648	0.9883	0.1525
2.5708	0.9880	0.1542
2.5768	0.9878	0.1559
2.5829	0.9875	0.1577
2.5889	0.9872	0.1596
2.5950	0.9869	0.1614

Freq. S₁₁ S₂₁

Figure B.11: Format of the file that contains the goal mask. The frequencies have to be introduced in GHz and the goal S_{11} and S_{21} in linear units.

GHz of the passband in **Freq. min (GHz)** and **Freq. max (GHz)**, respectively. Then, we have to specify the weight, between 1 and 0, for the passband in **Weight in-band**. The frequency band and weight can be different for the S_{11} and S_{21} parameters. Finally, we can set a different weight in the error computation for these parameters. This is done in the field **Weight S11 (vs S21)**.

Once all the fields are completed, we click on **DONE**. Then, we run the optimizer by clicking on **RUN** in the window of Fig. B.8.

For this example, we are going to load the mask, since we have the partial responses from the filter synthesis *Cameron et al.* (2007). The initial values of the variables are shown in Fig. B.12. The initial S-parameters response and the loaded mask are drawn in Fig. B.13.

While the optimizer is working, a curve with the error function, according to the number of iterations, is shown. This curve, for the first filter design step, is plotted in Fig. B.14.

As observed, the error is decreasing when more iterations are completed. Once it

Figure B.12: Layout of the geometry under design, plotted by the GUI. The initial values of the variables are also given.

Figure B.13: S-parameters of the component given in Fig. B.12, obtained by the MEN technique. The MEN result is compared to the partial matrix response (Goal).

finishes, according to the parameters introduced in ‘Optimizer Parameters’ of Fig. B.2, it plots the final geometry with the final values of the variables (shown in Fig. B.15) and the final S-parameters response (shown in Fig. B.16).

As we can see, the final length of the resonator is higher than the initial value (given in Fig. B.12), so then the resonant frequency has been shifted to lower frequen-

Figure B.14: Convergence of the optimization algorithm according to the number of iterations. The picture shows the value of the objective function that, in this tool, is defined as the error between the MEN S-parameters and the goal mask from the coupling matrix. This is the convergence rate for the first optimization step.

Figure B.15: Layout of the geometry after the first optimization step, plotted by the GUI. The final values of the variables are also given.

cies, and now it matches better the mask. The output coupling has been increased, while the input coupling has been decreased with respect to the initial values. These changes make the obtained response more similar to the goal mask, as observed in Fig. B.16.

Figure B.16: S-parameters of the component given in Fig. B.15, obtained by the MEN technique. The MEN result is compared to the partial matrix response (Goal).

According to *Martinez Martinez et al. (2017)*, when the second resonator is added, the response should match the final goal mask, since the order of the filter is two. However, as it will be observed, the response obtained when the second resonator is added, is not perfectly equal to the matrix response. For this reason and to show more examples about how to define the variables, we perform two more optimization steps with the complete geometry in Fig. B.3 (including both resonators).

To include the new resonator, we have to update the file ‘microstrip_original.asc’. The initial input/output coupling value of the new geometry is the final input coupling value obtained in the previous optimization step. The inter-resonator coupling value is initially fixed equal to the output coupling obtained in the previous optimizer step. For more details about this segmentation technique, see *Martinez Martinez et al. (2017)*.

For this second step, we modify the fields of the main window as shown in Fig. B.17. When we click on **Optimize**, the coordinate map in Fig. B.18 appears. According to this map, we complete the variable window of Fig. B.19.

As we can see, in this second step, we want to modify the variable x_3 , and the

Figure B.17: GUI for the analysis and design of shielded microstrip components through the MEN technique. It implements the MEN formulation of Chapter IV for the analysis of planar components excited by two lateral ports, together with an optimizer module. The fields have been completed with the parameters required in the second and third optimization steps.

Figure B.18: Map of coordinates given by the optimizer. The dimensions are in mm. It helps the user to visualize the optimization variables. In this case, the geometry is for the second and third optimization steps, where the two resonators are considered.

dependent variables are x_2 , x_{14} and x_{15} . This will modify the length of the resonators without modifying the input/output couplings. By modifying x_3 , the inter-resonator

OPT_MODULE

Num. of variables: 1

Num. dep variables: 3

CREATE

dy1: y12

dy2: y7

Dep. variable 1: x2 equal to x3

Dep. variable 2: x14 equal to -x3

Dep. variable 3: x15 equal to -x3

Variable 1: x3

Create Goal Mask

Load Goal Mask

RUN

Figure B.19: Optimizer window to define the variables and the dependent variables. For the second step of design, the necessary variables are specified.

coupling will be also modified.

After defining the variables for this step, we load the new mask and we define the passband and the weights for these frequencies. This is set as shown in Fig. B.20. It is important to click then on **DONE** to upload the new mask.

The initial value of x_3 together with the initial response are shown in Fig. B.21 and Fig. B.22, respectively. As observed, the filter response is not perfect when the new resonator is added. Thus, we run the optimizer. The convergence curve of the optimizer for this step is shown in Fig. B.23.

When it finishes, we can check the final value of the variable in Fig. B.24. The S-parameters response is shown in Fig. B.25. If we compare the responses given in Fig. B.22 and B.25, we can see that the resonant frequencies are now matching the matrix response better than before. We still see in Fig. B.25 that the bandwidth of the real filter is lower than the matrix response. To compensate this effect, we

Figure B.20: Window to load the mask. The mask is defined in the specified file. The frequencies of the passband and the weight given to the passband in the total error computation can be also defined in this window. Finally, different weights can be assigned to the S_{11} and S_{21} parameters.

Figure B.21: Layout of the geometry under design, plotted by the GUI. The initial value of the variable is also given.

should increase the input/output couplings. To adjust these couplings, we run a last optimization step.

As shown in Fig. B.26, we select as variable, the coordinate y_{10} . To main the filter symmetric and the input/output lines with rectangular shape and 2 mm of width,

Figure B.22: S-parameters of the component given in Fig. B.21, obtained by the MEN technique. The MEN result is compared to the partial matrix response (Goal).

Figure B.23: Convergence of the optimization algorithm according to the number of iterations. The picture shows the value of the objective function that, in this tool, is defined as the error between the MEN S-parameters and the goal mask from the coupling matrix. This is the convergence rate for the second optimization step.

the dependent variables are y_{11} , y_{12} , y_9 , y_6 , y_7 , y_5 , y_8 .

Then, we load the mask as shown in Fig. B.27.

The initial value of y_{10} can be seen in Fig. B.28. The initial response of this step is the final response from the previous one, shown in Fig. B.25.

Figure B.24: Layout of the geometry after the second optimization step, plotted by the GUI. The final value of the variable is also given.

Figure B.25: S-parameters of the component given in Fig. B.24, obtained by the MEN technique. The MEN result is compared to the partial matrix response (Goal).

We run the optimizer and we check the convergence rate in Fig. B.29.

The final value for this variable and the final structure for the filter are shown in Fig. B.30. As observed, the optimizer has increased the input/output couplings, as expected.

The final response of the optimizer is given in Fig. B.31. As we can see, the

OPT_MODULE

Num. of variables: 1

Num. dep variables: 7

CREATE

dy1: y12

dy2: y7

Dep. variable 1	y11	equal to	y10
Dep. variable 2	y12	equal to	y10-2
Dep. variable 3	y9	equal to	y10-2
Dep. variable 4	y6	equal to	-y10
Dep. variable 5	y7	equal to	-y10
Dep. variable 6	y5	equal to	-y10+2
Dep. variable 7	y8	equal to	-y10+2

Variable 1: y10

Create Goal Mask

Load Goal Mask

RUN

Figure B.26: Optimizer window to define the variables and the dependent variables. For the third step of design, the necessary variables are specified.

LOAD_F_OBJfig

Name File: S_step2.mat

S11

Freq. min (GHz): 3.026

Freq. max (GHz): 3.3

Weight in-band: 0.9

S21

Freq. min (GHz): 3.026

Freq. max (GHz): 3.3

Weight in-band: 0.9

Weight S11 (vs. S21): 0.5

DONE

Figure B.27: Window to load the mask. The mask is defined in the specified file. The frequencies of the passband and the weight given to the passband in the total error computation can be also defined in this window. Finally, different weights can be assigned to the S_{11} and S_{21} parameters.

Figure B.28: Layout of the geometry under design, plotted by the GUI. The initial value of the variable is also given.

Figure B.29: Convergence of the optimization algorithm according to the number of iterations. The picture shows the value of the objective function that, in this tool, is defined as the error between the MEN S-parameters and the goal mask from the coupling matrix. This is the convergence rate for the third optimization step.

response of the filter is now closer to the matrix response than before. It is important to mention that this is only a very simple example that has been shown to explain this optimization MEN tool. Furthermore, using this GUI, different microstrip circuits with the structure of Fig. B.1 could be designed, following a similar optimization

Figure B.30: Layout of the geometry after the third optimization step, plotted by the GUI. The final value of the variable is also given.

Figure B.31: S-parameters of the component given in Fig. B.30, obtained by the MEN technique. The MEN result is compared to the partial matrix response (Goal). This is the final response of the filter after the optimization process.

process, by means of the Multimode Equivalent Network technique.

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