Contributions to technology and techniques for ground-based and space-based astronomy: synthesis of EDAC codecs, AOLI and COELI

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In no particular order, I would like to dedicate this thesis to:

my mother, for her loving support in these times of hard work;

my father, for his useful advice all along the elaboration of this thesis;

my sister, for being my particular guide star in a wide range of targets;

my brother, for caring about my work though sometimes pretending not to;

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Thesis by publications

The present thesis is formed by a collection of research papers derived from work done during the doctoral period. The references of such publications are listed below.

1. C. Colodro-Conde and R. Toledo-Moreo. “Design and analysis of efficient synthesis algorithms for EDAC functions in FPGAs”. In: *IEEE Transactions on Aerospace and Electronic Systems* 51.4 (Oct. 2015), pp. 3332–3347.
2. C. Colodro-Conde, S. Velasco, J. J. Fernández-Valdivia, R. López, A. Oscoz, R. Rebolo, B. Femenía, D. L. King, L. Labadie, C. Mackay, B. Muthusubramanian, A. Pérez Garrido, M. Puga, G. Rodríguez-Coira, L. F. Rodríguez-Ramos, J. M. Rodríguez-Ramos, R. Toledo-Moreo, and I. Villó-Pérez. “Laboratory and telescope demonstration of the TP3-WFS for the adaptive optics segment of AOLI” In: *Monthly Notices of the Royal Astronomical Society* 467.3 (Jun. 2017), pp. 2855–2868.
3. M. P. Cagigal, P. J. Valle, C. Colodro-Conde, I. Villó-Pérez, and A. Pérez-Garrido. “Covariance of lucky images for increasing objects contrast: diffraction-limited images in ground-based telescopes” In: *Monthly Notices of the Royal Astronomical Society* 455.3 (Jan. 2016), pp. 2765–2771.

DECLARACIÓN DE CONFORMIDAD A TESIS POR COMPENDIO

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2. C. Colodro-Conde, S. Velasco, J.J.F. Valdivia, R.L. López, A. Oscoz, R. Rebolo, B. Femenía, D.L. King, L. Labadie, C. Mackay, B. Muthusubramanian, A. Pérez Garrido, M. Puga, G. Rodríguez-Coira, L.F. Rodríguez-Ramos, J.M. Rodríguez-Ramos, R. Toledo-Moreo, and I. Villó-Pérez. "Laboratory and telescope demonstration of the TP3-WFS for the adaptive optics segment of AOLI" In: *Monthly Notices of the Royal Astronomical Society* (in press).
3. M. P. Cagigal, P. J. Valle, C. Colodro-Conde, I. Villó-Pérez, and A. Pérez-Garrido. "Covariance of lucky images for increasing objects contrast: diffraction-limited images in ground-based telescopes" In: *Monthly Notices of the Royal Astronomical Society* 455.3 (Jan. 2016), pp. 2765–2771.

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Abstract

The present dissertation covers the contributions made by the author in the context of three astronomy-related projects, namely: the Euclid satellite, the AOLI (Adaptive Optics Lucky Imager) instrument for ground telescopes and the COELI (COvarianceE of Lucky Images) algorithm for the post-processing of astronomical images. All the contributions have been made in the field of engineering, solving technological problems that will help the astronomers to achieve the specific scientific goals of each project.

Euclid is a space mission by the European Space Agency (ESA) whose scientific objective is to map the geometry of the dark Universe. The author of this dissertation was involved in the design of the control unit of the infrared instrument on-board the satellite. Such involvement derived in the creation of a new line of research aimed to improve the reliability of digital circuits in space. The strategy that was followed to reach this objective was to optimize the implementation of the EDAC (Error Detection and Correction) codecs, a kind of very recurring element that is used to protect volatile memories from the hazardous radiation that exists in the space environment.

AOLI is an instrument for ground telescopes that has been designed to image the skies with resolutions even finer than the ones that can be achieved from space telescopes with the absence of the turbulent atmosphere. In order to make this possible, AOLI combines two well-known high-resolution techniques that had never been specifically combined in an instrument before: Adaptive Optics (AO) and Lucky Imaging (LI). The author of this thesis has been in charge of the real-time control of the AO subsystem of AOLI, which involved the use of a novel wavefront sensor that had not been used before in an astronomical application.

The last of the contributions, named COELI, is also related to high-angular resolution astronomy, but following a post-processing approach. This new algorithm takes advantage of the statistical properties of atmospheric distortion so as to avoid any residual halo in the resulting images, as opposed to what would happen with conventional Lucky Imaging. The results of the executed tests prove that this newer technique provides enhanced contrast capabilities for detecting faint objects very close to a host star.

Table of contents

List of figures	xxi
Nomenclature	xxiii
1 Introduction	1
1.1 The Euclid space mission	2
1.1.1 NI-ICU architecture	3
1.1.2 Research contributions	5
1.2 The AOLI instrument	6
1.2.1 Instrument description	7
1.2.2 Control of the adaptive optics subsystem	11
1.3 The COELI algorithm	13
2 State of art	17
2.1 Synthesis of EDAC cores in FPGA	17
2.1.1 Radiation-hardening techniques	17
2.1.2 Efficient EDAC implementations in scientific publications	20
2.1.3 Optimality of commercially-available synthesis tools	21
2.2 High-resolution imaging from ground telescopes	22
2.2.1 Speckle interferometry	23
2.2.2 Adaptive optics	24
2.2.3 Lucky imaging	26
3 Efficient synthesis algorithms for EDAC functions in FPGAs	31
4 The adaptive optics subsystem of AOLI	49
5 The COELI algorithm	67

6	Conclusions	77
	References	81
Appendix A	Journal Impact Factors	91

List of figures

1.1	NI-ICU internal architecture	4
1.2	Optical layout of AOLI	9
1.3	AOLI mounted at one the two Nasmyth focuses of the WHT	10
1.4	Screenshot of the RTC software during a closed-loop test at the WHT	12
1.5	Example extrasolar planetary system	14
2.1	TMR-based SEU-hardened flip-flop.	19
2.2	Read an write accesses in an EDAC-protected RAM.	20
2.3	Effect of atmospheric turbulence on the image of a star	23
2.4	Basic components of an adaptive optics system	25
2.5	Result of post-processing a stack of short-exposure images	27
2.6	Profile of a partially corrected image of a star	29

Nomenclature

ADC	Analog-to-Digital Converter (in analogue electronics)
ADC	Atmospheric Dispersion Corrector (in astronomical instrumentation)
AIV	Assembly, Integration & Verification
AO	Adaptive Optics
AOLI	Adaptive Optics Lucky Imager
ASIC	Application Specific Integrated Circuit
AU	Astronomical Unit
BCH	Bose-Chaudhuri-Hocquenghem
CCD	Charge-Coupled Device
CDMU	Command and Data Management Unit
CDPU	Central Data Processing Unit
CMB	Cosmic Microwave Background
COELI	COvariancE of Lucky Images
DAC	Digital-to-Analog Converter
DAS	Driver and Acquisition Support
DC/DC	Direct Current - Direct Current
DM	Deformable mirror
EDAC	Error Detection and Correction

EEPROM	Electrically Erasable Programmable Read-Only Memory
EMC	Electromagnetic Compatibility
EMCCD	Electron Multiplying Charge-Coupled Device
ERDF	European Regional Development Funds
ESA	European Space Agency
FF	Flip-flop
FOV	Field Of View
FPGA	Field Programmable Gate Array
FS-EDAC	Full Search EDAC
G-EDAC	Greedy EDAC
GPM	General Purpose Module
GRACE	GRound based Adaptive optics Controlled Environment
GTC	Gran Telescopio Canarias
HK	Housekeeping
HST	Hubble Space Telescope
IAC	Instituto de Astrofísica de Canarias
IoA	Institute of Astronomy
JWST	James Webb Space Telescope
LED	Light-Emitting Diode
LGS	Laser Guide Star
LI	Lucky Imaging
LUT	Look-Up Table
LVPS	Low Voltage Power Supply
MDPA	Multi-DSP microProcessor Architecture

MIMO	Many-Input, Multiple-Output
MISO	Many-Input, Single-Output
MTBF	Mean Time Between Failures
NI-CU	NISP Calibration Unit
NI-DPU	NISP Data Processing Unit
NI-FPA	NISP Focal Plane Array
NI-FWA	NISP Filter Wheel Assembly
NI-GWA	NISP Grism Wheel Assembly
NI-ICU	NISP Instrument Control Unit
NI-OMADA	NISP Opto-Mechanical Assembly and Detector Assembly
NISP	Near Infrared Spectrometer and Photometer
NOT	Nordic Optical Telescope
PCB	Printed Circuit Board
PID	Proportional-Integral-Derivative
PROM	Programmable Read-Only Memory
PWM	Pulse-Width Modulation
RAM	Random Access Memory
RHBD	Radiation Hardening By Design
RHBP	Radiation Hardening By Process
RS	Reed-Solomon
RTC	Real-Time Control
SEB	Single-Event Burnout
SEC-DED	Single Error Correction - Double Error Detection
SECOIA	SErial COntrol Interface

SEE	Single-Event Effect
SEU	Single-Event Upset
SH-WFS	Shack-Hartmann Wavefront Sensor
SOS	Silicon On Sapphire
SRAM	Static Random Access Memory
SpW	SpaceWire
TC	Telecommand
TCP/IP	Transmission Control Protocol / Internet Protocol
TID	Total Ionizing Dose
TM	Telemetry
TMR	Triple Modular Redundancy
TP3-WFS	Two Pupil Plane Positions Wavefront Sensor
TRL	Technology Readiness Level
UC	Universidad de Cantabria
ULL	Universidad de La Laguna
UPCT	Universidad Politécnica de Cartagena
VHDL	VHSIC Hardware Description Language
VHSIC	Very High Speed Integrated Circuit
VIS	VISible instrument
WFS	Wavefront Sensor
WHT	William Herschel Telescope

Chapter 1

Introduction

“El Universo es lo más grande que hay”

Rafael Toledo-Moreo

The present dissertation covers the contributions in the field of space electronics and astronomical instruments in which the author has participated as a PhD student. The studies have been performed in the context of three astronomy-related projects: the Euclid space mission, the AOLI instrument (Adaptive Optics Lucky Imager) and the COELI algorithm (COvarianceE of Lucky Images). Each one of them poses a challenge in its own way by pushing the current technology and techniques to new limits, all with the aim of creating the new tools that will allow us to keep satiating our thirst for knowledge about the Universe, at least for some considerable time.

Indeed, the acquisition of new knowledge may be the most evident recompense of a successful astronomical project, but the impact on society can actually be much more tangible than that. Throughout history, astronomy has been one of the main drivers of human progress. For example, understanding of the skies enabled ancient civilizations to create calendars that, among other uses, helped to optimize the agricultural schedules. Nowadays, astronomy is a highly active source of technology transfer with applications as diverse as telecommunications, medicine, defence and, of course, space exploration [1].

The contributions of this thesis are not focused on scientific discoveries but on advances in engineering applied to observational astronomy. Despite the fact the three projects mentioned at the beginning of this section originated independently, each with its own technological approaches and scientific goals, the junction of them all in this thesis gives an added value because it allows exploring three different points in the engineering process where research and innovation can be conducted:

1. The development of new design tools.
2. The development of new scientific instruments powered by novel technologies.
3. The development of new algorithms for reducing raw astronomical data.

The following subsections present an introduction to Euclid, AOLI and COELI, and how each project is linked to one of the innovative aspects listed above.

1.1 The Euclid space mission

Euclid is the Medium Class mission of the ESA Cosmic Vision 2015-2025 programme selected in 2011, with a foreseen launch slot in 2020. The main objective of Euclid is to understand the origin of the accelerating expansion of the Universe by studying the dark matter and the dark energy [2, 3].

The Euclid spacecraft is equipped with a 1.2 m Korsch telescope, which directs the light to two instruments: the VISible instrument [4] (VIS) and the Near Infrared Spectrometer and Photometer [5] (NISP). With these instruments physicists will probe the expansion history of the Universe and the evolution of cosmic structures by measuring the modification of shapes of galaxies induced by gravitational lensing effects of dark matter and the 3-dimension distribution of structures from spectroscopic redshifts of galaxies and clusters of galaxies [6].

The NISP instrument performs imaging photometry and slitless spectroscopy measurements in sequence inserting optical components in the light path by configuring a filter wheel [7] (NI-FWA) and a grism wheel [8] (NI-GWA). Besides the wheel cryomechanisms, the mechanical structure of the instrument (NI-OMADA, Opto-Mechanical Assembly and Detector Assembly) encompasses the Calibration Unit (NI-CU) and the Focal Plane Array [9] (NI-FPA). The process of acquiring and processing the science data is handled by the Data Processing Unit [10] (NI-DPU), while the overall control of the instrument is done by another piece of electronics: the Instrument Control Unit [11, 12] (NI-ICU).

Among all the institutions participating in this mission, the Universidad Politécnica de Cartagena (UPCT), in collaboration with the Instituto de Astrofísica de Canarias (IAC) and with a contract with Crisa (Airbus Defence and Space), is responsible for the design, manufacturing and validation of the control electronics of the NISP instrument, that is, the NI-ICU. In this context, the author of this dissertation is responsible of the processor board of the NI-ICU, called CDPU (Central Data Processing Unit).

1.1.1 NI-ICU architecture

The NI-ICU is constituted by two identical sections, one nominal, the other in cold redundancy. Internally, each section of the NI-ICU (nominal or redundant) is organized in four boards: LVPS (Low Voltage Power Supply), CDPU (Central Data Processing Unit), DAS (Driver and Acquisition Support) and backplane motherboard that interconnects the rest of boards by routing the necessary signal and power lines. The internal architecture of the NI-ICU is depicted in Figure 1.1, and the three main boards are described below.

LVPS board

The LVPS board takes the 28V power line from the spacecraft primary bus and produces all the necessary secondary power supplies by means of DC/DC (Direct Current - Direct Current) converters. These secondary lines are used to power the NI-OMADA elements under NI-ICU control, as well as the NI-ICU itself. The LVPS board allocates filters to meet the EMC (Electromagnetic Compatibility) requirements. Finally, a non-power-related feature is also provided by this board: the intra-instrument MIL-STD-1553B [13] transceivers, which do not fit in the CDPU board because of PCB (Printed Circuit Board) size restrictions.

CDPU board

The CDPU board is based on Crisa's General Purpose Module (GPM). It features a MDPA ASIC (Multi-DSP microProcessor Architecture Application Specific Integrated Circuit) which includes, among others, a 80MHz LEON2-FT microprocessor, two 1553 interfaces and eight SpaceWire [14, 15] links. Regarding memory resources, the board encompasses 64 KBytes of PROM, 4 MBytes of EEPROM and 8 MBytes of EDAC-protected SRAM. A supporting RTAX2000S FPGA provides additional functions mainly related to the processor system and internal/external interfaces (e.g., communication with the DAS board).

DAS board

The DAS board features a SECOIA (SERial CONTROL Interface) ASIC that controls all the driving and acquisition electronics that allow the NI-ICU to interface the NI-OMADA elements under its control (motors, reference position sensors, heaters, temperature sensors and calibration unit LEDs). Additionally, the DAS board also acquires NI-ICU internal telemetry such as temperature, voltage and intensity readings.

1.1.2 Research contributions

Being a Medium Class mission of the ESA Cosmic Vision Plan, the relatively short preparation time of Euclid (selected in 2011, scheduled to launch in 2020) forces the use of technologies which are almost available. In fact, a guiding principle of the Cosmic Vision Plan is that a mission must have reached a Technology Readiness Level [16] (TRL) of 5-6 to be adopted, which means that the critical base technology developments of the spacecraft and science instruments must be completed beforehand [17]. In the case of Euclid, the main technology developments that were foreseen are related to the cold gas propulsion system and the CCD detectors. The rest of components of the spacecraft are required to have a high level of maturity so as to minimize development risks.

At first sight, the described scenario does not seem to be best environment to promote the type of pioneering research that is usually part of a PhD thesis. However, the fact is that having taken part of the development of the NI-ICU was the perfect opportunity to obtain first-hand information about the current state-of-art of space technology and the areas where there could be a margin for improvement. Indeed, Euclid is not a mission where the results of such initial research could be applied, but the acquired knowledge would be available to be exploited in future missions once the required level of maturity has been reached.

The space environment is known to be particularly harsh, not only for humans but also for the electronic circuits on-board satellites such as Euclid. In contrast to most circuits designed for ground systems, the ones on-board satellites need to survive low temperatures, vacuum, high-energy electromagnetic radiation and particle radiation [18]. Among these characteristic conditions, radiation is a primary concern because circuits manufactured with radiation-hardening techniques are still vulnerable to some of the effects produced by it. This is the case of Single Event Upsets (SEUs), which may compromise the reliability of space systems if no action is taken to mitigate them [19].

A SEU consists on an alteration of the contents of a memory element such a flip-flop caused by a hit of a high-energy particle. Given the fact that SEUs cannot be completely eliminated just by applying radiation-hardened manufacturing processes, additional mitigation measures need to be considered during the design of digital circuits for space. Design techniques for SEU protection typically involve some kind of redundancy, which may be applied at different levels.

In the case of the NI-ICU of Euclid, redundancy is first applied at unit level by dividing the NI-ICU in two identical sections in cold redundancy, as explained before in Section 1.1.1. The RTAX2000S FPGA of the CDPU board provides built-in Triple Modular Redundancy (TMR) protection for every instantiated flip-flop. The LEON2-FT processor inside the MDP ASIC protects each 32-bit word of its register file by means of a 7-bit Error Detection and

Correction (EDAC) checksum that features single-error correction and double-error detection (SEC-DED) capabilities. The external SRAM that is used for storing instructions and data is also protected by EDAC, with the EDAC codec implemented in the MPDA ASIC itself. Several EDAC codecs are also implemented in the CDPU FPGA for protecting buffers and tables based on internal RAM resources.

Being such a recurring element, it was considered worth to investigate whether the currently available EDAC cores had a proper level of optimization in terms of area, processing speed, power consumption and, most importantly, reliability. Given that they are the parts that protect critical elements such as processor register files, instruction and data memories, it is clear that the reliability of EDAC cores deserves a special attention. This statement is even more true in recent research trends, which aim to make it viable to use SRAM-based FPGAs in space. The case of SRAM-based FPGAs is specially delicate because the configuration memory that defines the internal circuitry is vulnerable to SEU itself.

The first test revealed a surprising fact: the most widely-used FPGA synthesis tools have a suboptimal performance when synthesizing many-input logic functions such as the one that are present in Hamming EDAC codecs. This statement is true not only for the complex case of a full EDAC codec, but also for the simplest case of processing a single many-input, single output logic gate. This discovery motivated the research of novel synthesis algorithms that handle the specific case of EDAC codecs in a more efficient way.

1.2 The AOLI instrument

Since the invention of the telescope, the great advances in astronomy have been always tied to technological advances in telescopes and instruments, which either enabled the observation of fainter objects, provided an increased resolving power or opened new windows for observing the skies from a different perspective.

Clear examples of the close connection between astronomy and technology date back to the days of Galileo. It was him who first gathered conclusive observational evidence of the heliocentric model by pointing his own technologically advanced telescopes to Venus, revealing that it exhibited a full set of phases similar to that of the Moon [20]. More recently, the introduction of radio astronomy during the 20th century resulted in the discovery of new classes of astronomical objects such as quasars and pulsars [21], as well as the establishment of the Big Bang theory. In relation to this, the high resolution maps of the cosmic microwave background (CMB) produced by the Planck spacecraft during its four years of service (2009-2013) have vastly increased our knowledge about the past, present and future of the Universe [22]. On the other hand, recent technological advances in the acquisition of high-resolution

images in the visible bands have had a remarkable impact on the current astrophysical models. For example, the first resolved image of a star different from the Sun, obtained in 1996 by the Hubble Space Telescope (HST) [23], triggered the creation of new models of stellar physics that would explain those initial observations and the ones that followed.

The Adaptive Optics Lucky Imager (AOLI) is an astronomical instrument for ground telescopes that has been developed with the objective of obtaining images with extremely high angular resolution in the visible bands. Under this premise, AOLI aspires to be the tool that will enable astronomers to make remarkable scientific discoveries that otherwise would not be possible to make with the technology currently available in ground telescopes, where the influence of the atmosphere makes it very difficult to reach the maximum resolution that can theoretically be achieved for a given aperture size. More specifically, the scientific targets that AOLI is expected to study are multiple star systems with very small angular separation.

AOLI represents the convergence of various techniques that have been explored and extensively developed by teams from several institutions. Instituto de Astrofísica de Canarias (IAC) leads the project and provides not only the management but also its expertise and capabilities on AIV (Assembly, Integration & Verification) of state-of-the-art instruments, optical instrumentation and lucky imaging [24, 25]. Institute of Astronomy (IoA) (University of Cambridge) collaborates with the optics and detectors [26, 27]. Universidad Politécnica de Cartagena (UPCT) has developed the real-time control software for the adaptive optics subsystem [28], as well as the lucky imaging software [25]. Universität zu Köln is in charge of the science software [29, 30]. Finally, University of La Laguna (ULL) has contributed with the novel Two Pupil Plane Positions Wavefront Sensor (TP3-WFS) [31].

1.2.1 Instrument description

As its own name suggests, AOLI has been conceived as a double system that for the first time combines the resolving power of two well-known high angular resolution techniques: adaptive optics (AO) and lucky imaging (LI). Adaptive optics consists on undoing the effect of atmospheric distortions by sensing the incident wavefronts and actuating in real time over a deformable mirror (DM). On the other hand, lucky imaging selects and processes only the best images out of a series of short exposures, taking advantage of the fact that, statistically, there is a certain percentage of images that will have been acquired during periods of little turbulence. The combination of the two techniques in medium-sized or large telescopes is expected to produce images with resolutions that have never been seen before, even from space telescopes. Further explanations about AO and LI will be given later in Chapter 2.

Initially targeted for the 4.2m William Herschel Telescope (WHT) (Roque de Los Muchachos Observatory, Spain), AOLI has been designed with modularity in mind, in the sense

that it would require relatively low effort to adapt it to other telescopes such as the 10.4m Gran Telescopio Canarias (GTC), located at the same observatory. The modular concept of AOLI would also make it possible to change the science payload from LI to other functions or instruments so that they can take advantage of the benefits of AO [32].

AOLI has evolved throughout the years, always tending to a more compact and versatile design. Nevertheless, the definition of the four main modules that comprise the instrument has not changed since its conception. Such subsystems are described below:

- **Calibration module:** delivers a calibrated point-like source resembling the telescope f-ratio and exit pupil, thus allowing to perform tests on a laboratory. A motorized turbulence wheel with a pseudo-random phase screen optionally simulates the effect of the moving atmosphere.
- **Front-end module:** provides the interface with the telescope. The input beam is first collimated and then passed through an atmospheric dispersion corrector (ADC) before it strikes a 241-actuator deformable mirror (DM). The light is then reflected and reimaged onto a pick-off mirror, which divides the light so that the AO reference object (i.e., the one used to estimate the incident wavefront) passes through a pin-hole to the wavefront sensor, while the rest of the information deflects towards the science arm.
- **Science module:** consists on a 1024×1024 pixels Electron Multiplying CCD (EM-CCD) by Andor equipped with two different plate scales and fields of view (FOV). This part is in charge of acquiring the short-exposure images that are the input of the LI algorithm. A filter wheel is used to select the wavelength of interest.
- **Wavefront sensor module:** the last arm of the instrument corresponds to the optics of the TP3-WFS, which represents one of the key innovations of AOLI. A lateral prism separates the input beam and introduces a differential delay between the two optical paths. Then, a common lens system generates two defocused pupil images over a 512×512 EMCCD detector by Andor. This subsystem does not include any filter because the TP3-WFS operates in broadband. The union of the WFS and the DM, in spite of being part of different optical modules, is commonly called the *AO subsystem*.

The optical layout of the core components of AOLI (excluding calibration module) is depicted in Figure 1.2. On the other hand, Figure 1.3 shows a commented photograph of the instrument as installed at the WHT for its first commissioning run in May 2016. The elements before the telescope focal plane (derotator, guide star pick-off mirror) are not technically part of the instrument itself, but they have been identified in the image nevertheless because they are also part of the typical operations.

Fig. 1.2 Optical layout of AOLI. The pick-off mirror marks the common vertex between the front-end, science and wavefront sensor modules. The calibration module is not shown in the diagram because only the components that are active during the observations have been drawn.

Fig. 1.3 AO LI mounted at one the Nasmyth focuses of the WHT, inside the GRACE (GRound based Adaptive Optics Controlled Environment) structure. The main components have been identified with labels, and the path that the light rays would follow during observations (with a WFS pick-off mirror in position) have been drawn in red (background photograph by Roberto López-López).

In addition to the optics, AOLI comprises three computers:

- **Science computer:** runs the frame grabbing software on the LI camera. This software dumps the acquired images to disk and serves them to the local network for real-time preview at the control computer. In addition, this computer interfaces with the pick-off mirror assembly, ADC, FOV selector and the filter wheel assembly.
- **AO computer:** runs the frame grabbing software on the AO camera, the wavefront reconstruction software of the TP3-WFS and the real-time control software that actuates over the deformable mirror based on the reconstructed wavefronts.
- **Control computer:** centralizes all the functionality provided by the science computer, AO computer and telescope control system. Intended to be used during observations, this computer would be located at the telescope control room.

At the time of writing the present dissertation, AOLI was undergoing a series of commissioning runs at the WHT that were mainly focused on tuning the adaptive optics subsystem of the instrument, which had not been tested under a real environment until then. The conclusion of those tests will give the green light to nominal scientific observations.

1.2.2 Control of the adaptive optics subsystem

The author of this dissertation was responsible of the real-time control (RTC) software of the adaptive optics of AOLI. This involved not only the development of this piece of software, but also the characterization and testing of the whole AO subsystem both in the laboratory and in the telescope. Such tasks are of special interest from a research perspective because they require close contact with a new type of WFS that had never been implemented before for astronomical applications: the TP3-WFS. Indeed, the equations that define this wavefront reconstruction technique had been available for more than a decade [33], but AOLI is the first instrument to implement them in real-time as a physical device [34, 35], to the best of the author's knowledge. With a fully functioning DM purchased from a specialized company (ALPAO), the RTC software was thus the last piece of the puzzle that was required to employ this novel WFS in an actual adaptive optics system.

The RTC software receives the wavefronts reconstructed by the TP3-WFS, expressed as a set of Zernike modes [36], and calculates a vector of DM actuations that re-shape the incident wavefront so that it becomes more similar to the desired control target, normally a planar wavefront. The control algorithm is based on an array of proportional-integral-derivative (PID) controllers, where each controller is assigned to an individual Zernike mode and can be configured with independent parameters.

Fig. 1.4 Screenshot of the RTC software during a closed-loop test at the WHT, showing the following real-time plots: input vector of Zernike modes, output actuation vector and RMS error of the reconstructed wavefront with respect to the defined control target.

In order to properly actuate over the DM, the control algorithm needs information about the static and dynamic response of the system, understood as the relationship between particular combinations of DM actuations and the differential phase that they introduce on the incident wavefront. In spite of not being a real-time controlling function, the process of acquiring and processing this necessary information is also automated by the RTC software, as part of the available characterization procedures. This information was specially useful not only for configuring the control algorithm but also while debugging the TP3-WFS in earlier stages of the project.

A handy graphical front-end (Figure 1.4) exposes all the functionalities of the RTC software to the user of the instrument, including: configuration of the real-time control loop, real-time plotting and recording of AO-related data, manual operation of the DM, creation of a TCP/IP server for local/remote operation of the DM by other programs, automatic execution of static and dynamic characterization procedures and on-the-fly update of the control parameters (control target, PID parameters) during closed-loop operation.

In Figure 1.4, the first two modes (tip and tilt) in the Zernike representation have a high value because their control was disabled at that time. The 1-D representation of the actuation vector gives information about the stress that the DM is experiencing, around 22% of the maximum stroke in the example screenshot. If desired, a 2-D representation of this vector can be obtained if the position of each actuator over the DM surface is known, which can be obtained by means of the static characterization procedure or by referring to the manual of the DM. The root-mean-squared (RMS) error plot is a reliable indicator of the goodness and stability of the control loop. If the RTC software is configured correctly, this plot should always show a substantial drop at the time of closing the control loop, although this drop does not appear in the screenshot because it has already left the time span of the plot. Finally, an inspection of the last entries of the program log shown at the lower right quadrant of Figure 1.4 reveals the use of one common operation procedure: the adjustment of the control parameters in closed-loop mode.

The RTC software was designed with a focus on performance and flexibility. Low latencies are required in any AO system because otherwise the calculated DM actuations would correspond to input wavefronts that do not exist on the real world any more. What is more, the element in the AO control loop with the longest latency sets an upper limit to the AO sampling rate, which should be kept as high as possible so as to get the best closed-loop performance. Flexibility was also a desirable feature in the RTC software not only to counter the uncertainties that the new TP3-WFS arose, but also because AOLI was required to be adaptable for bigger telescopes such as the GTC.

1.3 The COELI algorithm

The search of planets orbiting stars different from the Sun (i.e., *exoplanets*) is a very recent and active field of research in the context of high resolution astronomy. Ever since the first confirmed detection of an extrasolar planetary system in 1992 [37], a lot of effort has been put on advancing in the technologies that enable the detection planets with characteristics similar to those of the Earth, possibly hosting life as we know it.

The current result of the effort put in exoplanet search is adequately represented by the Kepler space telescope, launched in 2009 with the objective of estimating the probability of a star having an Earth-like planet [38]. As of 2016, the analysis of the first four years of Kepler data has led to the discovery of more than a thousand exoplanets [39], using a detection technique based on the periodic drops in brightness that occur when planets transit their host stars (known as the *transit method*). Another common way of detecting exoplanets, currently applied to ground telescopes, is to measure their influence on the orbits of the host

Fig. 1.5 Example extrasolar planetary system, highlighting the difficulty of resolving this kind of systems from the Earth due to their small angular separation.

stars by means of high-resolution Doppler spectroscopy. This latter method has actually been known for several decades, [40] but its enabling technologies did not become available until relatively recently [41].

Unsurprisingly, the percentage of exoplanets discovered by direct imaging is still very reduced (around 2% of all discovered exoplanets as of March 2017 [42]). There are two main reasons that explain the extreme difficulty of imaging planets out of our solar system, conveniently summarized in two words: resolution and contrast. The requirement of high resolution becomes evident if one thinks of a hypothetical Earth-like exoplanet orbiting a Sun-like star at a distance of one astronomical unit (AU) (i.e., the same distance that separates the Earth from the Sun). If such system was located in the vicinity of our Solar System at a distance of 10 parsec (32.6 light years), the angular separation of the two objects as seen from the Earth would be as low as 0.1 arcsec in the best case of the orbit (see Figure 1.5). When one compares this angular separation with the ≈ 0.05 arcsec resolution in the visible bands provided by the state-of-art Hubble Space Telescope (HST), it becomes clear that imaging exoplanets represents a particularly demanding challenge. As a matter of fact, resolutions comparable to that of the HST have only been obtained from ground telescopes by means of specific high resolution techniques such as adaptive optics or lucky imaging.

Besides providing high resolution, instruments willing to image Earth-like exoplanets must be capable of discerning between objects that have radically different apparent magnitudes. In other words, they must also provide high contrast. This idea may once again be illustrated with the example of Figure 1.5. Knowing the flux radiated by the Sun and reflectivity of the Earth, it can be calculated that the apparent magnitudes of a similar system as seen from 10 parsec would be around 5 and 30, respectively. The conversion of these magnitudes to a linear scale leads to the conclusion that there is a factor of 10^{10} in the relation between both fluxes. Of course, the sole detection of a planet as faint as magnitude 30 is a considerable challenge, but it becomes even more challenging if one considers the fact that even the diffraction pattern caused by the host star may completely hide the planet.

The COELI algorithm was the result of a collaboration between Universidad Politécnica de Cartagena (UPCT) and Universidad de Cantabria (UC) in the search of new methods to improve the resolution and contrast of astronomical images acquired from ground telescopes. Among other uses, the improvements in these two parameters would enable astronomers to identify fainter companions that are closer to their host stars, thus improving the current technology for direct detection of exoplanets.

There is one common characteristic in the images acquired by adaptive optics or lucky imaging that limits their capability to resolve faint and close companions. The fact that neither of these methods perfectly correct the effect of atmospheric turbulence results in the appearance of an halo around the target star(s) that may hide such companions, as will be explained later in Chapter 2. By applying a smart processing technique over a series of short exposure images, COELI is capable of removing that halo almost completely, thus revealing objects located at the minimum distance that the telescope can resolve, just as if it was not affected by the atmospheric turbulence. One downside of COELI, however, is that the obtained images are actually covariance maps, which from a scientific perspective means that they can be used for astrometry but not for astrophotometry.

Being an algorithm based on the concepts of lucky imaging, COELI shares some other limitations with that technique, such as the impossibility of applying it to telescopes larger than a given aperture size (refer to Section 2.2.3 for a complete explanation about this issue). On the other hand, just as AOLI implements adaptive optics to enable lucky imaging on large telescopes, the same principle would apply to the COELI algorithm. With COELI already tested and characterized at the 2.2m telescope in Calar Alto Observatory (Spain), further improvements in performance are expected when COELI is executed over the images produced by AOLI at the 4.2m William Herschel Telescope (WHT), whose larger aperture size would enable the detection of fainter objects at closer distances.

Chapter 2

State of art

This chapter contains a background study about the two main fields that define the present dissertation: the optimization of the synthesis of EDAC cores in FPGA devices (Section 2.1) and the acquisition of high-resolution images from ground-based telescopes (Section 2.2).

2.1 Synthesis of EDAC cores in FPGA

Before delving into the problem of optimizing the implementation of EDAC cores in FPGAs, this section will first present the techniques that are most commonly used to protect the electronic circuits from the harmful ionizing radiation. After that, the state of art of efficient EDAC implementations will be identified and analyzed in comparison to the algorithms that represent the contribution of the present dissertation on this field, described later in Section 3.

2.1.1 Radiation-hardening techniques

At ground level, life beings and circuits are protected by the Earth's magnetosphere, which deflects away the harmful high-energy particles that are generated all across the Universe, from solar wind to galactic cosmic rays. In contrast, the ionizing radiation present in the space environment is a major concern during the design of integrated circuits, which may suffer from temporal or permanent faults unless proper mitigation measures are implemented.

Ionizing radiation degrades the performance of transistors by altering their threshold voltages and leakage currents, up to a point where one or more transistors enter a permanent open or closed state, thus leading to device failure. The amount of accumulated radiation that a devices can tolerate before becoming permanently damaged is called the Total Ionizing Dose (TID). Besides this low-paced degradation, radiation can also produce unwanted effects at specific moments in time as a result of single high-energy particles hitting the

circuits. These effects fall under the category of Single-Event Effects (SEE), a category which includes reversible failures such as Single-Event Upsets (SEU) and destructive failures such as Single-Event Burnouts (SEB).

Radiation tolerance is a requirement for any space mission, as well for projects operating in other radiative environments at ground level such as the vicinity of radiology equipment, nuclear reactors, or particle accelerators. The strategies for increasing the resilience of integrated circuits to radiation may be classified as follows:

- Radiation Hardening By Process (RHBP): involves modifying the fabrication process.
- Radiation Hardening By Design (RHBD): involves special design techniques.

One classical example of RHBP is to increase the size of the transistors. Large gates have higher drive capabilities and also higher node capacitances, which increases their immunity to radiation when compared to smaller gates. The use of silicon on sapphire (SOS) substrates instead of the regular silicon ones can prevent stray currents caused by radiation from spreading to nearby circuit elements. It is also typical to use a shielding material for the device package, thus adding an extra layer that reduces the intensity of the radiation over the semiconductor die up to some extent [43–45].

Unfortunately, RHBP alone is not enough to provide an adequate protection against SEU [46]. A SEU consists on the alteration of the value stored in one memory element of a digital circuit (flip-flops, RAM cells) caused by the hit of high-energy particles such as alpha particles, protons and ions. The corruption of memories on-board a spacecraft can result in critical or even catastrophic consequences if not handled properly. In order to mitigate SEU, RHBD needs to be applied in addition to RHBP.

SEU protection in digital circuits is accomplished by combining various redundancy techniques at different levels. At the lowest level, it is common for all flip-flops to implement Triple Modular Redundancy (TMR), as shown in Figure 2.1. If the bit stored in one of the three flip-flops gets corrupted because of SEU, the voting logic will still provide the correct value as indicated by the other two unaltered flip-flops. It is thus common for basic “SEU-hardened flip-flop” primitives to be part of radiation-hardened component libraries [47, 48]. The same TMR principle can be applied at a global level by replicating large portions of sequential and combinational logic. While being an effective method for improving the reliability of digital circuits, the downside of TMR is that has a considerable impact on area occupation, timing performance and power consumption.

In the case of large memory areas such as RAMs, the preferred method for SEU protection is based on Error Protection and Detection (EDAC) functions. For each data word stored in a RAM memory, a corresponding checksum calculated by an EDAC encoder is stored

Fig. 2.1 TMR-based SEU-hardened flip-flop.

along, which is a much more efficient method than triplicating every stored data word as in TMR. When the data needs to be recovered, an EDAC decoder reads the previously stored codeword (data + checksum) and tries to provide the piece of data that was originally stored, which may have suffered from undesired bit alterations as a result of SEU. Depending on the mathematical properties of the chosen EDAC function, the decoder may be able to detect and/or correct a given number of errors in a single piece of read data.

The process of storing/retrieving information to/from an EDAC-protected RAM is illustrated in Figure 2.2. An EDAC-protected RAM would require one EDAC encoder for each write port and one EDAC decoder for each read port. In the case a shared read/write port, it is convenient to implement the EDAC encoder and decoder as a single “EDAC codec” block. By looking at the defining equations of EDAC codes, it is easy to realize that the core functions of the EDAC encoder and decoders are practically the same, thus making it possible to merge them in a single block that implements the common equations only once.

There are several types of EDAC codes proposed in the literature, each with different properties and applications. In the context of space applications, the most common ones are the Reed-Solomon (RS) codes [49], the Bose-Chaudhuri-Hocquenghem (BCH) codes [50] and the SEC-DEC Hamming codes [51]. The present dissertation focuses on SEC-DEC Hamming codes, which have been traditionally used to protect volatile memories from SEU [52–54]. The rest of codes (RS, BCH) are more typically oriented for satellite communications [55–57].

Fig. 2.2 Read and write accesses in an EDAC-protected RAM.

2.1.2 Efficient EDAC implementations in scientific publications

One of the objectives of the present dissertation is to develop new FPGA synthesis algorithms specialized in the specific case of SEC-DEC Hamming EDAC functions. The ultimate goal is to improve the reliability of the implemented EDAC cores themselves under a radiative environment and consequently of the elements that are protected by those cores.

Previous works in the literature had already studied the problem of optimizing the implementation of EDAC functions, but in the field of ASIC devices [58–60]. Clearly, the assumptions made for ASIC devices cannot be applied directly in FPGAs. Despite of being reprogrammable, the architecture of a specific FPGA model is fixed, making it impossible to design custom cells at layout level as would happen with ASICs. From the point of view of a synthesis algorithm, the underlying technology determines the nature of the optimization parameters. In the case of FPGA devices, parameters such as the area utilization is measured as the number of utilized look-up tables (LUTs) and flip-flops, rather than the number of transistors. As opposed to ASICs, the area and performance cost of implementing any logic function of up to K inputs is virtually the same for a K -input LUT FPGA.

Other works in the literature do explore the problem of optimizing the implementation of EDAC cores in FPGAs [61, 62], but only for RS codes, which are not the subject of the present dissertation. In any case, previous works do not always provide results of all the optimization parameters analysed here, namely the area utilization, minimum path delay, power consumption and reliability.

Given the non-existence of FPGA synthesis algorithms specialized in SEC-DEC Hamming EDAC functions, the latest versions of a set of commercially available synthesis tools were considered to represent the state-of-art of the proposed research topic.

2.1.3 Optimality of commercially-available synthesis tools

The studies about the state-of-art of the synthesis of EDAC functions were performed over the following commercial synthesis tools: Xilinx Vivado 2014.2 (for Xilinx Series 7 FPGAs), Xilinx XST 14.7 (for other Xilinx FPGAs), Mentor Graphics Precision RTL 2014.09 and Synopsys Synplify 2013.03.

The first tests demonstrated that commercial synthesis algorithms have a sub-optimal performance even for the simplest case of trying to synthesize just one of the outputs of an EDAC decoder. To reach this conclusion, the commercial synthesis tools were made to process a n -input XOR gate expressed as a simple VHDL statement, varying n from 2 to 500 in each run. Then, the results were compared with the MISO (Many-Input, Single-Output) algorithm, a custom synthesis algorithm developed by the author of this dissertation, capable of providing an optimal solution for a given n -input logic gate in terms of number of inferred LUTs and maximum level (i.e., maximum number of LUTs that a path goes through) [63].

The results of these initial tests showed that the MISO algorithm always provided equal or better results than the rest of synthesis methods for any n , both for the number of inferred LUTs and the maximum level of the resulting network. Precision RTL was the commercial synthesis tool that performed best, and still, it provided suboptimal results in 19% of the cases. From the results it was also discovered that the use of architecture-dependent strategies such as the combination of two K -input LUTs to form a single LUT with $K + 1$ inputs was actually unsuitable for the specific application of the many-input logic gates.

Further tests were executed and later published in [63], this time focusing on applying the MISO algorithm to actual EDAC decoders instead of single many-input gates. The tests were performed in two models of radiation-hardened FPGAs, one from the RT ProASIC3 series (flash-based) and another from the RTAX-S series (antifuse-based). Despite the fact that MISO needs to process each output of the EDAC function independently, as opposed to commercial synthesis tools, the results showed that the circuits generated by MISO use fewer combinational cells (up to 23.5% less) and generally deliver better timing performances (up to 23.6% faster maximum path delays). What is more, these studies confirmed that the use of combined LUTs was unsuitable not only for the case of the many-input logic gates but also for the case of full EDAC codecs.

The results of the initial studies that have just been described motivated the development of new synthesis algorithms more sophisticated than MISO that could process several outputs in a joint manner, that is, following a Many-Input, Multiple-Output (MIMO) approach. The resulting algorithms, named FS-EDAC (Full Search EDAC) and G-EDAC (Greedy EDAC), represent the main contributions of the present dissertation in the field of FPGA synthesis algorithms specialized in EDAC codecs [64].

2.2 High-resolution imaging from ground telescopes

From the first telescopes in the XVII Century to the latest and largest ground-based telescopes, astronomers have always suffered the limitations imposed by the Earth's atmosphere. On the one hand, the blurring effect caused by the random aberrations introduced by the always-changing atmosphere (an effect known as *seeing*) limits the angular resolution to a value much lower than the theoretical limit of the telescope itself (i.e., the diffraction limit). On the other hand, our atmosphere absorbs many parts of the electromagnetic spectrum, which makes it very difficult or even impossible to observe at some specific wavelengths from the surface of the Earth.

Ever since the launch of the first artificial satellites in the 1960s, a number of space telescopes operating in different wavelength ranges have been put into orbit with the objective of overcoming the limitations that the atmosphere imposes on ground. One of the most notable examples is the 2.4-meter Hubble Space Telescope (HST), launched in 1990 and still providing images with extremely high angular resolution in the infrared, optical and ultraviolet bands, only limited by the optics of the telescope itself.

During the last decades a lot of effort has been put in developing the technologies that would enable ground-based telescopes to achieve angular resolutions comparable to those that can be achieved from space in absence of the atmosphere. This would mean a considerable technological advance, as ground-based telescopes would then gain one of the main advantages of space-based telescopes (high resolution) while keeping their own inherent qualities (lower cost, easier to maintain and upgrade).

As an example of the margin for improvement that would be available for high-resolution techniques, let us consider the case of the 4.2-meter William Herschel Telescope (WHT) in Roque de los Muchachos Observatory (Spain), which is the telescope for which the AOLI instrument is initially targeted. Following Rayleigh's criterion for determining the resolving power of a circular aperture [65], the diffraction limit of a telescope can be approximated as:

$$\Delta\alpha = 1.22 \frac{\lambda}{D} \quad (2.1)$$

It can be seen from Equation 2.1 that the maximum theoretical resolution of a telescope ($\Delta\alpha$) depends on the wavelength that is being observed (λ) and the diameter of the aperture (D). Thus, particularizing for the case of the WHT ($D = 4.2$ m) operating in the *I* band ($\lambda = 806$ nm), the diffraction limit would be around 0.05 arcsec. The maximum angular resolution would be limited in practice by the typical diameter of the seeing disk (long exposure image of a star as seen through the turbulent atmosphere), which rarely goes below 0.5 arcsec even in the best observing sites on Earth, including the Roque de los Muchachos

Fig. 2.3 Effect of atmospheric turbulence on the image of a star, as seen by AOLI at the WHT. Random speckle interference patterns (a) appear when the exposure time is shorter than the speed of variation of the atmosphere. Longer exposure times cause the appearance of the seeing disk (b), whose width may hide high-resolution details such as the existence of multiple systems with low angular separation. The appearance of a diffraction-limited image (c) was obtained experimentally by means of the instrument calibration source.

Observatory [66]. Therefore, a high-resolution technique that leads to a diffraction-limited image would improve the angular resolution in a factor greater than 10 for the particular case of the WHT (see comparison in Figure 2.3). Following Equation 2.1, the margin for improvement would be larger for the case of bigger telescopes or shorter wavelengths.

In the last century, three main techniques have been developed to counteract the effects of atmospheric turbulence in order to achieve high-resolution imaging in ground-based astronomy, namely: speckle interferometry, adaptive optics (AO) and lucky imaging (LI). These techniques are further explained in the following subsections, with a special focus on AO and LI, which are essential parts of either AOLI or COELI.

2.2.1 Speckle interferometry

Speckle interferometry derives high resolution information of astronomical objects from series of short-exposure images containing speckle patterns. Such interference patterns appear on the images of point sources as random clouds of small dots when the exposure time is low enough to freeze the effect of the turbulent atmosphere (see Figure 2.3a). Indeed, the speckle patterns contain the information of the original objects, though convolved by the transfer function of the atmosphere at the time each acquisition. Speckle interferometry exploits the statistical properties of the speckle images and recovers information of the original objects by performing operations in the frequency domain.

The first speckle interferometry techniques only recovered auto-correlation maps [67], but further studies yielded to the recovery of true diffraction-limited images with the help of

aperture masks [68]. This latter technique, generally referred to as *aperture masking*, is still providing results in astronomical research despite its age [69]. The fact of blocking part of the incoming light, however, means that only relatively bright stars can be observed. What is more, the speckle patterns of faint stars usually have a low signal-to-noise ratio because the light of the original star is distributed over a relatively wide area in the image sensor.

2.2.2 Adaptive optics

Adaptive optics (AO) consists on continuously actuating over a deformable mirror (DM) in such a way that the distortions of the incident wavefront, measured by a wavefront sensor (WFS), are corrected. Diffraction limited images can be obtained at the image plane with no further processing by performing high-precision AO at a faster rate than the speed of variation of the atmosphere. This fact represents an important improvement with respect to speckle interferometry, as the incoming light is better concentrated in the image plane with AO, thus enabling to do science over fainter astronomical objects.

Implementation

Figure 2.4 depicts the diagram of a generic AO system. The light of the astronomical object under interest travels a very long distance through space and arrives to the atmosphere as a planar wavefront. Then, the wavefront is distorted as a result of its pass through the turbulent atmosphere, finally reaching the telescope aperture. At this point, the AO system comes into action. The light of the AO reference object, which may be the object of interest itself or another star within its isoplanatic area (that is, the area in the sky in which wavefront aberrations are approximately equal) enters the WFS and the shape of the incident wavefront is estimated. Using this information, the control algorithm (implemented in a general purpose computer or as a custom device) commands the DM in such a way that the distortions that had been introduced by the atmosphere are undone, thus producing a high-quality image at the scientific instrument placed at the output of the AO system. This process is done continuously so that in nominal closed-loop operation the control algorithm just corrects small deviations from the ideal wavefront as read by the WFS.

Challenges and limitations

Although the idea behind AO may appear conceptually simple, this high-resolution technique actually poses a number of technical challenges that make it very difficult to reach the diffraction limit under certain circumstances. To begin with, telescopes with larger apertures are theoretically capable of achieving higher resolutions, but the components of the adaptive

Fig. 2.4 Basic components of an adaptive optics system (further explanations in the text)

optics system need to be dimensioned accordingly. A large telescope aperture means that the WFS needs to sample the incoming wavefronts with better spatial resolution so that the higher-order aberrations present in the aperture can be detected and later corrected by a DM with a sufficient number of actuators [70]. While the current state-of-art AO systems for existing telescopes use several hundreds of wavefront samples and actuators [71, 72], the next generation telescopes are expected to require several thousands of them [73, 74]. This does not only require the development of newer DMs with more actuators, but also further improvements in the field of wavefront sensing.

Wavefront sensing at high spatial resolutions means to divide the incoming light in a greater area over the image sensor, which sets a limit to the magnitude of the star that can be used to sense the incoming wavefront [75]. This fact severely affects the sky coverage of an AO-enabled instrument because it is not always possible to find a bright AO reference star sufficiently close to the object of interest. Fortunately, this problem can be effectively addressed by projecting an artificial guide star in high layers of the atmosphere with a laser, although sub-optimal performance can be expected due to the fact that the laser beam follows a different path through the atmosphere than the one that a natural star would follow [76–78].

In addition to the issues related to the spatial domain, temporal aspects of atmospheric turbulence also have a big impact on the design of AO systems. An optimal correction can be achieved when the rate of the real-time control loop is faster than the speed of variation of the atmosphere, commonly known as the Greenwood frequency (f_G) [79]. However, the fact that this frequency scales with the wavelength ($f_G \propto \lambda^{-6/5}$) makes it very difficult to perform high-quality AO in the visible wavelengths. Indeed, adaptive optics has enabled astronomers to reach the diffraction limit of ground-based telescopes in the infrared bands for more than twenty years [80, 81], but comparable performances have not been achieved in the visible bands until very recently [82].

In relation to these challenges and limitations, AOLI expects to deliver diffraction-limited images in the visible bands by combining a low-order adaptive optics system with the high-order resolving power of lucky imaging [83]. None of the two parts alone could provide the desired performance in large ground-based telescopes, but their joint use has proven to be effective in recent, yet preliminary on-sky tests [84, 85].

2.2.3 Lucky imaging

Similarly to speckle interferometry techniques, the input of the Lucky Imaging (LI) algorithm is a series of short-exposure images containing speckle patterns, but the operations follow a completely different approach. In LI, the images that suffered less atmospheric distortion (called *lucky images*) are detected, then re-centered and added together into a single image. This technique has proven to be a simple, cost-effective way to reach the diffraction limit in the visible bands at mid-sized telescopes (up to 2.5m), with several astronomical instruments implementing it in the past few years [25, 86, 87]. The effect of applying LI over an example series of short-exposure images taken at the 2.4m Nordic Optical Telescope (NOT) is illustrated in Figure 2.5.

Implementation

The identification of the lucky images among a series of short-exposure images (commonly referred to as a *cube* or *stack*) must be done according to some quality criterion. Images whose quality are below a defined threshold are discarded and no longer processed. In the case of punctual targets like stars, the most common quality estimator is the peak brightness of each image [88], though recent studies suggest that performing the frame selection in the frequency domain may provide a better performance [89]. When the LI reference is an extended object such as a planet, the quality estimators are based on other parameters such as image contrast [90] or gradient-based sharpness measurements [91].

Fig. 2.5 Result of post-processing a stack of 1000 short-exposure images of GG Tau A taken at the NOT (x axis: pixel position along profile, y axis: pixel value). The direct accumulation of all the images of the stack gives a seeing-limited image (a). Aligning the images as a previous step to accumulation reveals a close companion (b). The sharpest output images are obtained by applying the full LI technique (c) (images provided by Sergio Velasco).

Regarding the alignment of the images, the method once again depends on the nature of the object being observed. Punctual objects are typically re-centered with respect to the brightest pixel in each image in the stack [88]. This simple technique has proven to be more effective than other more complex ones such as centroid-based alignment [92]. When there are several stars of similar magnitude in the field, a limited search region must be set with care so as to ensure that the same reference star is always used throughout the whole stack. The method of the brightest pixel, however, cannot be applied to extended objects because such pixel may appear at completely unrelated positions over the surface of the object in each image of the stack. In these cases, other techniques such as the correlation with respect to a reference image or region may be used instead [91].

As a previous step to the alignment process, it is highly recommendable to upsample the input images so as to achieve sub-pixel precision, which ultimately provides an increase of resolution at the output of the LI algorithm of about 5% [93]. The need of sub-pixel precision is justified by the fact that the atmosphere produces random, non-integer displacements over the pixel array. This preprocessing stage may be applied using common upscaling methods for astronomy such as the Drizzle algorithm [94].

Finally, it is worth mentioning that, even though the LI algorithm is intended to be executed off-line as part of the image reduction pipeline, a real-time preview of the output of the LI algorithm may be helpful for the operator of a LI-enabled instrument. The frame grabbing software of AOLI, for example, displays the result of applying LI to the best recent images. Other instruments like FastCam opted for implementing the real-time previews on custom hardware [95].

Challenges and limitations

The principles of LI have been known for a long time [96], but it was not until the availability of fast, very low noise cameras that this technique started to be applicable to astrophysics [97]. The requirement of short exposure times is set by the speed of variation of the atmosphere itself, which determines how long the speckle patterns remain correlated over time. For example, LI in the *I*-band (806 nm) would require exposure times around 30 ms [98]. The observation of faint astronomical objects with sufficient signal-to-noise ratio at these short exposure times is only possible with very low noise image detectors such as electron-multiplying CCDs (EMCCDs). Common CCDs are not suitable for this application because their lower signal-to-noise ratio would set an important limitation to the sky coverage of a LI-enabled instrument [88].

Though it is indeed possible to reach the diffraction limit of the telescope with LI, the required number of input images becomes unfeasibly high for large telescopes. The explanation that supports this statement can be found on the following formula, which estimates the probability of obtaining a single lucky image [96]:

$$P \approx 5.6 \exp \left[-0.1557 \left(\frac{D}{r_0} \right)^2 \right] \quad (2.2)$$

where D is the size of the aperture and r_0 is the coherence length, a value that depends on the quality of the observing site and on the observing wavelength ($r_0 \propto \lambda^{6/5}$). This estimation, assumed to be valid for $D/r_0 \geq 3.5$, considers that an image is lucky when it was formed with a squared wavefront distortion over the aperture of $\sigma^2 \leq 1 \text{ rad}^2$.

It is known that good observing sites have typical values of $r_0 = 0.2$ to 0.4 m in the *I*-band [99]. Taking $r_0 = 0.35$ m, the probability of getting a lucky image in a 2.5-meter telescope such as the Nordic Optical Telescope (NOT) would be around $P = 1/500$, or equivalently, one lucky image every 15 seconds, assuming an exposure time of 30 ms. In spite of having to discard a large quantity of frames, it would still be feasible to accumulate a sufficiently high number of frames in order to get a good signal-to-noise ratio at the output of the LI

Fig. 2.6 Profile of a partially corrected image of a star, showing a diffraction-limited central core and the residual halo corresponding to the atmospheric turbulence that could not be corrected. This profile has been obtained by simulation for $\lambda = 806$ nm, $D = 2.5$ m and $r_0 = 0.35$ m, and is representative of the typical images that would be given by AO or LI.

algorithm, depending on the magnitude of the object under study. On the other hand, for the case of a large telescope like the 4.2m William Herschel Telescope (WHT), the probability becomes as low as $P = 1 \times 10^{-9}$, which would make the use of LI completely impractical.

Another limitation of LI, shared also by AO, is that the resulting images of punctual objects possess a diffuse halo of diameter $\approx \lambda/r_0$ around the central core(s), corresponding to the time average of the residual speckle patterns [98, 100] (see example in Figure 2.6). Even if the central cores are diffraction limited (diameter $\approx \lambda/D$), the surrounding halo can pose a problem in the field of high-contrast imaging (e.g., the study of brown dwarfs or exoplanets), as it may hide close companions that are comparatively faint with respect to the host star. Given that the flux contained in the residual halo increases with the wavefront distortion σ^2 [101], the detectability of faint companions may be improved in the case of LI by selecting a low percentage of images (that is, the ones with lower σ^2). Unfortunately, the presence of the atmosphere makes it extremely unlikely to obtain perfect images with no residual speckles ($\sigma^2 \approx 0$) when D is several times higher than r_0 [98], meaning that the halo will always appear on mid-size and large telescopes.

The AOLI instrument and the COELI algorithm were designed to overcome some of the limitations of standalone LI. The use of low-order AO in AOLI would effectively reduce the wavefront error at the aperture (σ^2), which would increase the probability of getting a lucky image and thus may enable the application of LI to large telescopes. On the other hand, COELI improves the detectability of faint objects falling inside the residual halo by analyzing the correlation between the peak pixel and the rest of the pixels of each lucky image. This method takes advantage of the fact that the intensities of astronomical objects over the image plane vary in phase, as opposed to the surrounding halos, which vary in counter phase.

Chapter 3

Efficient synthesis algorithms for EDAC functions in FPGAs

This chapter presents a summary of the first article that is part of the present thesis by publications, which has the following reference:

C. Colodro-Conde and R. Toledo-Moreo. “Design and analysis of efficient synthesis algorithms for EDAC functions in FPGAs”. In: *IEEE Transactions on Aerospace and Electronic Systems* 51.4 (Oct. 2015), pp. 3332–3347.

This work is devoted to the development of new synthesis algorithms for FPGAs that are specialized in the case of Hamming SEC-DED EDAC codecs. The objective to generate circuits with better figures of merit than the ones produced by commercial synthesis tools, which represent the current state of art as explained in Section 2.1. This objective was motivated by the results of the preliminary studies presented in Section 2.1.3, which indicated that there was room of improvement in the synthesis of EDAC codecs, mainly due to the fact that commercial synthesis tools are focused on providing high flexibility, sometimes at the expense of optimality. Particularly, the figures of merit to be optimized by the proposed algorithms are the following post place and route parameters:

- **Area occupation**, measured as number of used LUTs.
- **Maximum speed**, measured as the inverse of the maximum path delay.
- **Total power consumption**, estimated by means of vendor-specific tools.
- **Mean Time Between Failures (MTBF)**, known to be proportional to the number of sensitive bits of the resulting FPGA programming files (i.e., bits that are associated to the circuitry of the design).

At the synthesis stage it is not possible to obtain a direct estimation of all the parameters above. As a matter of fact, only the number of used LUTs can be determined during synthesis. Therefore, the proposed algorithms do not directly optimize the post place and route parameters listed above, but other characteristics of the synthesised circuits that are known to be tightly related to them.

- **Number of used LUTs:** not only determines the area occupation, but also affects the circuit speed (fewer LUTs facilitate the place and route process), the power consumption (every LUT consumes energy) and the MTBF (the memory that defines a LUT is actually a set of sensitive bits).
- **Maximum level,** where the *level* of a path is understood as the number of LUTs that it goes through. This parameter affects the circuit speed because each LUTs adds a certain delay to the path, as well as the nets used to connect two consecutive LUTs.
- **Number of nets:** having fewer nets simplifies the routing process, which usually results in shorter delays. In addition, each net has an associated static and dynamic power consumption, depending on its length. The number of nets also affects the MTBF because the active FPGA switching matrices are encoded with sensitive bits.
- **Maximum fan-out:** mainly affects the circuits speed, because high fan-outs result in longer delays when charging the input capacitances of the LUTs where the output of other LUTs are connected to.

Given that not all the parameters can be optimized at the same time, the proposed algorithms establish a priority mechanism that proved to be effective for optimizing the post place and route figures of merit that were listed before.

The article presents three new synthesis algorithms:

- **Backtracking version of FS-EDAC (Full Search EDAC):** this is a straightforward implementation of a full search of the whole space of solutions that implement a given EDAC function in a K -input LUT FPGA. Though useful during the initial stages of the research, it is not practical to apply this algorithm even to medium-sized EDAC functions (e.g., EDAC for 24-bit data words) because of its long execution time.
- **Dynamic programming version of FS-EDAC:** this version is functionally equivalent to the backtracking one, but significantly faster to execute in spite of also exploring the whole space of solutions. Every step of the algorithm is accelerated by means of dynamic programming techniques, which basically consist in performing the algorithm

steps in an incremental way, without recalculating everything from scratch with each recursive iteration. This improved version makes it viable to process medium-sized EDAC functions, but not the largest ones (e.g., EDAC for 64-bit data words).

- **G-EDAC (Greedy EDAC):** this algorithm represents the most valuable contribution of the paper, as it allows obtaining quasi-optimal results with fast execution times, even for the largest designs. Instead of searching and analysing all the possible solutions, G-EDAC only explores a small subset of the space of solutions, iteratively selecting only the ones that apparently help to produce a better final network.

All the algorithms are based on manipulating an input matrix that defines the target EDAC function, producing a candidate output netlist in each iteration. An iteration consists on grouping one of the available common terms into a new LUT. A common term is defined as a group of two or more terms in one output that are shared at least with another output. Every generated netlist is evaluated and compared to the netlist that is currently considered the best one according to the defined optimization criteria. If the netlist under study is found to be better than the one currently considered the best, then the old solution is discarded and replaced with the current one. The algorithms end once all the networks of interest have been evaluated (every possible network in the case of FS-EDAC).

After fully describing the three algorithms, with all the related equations and some illustrative examples, the paper presents a detailed breakdown of the execution time of the algorithms. The obtained timing results confirm that G-EDAC is much faster to execute than the dynamic version of FS-EDAC, which in turn is faster to execute than the backtracking version. Even for a small 10-bit EDAC function, which represents an advantageous case for the slower algorithms, G-EDAC performs > 64 times faster than the dynamic version of FS-EDAC. The factor of improvement obtained with this latter algorithm with respect to the backtracking version is $\times 5.321$ for the case under study.

Finally, the paper introduces the hardware implementation results, comparing the proposed algorithms with three commercially available synthesis tools, choosing different FPGA architectures and target EDAC functions. These results indicate that the proposed algorithms outperform the commercial synthesis tools in all the evaluated parameters and cases, both at synthesis time and after place and route. The improvements obtained for the area occupation, maximum speed, total power consumption and MTBF after place and route were as high as 34.48%, 17.72%, 34.37% and 10.91% respectively in some of the tested cases. These numbers clearly indicate a significant advance in the state of art of FPGA synthesis algorithms applied to EDAC functions.

Design and Analysis of Efficient Synthesis Algorithms for EDAC Functions in FPGAs

Carlos Colodro-Conde, Rafael Toledo-Moreo

Abstract—Error Detection and Correction (EDAC) functions have been widely used for protecting memories from single event upsets (SEU), which occur in environments with high levels of radiation or in deep submicron manufacturing technologies. This paper presents three novel synthesis algorithms that obtain area-efficient implementations for a given EDAC function, with the ultimate aim of reducing the number of sensitive configuration bits in SRAM-based Field-Programmable Gate Arrays (FPGAs). Having less sensitive bits results in a lower chance of suffering a SEU in the EDAC circuitry, thus improving the overall reliability of the whole system. Besides minimizing area, the proposed algorithms also focus on improving other figures of merit like circuit speed and power consumption. The executed benchmarks show that, when compared to other modern synthesis tools, the proposed algorithms can reduce the number of utilized look-up tables (LUTs) up to a 34.48%. Such large reductions in area usage ultimately result in reliability improvements over 10% for the implemented EDAC cores, measured as MTBF (Mean Time Between Failures). On the other hand, maximum path delays and power consumptions can be reduced up to a 17.72% and 34.37% respectively on the placed and routed designs.

Index Terms—field programmable gate arrays (FPGA), synthesis tools, efficient synthesis, error detection and correction (EDAC), single event upsets (SEU)

I. INTRODUCTION

WITH the continuous downscaling of the VLSI fabrication technologies, radiation induced errors have become a major concern in modern digital electronics. Even at ground level, high-energy particles like neutrons coming from the cosmic background create undesired current pulses that may invert the value stored in a memory element such as a flip-flop [8], [18], [32]. This kind of errors, called single event upsets (SEU), compromise the reliability of the systems if no action is taken to mitigate them. In the space environment, outside the protection of the magnetosphere of the Earth, SEUs become a critical concern because of the high radiation levels. SEUs can have serious consequences for the spacecraft, including loss of information, functional failure or loss of control [5].

Due to their flexibility and high computing capability, Field-Programmable Gate Arrays (FPGAs) are nowadays widely used to implement digital systems. In aerospace applications, antifuse FPGAs have been traditionally preferred due to their high tolerance to radiation [39]. Nevertheless, there is a growing interest in using SRAM-based FPGAs in space embedded systems because of their lower costs, higher performances and in-flight reconfigurability [28], [36], [43].

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SRAM-based FPGAs are specially sensitive to radiation because of the fact that the configuration memory that defines the circuitry implemented inside them is vulnerable to SEUs. Regarding this matter, Hamming EDAC (Error Detection And Correction) cores with SEC-DED (Single-Error-Correction, Double-Error-Detection) capabilities have proven to be an effective way to protect the internal configuration memory [1], [17], [25], [29], [30], as well as other external volatile memories [6], [23], [24]. However, the fact of such cores being placed inside the FPGA makes them less reliable, as they can also be affected by SEUs in configuration memory.

The aim of this work is to develop new synthesis algorithms capable of minimizing the area utilization of SEC-DED EDAC functions in FPGAs without affecting negatively other parameters like processing speed and power consumption. A reduction in area means a lower number of sensitive configuration bits (i.e., bits which are associated with the circuitry of the design), and therefore more reliability of the EDAC cores themselves when they are implemented in SRAM-based FPGAs. This statement is justified by the fact that, under the same radiation environment (modelled by the particle flux) and FPGA model (which determines the area cross-section per bit), the *Mean Time Between Failures* (MTBF) is directly proportional to the number of sensitive bits of the design [22], [41], [42]. The EDAC cores can also get an increase in reliability when implemented in other FPGA technologies (e.g., flash-based), as every instantiated component has an intrinsic rate of failure [26]. On the other hand, having higher processing speeds not only has a positive impact on memory throughput, but also on the reliability of the whole system, as higher scrubbing rates would be possible [7], [38]. Efficient implementations are desirable from an economic perspective as well because they allow choosing smaller FPGAs with lower speed grades.

The algorithms presented in this paper achieve a significant reduction in area, maximum path delay and power consumption figures of the implemented circuits (up to 34.48%, 17.72% and 34.37%, respectively) when compared to the ones obtained by means of commercial synthesis tools. The resulting increase of the MTBF (up to 10.91%) gives an extra protection to these critical functions, which can be easily combined with other well-known techniques like TMR for further increasing the immunity to SEU.

The paper is structured as follows. Section II presents the related works. Section III briefly explains the theory behind EDAC codes. Section IV describes the proposed algorithms formally. Section V compares the performance of the developed methods with other modern synthesis tools. Finally, Section VI draws the main conclusions.

II. RELATED WORKS

Most of the previous work about optimizing the implementation of SEC-DED EDAC has been aimed to ASIC (Application-Specific Integrated Circuit) devices. The optimization goal is usually the number of transistors, although the speed of the resulting circuit is also considered in some cases [4], [23], [24]. In FPGA devices, as opposed to ASICs, one does not have the freedom to create custom cells at layout level, so the optimization cannot be done following this approach. Another difference is that the main parameters that determine area utilization in FPGAs are usually the number of utilized LUTs and flip-flops, rather than the number of transistors. In a 6-input LUT FPGA, the hardware cost of instantiating any logic function up to 6 inputs is virtually the same, while in ASICs the number of inputs of a gate makes a great difference. Because of these reasons, specific optimization methods were developed for this paper, with the focus on FPGA devices.

Other works in the literature do focus on optimizing the implementation of EDAC codes in FPGAs, but not on the kind of EDAC codes that will be discussed in our paper. For example, [3] introduces an alternative implementation of Reed-Solomon (RS) codes [40] with reduced area utilization, in the context of satellite communications. Despite mentioning the importance of achieving high processing speeds and low power consumption in space applications, the authors of that work do not include results about these two parameters.

There is another work [37] whose main aim is to obtain small and fast implementations of RS codes in FPGAs by choosing the most suitable generator polynomials and multiplication constants. In that work, the optimized RS codes are intended to be used in combination with SEC-DED Hamming codes for protecting memories from SEU, but the optimization of the latter part is not considered. The SEC-DEC Hamming codes are just the ones that the present paper will focus its interest on.

In addition to the novelty of optimizing the implementation of SEC-DEC Hamming codes on FPGAs, the present work proposes new custom synthesis algorithms as an alternative way to obtain such optimized implementations. The analyses and in-depth descriptions presented in the paper allow to obtain a comprehensive knowledge of the problem of synthesizing EDAC functions efficiently from a hardware, a mathematical and a computational perspective.

In the literature on the field, the synthesis process for FPGAs has been traditionally divided in two stages: logic optimization [11], [14] and technology mapping [9], [16]. Recent research on this subject has been focused on combining both stages in order to provide better results [10], [35], although according to [15], [31] there is still much room for improvement. The proposed algorithms exploit this latter idea to its fullest, making no distinction between logic optimization and technology mapping stages.

In contrast to most of the synthesis tools and algorithms, which provide high flexibility at the expense of optimality, this paper focuses on a particular type of problem and tries to find the best possible solution for a given optimization criterion,

which can be configured as desired. This strategy will allow a significant reduction in area, maximum path delay and power consumption of the implemented circuits when compared to the ones obtained by means of commercial synthesis tools, which represent the state of art of FPGA synthesis algorithms. Additionally, the obtained solutions can be used to measure the quality of faster, heuristic approaches. In particular, this work presents a greedy algorithm whose use is strongly justified in spite of not being completely optimal, as it has viable execution times even for the largest designs.

III. SEC-DED EDAC CODES

The present study will focus on the odd-weight column EDAC codes proposed in [26]. These codes are extensively used in many applications as a result of their SEC-DED (Single-Error-Correction, Double Error Detection) capabilities and their relatively low hardware needs [4], [12], [27].

According to coding theory [21], a SEC-DED EDAC code can be defined via its parity-check matrix $\mathbf{H}_{r \times n} = \{h_{ij}\}$. This matrix has r rows and $n = r + k$ columns, being r the number of parity bits, k the number of data bits and n the codeword length. The codeword $\mathbf{m} = \{m_i\}$ is formed by concatenating the input data bits $\mathbf{d} = \{d_i\}$ and the calculated check bits $\mathbf{c} = \{c_i\}$, and it is the one that is actually stored in memory for protecting the original data.

For calculating the bit i ($1 \leq i \leq r$) of the check bits $\mathbf{c}$, one has to take the row i from the $\mathbf{H}$ matrix and check the positions where $h_{ij} = 1$, with $1 \leq j \leq k$. These positions indicate the elements of the data bits vector $\mathbf{d}$ that have to be XOR'ed between themselves in order to obtain c_i .

In most applications, the codeword $\mathbf{m}$ is saved in a memory which may suffer from undesired bit alterations caused by SEU. When the data needs to be recovered, the syndrome vector $\mathbf{s} = \{s_i\}$ has to be calculated in order to know if the codeword has been altered. With SEC-DEC codes, the syndrome vector can be used to spot the location of single-bit errors so that they can be corrected with a bit flip. If the obtained syndrome is equal to $\mathbf{0}$, it means that the retrieved codeword is the same as the one that was originally saved. If the syndrome is not equal to $\mathbf{0}$ and the parity of the syndrome is even, a double-bit error flag can be raised.

For calculating the bit i ($1 \leq i \leq r$) of the syndrome vector $\mathbf{s}$, one has to take the row i from the $\mathbf{H}$ matrix and check the positions where $h_{ij} = 1$, with $1 \leq j \leq n$. These positions indicate the elements of the codeword $\mathbf{m}$ which have to be XOR'ed between themselves in order to obtain s_i .

Let us illustrate the procedure explained above with the following example $\mathbf{H}$ matrix:

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The $\mathbf{H}$ matrix above represents the only odd-weight-column SEC-DEC code that generates an 8-bit length codeword ($n = 8$) for a 4-bit data word ($k = 4$). In coding theory, this is

denoted as an (8,4) code. For $\mathbf{H}$ under consideration, the check bits shall be calculated as follows:

$$\begin{aligned} c_1 &= d_1 \oplus d_2 \oplus d_3 \\ c_2 &= d_1 \oplus d_2 \oplus d_4 \\ c_3 &= d_1 \oplus d_3 \oplus d_4 \\ c_4 &= d_2 \oplus d_3 \oplus d_4 \end{aligned} \quad (2)$$

Assuming an input data vector $\mathbf{d} = [1011]$ and applying (2), the check bits would be $\mathbf{c} = [0010]$. The codeword, that is, the actual bits that will be saved in memory would be $\mathbf{m} = [10110010]$.

According to the procedure described previously, the syndrome shall be calculated with the following equations:

$$\begin{aligned} s_1 &= m_1 \oplus m_2 \oplus m_3 \oplus m_5 \\ s_2 &= m_1 \oplus m_2 \oplus m_4 \oplus m_6 \\ s_3 &= m_1 \oplus m_3 \oplus m_4 \oplus m_7 \\ s_4 &= m_2 \oplus m_3 \oplus m_4 \oplus m_8 \end{aligned} \quad (3)$$

If neither the data bits or the check bits have been altered, the resulting syndrome is $\mathbf{s} = [0000]$, as expected. However, if we flip m_3 , for example, we would get $\mathbf{s} = [1011]$. By inspecting $\mathbf{H}$, we can spot that the error occurred at d_3 , as $\mathbf{s} = [1011]$ matches with the third column of $\mathbf{H}$.

IV. DESCRIPTION OF THE ALGORITHMS

The purpose of the proposed algorithms is to synthesize a given EDAC function in such a way that it allows an efficient implementation on the desired FPGA technology. The optimization goals followed in this study are listed and justified in Section IV-A.

The algorithm presented in Section IV-C guarantees optimality because the whole space of solutions is explored, avoiding to make assumptions based on heuristics. This approach is often categorized as backtracking, and it has the drawback of having long processing times. In order to reduce the cost of blind backtracking, a dynamic programming version of the same algorithm will be presented in IV-D, and its processing speed will be measured later in V-B. In spite of being implemented differently, both versions of the algorithm are functionally equivalent, and they will be referred to with the same name: *FS-EDAC* (Full Search EDAC).

The results provided by any of the versions of FS-EDAC are useful to set an optimal reference that can be used measure the performance of the available synthesis tools, and to develop faster algorithms that try to reach the best possible solution. Following this idea, Section IV-E presents *G-EDAC* (Greedy EDAC): a greedy variant of FS-EDAC whose performance will be evaluated later in Section V, both in terms of processing speed and hardware implementation results.

A. Optimization goals

The algorithms will focus their effort in reducing area occupation, although some attention will also be put in optimizing processing speed and power consumption.

The area occupation can be calculated as the number of utilized LUTs, which are the basic combinational resources available in every FPGA technology. Other specialized FPGA

resources such as carry logic or embedded multipliers will be ignored as they cannot be exploited for implementing the circuits under study. For measuring processing speed, path delays will be considered instead of clock periods, as the inferred circuits will be purely combinational. Optionally, higher speeds can be achieved by inserting flip-flops within the internal stages to form a pipeline, but at the cost of more area, specially if Triple Modular Redundancy (TMR) is used to protect such flip-flops against radiation.

The actual maximum path delay, which will determine the processing speed, can only be obtained by the timing analysis tools provided by the selected FPGA vendor after the synthesis, place and route stages. Because of this reason, the parameter to minimize will not be the maximum path delay itself. Instead, other variables which are directly related to the maximum path delay will be considered, namely:

- The maximum number of LUTs that a path has to go through (from now on, *levels*). Each LUT adds a certain delay to the path, as well as the nets used to connect two consecutive LUTs.
- The number of nets or signals that are part of the circuit. The higher the number of nets, the more complex will get the routing process, which generally results in longer delays.
- The maximum fan-out of all the LUTs present in the circuit. A higher fan-out in an output means that it takes more time for this output to charge the input capacitances of the inputs it is connected to, as the current is divided among those inputs.

Both the number of utilized LUTs and the number of nets have an impact on power consumption. As these parameters will be minimized by the algorithms, a reduction on power consumption can also be expected for a given clock frequency.

Given that not all of the mentioned parameters can be minimized at the same time, some type of priority mechanism has to be established. The proposed algorithms prioritize the minimization of the parameters with the following order (highest priority first):

- 1) Number of LUTs,
- 2) Maximum level.
- 3) Number of nets.
- 4) Maximum fan-out.

In all cases, the optimization consists on minimizing the value of the parameter.

B. Algorithm interface

The inputs to all the algorithms presented in this section are $\mathbf{H}_{r \times n} = \{h_{ij}\}$, $l(j)$ and K , while the output is the optimized netlist. Each one of the inputs will be described in the paragraphs below.

The parity-check matrix $\mathbf{H}$ defines the EDAC function to implement. It consists of a binary matrix ($h_{ij} = 0$ or 1) where each of the r rows represents an output, and each of the n columns represents an input. As explained in Section III, an output i is calculated as the XOR combination of all the inputs of the corresponding row that satisfy the condition $h_{ij} = 1$.

The function $l(j)$ establishes the level of the j th input ($l(j) \geq 0$). It is common that the level of every input signal comes directly from the output of a flip-flop. For that particular case, we would have $l(j) = 0$ for $1 \leq j \leq n$.

The constant K is the maximum number of inputs that a LUT of the target technology has, so the algorithms may instantiate LUTs with a number of inputs between 2 and K ($K \geq 2$). Other characteristics of the selected FPGA technology like the routing architecture are not taken into account.

C. Backtracking version

This section describes the backtracking version of FS-EDAC. The steps of the algorithm are summarized in the list below, and they will be explained extensively throughout the subsequent subsections:

- 1) Initialization
- 2) Search for common terms
- 3) Update of the current network
- 4) Parameter calculation
- 5) Solution evaluation
- 6) Construction of the final network

Steps 3 to 5 will be applied to each *common term* found in Step 2. A common term is defined as a group of two or more terms in one output (i.e., a column of $\mathbf{H}$) that are shared at least with another output. At the same time, Steps 2 to 5 will be applied to each network obtained in Step 3. This means that FS-EDAC is a highly recursive algorithm, as it enters a new level of recursion with each found common term, including the ones obtained as a result of having grouped previous common terms. Therefore, a long execution time can be expected when the input functions have a high number of inputs or outputs. Nevertheless, this approach ensures that every possible solution is evaluated, which guarantees that the result is optimal regardless the defined optimization goal or cost function.

1) *Initialization*: As we will see later in Section IV-C3, in each recursive iteration the $\mathbf{H}$ matrix will be extended with a new column, representing the common term that was selected. The extended version of $\mathbf{H}$ will be named $\mathbf{H}_{r \times n^*}^* = \{h_{ij}^*\}$.

In the first iteration we need to work with the original $\mathbf{H}$ matrix, because no common term has been found yet. Therefore we establish $\mathbf{H}^* \leftarrow \mathbf{H}$ and $n^* \leftarrow n$.

It will also be useful to keep track of the number of common terms found so far. We define a new variable called *gates_ct* and set it to 0, as initially there are no common terms.

Given that each tested solution will be compared to the one that is currently considered the best, an initial solution needs to be calculated to serve as a first reference. For this purpose, we will obtain the network that results from applying the MISO algorithm [13] to each output (or row) defined by $\mathbf{H}$. If the optimization parameters under consideration are the ones listed in Section IV-A, they can be calculated with the formulas included in [13]. In any case, these parameters shall be stored in memory for later comparison with other solutions.

2) *Search for common terms*: The first step of every iteration is to find all the possible groups of two or more terms in one output that are shared at least with another output. This is done by trying all the combinations of 2 or more columns and 2 or more rows, and checking if the elements of $\mathbf{H}^*$ defined by those combinations are equal to 1.

First, we define:

$$\mathcal{P}_{=q}(S) = \{P \in \mathcal{P}(S) \mid |P| = q\} \quad (4)$$

where $\mathcal{P}(S)$ is the power set (i.e., the set of all subsets) of the set S , and $|P|$ is the cardinality (i.e., the number of elements) of the set P .

We can define Q^R and Q^C as the set of all possible combinations of rows and columns that may together form a common term, respectively:

$$Q^R = \bigcup_{s=2}^r \mathcal{P}_{=s}(\mathbb{N}_{\leq r}) \quad (5)$$

$$Q^C = \bigcup_{t=2}^K \mathcal{P}_{=t}(\mathbb{N}_{\leq n^*}) \quad (6)$$

We can check if those combinations indeed form a common term by inspecting the $\mathbf{H}^*$ matrix:

$$G = \{(A, B) \in Q^R \times Q^C \mid \mathbf{H}^*[A, B] = \mathbf{1}\} \quad (7)$$

The result is G : a set of ordered pairs where the elements of the pair are the sets of rows and columns that together form a common term in the current $\mathbf{H}^*$ matrix. In (7), $\mathbf{H}^*[A, B]$ is the submatrix of $\mathbf{H}^*$ that results from selecting the rows indicated by A and the columns indicated by B .

Finding G is a computationally intensive operation, which requires to test a high number of combinations, with their respective memory accesses. For a given $\mathbf{H}^*$, two nested sweeps are needed: one for s ($2 \leq s \leq r$) and another one for t ($2 \leq t \leq n^*$). This implies $\left[\sum_{s=2}^r \binom{r}{s}\right] \cdot \left[\sum_{t=2}^K \binom{n^*}{t}\right]$ combinations. For each combination, there are $s \cdot t$ accesses to the elements of $\mathbf{H}^*$ in order to check if they are all equal to 1. This means a total of $\left[\sum_{s=2}^r \binom{r}{s} s\right] \cdot \left[\sum_{t=2}^K \binom{n^*}{t} t\right]$ memory accesses. Note that as n^* increases, the number of tested combinations and memory accesses increase as well. This fact is specially undesirable for our algorithm, because n^* is incremented after each recursive iteration, as will be explained later in Section IV-C3. This issue becomes even more problematic when one takes into account that a new G will have to be obtained for every (A, B) in the current G , reaching several levels of recursion depending on the size and contents of the original $\mathbf{H}$ and also on the value of K . This is just the main problem that will be mitigated later in Section IV-D by means of dynamic programming.

3) *Update of the current network*: For each found group $(A, B) \in G$, we need to update $\mathbf{H}^*$ so that we can perform a new iteration of the recursive algorithm later. First, the chosen group has to be removed from the current $\mathbf{H}^*$. We do so by filling with zeros the array positions determined by (A, B) :

$$h_{ij}^* \leftarrow \begin{cases} 0 & i \in A, j \in B \\ h_{ij}^* & \text{else} \end{cases} \quad (8)$$

Then, a new column is appended to $\mathbf{H}^*$, representing the new common term, which is in fact a FPGA LUT that implements a XOR function with $|A|$ inputs:

$$\mathbf{H}^* \leftarrow [\mathbf{H}^* | \mathbf{c}] \quad (9)$$

with $\mathbf{c} = \{c_i\}$ being a $r \times 1$ column vector such that:

$$c_i = \begin{cases} 0 & i \in A \\ 1 & i \notin A \end{cases} \quad (10)$$

Note that the fact of adding a column to $\mathbf{H}^*$ makes:

$$n^* \leftarrow n^* + 1 \quad (11)$$

Given that such column represents a common term:

$$\text{gates_ct} \leftarrow \text{gates_ct} + 1 \quad (12)$$

Also note that the number of elements equal to 1 in $\mathbf{H}^*$ is always reduced with each update, due to the fact that $|B| \geq 2$.

The output of the common term that has just been added needs to be given a level, just like any of the original inputs. It is easy to realize that the level of the new common term is equal to the greatest level of the selected terms (which can be either inputs or common terms) plus 1:

$$l(n^*) \mapsto \max \{l(j) | j \in B\} + 1 \quad (13)$$

The information about the rows that comprise the new group (A) has already been saved in $\mathbf{H}^*$ with the addition of the new column. But we may also have to save the information about the columns (B) elsewhere for later access. Such information will be needed for reconstructing the resulting network when the algorithm finishes. For this purpose, we can define the function $c(t)$ as the set of selected columns for the t th common term. In each iteration, this function has to be updated with a new element:

$$c(\text{gates_ct}) \mapsto B \quad (14)$$

4) *Parameter calculation:* Now that we have updated $\mathbf{H}^*$, n^* , $l(j)$ and gates_ct , the next step is to obtain parameters that characterize the current solution in order to evaluate its quality according to the defined optimization goals. As stated in Section IV-A, the proposed implementation of the algorithm will consider the following optimization parameters: the number of gates, the maximum level, the number of nets and the maximum fan-out.

Such parameters must be calculated for the network that results from applying the MISO algorithm to each of the outputs defined by $\mathbf{H}^*$, so that they correspond to the final network that FS-EDAC would output in the case that it was indeed the best overall one. In spite of being a fast algorithm, applying MISO in each iteration of the FS-EDAC can take a very long time because there is usually a high number of iterations. This section proposes alternative ways for obtaining the optimization parameters without the need to apply the MISO algorithm at all, derived on the equations presented in [13]. This way, the execution time is significantly reduced.

The number of gates of the final network can be easily calculated using the following formula:

$$\text{gates} = \text{gates_ct} + \sum_{i=1}^r \left\lceil \frac{\left[\sum_{j=1}^{n^*} h_{ij}^* \right] - 1}{K - 1} \right\rceil \quad (15)$$

The first addend corresponds to the common terms found up to this point. The other one is related to the gates that would be instantiated by the MISO, should it have been applied.

For calculating the maximum level of the final network, we first need to find the number of inputs and common terms that have a level equal to m for a given output i :

$$l_{=m}(i) = |\{j \in \mathbb{N}_{\leq n^*} | l(j) = m, h_{ij} = 1\}| \quad (16)$$

The level of the output i of the final network is given by:

$$\text{out_level}(i) = \left\lceil \log_K \left(\sum_{m=0}^{\infty} l_{=m}(i) \cdot K^m \right) \right\rceil \quad (17)$$

The global maximum level is therefore:

$$\text{max_level} = \max_i \{\text{out_level}(i)\} \quad (18)$$

There is a compact formula for the number of nets of the final network:

$$\text{nets} = \sum_{t=1}^{\text{gates_ct}} |c(t)| + \sum_{i=1}^r \left\lceil K \frac{\left[\sum_{j=1}^{n^*} h_{ij}^* \right] - 1}{K - 1} \right\rceil \quad (19)$$

The first addend corresponds to the nets used for connecting the common terms that have been found up to this point, while the other part of the equation refers to the network produced by the MISO algorithm.

For obtaining the maximum fan-out, we first need to obtain the fan-out of every input or common term of the current solution. There is no need to calculate the fan-outs of the gates instantiated by the MISO algorithm, as they are all equal to 1 (that is, the minimum possible value).

The fan-out of the j th column of $\mathbf{H}^*$, which could correspond to either an input or a common term, is simply calculated as:

$$\text{fanout}(j) = \sum_{i=1}^r h_{ij}^* \quad (20)$$

Therefore, the maximum fan-out is:

$$\text{max_fanout} = \max_j \{\text{fanout}(j)\} \quad (21)$$

5) *Solution evaluation:* In this step, the solution that is currently considered the best is compared to the one corresponding to this iteration. For this purpose, we will take the optimization parameters calculated in the previous step and apply the procedure defined in Fig. 1, which is used to determine which solution is better.

Following the strategy defined in Section IV-A, the criteria defined in Fig. 1 minimizes the area of the inferred circuit, while trying to maintain good values for the rest of parameters. Nevertheless, any other optimization goal is possible, as long as the associated parameters or cost functions can be obtained or estimated during the synthesis phase.

Fig. 1: Priority mechanism to select the best out of two possible solutions.

If this step discovers that the current solution is better than the one that was being considered the best, then the old solution has to be discarded and replaced with the current one. The associated optimization parameters need to be stored in memory for the comparisons that will be performed in later iterations of the algorithm. All the variables that define the new solution ($\mathbf{H}^*$, n^* , $l(j)$, etc) need to be stored as well in case it ends up being the best overall one, so that the final network can be constructed in the last step.

6) *Construction of the final network*: Once all the groups have been processed, the last step is to apply the MISO algorithm to each output (or row) defined by the $\mathbf{H}^*$ that was considered to be the best among all. The result is a circuit that represents the optimal solution according to the defined optimization criteria.

D. Dynamic programming version

This section describes an alternative implementation of FS-EDAC which is equivalent in terms of functionality to the backtracking version presented in Section IV-C, but considerably faster because it takes advantage of memoization techniques in order to avoid recalculating everything from scratch in each recursive iteration. It does so by either precomputing parameters at the beginning of the algorithm or by reutilizing partial results obtained in previous iterations. In computer science, this type of techniques are usually referred to as *dynamic programming*.

The definition of the steps of this algorithm is the same as in Section IV-C. Therefore, the procedure will be explained in an analogous way, using the same subsection structure.

1) *Initialization*: First, we get an initial G using (7), just as explained in Section IV-C2. This will be the only time that we will have to apply this computationally intensive operation, as the subsequent G s will be obtained using a dynamic approach.

In Section IV-C1, we prepared an $\mathbf{H}^*$ matrix that would be updated and extended in every recursive iteration. Now, by means of a custom dynamic programming technique, it will be possible to avoid the need to maintain $\mathbf{H}^*$ at all, thus saving lots of read and write accesses to it. That means that the subsequent searches of groups will have to be performed in an alternative, indirect way. The same idea applies to the calculation of the optimization parameters, which used to depend a lot on $\mathbf{H}^*$ as well.

In spite of not having to maintain $\mathbf{H}^*$, we must keep track of the number of columns that it would have had in the case that it existed. Therefore, we initialize $n^* \leftarrow n$.

In later stages of the algorithm, we will want to know the sum of elements of $\mathbf{H}^*$ equal to 1 at each row. For this purpose, we define a vector $\mathbf{a}_{r \times 1} = \{a_i\}$ that will be updated in each iteration so as to keep track of the row sums:

$$a_i = \sum_{j=1}^n h_{ij} \quad (22)$$

Now we prepare the LUTs (software LUTs, not hardware LUTs of the FPGA) that will replace some of the calculations later in the algorithm with simple memory accesses.

There will be a software LUT for the number of gates (or FPGA LUTs) that are needed to merge together a given number of signals s , and an analogue LUT for the number of nets:

$$\text{gates_LUT}(s) = \left\lceil \frac{s-1}{K-1} \right\rceil, \quad 1 \leq s \leq n \quad (23)$$

$$\text{nets_LUT}(s) = \left\lceil K \frac{s-1}{K-1} \right\rceil, \quad 1 \leq s \leq n \quad (24)$$

There is no need to calculate (23) and (24) for $s > n$, as the worst case would be $s = n$ for any $\mathbf{H}$ with n columns. The worst case happens when all the elements of a row in $\mathbf{H}$ are 1, so all of them are to be merged together in order to produce the output corresponding to that row. Due to the fact that $K \geq 2$, the number of elements in a row equal to 1 will always be reduced with each recursive iteration, so it is impossible to get out of the bounds of the LUTs if they are dimensioned as explained above.

Note that both (23) and (24) could have been obtained by applying the MISO algorithm to an imaginary network with s inputs, but that would have taken more execution time. Also note that those equations do not include the gates and nets that may have been instantiated as a result of having found a number of common terms. We need to prepare separate counters for that purpose: gates_ct and nets_ct , and set their values to 0.

For the levels, we initialize a new matrix $\mathbf{L}_{r \times (m_{max}+1)} = \{l_{im}\}$ with the values given by (16):

$$l_{im} = l_{=m}(i) \quad (25)$$

As usual, the matrix has been dimensioned for the worst case, which occurs when all the elements in two or more rows of $\mathbf{H}$ are equal to 1, all input levels $l(j)$ are equal to each other, and all the common terms are formed by pairs of signals grouped in a stairwise manner. Knowing n and $l(j)$, but not the contents of $\mathbf{H}$ itself, an upper bound for the maximum expected level is:

$$m_{max} = n + \max \{l(j) | 1 \leq j \leq n\} - 1 \quad (26)$$

Finally, we have to compute the initial fanouts using (20) so as to be able to update them in subsequent steps. The fanouts can be saved in a new $\mathbf{f}_{n \times 1} = \{f_j\}$ vector, which will be extended later for every new common term.

2) *Search for common terms*: At this stage, the set of found groups for the current solution is already located in G , as we calculated it either in the initialization step or in the previous network update step. Therefore, there is no need to perform this time-consuming process any more.

3) *Update of the current network*: In the backtracking version of the algorithm, this step was devoted to select a group $(A, B) \in G$ and update $\mathbf{H}^*$ accordingly. However, it was explained in Section IV-D1 that we would no longer need to maintain $\mathbf{H}^*$. Instead, now we use a dynamic approach for obtaining the new G that will be used in the next recursive iteration, based on the information currently stored in G .

First of all, we have to apply equations (11), (12) and (14), just as in the non-dynamic version of the algorithm. The row sums will be updated in an incremental way, reflecting the changes that would have been made to $\mathbf{H}^*$ in the case that it existed:

$$a_i \leftarrow \begin{cases} a_i - |B| + 1 & i \in A \\ a_i & \text{else} \end{cases} \quad (27)$$

Now we are ready to start building a brand-new G^* :

$$G^* \leftarrow \emptyset \quad (28)$$

For each $(A', B') \in \{G - (A, B)\}$, we need to calculate a new group to be added to the group set G^* :

$$(A'', B'') = \begin{cases} (A', B') & A' \cap A = \emptyset \vee \\ & B' \cap B = \emptyset \\ (A', (B' - B) \cup n^*) & A' \subseteq A \wedge \\ & B' \cap B \neq \emptyset \wedge \\ & B' \not\subseteq B \\ (\emptyset, \emptyset) & \text{else} \end{cases} \quad (29)$$

We add each new group (A'', B'') to G^* as indicated below:

$$G^* \leftarrow G^* \cup (A'', B'') \quad (30)$$

After all groups $(A', B') \in (G - (A, B))$ have been processed, we can replace the previous G with the one that has just been built:

$$G \leftarrow G^* \quad (31)$$

In the following paragraphs, we will explain equation (29), which reassembles one of the most important contributions that will help to accelerate the algorithm.

$$\mathbf{H}^* = \begin{matrix} & (A, B) \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 0 & 1 & 1 & \cdots & 0 \end{bmatrix} \end{matrix}$$

$$(A', B') = (A'', B'')$$

(a) Independent groups

$$\mathbf{H}^* = \begin{matrix} & (A, B) \\ \begin{bmatrix} 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 1 & \cdots & 1 \\ 0 & 0 & 1 & \cdots & 1 \end{bmatrix} \end{matrix}$$

(b) Updatable group

$$\mathbf{H}^* = \begin{matrix} & (A, B) \\ \begin{bmatrix} 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 1 & \cdots & 1 \\ 0 & 1 & 1 & \cdots & 0 \end{bmatrix} \end{matrix}$$

(c) Discardable group

$$\mathbf{H}^* = \begin{matrix} & (A, B) \\ \begin{bmatrix} 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 1 & \cdots & 1 \\ 0 & 0 & 1 & \cdots & 1 \end{bmatrix} \end{matrix}$$

(d) Duplicated group

Fig. 2: Example scenarios when updating G .

The first condition of (29) is the one in which the selected group (A, B) and the group to be updated (A', B') do not have any element in common. This means that the fact of grouping the terms defined by (A, B) does not affect the group (A', B') at all, because they are totally independent. Therefore, we keep (A', B') untouched.

The scenario described in the paragraph above is depicted in Fig. 2a. In this example, $\mathbf{H}^*$ is shown only for illustrative purposes, because in fact it is not calculated. In Fig. 2a, the group (A, B) has been already blanked out as a result of applying (8), and the corresponding common term has been added to the last column of $\mathbf{H}^*$ as in (9). For clarity, the example A, B, A' and B' vectors only contain contiguous elements, but it does not have to be always this way.

The second condition of (29) occurs when there is an overlapping between (A, B) and (A', B') in such a way that a new group can be created using the new common term that was added as a result of having chosen (A, B) . Fig. 2b shows an overlapping that has broken the group (A', B') , which can no longer exist in its current form because not all of its elements are equal to 1 ($\mathbf{H}^*[A', B'] \neq 1$). In spite of that, the characteristics of the overlapping allows to define a new group based on (A', B') which uses the recently added common term. In (29), the condition for updating (A', B') has been designed so that $2 \leq |A''| \leq r$ and $2 \leq |B''| \leq K$. Otherwise, the resulting (A'', B'') could be an invalid common term.

In the last case of (29), the group (A', B') is discarded because it has been broken in such a way that it cannot be

updated to use the recently added common term. Fig. 2c shows an example of this last scenario.

It has to be noted that two different (A', B') groups can lead to the same (A'', B'') for a given (A, B) and G . Such is the case of Fig. 2d, if one compares it with Fig. 2b. After applying (30), the fact that G^* is defined as a set avoids any possible duplication.

4) *Parameter calculation*: In this stage, we will obtain the optimization parameters of the current solution, taking advantage of partial results from previous recursive iterations.

As we have already updated `gates_ct` in Section IV-D3, we just need to apply the following equation to get the total number of gates:

$$\text{gates} = \text{gates_ct} + \sum_{i=1}^r \text{gates_LUT}(a_i) \quad (32)$$

The common term selected in Section IV-D3 must be given a level $l(n^*)$ using (13). Then, $\mathbf{L} = \{l_{im}\}$ is updated with:

$$l_{im} \leftarrow \begin{cases} l_{im} + 1 & i \in A, m = l(n^*) \\ l_{im} - 1 & i \in A, m \in \{l(j) | j \in B\} \\ l_{im} & \text{else} \end{cases} \quad (33)$$

Basically, we decrease the level counter of the elements that would have been removed from $\mathbf{H}^*$ in the current recursive iteration, and increase the counter for the level of the new common term. The maximum level is then calculated as in (17) and (18), but replacing $l_{=m}(i)$ with l_{im} .

In order to know the total number of nets, first we need to update `nets_ct` with the number of nets of the new common term:

$$\text{nets_ct} \leftarrow \text{nets_ct} + |B| \quad (34)$$

The number of nets of the final network is calculated as:

$$\text{nets} = \text{nets_ct} + \sum_{i=1}^r \text{nets_LUT}(a_i) \quad (35)$$

The fan-out vector $\mathbf{f} = \{f_j\}$ is updated according to the following formula:

$$f_j \leftarrow \begin{cases} f_j - |A| + 1 & j \in B \\ f_j & \text{else} \end{cases} \quad (36)$$

After that, the fan-out of the new common term must be appended to $\mathbf{f}$:

$$f_{n^*} \leftarrow |A| \quad (37)$$

The maximum fan-out is obtained with (21), but replacing $\text{fanout}(j)$ with f_j .

E. Greedy variation

This section defines the greedy variant of FS-EDAC, called G-EDAC. The steps of this last algorithm are the same than the ones listed at the beginning of Section IV-C, but the iteration process is different.

In FS-EDAC, Steps 2 to 5 had to be applied to each network obtained in Step 3. With G-EDAC, this new level of recursion is only entered for the network with the best cost function for the current set of common terms under consideration. It is

easy to identify the best network because by the time the new recursion level has to be entered, Step 5 would have already been performed for all the networks of the current level, as part of the process of finding the best overall solution.

The process explained above reduces dramatically the number of algorithm branches, and therefore a huge improvement in processing time can be expected. However, the result is not guaranteed to be optimal any more, unlike happened with FS-EDAC. The degree of optimality achieved by FS-EDAC, as well as its processing time, will be evaluated in Section V.

V. RESULTS

In this section, the results of applying the algorithms proposed in this paper will be compared between themselves and to those obtained by commercial synthesis tools, namely *Xilinx Vivado 2014.2*, *Mentor Graphics Precision RTL 2014.09* and *Synopsys Synplify 2013.03*.

Every tested synthesis algorithm or tool has been provided with the same input, although expressed in different forms. The inputs will be passed to FS-EDAC and G-EDAC as explained in Section IV-B. In the case of commercial synthesis tools, the input is usually specified with hardware description languages (HDLs) like VHDL or Verilog. In this work, VHDL has been selected.

A. Execution time

The section is devoted to perform a deep analysis about the execution time of both versions of FS-EDAC: the one based on backtracking and the one based on dynamic programming. The later algorithm was designed for accelerating the first one while maintaining exactly the same functionality and overall structure, which allows to perform a detailed comparison for each algorithm step, finally leading to some interesting conclusions. On the other hand, the same type of analysis cannot be made for G-EDAC as well because the iteration process is not comparable. This is just the reason why the execution time of G-EDAC becomes several order of magnitude faster than any version of FS-EDAC, as we will see later in this section.

The two software implementations of FS-EDAC were implemented in MATLAB, following the equations presented in Sections IV-C and IV-D. The objective of this analysis is to quantify the amount of acceleration obtained by applying dynamic programming techniques to this specific algorithm.

Table I shows the results of the benchmark that was performed in order to compare the two different implementations. The algorithms were executed 10 times in a Intel Core 2 Quad processor at 2.66 GHz, one run after another and forcing them to run in a single core. The time estimates presented in the table are median values of the 10 runs.

The parity-check matrix that was selected as the input for the algorithms was the only possible (15,10) minimum odd-weight EDAC code that can be built according to the method explained in [26]. The size of the associated $\mathbf{H}$ matrix ($n = 15$ and $r = 5$) is big enough for the algorithms to take a reasonable amount of time in being executed, which provides an adequate precision for the benchmark. For this matrix, we will assume that the n inputs come directly from the output

	Backtracking		Dynamic		Acceleration (dir/dyn)
	Absolute time (s)	Relative time (%)	Absolute time (s)	Relative time (%)	
Initialization	0.000	0.000	0.003	0.027	0.048
Search for common terms	33.529	63.278	2.901	23.543	11.558
$\mathbf{H}^*$ building	3.672	6.930	0.000	0.000	∞
Row sums	0.855	1.614	0.416	3.376	2.055
Number of gates	0.224	0.423	0.207	1.680	1.082
Maximum level	3.390	6.398	2.522	20.467	1.344
Number of nets	0.577	1.089	0.206	1.672	2.801
Maximum fan-out	0.689	1.300	0.344	2.792	2.003
Globals declaration	6.577	12.412	3.601	29.223	1.826
Function overhead	3.237	6.109	1.885	15.297	1.717
Solution evaluation	0.237	0.447	0.237	1.923	1.000
Total	52.987	100.000	12.322	100.000	4.300

TABLE I: Execution time breakdown for the two software implementations with an example $\mathbf{H}$

of n flip-flops, so $l(1 \leq j \leq n) = 0$. The maximum number of inputs per LUT will be set to $K = 3$.

In Table I, the first column lists the different tasks in which the software implementations spend a measurable CPU time, including the total execution time. Most of the listed tasks are directly related to the different steps of the algorithm as described in Sections IV-C and IV-D. Others, like *Globals declaration* and *Function overhead*, are software-specific issues, not closely related with the algorithm itself.

The first and most important conclusion that can be extracted from Table I is that, indeed, the dynamic programming techniques have permitted to accelerate the total execution of the algorithm. With the example inputs that were established, the measured acceleration factor was 4.3.

A closer inspection of Table I reveals that every single step of the algorithm is accelerated, with the exception of the *Initialization* step, which is something that could be expected because a full search of groups in the initial $\mathbf{H}$ has to be performed in the dynamic version of the algorithm. However, the impact of this delay in the total execution time is insignificant, as this is a task that is executed only once. The rest of tasks listed in Table I are executed once for every possible solution. With $\mathbf{H}$ under consideration, the number of tested solutions is 236321 in both versions of the algorithm, as they are functionally equivalent.

Table I shows that the step which has benefited most from the dynamic approach is the search for common terms. Also, it is interesting to note that the dynamic version does not spend any time in building the $\mathbf{H}^*$ matrix. This behaviour is consistent to what was explained in Section IV-D1 about the non-maintenance of $\mathbf{H}^*$.

Up to this point, only the *Absolute time* and *Acceleration* columns have been analysed. But there are also many interesting conclusions that can be extracted from the *Relative time* columns. With them, we can discover which tasks have a greater impact on the total execution time, and think of some ideas to make them faster.

The relative time spent in the *Globals declaration* entry shows that it is the main component that is decelerating the dynamic algorithm. This component can be eliminated by using another programming language that makes an efficient handling of the global variables, with no unwanted delays

because of context changes. Such is the case of the C/C++ language, for example. If we removed the row *Globals declaration* from Table I, we would get a overall acceleration of $\times 5.321$. A proper C/C++ implementation would also decrease the execution time of every single algorithm stage.

The second most important component in the execution time of the dynamic implementation is the *Search for common terms* step. After all the optimizations that have been made, the only effective way of reducing the time spent at this stage would be to reduce the number of tested groups, in such a way that the best solution is not discarded.

Further analysis of the 236321 tested solutions reveals that there are only 35 unique solutions. The rest of them can be considered duplicates, with exactly the same structure as other solutions but combining different inputs, which in the end makes no difference at all. If the algorithm had been able to discard those 236286 duplicates, its execution time would be dramatically reduced, getting as fast as a few milliseconds for the present benchmark considering the fact that the execution time is proportional to the number of tested cases. Note that such huge boost would be possible because of the regularity of the selected $\mathbf{H}$ matrix, which contributes to increase the number of duplicates. Other $\mathbf{H}$ matrices can be less regular, but still, processing speed will surely be reduced if the duplicates are identified in an efficient way.

Another way to accelerate the algorithm is to change the strategy and follow a greedy approach like G-EDAC. However, this type of techniques do not always ensure the best possible solution, and sometimes they have to be completely redesigned if the optimization parameters or their priority are changed. By avoiding duplicate solutions in brute-force implementations, as explained in the paragraph above, one would theoretically be able to get speeds comparable to those of a greedy algorithm and still obtain optimal results, keeping the flexibility to alter the optimization parameters at will. As a reference, G-EDAC was made to process the same $\mathbf{H}$ of the present benchmark, and the resulting processing time was 190 milliseconds.

B. Hardware implementation results

In this section, the hardware implementation results of the algorithms described in this paper will be analysed and

TABLE II: Post place and route results of the Hsiao (22,16) function in an A3P030 FPGA ($K = 3$)

Synthesis tool	LUTs	Max. level	Nets	Max. fan-out	Common terms	Max path delay (ns)	Total power (mW)
Synplify	29	3	76	3	1	3.640	1.961
Prec-RTL	28	3	74	3	2	3.551	1.737
MISO	24	2	72	3	0	3.099	1.660
G-EDAC	20	2	60	3	4	3.052	1.433
FS-EDAC	19	2	57	3	5	2.995	1.287

compare to those obtained by several commercial synthesis tools. Additionally, the standalone version of MISO will be used for processing a full EDAC function, not just as a part of the present algorithms. For an explanation about how to apply MISO to a multi-output function like EDAC, please see [13].

The EDAC functions will be passed to the commercial synthesis tools as VHDL source files containing the defining equations with no further manipulations. For example, the (8,4) SEC-DED code defined in equation (3) would be translated to VHDL as follows:

```

s(1) <= m(1) xor m(2) xor m(3) xor m(5);
s(2) <= m(1) xor m(2) xor m(4) xor m(6);
s(3) <= m(1) xor m(3) xor m(4) xor m(7);
s(4) <= m(2) xor m(3) xor m(4) xor m(8);

```

Signals m and s are connected to a set of flip-flops, which in turn are connected to the input and output ports of the top level entity, so they are associated with physical pins of the FPGA. According to the nomenclature presented in this paper, this means that all the inputs will have a level equal to 0. Given that no registers have been instantiated between signals m and s , the core of the circuit will be purely combinational.

When a synthesis tool is given a piece of VHDL code like the one shown above, it generates a netlist in EDIF format which is specific for the target technology. The netlist defines a circuit that can only contain the components available in the target FPGA, without specifying where they will be placed or how the connections will be routed throughout the die. Those tasks are in charge of the place and route tools provided by the FPGA vendor (available as part of *Libero SoC* for *Microsemi* FPGAs or as part of *Vivado* for *Xilinx* FPGAs).

FS-EDAC produces a simplified circuit, but such circuit has to be inserted somehow in the design flow, so that the place and route tools can finish implementing it. One possible way is to generate an EDIF netlist, emulating a normal synthesis tool. However, the authors chose to generate a technology-dependent VHDL file, which is a VHDL file that includes specific libraries for the target technology and only instantiates components that are present in such technology. Using this technique, the generated VHDL cores can be easily integrated in larger designs by adding them into the design flow just like any other HDL source file. If the VHDL file is defined correctly, a commercial synthesis tool like Synplify or Vivado will not modify the circuits defined in those files. Instead, it will perform a direct translation from VHDL to EDIF.

Several tests have been executed for this study, evaluating the selected synthesis tools for two different FPGA architectures: one from Microsemi and the other from Xilinx.

The synthesis, place and route tools have been configured to

reduce the area utilization as much as possible, so as to match the first optimization objective that was established for FS-EDAC (see Section IV-A and Fig. 1). For this purpose, the only tool that needed a change of the default options was Precision RTL, whose *Optimization goal* property was set to *Compile for area*. On the other hand, the techniques that were applied in order to obtain the best possible timing performances (timing was a lower-priority optimization goal) will be explained in subsequent subsections, because each tool has its own way to do so. The results shown in the tables of this section have been extracted from the reports generated by the respective tools, after completing the process of placing and routing the synthesized circuits.

The maximum path delay is measured from the input to the output of the EDAC entity. Each input and output signal of this combinational block has been connected to one flip-flop so as to minimize the effects of widespread placing, which happens when the EDAC entity is directly connected to the FPGA I/O pins. The existence of the flip-flops allows the placing tools to make the circuits more compact, so the comparison of the maximum path delay is more fair. Besides, the results will be more similar to those that would be found in a real application, as it is a usual practice to register the inputs and outputs of every FPGA core.

In the present study, the total power (static power plus dynamic power) has been calculated as accurately as possible, feeding the power estimation tools of the respective manufacturers with the output of a post place and route simulation of the circuit. The test bench switches the inputs at every clock cycle with a probability of 50%. On the other hand, the clock frequency has been established to 100 MHz. The computed total power only includes the elements that belong to the EDAC entity itself (i.e., the consumption of clock nets, I/O blocks and flip-flops are not considered).

1) Results with a 3-input LUT FPGA: In the first set of tests, the selected FPGA model is an A3P030, from the Microsemi ProASIC3 family. The core of this FPGA consists of a sea of cells called *VersaTiles*. Each cell can be configured either as a 3-input LUT ($K = 3$), a D-flip-flop or a latch, and they may be connected between themselves through any of the four levels of routing hierarchy.

The results obtained for the A3P030 are specially relevant, as the ProASIC3 family is widely used for developing and prototyping FPGA designs for space missions [2], [19], [20], [33], [34], [44], where EDAC functions are usually implemented for protecting memories from SEU. Once the designs have been finished, the ProASIC3 netlists can be exported to a RTAX FPGA. This latter family of FPGAs is very popular in space

TABLE III: Post place and route results of the Hsiao (72,64) function in an XC7A100T FPGA ($K = 6$)

Synthesis tool	LUTs	Max. level	Nets	Max. fan-out	Common terms	Max path delay (ns)	Total power (mW)	Sensitive bits
Vivado	48	3	256	5	0	2.580	1.404	23222
Synplify	52	2	260	5	0	2.486	1.496	23824
Prec-RTL	47	2	254	5	1	2.519	1.445	23388
MISO	48	2	256	5	0	2.476	1.464	23284
G-EDAC	35	2	192	3	12	2.332	1.103	21223

applications due to their high reliability against radiation.

Because of it being flash-based, it is not possible to re-program a ProASIC3 in run-time, so the EDAC cores cannot be used to protect the configuration memory of the FPGA itself. Still, the use of the proposed algorithms will be beneficial for the EDAC cores and for the reliability of the systems as a whole because of the reasons outlined in Section I.

All the algorithms under study were fed with the (22,16) EDAC function presented in [26]. Table II shows that the proposed algorithms (G-EDAC and FS-EDAC, in any of its versions) provide better results than Synplify and Precision RTL for every evaluated parameter with the exception of the maximum fan-out, which the same for all the tested tools. The performance of Vivado could not be evaluated in this case because this tool does not support FPGAs by Microsemi.

Since the FS-EDAC algorithm tries all possible combinations of common terms among every output, it is natural that it produces a circuit with less area than any other algorithm, as it was configured to find the solution with the fewer number of LUTs. In this case, FS-EDAC obtains around $1/3$ less LUTs than the commercial synthesis tools.

It is specially relevant the fact that G-EDAC produces very similar results to FS-EDAC in spite of being several orders of magnitude faster. Still, it has to be checked in the next subsection whether such degree of optimality is also achievable with bigger input designs and different FPGA architectures.

It can happen that FS-EDAC and G-EDAC get a high value in the maximum level, as this parameter has been given a lower optimization priority than the number of LUTs. However, Table II shows that both FS-EDAC and G-EDAC have outperformed the rest of algorithms for this parameter as well. One could think that the higher number of common terms has allowed this behaviour, however, the truth is that it all depends on how well these terms are grouped. All of the proposed algorithms take a great care in grouping the common terms properly.

It is surprising to realize that the MISO algorithm obtains better results than other commercial tools for this case, in spite of being a simple algorithm which processes each output separately, completing its execution in a matter of milliseconds even for the biggest designs. According to the *Common terms* column, both Synplify and Precision RTL have processed the EDAC function as a whole (therefore identifying common terms), but still, the area utilization is worse than MISO.

The seventh column of Table II confirms that the assumption about the relationship between the maximum level and the maximum path delay has been correct. Assuming that the place and route tools perform well, the circuits with lower maximum

level are prone to be faster, as explained in Section IV-A. In order to extract the maximum effort of the Libero IDE place and route tools, a multi-pass place and route process was performed with a strict clock period constraint (1 nanosecond). After 20 passes, the solution with lower maximum path delay was selected. In this case, the speed improvement of the proposed algorithms has been between 14.05% and 17.72% when compared to the commercial synthesis tools.

The estimated total power has proven to be approximately proportional to the number of used resources, that is, the number of LUTs and nets. The improvement is equal to 34.37% for the most favorable case. Nevertheless, it is important to note that the resulting total power also depends a lot on the performance of the place and route tools, just like happens with the maximum path delay. It could occur, for example, that an inferred net goes through a lot of FPGA nets and switching matrices or multiplexers, resulting in longer path delays and increased total power consumption. These effects are minimized by using the technique explained in the previous paragraph, which has allowed to obtain consistent results.

2) *Results with a 6-input LUT FPGA:* The other battery of tests was obtained with the Vivado software suite, using the XC7A100T, a 6-input LUT FPGA ($K = 6$) from the Artix-7 family. The architecture of this family is very different from the ProASIC3, and the differences are not limited to the number of inputs of each LUT. The main logic resources of this FPGA are the *Configurable Logic Blocks* (CLBs). Each one consists on a pair of slices, and every slice contains four 6-input LUTs plus four storage elements (latches or flip-flops). The CLBs are arranged in a regular array of rows and columns, connected between them by means of switching matrices and general-routing resources that run vertically and horizontally around the CLBs. The configuration memory of this FPGA is stored in a SRAM memory, which is specially sensitive to SEU, so it will be highly interesting to check the increase of reliability of the EDAC cores themselves when the proposed synthesis algorithms are applied.

In this section, the algorithms were fed with a larger EDAC function than in Section V-B1, namely the (72,64) EDAC function presented in [26]. It was considered convenient to choose a bigger input function so as to allow some margin of optimization to the synthesis tools now that K is also higher than before. A general rule is that a higher value of K usually allows to infer smaller circuits, given that each LUT will be able to host a bigger portion of the complete logic function. Having smaller synthesized circuits would not permit to draw firm conclusions, as the results would be more similar between themselves.

FS-EDAC could not be tested in this new analysis because the execution times were unacceptably long. An estimation showed that it could take several years to execute FS-EDAC for the EDAC function under consideration, in contrast to the 9.33 seconds taken by G-EDAC to process the same function.

Table III shows the results for the XC7A100T. G-EDAC is clearly the algorithm that performs best, in spite of the fact that greedy approaches do not always guarantee optimal solutions. The reduction of area utilization with the other tools is now between 25.53% and 32.69%. Regarding the maximum level, only Vivado has obtained a value of 3, which explains the worse value in the maximum path delay for this tool (9.61% higher than G-EDAC). For the rest of the tools, the maximum path delays are more similar between themselves, as all of them have obtained a maximum level equal to 2.

Neither Synplify nor Vivado have been able to find any common term, and still, MISO obtains better figures of merit than the other two without even trying to find a common term. This is because MISO guarantees that the solution minimizes all the optimization parameters defined in Section IV-A at the same time, as long as LUT output sharing is not allowed (i.e., no common terms). In this case, Precision RTL has beaten MISO because the first one has grouped a proper common term, although the mere fact of selecting one or more common terms does not always mean an improvement over selecting none (see Table II).

It can be seen in Table III that the relationship between the post-place and route maximum path delay and the total power with the optimization parameters is not as strong as in Table II. This is because the method of running multiple passes is not possible in Vivado, as its place and route algorithms seem to be deterministic (i.e., they do not rely on random seeds). In order to extract the maximum effort from this tool, the chosen strategy was to iteratively reduce the clock period constraint until such constraint cannot be met by Vivado.

Lastly, an additional column was appended to Table III, which was not present in Table II. This new column gives the number of sensitive bits calculated by the Vivado bitstream generation tool. Unfortunately, such information is not reported by Libero SoC, so it could not be included in Table II.

Table III confirms the idea that a lower area usage results in a smaller number of sensitive bits. The reduction obtained with G-EDAC with respect to the other algorithms and tools is between 8.61% and 10.91%. This is just the percentage of improvement in circuit reliability that can be expected for an EDAC core implemented in a SRAM-based FPGA, as the number of sensitive bits is directly proportional to the MTBF (Mean Time Between Failures) for a given radiation environment and FPGA model. This means that the ultimate goal of improving the reliability of EDAC cores implemented in SRAM FPGAs has been successfully accomplished.

VI. CONCLUSIONS

In this paper, three algorithms specialized in synthesizing EDAC functions on FPGAs have been extensively described and evaluated. By exploiting the properties of this particular problem, the algorithms are able to obtain optimal (FS-EDAC)

or quasi-optimal (G-EDAC) solutions given a predefined optimization criteria.

When compared to the results obtained by means of commercial synthesis tools, the proposed algorithms can reduce the area, maximum path delay and power consumption of the resulting circuits up to 34.48%, 17.72% and 34.37%, respectively. These numbers confirm the fact that there is still a wide margin of improvement for the synthesis tools that are currently available in commercial FPGA design suites.

In addition, the EDAC cores were effectively hardened against radiation, achieving an increase of 10.91% in the MTBF. The higher reliability becomes even more useful in the cases where the EDAC cores are in charge of protecting the configuration memory that defines the circuitry implemented in the FPGA, including the EDAC cores themselves.

Even though the algorithms proposed in this paper were designed specifically for FPGA devices, it is easy to adapt FS-EDAC so that it minimizes other parameters that are specific to ASIC devices, e.g., the number of transistors and/or the die area. On the other hand, the greedy version of the algorithm (G-EDAC) is also adaptable, but a new battery of tests should be executed in order to evaluate its degree of optimality.

Being a highly recursive algorithm, FS-EDAC has the drawback that it is not practical for dealing with large functions, as the number of possible combinations increases exponentially with every added input or output. It was shown in Section V-A that dynamic programming techniques produce a significant performance boost (5 times faster than backtracking), but still, the long processing times prevented the authors from determining the optimal results for the largest tested circuit. As future work, it will be analysed whether more intelligent searches can be performed for finding common terms, as the current implementations try many combinations that are identical between them (see Section V-A). A smarter algorithm which is able to skip some or all of the duplicates may have a reasonable execution time even for the largest functions.

For all the designed algorithms, the MATLAB language and software suite was selected because it allowed fast development and easy debugging, reducing the risk of programming errors. However, it was said in Section V-A that C/C++ implementations would be very beneficial for accelerating the algorithms. Software ports to the C/C++ language will consequently be scheduled as future work for this paper.

Finally, another line of research could be opened regarding the implementation of heuristic or greedy algorithms more sophisticated than G-EDAC. Again, having developed FS-EDAC will be useful for this task, as the degree of optimality will be subject to be measured by comparing the results of the new algorithms with the ones obtained by FS-EDAC.

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Chapter 4

The adaptive optics subsystem of AOLI

This chapter summarizes the second of the articles that are part of the present dissertation, which has the following reference:

C. Colodro-Conde, S. Velasco, J. J. Fernández-Valdivia, R. López, A. Oscoz, R. Rebollo, B. Femenía, D. L. King, L. Labadie, C. Mackay, B. Muthusubramanian, A. Pérez Garrido, M. Puga, G. Rodríguez-Coira, L. F. Rodríguez-Ramos, J. M. Rodríguez-Ramos, R. Toledo-Moreo, and I. Villó-Pérez. “Laboratory and telescope demonstration of the TP3-WFS for the adaptive optics segment of AOLI” In: *Monthly Notices of the Royal Astronomical Society* 467.3 (Jun. 2017), pp. 2855–2868.

This work provides a thorough description of the AO subsystem that we have developed for AOLI, including all the related hardware, software, calibration and operational procedures. Previous publications by the AOLI consortium did describe the instrument, but under a different scope: they either referred to former designs which are no longer applicable [102] or they provided little detail about the AO part [103]. Another unique contribution of this newer manuscript, and probably the most relevant one, is the characterization and first closed-loop results of the novel TP3-WFS, whose underlying wavefront reconstruction algorithm had only been demonstrated by simulation until its physical implementation for AOLI [33].

As explained in the article, the availability of wavefront sensors with improved sensibility would increase the sky coverage of AO-enabled instruments relying on such sensors, and that is exactly what the TP3-WFS aims to achieve. Similarly to the widely-used Shack-Hartmann WFS (SH-WFS) [104], the TP3-WFS bases its calculations on geometrical optics, but following a completely new approach: instead of dividing the light in an array of subapertures, this newer WFS works with the intensities of two defocused pupil images taken at two different planes, using an algorithm that theoretically allows attaining good reconstructions under extremely low light levels.

After introducing the basic concepts of AO and LI, the article starts describing the hardware of AOLI, with a focus on the TP3-WFS. The optical design of the TP3-WFS is of special interest because AOLI is the first instrument ever to implement this type of sensor as a physical device, and later use it in an actual astronomical application. Then, the article continues with the description of all the AO-related software, namely: frame grabbing software, wavefront reconstruction software and real-time control software. In addition to the description of the functions and internal architecture of each piece of software, the article gives some practical considerations about their implementation, configuration and on-the-field use. All this information will be helpful for future systems willing to take advantage of the technologies developed for AOLI.

Due to the fact that AOLI was based on a new type of WFS, a lot of effort was put on its validation, first on the laboratory (using the instrument calibration source) and later on the telescope (with natural reference objects on the sky). The correctness of the reconstructions given by the TP3-WFS were tested under different conditions using a set of characterization procedures that were developed to measure the response of the control plant, spanning from the DM to the WFS. The conclusions of such analyses was that the TP3-WFS was indeed capable of recovering the wavefront with at least the same spatial resolution than the one given by the DM itself, regardless of whether the AO reference was a punctual object like a star or an extended object like a planet.

With the TP3-WFS already validated, the following step was to test the on-sky closed-loop operation of the complete AO subsystem. The results included in the article demonstrate the proper closure of the AO control loop using several reference stars of different magnitudes, as well as an extended reference object (Neptune). This statement is supported both by the analysis of the recorded AO-related data (e.g., wavefront error statistics) and by the inspection of the resulting images at the science camera. As expected, the best performances were obtained with the brightest reference objects, which provided better signal-to-noise ratios at the WFS camera. Unfortunately, it was not possible to measure the faintest magnitude of the reference object with which the AO subsystem could operate reliably because of a failure in the camera hardware that was discovered during the off-line analysis of the acquired data.

In addition to the demonstration of the use of the TP3-WFS in the AO subsystem of AOLI, which represents the main objective of the article, a final set of preliminary results about the science subsystem was also included as an advance of future works that will investigate the effect of combining AO with LI. Such results are encouraging because they point to a significant increase in the probability of getting a lucky image, and thus may enable AOLI to reach its final goal: to obtain diffraction-limited images in the visible bands on large ground-based telescopes.

Laboratory and telescope demonstration of the TP3-WFS for the adaptive optics segment of AOLI

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ABSTRACT

AOLI (Adaptive Optics Lucky Imager) is a state-of-art instrument that combines adaptive optics (AO) and lucky imaging (LI) with the objective of obtaining diffraction limited images in visible wavelength at mid- and big-size ground-based telescopes. The key innovation of AOLI is the development and use of the new TP3-WFS (Two Pupil Plane Positions Wavefront Sensor). The TP3-WFS, working in visible band, represents an advance over classical wavefront sensors such as the Shack-Hartmann WFS (SH-WFS) because it can theoretically use fainter natural reference stars, which would ultimately provide better sky coverages to AO instruments using this newer sensor. This paper describes the software, algorithms and procedures that enabled AOLI to become the first astronomical instrument performing real-time adaptive optics corrections in a telescope with this new type of WFS, including the first control-related results at the William Herschel Telescope (WHT).

Key words: instrumentation: adaptive optics – instrumentation: high angular resolution

1 INTRODUCTION

Reaching the diffraction limit in the visible wavelength is one of the main reasons to place optical telescopes on-board satellites such as the Hubble Space Telescope (HST), thereby avoiding the distortions and blurring that the atmosphere introduces on the unaltered wavefronts. With its life-cycle coming to end and without any oncoming substitute in visible bands, it is crucial to provide the scientific community with tools capable of providing similar resolutions from

ground. In the era of the extremely large telescopes, and due to the increase of the atmospheric distortion as the diameter of the aperture grows, this has become a world top engineering challenge.

There are two main techniques which lead to diffraction-limited imaging. On the one side, Lucky Imaging (LI) (Hufnagel & Stanley 1964; Fried 1978; Brandner & Hormuth 2016) offers an excellent and cheap method for reaching diffraction limited spatial resolution in the visible band in small and mid-size ground-based telescopes (Oscoz et al. 2008). However, this technique suffers from two important limitations. First, resolutions similar to the HST can only be

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achieved in telescopes with sizes below 2.5m (Femenía et al. 2011; Labadie et al. 2011). Second, most of the images are discarded, meaning that only relatively bright targets can be observed.

The other technique, Adaptive Optics (AO), has been the main procedure to improve the quality of the largest ground-based telescopes during the last twenty years (Rousset et al. 1990; Beckers 1993; Milli et al. 2016). The use of AO systems in infrared observations provides an adequate performance due to the reduced effects of turbulence in this wavelength range, thus achieving excellent results (Guyon et al. 2010; Macintosh et al. 2014; Law et al. 2014). Unfortunately, the scarce number of AO systems developed for the visible bands (Beuzit et al. 2008; Close et al. 2012) have not yet achieved the versatility and image quality already achieved in NIR (Close et al. 2013; Guyon 2005), except for solar telescopes (Berkefeld et al. 2012). The difficulty of performing AO in the visible wavelengths is explained by the fact that the correlation time of the atmospheric turbulence scales with $\lambda^{6/5}$ (Greenwood 1977; Fried 1990), which means that an AO control loop operating in visible bands needs to be faster than in the NIR bands in order to provide the same degree of correction.

The Adaptive Optics Lucky Imager (AOLI) is a state-of-the-art instrument which was conceived to obtain extremely high resolution at optical wavelengths on big-sized telescopes by combining the two techniques presented in the paragraphs above (AO + LI) (Velasco et al. 2015, 2016). Initially targeted for the 4.2m William Herschel Telescope (WHT, Observatorio del Roque de los Muchachos, Spain), the instrument is designed as a double system that encompasses an adaptive optics control system before the science part of the instrument, this last implementing LI (Mackay et al. 2016). Each of the two parts of AOLI can behave as a standalone system, meaning that the AO subsystem might be used with any other science instrument, performing imaging, spectroscopy or even coronagraphy or polarimetry.

Besides the fact of combining AO and LI for the first time in an astronomical instrument, the other key innovation of AOLI is the development and use of a new type of wavefront sensor (WFS) in its AO subsystem: the Two Pupil Plane Positions Wavefront Sensor (TP3-WFS). The implementation of the TP3-WFS was motivated by the will of being able to use fainter AO reference stars than the ones that classical wavefront sensors such as the Shack-Hartmann WFS (SH-WFS) can use, all with the aim of increasing the sky coverage of the instrument without the need of laser guide stars.

The present work gives a comprehensive description of the AO subsystem of AOLI, focusing on the TP3-WFS and all the related algorithms and procedures that were developed for its characterization and testing. The first control-related results obtained with AOLI at the WHT, also included in the paper, confirm the viability of the TP3-WFS as part of a fully-functional adaptive optics system.

The paper is organized as follows. Section 2 introduces the AO subsystem of AOLI. Section 3 describes the AO real-time processing pipeline. Section 4 describes the procedures that were used to get information the AO subsystem, thus enabling the configuration the control algorithm. Section 5 presents the results obtained both in laboratory and telescope tests. Finally, Section 6 draws the main conclusions.

2 THE ADAPTIVE OPTICS SYSTEM OF AOLI

AOLI has been built putting together the expertise of several institutions, each group specialized in a different subject. To face the challenge that AOLI represents we have implemented a new philosophy of instrumental prototyping by modularizing all its components (López et al. 2016): simulator/calibrator (Puga et al. 2014), science module (performing LI) and AO module. Figure 1 depicts the optical layout of AOLI, along with a description of the setup. On the other hand, Figure 2 shows a photograph of the instrument as mounted on the WHT.

The AO subsystem of AOLI comprises a 241-actuators deformable mirror (DM) by ALPAO, a pick-off guide-star subsystem and the Two Pupil Plane Positions Wavefront Sensor (TP3-WFS). The TP3-WFS operates with the images provided by an Andor Ixon DU-897 camera, which is based on a sub-photon noise 512x512 e2v EMCCD (Electron Multiplying Charge-Coupled Device) detector. The heart of the AO system is its Real-Time Control Software (RTC), which allows the control of 153 Zernike modes with a delay under 40 μ s. The delay of the calculations performed in the TP3-WFS itself is around 1 ms for the same number of reconstructed modes.

AOLI was required to perform wavefront sensing using faint reference stars up to magnitude 16 in the *I* band with a seeing of 1 arcsec and at a wind speed of 8 km/s, which corresponds to the median value of the wind speed at the Roque de los Muchachos observatory. For the WHT this means sensing with up to four magnitudes fainter stars than the limit reached with a classical Shack-Hartmann WFS (SH-WFS). On the other hand, it was estimated that performing low-order AO corrections at a rate of 100 Hz in combination with LI would provide the desired level of correction at the science camera.

2.1 The TP3-WFS

The leading position for sensing the wavefront on AO systems has been occupied so far by the Shack-Hartmann WFS (SH-WFS), (Hartmann 1900; Shack & Platt 1971; Platt & Shack 2001). One important disadvantage of the SH-WFS is that the incoming photons are distributed among all the illuminated lenslets. This sets a limitation for the magnitude of the reference stars whose wavefronts can be reconstructed, and consequently to the sky coverage of an instrument based on this type of sensor. We have developed the TP3-WFS with the aim of overcoming this disadvantage.

The TP3-WFS bases its calculations on the intensity of the images of two defocused pupil images taken at two different planes. Computer simulations predict that this way of measuring wavefronts allows attaining good reconstructions with down to 100 photons falling within each pupil image (van Dam & Lane 2002). Although this statement is yet to be thoroughly tested under a real environment (not only by simulations), if confirmed it would mean a considerable improvement in the sensitivity when compared to the SH-WFS. Another advantage of the TP3-WFS with respect of previous wavefront sensors such as the SH-WFS is that it is capable of working on extended targets, as demonstrated later in Section 5.

The TP3-WFS is composed of the wavefront reconstruc-

Figure 1. AOLI optical layout. The system is divided in three modules: Deformable Mirror (DM), WaveFront Sensing (WFS) and SCIENCE (SCI). The common vertex is a Pick-off mirror that selects the reference star for wavefront sensing through a pin-hole in the mirror. This pin-hole can be selected as a real hole or with a splitting ratio $R/T = 30/70$ and several sizes depending on the current seeing. The science arm can select two possible collimators to obtain two different Fields Of View (FOV). The WFS arm uses a lateral prism to introduce a differential delay between the two optical paths, using a common lens system to defocus the pupil image over the detector. The DM module is a typical 1:1 system with an Amici-biprims Atmospheric Dispersion Corrector (ADC).

Figure 2. AOLI mounted at one of the two Nasmyth focuses of the WHT, inside the GRACE (Ground based Adaptive optics Controlled Environment) structure. The main components have been identified with labels.

tion software (WFR), the WFS camera and the surrounding optics (see Figure 1). The main function of the TP3-WFS optics is to obtain the two defocused images near the pupil plane, which are later acquired by the WFS camera and finally processed by the WFR software. The WFR software is founded on the algorithm proposed by [van Dam & Lane \(2002\)](#), and it operates in real-time thanks to its GPU-accelerated implementation ([Fernández-Valdivia et al. 2013](#)). More detail about the internals of the WFR software will be given later in Section 3.2.

3 AO PROCESSING PIPELINE

Had the AO system of AOLI been based on well-known technologies such as the Shack-Hartmann WFS, the team could have benefited from the developments and knowledge from previous projects, or even from commercially available solutions. However, the use of a type of WFS that had never been implemented before for astronomical applications forced the development of not only the WFS itself, but also of the surrounding methods and software. The new system had to be

designed for the specific needs of the TP3-WFS, with a focus on flexibility so as to counter uncertainties that this new method arose.

In the end, the AO part of AOLI was implemented with three new pieces of software: the frame grabbing software (FG), the WFR and the RTC. The three pieces of software are interconnected in a processing pipeline as shown in Figure 3. This processing pipeline is triggered every time a new frame is produced by the WFS camera (which sets the AO sampling rate), and it re-enters the idle state once a new actuation vector is sent to the DM.

The target environment for all the AO-related software would be a single computer operating with GPUs and under Windows. We selected an Intel Core i7-4790K CPU and an nVidia GeForce GTX Titan Z GPU running the Windows 7 operative system. The following subsections will provide details about each of the three components of the AO processing pipeline.

3.1 Frame grabbing software

The main objective of the FG software is to continuously acquire images from the WFS camera and send them to the WFR software as soon as they are being received, with no further processing in between. Additionally, this software allows configuring the camera parameters and provides real-time information once the acquisition process has started.

Reference AO stars with bright apparent magnitudes may produce images in the EMCCD sensor with unnecessary large signal-to-noise ratios (S/N), with no actual improvement in the accuracy of the reconstructed wavefronts. In those cases, the operator of the instrument would normally decrease the exposure time of the WFS camera so as to get a faster sampling rate, which would enable the RTC software to produce DM actuations more frequently and thus improve the quality of the AO corrections even with bad atmospheric conditions. The upper limit of the sampling rate is set by the camera hardware itself and by the element of the AO processing pipeline whose execution time is longer.

On the other hand, the S/N of the pupil images of faint reference AO stars can indeed be improved by lowering the sampling rate of the camera, but this may have a negative impact on the quality of the AO corrections because the RTC software may not be able to keep up with the speed of variation of the atmospheric turbulence at that specific period of time. In those cases, the recommended solution is to activate the binning function of the EMCCD, which will increase the S/N of the image by combining the charges of adjacent pixels, at the expense of a lower spatial resolution in the acquired pupil images. This would improve the accuracy of the wavefront reconstructions in the cases of low light, although it will not be possible to reconstruct the higher-order modes because of the loss of image resolution.

For an optimal configuration of the camera, it is important to know that its sampling rate does not only depend on the exposure time and the size of the readout region, but also on other parameters such as the frequency of the EMCCD clocks, specially the clock that drives the ADC (analog-to-digital converter). This type of clock fine-tuning has to be done carefully in order not to degrade the S/N.

3.2 Wavefront reconstruction software

The WFR software takes the two defocused images formed by the TP3-WFS optics and calculates the photon displacements between those two planes by applying the Radon transform (Radon 1917) over a set of projection angles. The photon displacements are then used to produce an estimation of the slopes of the incident wavefront. Finally, the algorithm outputs a Zernike representation of the reconstructed wavefront (von F 1934), calculated as the least-squares fit between the calculated slopes and the ones that each individual Zernike mode would produce.

The WFR algorithm is clearly the one that has a higher computational cost in the whole AO processing pipeline, so a huge effort was put in optimizing its implementation in order to achieve real-time performance. Among all the possible acceleration methods, it was decided that the WFR would take advantage of GPUs in order to achieve real-time operation, more specifically, by means of the CUDA language. When compared to other FPGA or CPU-based approaches, the GPU implementation was considered a good trade-off from the point of view of development costs, flexibility and re-usability (Rodríguez Ramos et al. 2015).

Figure 4 provides a graphical summary of the steps of the WFR algorithm, whose equations are further described in van Dam & Lane (2002). Even though the description of the algorithm itself is out of the scope of this paper, the following subsections will give practical considerations about some of the blocks in Figure 4, which can be helpful for both the development and use of future wavefront sensors using this reconstruction technique, besides the TP3-WFS itself.

3.2.1 Extract pupils

The first step of the algorithm is to extract the two regions of the input image that correspond to each pupil image. In the case of AOLI, this separation has to be done by software because the optics of the TP3-WFS were designed in such a way that both defocused pupil images fall within the area of a single WFS detector.

In earlier versions of AOLI, each pupil image was sent to one different camera, but in the end this proved to be a bad idea because of two reasons: first, because it was very hard to synchronize the camera hardware and the related software in order to acquire from the two cameras with a high level of synchronism. Synchronism is obviously a requirement because it only makes sense to process pupil that come from the same instant of time. The second reason is that even small differences between the two cameras (quantum efficiency, dynamic range, bias level, noise, etc.) have a considerable impact on the reconstruction quality. Using a single camera is thus an effective way to solve both problems.

3.2.2 Bias and flat correction

The simulations made with different configurations showed that small deviations between the mean bias level of the two pupil regions has a considerable impact on the accuracy of the reconstructed wavefronts. Being an algorithm that bases its calculations on the differences between the two input images, this behaviour was actually expected. Therefore, it is

Figure 3. AO processing pipeline, consisting in three pieces of software running on the AO computer. The pipeline is triggered on the reception of a frame from the WFS camera, and re-enters an idle state after a new vector of actuations has been sent to the DM. All the elements are required to have a low latency so as to ensure that the actuation vectors sent to the DM correspond to wavefronts that still exist in the real world.

Figure 4. WFR software architecture. For each input image, containing two defocused pupil images, a vector of Zernike modes is calculated. The whole algorithm is implemented on GPU, with the exception of the *Extract pupils* block. This is justified because sending full images to the GPU would create a bottleneck in the CPU-GPU communication channel.

considered mandatory to apply bias corrections as a pre-processing step.

In the same way, it is highly recommended to apply flat-field corrections to the input images, specially to reduce the effect of dust grains on the WFS sensor and optics. During a laboratory test, there was one big particle of dust falling on one pupil image that caused the control loop to enter a peculiar oscillatory regime due to the non-linearity caused by the presence of that particle.

From the point of view of a real-time implementation, it is very convenient to perform the bias and flat-field corrections to each individual pupil image, after they have been extracted to the full image. The rest of the pixels of the original image are not processed anyway, so it is a loss of processing time to correct them as well.

3.2.3 Radon transform

One important parameter to configure in the WFR algorithm is the number of projection angles for which the Radon transform will be calculated. This parameter is closely related to the number of Zernike modes to be reconstructed, as the last step of the algorithm is a least-squares adjustment between the measured wavefront slopes and the ones that the different Zernike modes would produce. This means that, for a given number of reconstructed Zernike modes, there is a minimum number of angles that need to be calculated so as to ensure that the least-squares fit is being executed correctly. A larger number of angles means a better ability to represent high-order aberrations in the Radon transforms themselves.

Of course, even though a large number of projection angles would increase the probabilities of performing a correct fit, in practice setting a very high number of angles is a bad idea because the execution time of the algorithm has a linear dependence with the number of angles. For a given number of reconstructed Zernike modes, a trade-off can be found by performing a sweep over the number of angles and

determine by inspection whether the result of the static characterization is correct. Section 5.1 explains how to analyze the goodness of the static characterization results.

3.2.4 Least-squares fit

The final step of the algorithm actually requires to have pre-calculated the mean wavefront slopes that each Zernike mode would produce at perpendicular directions of each projection angle. This operation is particularly expensive from a computational perspective, so it was also GPU-accelerated in spite of not being part of the real-time AO processing pipeline. This enabled us to quickly experiment the effect of selecting different combinations of number of Radon projection angles and Zernike modes.

It is interesting to note that the input to the least-squares fit block in Figure 4 already contains all the information of the reconstructed wavefront, more specifically, the slopes along a set of projection angles. The least-squares fit is just a way of representing that wavefront in a more familiar way, which additionally enables separating some modes of special interest such as tip, tilt and defocus. It remains to be evaluated whether applying the control algorithm to the estimated slopes directly or to their 2-D integral results in a better level of correction.

At the time of writing the present work, the Zernike modes outputted by the TP3-WFS are not expressed in any particular units. While this could be a problem for other applications, it does not have any impact on the RTC control algorithm of AOLI, as this algorithm does not need to know the physical units of the measured wavefronts. Similarly, the results presented in this paper do not lose their validity because they are always intended to be analysed in a differential way (e.g., open loop vs. closed loop).

As a reference for future wavefront sensors willing to implement this reconstruction algorithm, Figure 5 shows the appearance of the slope maps of the 10 lowest-order Zernike modes (excluding piston, as it cannot be reconstructed).

Figure 5. Precalculated Zernike slope maps, used during the conversion of the calculated wavefront slopes to vectors of Zernike modes. The X axis corresponds to the projection angle θ ($0 \leq \theta < \pi$), while the Y axis corresponds to the Radon coordinate r ($-R \leq r \leq R$, where R is the radius of the aperture).

3.3 Real-time control software

The RTC software is the one responsible of actuating over the DM in such a way that the effect of atmospheric turbulence on the science camera images is minimized. The control algorithm is based on an array of proportional-integral-derivative (PID) controllers. Besides the real-time functionality itself, the RTC software implements the AO characterization procedures explained later in Section 4.

The computations in the RTC software were accelerated by means of the Blaze library (Iglberger et al. 2012). The use of this library enabled easy exploiting of the AVX2 instruction set (Advanced Vector Extensions 2) available on the target CPU, which is specially useful for accelerating vector and matrix operations like the ones that are executed in the RTC algorithm. The proper use of these instructions ultimately allowed obtaining better performances than other highly-optimized GPU libraries like cuBLAS, as the latency of the CPU-GPU link constitutes a tight bottleneck for the RTC algorithm.

Figure 6 depicts the architecture of the real-time part of the RTC software. It consists on a processing pipeline that is triggered on reception of a new reconstructed wavefront, and finishes when the new set of actuations is sent to the DM. Below is a brief description of the blocks appearing in Figure 6:

- **Mask:** Extracts the elements of interest from the input wavefront. In AOLI it is used to ignore the tip and tilt modes in the AO control loop, as the LI algorithm would remove them anyway in its shift-and-add stage, where the input images are re-centered using a predefined criterion such as the position of the peak pixel.
- **W2C (“wavefront to control”):** Takes the input wave-

front and transforms it to a different format just before entering the actual control stage. In AOLI, the W2C block is configured as pass-through.

- **Target:** Allows setting a control target that is different from 0. This is useful for compensating non-common path between the WFS camera and the science camera (NCPA, Non-Common Path Aberrations).
- **PID array:** As its name suggests, this block is composed of a set of independent PID controllers, one for each element present at its input vector (actually, an error vector).
- **C2A (“control to actuations”):** transforms the output of the PID array into the specific set of actuations that produce the desired correction. In AOLI, this block is configured to transform Zernike modes into actuation vectors.

4 SYSTEM ANALYSIS

In order to have a fully working AO instrument one first needs to obtain information from the control plant (i.e., the elements that are part of the control loop) to configure the parameters of the RTC processing pipeline that were explained in Section 3.3. For that purpose, we developed two different procedures, each of them characterizing the AO system under different conditions.

On one hand, the *static characterization* procedure is used to obtain the C2A matrix by inverting the obtained influence matrix. On the other hand, the timing information given by the *dynamic characterization* procedure allows setting proper values to the P , I and D parameters of the PID array. The following subsections will explain these two procedures in detail.

Figure 6. RTC software architecture, showing all the configurable blocks and the nature of the signals traveling throughout them.

4.1 Static characterization

The static characterization procedure allows obtaining the influence function of the AO system. This function represents the effect of actuating over the DM as seen by the WFS once the DM has reached its steady state.

The influence function is usually represented as the so-called *influence matrix* $\mathbf{I}_{M \times A} = [\mathbf{i}_1, \dots, \mathbf{i}_A]$, where M is the number of elements at the output of each WFS sample (normally a set of Zernike modes) and A is the number of actuators of the DM. In this matrix, each row $\mathbf{i}_a$ contains the response of the WFS when a single actuator is pushed with an actuation value equal to 1, whichever the actuation units are.

From the static point of view, the influence matrix contains all the information that is needed to correct an aberrated input wavefront by actuating over the DM. But that would only be possible if the control algorithm knew the vector of actuators that compensates a given input wavefront, which is just the opposite information that the influence matrix provides. Fortunately, it is possible to invert the influence matrix so that it can be used to convert from wavefronts to actuation vectors, even if the matrix is not square. If the matrix is not square, one can apply the Moore-Penrose pseudoinverse (Penrose 2008) instead of performing a regular matrix inversion.

The acquisition of the influence matrix was performed using the algorithm described in Appendix A. There is a set of parameters that is used to configure the algorithm, namely: n_{sa} (stabilize samples), n_{pp} (push-pull iterations), n_{avg} (average samples) and a_{val} (actuation value).

The parameter n_{sa} sets the amount of time (measured in WFS samples) to wait for the AO system to stabilize after a DM actuation. This does not only depend on the response time of the mirror, but also on the rest of elements on the AO chain. The best way to set a proper value to n_{sa} is to measure the dynamic response of the system (Section 4.2) so as to discover the amount of WFS samples that takes for the system to reach a steady state after a step in the input.

One could expect that higher values in n_{pp} and n_{avg} should enable getting measurements with a lower level of noise. However, setting $n_{avg} > 1$ can have a negative effect on the measurement if there is a non-zero mean drift in the input wavefront (either from a star in the sky or from a calibration source). Therefore, it is recommended to set $n_{avg} = 1$ and increase the value of n_{pp} in order to obtain a good signal-to-noise ratio.

Regarding a_{val} , it should be set with a value such that the signal that it produces on the WFS is bigger than the noise, and at the same time ensuring that the AO system (DM, optical path and WFS) does not exit its linear zone during the whole characterization process. The initial position of the DM is not relevant as long as it enables complying with this latter restriction.

The amount of time required to execute the static characterization procedure (t_{sa}) depends on the sampling rate of the WFS (f_{WFS}), the number of actuators (A) and on the following algorithm parameters: n_{sa} , n_{pp} and n_{avg} . By analyzing the steps of the algorithm, it can be shown that the duration of the static characterization process follows the equation below:

$$t_{sa} = \frac{A(n_{sa} + 2n_{pp}(n_{sa} + n_{avg}))}{f_{WFS}} \quad (1)$$

4.2 Dynamic characterization

The dynamic characterization procedure is used to obtain the impulse response of the AO system. Unlike the influence function obtained during the static characterization, the impulse response characterizes the system also in the time domain.

The information provided by the impulse response can be useful for designing a proper PID control loop, that is, setting adequate values to the K_p , K_i and K_d parameters. Another application of the measured impulse responses is to determine the amount of WFS samples that the AO system needs to reach a stable state after an actuation vector is sent to the DM, which sets a lower limit to the n_{sa} parameter of the static characterization procedure.

The dynamic characterization process consists on two stages. In the first stage, a pre-defined sequence of actuations $d_{in}[j]$ (with $1 \leq j \leq n_{in}$) is introduced into one actuator of the DM, and the resulting WFS output $\mathbf{d}_{out}[j] = [d_{out,1}[j], \dots, d_{out,M}[j]]$ (where M is the number of elements at the output of each WFS sample) is recorded as each element of the input sequence is introduced. After that, a post-processing stage is executed in order to obtain the impulse response of the system $\mathbf{h}[j] = [h_1[j], \dots, h_M[j]]$. The specific post-processing that needs to be applied depends on the nature of the selected input sequence.

4.2.1 On-line stage

The steps required to perform the on-line stage of the dynamic characterization over one DM actuator are explained in Appendix B. There are three configurable parameters in this algorithm: n_{sa} (stabilize repetitions), n_{avg} (average repetitions) and t_{RTC} (duration of each iteration of the RTC algorithm).

It is required that $n_{sa} > 0$ because otherwise the system will not have entered a stationary state by the time the first output sample is read. Setting $n_{avg} > 1$ allows improving the signal-to-noise ratio, though the same effect can be achieved

just by increasing the length of the input signal (i.e., increasing n_{in}), provided that the selected input signal is not a plain impulse. The parameter t_{RTC} must represent the amount of time that each iteration of the RTC algorithm would take, otherwise there would be a discrepancy between the complete AO system and the system which is being measured. The output signal $\mathbf{d}_{out}$ is the one that will be processed during the off-line stage so as to obtain the impulse response $\mathbf{h}[j]$.

As happened in the static characterization, the input parameters of the dynamic characterization have an effect on its execution time t_{dyn} , which may be calculated as follows:

$$t_{dyn} = \frac{n_{in}(n_{sta} + n_{avg})}{f_{WFS}} \quad (2)$$

4.2.2 Off-line stage

The calculations to be done on the off-line stage depend on the nature of the chosen input sequence. Among the different types of input sequences, we tested three of the most well-known ones: an impulse, white noise and maximum length sequences (MLS) (Borish & Angell 1983; Rife & Vanderkooy 1989). The tests were executed both in computer simulations (Colodro-Conde et al. 2015) and with the actual AOLI instrument. The authors finally chose the MLS method because it proved to perform better than the others, meaning that it produced more accurate, less noisy impulse responses with lower measurement durations.

The MLSs were generated with LFSRs (Linear Feedback Shift Registers) using the taps proposed by Ward & Molteni (2012). These LFSRs work with the values -1 and 1 instead of 0 and 1 , thus producing a pseudo-random sequence containing positive and negative pulses. The input sequence is multiplied by a constant a_{val} in order to control the magnitude of the actuation, in a similar way as it was done during the static characterization. The length of the MLS sequence n_{in} depends on the MLS order m as indicated in the following equation: $n_{in} = 2^m - 1$.

When the input sequence is a MLS, one can apply the cross-correlation between the MLS input and output in order to get the impulse response of the i th element of the WFS output vector (normally a Zernike mode):

$$h_i[j] = \frac{\mathcal{F}^{-1}(\mathcal{F}(d_{out,i}[j])\mathcal{F}(d_{in}[j])^*)}{n_{in}(a_{val})^2} \quad (3)$$

5 RESULTS

The AO system described in the previous sections was extensively tested under different conditions before going to telescope. Finally, on May 2016 the full AOLI instrument was moved to the William Herschel Telescope, where it saw first light on 22nd May 2016. At that night, we managed to close the AO control loop with a natural sky star with the new TP3-WFS, though with limited performance due to unexpected alignment issues. With the lessons learned during that night, further AO-related results were gathered on another commissioning run in October 2016, closing the loop once again but that time with several targets. In this section we will present the AO-related results, obtained both during laboratory and telescope tests.

5.1 Static characterization

During laboratory tests, the influence matrix itself proved to be an invaluable tool to learn how to configure the TP3-WFS, which had never been used on an AO system before. Given the fact that the WFS software was designed to also output the measured wavefronts as 2-D surfaces (calculated from the 1-D Zernike vectors), it was easy to determine whether the measurements were being done correctly just by representing each actuator response obtained during the static characterization (Section 4.1) and comparing it with the expected response.

In a system that works correctly, the 2-D representation of each actuator response should contain a peak that represents the position of the actuator as seen by the WFS. In the case of the TP3-WFS, one just has to ensure that the number of reconstructed Zernike modes is large enough to achieve a resolution that allows identifying such peaks. The specific number of modes that produce such resolution can be obtained either by computer simulations or by performing a sweep over the number of reconstructed modes with the test equipment. In the case of AOLI, the second option was used.

The TP3-WFS was configured as shown in Figure 7. The processed pupil regions measure 80×80 pixels each, the radius of the Zernike functions used during the precalculations was set to 25 pixels, the number of reconstructed Zernike modes was 153 and the number of Radon angles was 31. Given that the area of interest is only that of the pupils, the frame grabbing software was configured to read only the 90 scan lines where the pupils were located, in an attempt to maximize the sampling rate.

The rationale for configuring the pupil regions, the Zernike radius and the Radon angles the specified way can be found in Section 3.2. Regarding the 153 reconstructed modes, it was tested that closing the loop in laboratory with a lower number of modes led us to a worse PSF (Point Spread Function), while a higher number of modes did not result in a noticeable improvement. Ignoring the fact that LI would eliminate the need of controlling high-order modes, we decided to keep the specified number of modes with the objective of getting the best out of the AO loop alone.

On the other hand, the static characterization parameters were established as follows: $n_{sta} = 2$, $n_{pp} = 25$, $n_{avg} = 1$ and $a_{val} = 0.1$. This combination of parameters produced a good signal-to-noise ratio even on sky tests, with a reasonable execution time. The sampling period of the WFS camera was set to 16.243 milliseconds, and the number of actuators of the DM was 241. As a result, static characterizations in AOLI took almost 10 minutes, which is coherent with equation (1).

Figure 8 shows the results of performing the static characterization both in the laboratory (using the instrument calibration source, which simulates the WHT telescope) and with a natural star in the WHT. Instead of representing all the Zernike modes for all the actuators, this figure shows the 2-D surface representation of the modes for a few actuators, in order to ease the interpretation of the result. In the case of the laboratory tests (first row of Figure 8) there was no simulated turbulence, while during the measurements with a natural star (second row of Figure 8) the atmosphere created a natural turbulence giving a seeing of about 0.9 arcsec.

Figure 7. Configuration of the TP3-WFS, represented over an actual image read from the WFS camera while pointing to a natural star. The red squares correspond to the extracted pupil regions, while the green circles represent the size of the Zernike functions used for precalculating the slope maps. The background image does not correspond to a full frame but to a limited number of scan lines, in an attempt to maximize the camera sampling rate by reading only the region of interest.

Figure 8. Static characterization results for a selection of actuators, using either a laboratory calibration source or a natural star as AO reference. The actuators are detected at the expected positions. The center actuator is a special case because it is hidden by the central obscuration of the aperture, making it impossible to detect.

The results show that the influence functions obtained in the laboratory and in the sky were approximately equal. Of course, the ones obtained in the laboratory were more accurate because there was no turbulence, so they could be used as a reference to estimate the quality of the measurements performed with natural stars.

The 2-D surface representations in Figure 8 show the peak caused by each actuator in the cases where one would expect a peak. For example, one would expect a peak in Figures 8a and 8d because these actuators correspond to the visible area of the WFS (that is, the areas in grey in Figure 7). In Figures 8b and 8e one can see the response of an actuator located just at the outer border of the Zernike area marked in green in Figure 7, which can still be detected because it falls within the pupil region (red square in Figure 7). Lastly, Figures 8c and 8f show no peaks because the location of this actuator corresponds to the obscured area at the center of the pupils.

The main conclusion of the static characterization tests is that the TP3-WFS is able to reconstruct the high-resolution wavefronts generated by the movement of each individual actuator, even with pupils that have a central obscuration area as in the WHT. Knowing that the pupils in AOLI would have a central obscuration, we prepared an alternative version of the software which calculates annular Zernike modes (Mahajan 1981) instead of the regular ones.

However, it was not necessary to use it during the commissioning at the telescope because the main software behaved perfectly.

After verifying that the TP3-WFS apparently produced correct measurements with natural reference stars, it remained to be checked whether the same thing could be said about extended objects such as planets. For this purpose, the telescope was pointed to Neptune (apparent diameter = 2.3 arcsec) and the characterization procedure was executed again. The pupil images acquired by the WFS camera (Figure 9) were a good initial sign because they looked very similar to the ones that had been obtained while pointing to stars of similar apparent magnitudes. Figure 10 shows the result for a single actuator that is neither in the border nor in the center of the pupil region. The fact that the actuator was correctly detected suggests that the TP3-WFS may allow closing the AO loop with extended objects. This hypothesis will be confirmed later in Section 5.3.

5.2 Dynamic characterization

The dynamic characterization procedure described in Section 4.2 was performed on laboratory. The same results can be expected under all conditions, including sky observations.

The parameters of the dynamic characterization were established as follows: $n_{sta} = 2$, $n_{avg} = 10$, $a_{val} = 0.05$ and

Figure 9. A pair of pupil images while pointing at Neptune, extracted from a single camera frame. These images resemble those obtained while pointing to a natural star.

Figure 10. Static characterization result of a single visible actuator, using Neptune as the AO reference object. The actuator is detected at the expected position, just as happened when referencing with stars.

$t_{RTC} = 0$ seconds. These parameters were set this way because of the reasons already outlined in Section 4.2. The simulated delay of the RTC algorithm (t_{RTC}) was set to 0 because, in AOLI, this delay is several orders of magnitude lower than the rest of the delays of the system (just a few tens of microseconds). The sampling period of the WFS camera was set to 16.243 milliseconds, and only one actuator was measured: the one at the centre of the DM. The MLS order was set to $m = 9$, so the length of the input sequence was $n_m = 511$. As a result of the chosen parameters, dynamic characterizations in AOLI took almost 100 seconds, which is coherent with equation (2).

Figure 11 shows the impulse response of the actuator selected for this test. The impulse response of an actuator is actually a vector of Zernike modes, but in this figure only the defocus mode was represented. This choice is justified by the fact that, for an actuator located at the center of the pupil, only the circular modes (defocus, primary spherical, secondary spherical, etc.) have enough signal level over the noise. From the point of view of the timing analysis, the specific mode that is chosen does not matter as long as it provides enough S/N ratio. For other purposes different than this analysis, one could measure the response of any actuator over any mode by executing the dynamic characterization over a long enough time, but the shape of the resulting impulse response would be the same.

In Figure 11, the first feature that attracts the attention is the fact that the impulse response only has signal in two of the samples, the rest of the samples only contain noise. The rationale behind this behaviour is hard to realize unless one has some extra information about the timing of the system. That is exactly what Figure 12 tries to illustrate. This figure represents a timeline of the tasks that occur during the dynamic characterization of the system, using actual timing data from AOLI.

The first conclusion that can be extracted from Figure 12 is that transients of the DM cannot be observed by the WFS cameras. Given the fact that the integration period is much longer than the settling time of the DM, transients

Figure 11. Impulse response of the center actuator, defocus mode. The response is distributed over the first two samples. This behaviour can be explained by carefully analyzing the timing of each element in the AO processing pipeline.

are somehow averaged during the integration process, and the camera sees approximately the same signal that it would see with a perfectly steep input. That is, from the point of view of the WFS, the DM responds instantly.

Further inspection of Figure 12 reveals that the reason why the response to an impulse is divided in two samples is related to the instant of time along the integration period in which the actuation occurs, which in turn depends on the rest of delays of the system. For a system with timings similar to those of AOLI, it makes sense that the impulse response has two samples different from zero at most. The actual position and relative amplitude of these samples only depend on the accumulated delay of the tasks that are part of the plant of the system under control.

5.3 Closed loop operation on the telescope

After gathering the required information and expertise to close the control loop with the TP3-WFS on laboratory tests, we managed to successfully close the control loop with a natural star in the first commissioning night of the complete AOLI instrument in May 2016, in spite of the bad atmospheric conditions (between 1 and 2 arcsec) and an unexpected alignment problem that was discovered just as the telescope was pointed to the first star. The effects of this alignment problem, located at the instrument-telescope interface, accumulated during the night and made it impossible to close the loop reliably with other targets. In addition, the conclusions that could be extracted from the first target were limited because the acquisition parameters of the science camera had been accidentally set in such a way that the detector got saturated on closed loop operation, causing the so-called blooming effect.

The following commissioning run took place in October 2016. This time a lot of effort was put on ensuring a good alignment between the optical axes of the instrument and the telescope. One of the methods used to align the AO subsystem with the telescope during daytime was to open its petals, turn on the interior dome lights, and then perform an iterative re-adjustment of the instrument so that both the telescope and the calibration unit produced the defocused pupil images at the same positions of the WFS detector, regardless the defocus distances.

Despite the improved alignment, it was determined that static characterizations performed with the instrument calibration unit can degrade the control performance when used

Figure 12. Timing diagram of the insertion of an impulse into the AO system, which explains the obtained impulse response. Only the top four delaying elements (mirror response time, integration period, CCD readout time and wavefront reconstruction latency) have been depicted in this diagram. Other elements such as the communication latency between the programs of the AO processing pipeline have not been drawn because they are not significant when compared to the rest of delays. In the diagram, each color in the intermediate lines corresponds the events that are triggered with the arrival of each sample (i.e., an image from the WFS camera).

with natural stars. The reason is that the slightest discrepancies between the optical axis at the output of the calibration unit and at the output of the telescope make the incoming light fall within different areas of the DM surface and the WFS detector, thus changing the static response of the AO system. Therefore, it is highly recommended to be cautious about this and spend some minutes calibrating with a natural star at least once at the beginning of the night. The calibration process can be accelerated by choosing a bright reference star, which allows setting a fast AO sampling rate and still obtain a good S/N.

During the telescope tests, the TP3-WFS was configured as explained in Section 5.1. Depending on the magnitude of the AO reference object, the exposure time of the WFS camera and the number of scan lines was tuned to set a proper AO sampling rate, following the guidelines given in Section 3.1. Regarding the configuration of the control loop, most of the tests were performed discarding the tip-tilt out of the 153 measured modes. The rationale behind this decision can be found in Section 3.3. Even though the PID parameters had already been tuned in the laboratory using the information obtained during the dynamic characterization, the PID parameters were further tuned using the real turbulence from the sky, seeking a fast and stable response. The PID parameters were finally set as follows: $0.4 \leq K_p \leq 0.8$ (depending on the atmospheric conditions), $K_i = 0.4 * f_{WFS}$ and $K_d = 0$ for all the 151 controlled modes.

Figure 13 shows the root-mean-squared (RMS) plots of the error of the reconstructed wavefronts when using HD207470 (I magnitude = 7.5) as AO reference star. The error is calculated with respect to the control target, always set to a 0-vector of Zernike modes with the exception of a manually-tuned defocus term that compensates the NCPA between the WFS camera and the science camera. When comparing open loop and closed loop operation, Figure 13

Figure 13. RMS error of measured wavefront along time, using HD207470 as AO reference. Closing the control loop reduces both the average RMS error and its standard deviation.

shows an improvement factor of 5.23 in the average RMS, as well as an improvement factor of 2.03 in the standard deviation, which constitutes clear evidence that the control loop is operating correctly.

A more in-depth analysis was performed over the same AO reference star, obtaining statistics for each measured Zernike mode. The results are shown in Figures 14. These figures represent the average values and the standard deviation of the error in the first 50 modes (excluding tip and tilt) in open loop and closed loop operation, respectively. The modes have been ordered using the Noll notation (Noll 1976). Once again, these plots confirm the correct operation of the control loop, as the average value and standard deviation of each mode is reduced due to the closing of the loop.

Figure 14. Mode-by-mode analysis of HD207470, open loop.

Figure 15. Mode-by-mode analysis of HD207470, closed loop.

AO ref.	Mag.	t_{WFS}	RMS avg.	RMS std.
HIP10644	4.2 (<i>I</i>)	15.2 ms	1.35×10^{-3}	2.16×10^{-4}
HDS389AB	≈ 5.5 (<i>I</i>)	15.2 ms	1.71×10^{-3}	3.84×10^{-4}
HD207470	≈ 7.5 (<i>I</i>)	21.5 ms	3.57×10^{-3}	8.46×10^{-4}
Neptune	7.7 (int.)	15.2 ms	2.93×10^{-3}	7.09×10^{-4}

Table 1. Control performances for some AO reference objects. The table expresses the dependence between the magnitude of the reference object and the resulting control performance.

It was also checked that modifying a mode in the control target vector indeed changed the average value measured for that mode in the specified amount.

Similar results were obtained from other AO reference targets different from HD207470. Such results are summarized in Table 1. The RMS error plots for each target are analogous to the ones presented in Figure 13, but modulated by the different RMS averages and standard deviations. For example, the closed-loop error plot for HIP10644 would be shifted down with respect to the plot for HD207470, and it will be less noisy. On the other hand, the lower noise would result in shorter error bars in the mode-by-mode analysis.

One conclusion of Table 1 is that the performance of the

Figure 16. Coordinates of Neptune over the science camera. The dispersion of such coordinates is clearly reduced when applying AO with tip-tilt control activated.

control loop depends of the magnitude of the reference object, just as expected from any AO system. Unfortunately, due to a hardware failure that was discovered after the last commissioning run, it was not possible to perform an empirical study about the faintest magnitude of the reference star with which the AO subsystem can operate reliably. One of the EMI (electromagnetic interference) filters of the acquisition card of the WFS camera was physically damaged, resulting in a considerable degradation of the sensibility of the camera. The field study about the performance of the control loop under extremely low light levels is therefore postponed for future commissioning nights where this problem will be hopefully fixed.

Even though AOLI was not originally intended to do science with extended objects, Table 1 also presents the obtained control performances while pointing to Neptune, which confirm the ability of the TP3-WFS to work with extended objects. Further confirmation is presented in Figure 16, which plots the coordinates of Neptune on the science camera when the tip and tilt modes are controlled along with the rest of the modes. This figure is a clear evidence that the tip and tilt modes are being controlled correctly: the standard deviation of the euclidean distance to the mean position is 5.02 pixels in open loop operation, while in closed loop it is only 0.66. Regarding the rest of the modes, due of the lack of features in the surface of Neptune, the only possible test was to adjust the control target of the defocus mode and visually check how it indeed had a proportional effect on the blurriness of the science image.

The results presented so far demonstrate the suitability of the TP3-WFS as the input of an adaptive optics control loop, which was the main purpose of this paper. Nevertheless, further preliminary results will be presented in the following paragraphs regarding the improvement obtained in the images acquired by the science camera. A more comprehensive analysis of the results at the science segment of AOLI, including the application of the lucky imaging algorithm, is scheduled as future work.

Figure 17 shows a set of science images of HIP10644, in open and closed loop, all acquired with 30 ms exposure time. By looking at the speckles, it is clear that the RTC algorithm is effectively reducing the effect of atmospheric aberrations on the science image. The average value of the maximum pixel of each image is 846 in the case of open loop operation, while in closed loop the average value is 1577. This means that the average of the improvement factor of the Strehl ratio is 1.864.

Finally, Figure 18 plots the probability of obtaining an image whose peak pixel value is higher than a given value. The probabilities have been estimated from the images of HIP10644. This figure is specially interesting from the point of view of lucky imaging because it somehow represents the odds of getting a lucky image. For example, for a given number of images, the percentage of images whose maximum pixel value is over 2000 would be around 1% in open loop, while in closed loop it would be as high as 46.5%. This could mean an improvement from the point of view of the LI algorithm, which may be able to produce similar results with a significantly lower amount of processed images. This hypothesis, among others, will be thoroughly tested in future works that focus on the science segment of AOLI.

6 CONCLUSIONS

This paper has presented the first use of the TP3-WFS in a real-time adaptive optics system, with successful results. The analyses, practical considerations and results presented in this paper thus pave the way for new developments besides AOLI willing to use this new type of wavefront sensor, which theoretically provides better sky coverage than previously existing wavefront sensors. The empirical study needed to confirm this latest statement is scheduled for future commissioning runs.

The closure of the control loop improved the average RMS of the 151 controlled modes in a factor of 5.23, which resulted in a clear improvement in the quality of the acquired speckle images, whose average Strehl ratio was increased by a factor of 1.864. In addition, it was confirmed that the reconstruction technique used by the TP3-WFS is able to work with extended objects such as planets, even though such targets fall out of the initial purpose of AOLI.

The presented AO-only results are encouraging due to the fact that they represent just a fraction of what can be achieved by the instrument with the combination of the AO+LI parts. In this regard, future works will focus on thoroughly analyzing the LI performance (open loop vs. closed loop). The preliminary analyses presented in this paper point to a significant improvement of the percentage of lucky images, anticipating a positive outcome also on the science segment.

Regarding the future tests that will be executed to measure the faintest magnitude with which the AO control loop can be closed reliably, two main strategies will be used to maximize the sensibility of the TP3-WFS. First, the radius of the pupil images over the detector will be reduced so as to concentrate better the incoming flux. As explained previously in the article, this would improve the S/N ratio at expense of a reduction in the spatial resolution of the sensed wavefront. The loss of resolution will not pose a problem for

AOLI because it was calculated that compensating just the lowest-order modes would enable the LI algorithm to work as expected. The second strategy for maximizing the sensibility would be to use the EMCCDs in photon counting mode instead of the regular imaging mode that was used during the tests described in this article, thus matching the worst case scenario that was assumed in computer simulations. We expect to reach the required limiting magnitude for AO (mag. 16 in the *I* band) by the combination of these two ideas.

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Figure 17. Images of HIP10644 with and without AO corrections, showing worst, typical and best images according to the maximum pixel value. It seems clear that the closure of the control loop has a positive impact on the quality of the acquired images.

Figure 18. Probability of obtaining an image whose maximum pixel value is higher than a given value, calculated from HIP10644. According to this plot, closing the AO loop gives a higher probability of getting an image of a given quality.

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APPENDIX A: STATIC CHARACTERIZATION ALGORITHM

The static characterization algorithm described in Algorithm 1 pushes and pulls each actuator with respect to a reference position in order to calculate the slope of the function that relates the WFS readout with the movement of that actuator. The push-pull technique is applied so as to avoid hysteresis effects to affect the measurement. Additionally, all measurements are averaged a given number of times in order to improve the S/N ratio.

Algorithm 1 Static characterization.**Require:** $n_{pp} > 0$ and $n_{avg} > 0$ and $a_{val} > 0$ $\mathbf{I} \leftarrow \mathbf{0}$.**for** $a = 1$ to A **do**

Set DM to initial position.

Discard n_{sta} samples from the WFS.**for** $i = 1$ to n_{pp} **do**Using the DM initial position as a reference, push the a th actuator the amount set by a_{val} (i.e., add a_{val}).Discard n_{sta} samples from the WFS.Accumulate n_{avg} samples from the WFS.Add the accumulated result into $\mathbf{i}_a$.Using the DM initial position as a reference, pull the a th actuator the amount set by a_{val} (i.e., subtract a_{val}).Discard n_{sta} samples from the WFS.Accumulate n_{avg} samples from the WFS.Subtract the accumulated result from $\mathbf{i}_a$.**end for****end for** $\mathbf{I} \leftarrow \mathbf{I} / (2 \times n_{avg} \times n_{pp} \times a_{val})$.

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APPENDIX B: DYNAMIC CHARACTERIZATION ALGORITHM.

The dynamic characterization algorithm described in Algorithm 2 allows obtaining the impulse response of the desired actuator. In a first stage, the system is put in a steady state by introducing the input sequence at least once. Then, the same input sequence is introduced a given number of times, but also recording the output of the system every time a new sample is introduced, ultimately producing an averaged output sequence.

Algorithm 2 Dynamic characterization**Require:** $n_{sta} > 0$, $n_{avg} > 0$ and $t_{RTC} < 1/f_{WFS}$ $\mathbf{d}_{out}[j] \leftarrow \mathbf{0}, \forall j$ **for** $i = 1$ to n_{sta} **do****for** $j = 1$ to n_{in} **do**

Discard a sample from the WFS.

Wait t_{RTC} .Using the DM initial position as a reference, push the selected actuator the amount set by $d_{in}[j]$ (i.e., add $d_{in}[j]$).**end for****end for****for** $i = 1$ to n_{avg} **do****for** $j = 1$ to n_{in} **do**

Read a sample from the WFS.

Accumulate the new sample into $\mathbf{d}_{out}[j]$.Wait t_{RTC} .Using the DM initial position as a reference, push the selected actuator the amount set by $d_{in}[j]$ (i.e., add $d_{in}[j]$).**end for****end for** $\mathbf{d}_{out}[j] \leftarrow \mathbf{d}_{out}[j] / n_{avg}, \forall j$

Chapter 5

The COELI algorithm

This chapter summarizes the latest of the contributions of the present dissertation, published as a research article with the following reference:

M. P. Cagigal, P. J. Valle, C. Colodro-Conde, I. Villó-Pérez, and A. Pérez-Garrido. “Covariance of lucky images for increasing objects contrast: diffraction-limited images in ground-based telescopes” In: *Monthly Notices of the Royal Astronomical Society* 455.3 (Jan. 2016), pp. 2765–2771.

This work presents a novel high-resolution technique for ground-based telescopes called COELI (COvarianceE of Lucky Images), an algorithm that is capable of revealing faint companions to main stars located at angular distances as close as the diffraction limit of the telescope. This ability represents an important advantage with respect to conventional LI, where the intensity of the residual halo that appears on the processed images may hide secondary objects whose signal is very weak in comparison to the halo.

The manuscript starts with a review about the theory of atmospheric effects on astronomical images, particularizing on the case of the short-exposure images on which the LI algorithm is based. This is justified by the fact that COELI algorithm shares some of the concepts of conventional LI, such as the selection of the best images out of a series of short exposures that freeze the temporal evolution of the atmosphere. The need of lucky images sets a restriction about the aperture sizes and wavelengths that make this kind of techniques feasible. In the article, the tests are performed over images taken at the *I* band in the 2.2-meter telescope in Calar Alto (Spain) using the LI-enabled Astralux instrument.

The specific concepts that constitute the theoretical basis of COELI are presented in a dedicated section in the article. In that section, it is demonstrated that the pixels of an image that correspond to astronomical object (not just random speckles) vary in phase throughout a series of lucky images. In contrast, the pixels that do not correspond to any object (those

that would form the typical LI halo) vary in counter phase. This correlation between the intensity of the host star and their companions is the key principle that COELI exploits to detect astronomical objects that would otherwise remain hidden.

The COELI algorithm shares the first steps of conventional LI, namely the selection of the best images and their alignment. After that, the images are pre-filtered to reduce the readout noise. Finally, the normalized covariance between the most intense peak of the reference star and the rest of pixels of each frame is calculated along all the lucky images. The result is a normalized covariance map that on one hand has lost the photometric information, but on the other hand improves the contrast at resolutions as fine as the diffraction limit of the telescope, and thus the ability to detect faint objects very close to the main star.

Subsequent sections of the article present mathematical and empirical studies about the contrast and resolving power that can be expected from COELI. The provided formulas highlight the importance of using cameras with very low readout noise such as EMCCDs, as this noise clearly affects the detectability of faint companions. Further reduction of the readout noise is achieved by means of the aforementioned pre-filtering stage. In the article, several filtering techniques are tested, finally selecting the one that provides the best performance: two passes of a Laplacian mask with a radius of one pixel.

Tests performed over actual telescope data show that the contrast of secondary objects falling within the halo around the main star is greatly improved by COELI when compared to conventional LI. As a matter of fact, the halo is practically inexistent in the covariance maps given by COELI, clearly revealing close companions that otherwise would have been partially or totally hidden by the halo.

In order to know the maximum resolving power of COELI, further tests were performed over pseudo-empirical data. Images of binaries with various angular separation and flux ratios were generated by duplicating actual images of a single star, then translating and multiplying them by a reducing coefficient. The results indicate that the obtained contrast improves when the intensity of the companion or the distance between the objects increases. The last of the tests in the article demonstrates that COELI is capable of distinguishing between two objects separated by an angular distance equivalent to the diffraction limit.

All in all, COELI is considered an effective way to reach the diffraction limit from ground-based telescopes at a much lower cost when compared with AO. As happened with LI, COELI is not adequate for telescopes bigger than 2.5 meters because the probability of getting a lucky image would be impractically small. This limitation, however, can be overcome by acquiring the lucky images in combination with AO. The study of such hybrid system is actually one of the future works that are foreseen.

Covariance of lucky images for increasing objects contrast: Diffraction limited images in ground based telescopes

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ABSTRACT

Images of stars adopt shapes far from the ideal Airy pattern due to atmospheric density fluctuations. Hence, diffraction limited images can only be achieved by telescopes without atmospheric influence, e.g. spatial telescopes, or by using techniques like Adaptive Optics or Lucky Imaging. In this paper, we propose a new computational technique based on the evaluation of the COvarianceE of Lucky Images (COELI). This technique allows to discover companions to main stars by taking advantage of the atmospheric fluctuations. We describe the algorithm and we carry out a theoretical analysis of the improvement in contrast. We have used images taken with 2.2 m Calar Alto telescope as a testbed for the technique resulting that, under certain conditions, telescope diffraction limit is clearly reached.

Key words: OCIS codes: (350.1260) Astronomical Optics; (110.6770) Telescopes; (100.2980) Image enhancement; (110.2970) Image detection systems.

1 INTRODUCTION

Atmospheric effects affecting the image quality of a ground-based telescope has been a common topic in astronomy for years. The angular resolution of astronomical images from large optical telescopes is usually limited by the blurring produced by refractive index fluctuations through Earth's atmosphere. The development of different techniques like Speckle Interferometry (Weigelt 1983) and Speckle Masking first or Adaptive Optics (AO) later has allowed us to almost recover the telescope diffraction limit (Hardy 1998).

An alternative to these techniques is the Lucky Imaging (LI) technique which was first discussed in depth by David Fried (Fried 1978). The technique consists on taking a series of short exposures images and then selecting the best ones, i.e. those images with best Strehl ratio. As the atmospheric fluctuations are random, one expects that these fluctuations to be occasionally arranged in such a way as to produce a diffraction-limited image, being of main importance to chose a good criterion for the selection of the best images from the serie.

For medium-sized telescopes the LI technique seems to be very promising because of its low complexity and costs in terms of hardware. Furthermore, LI works with reference stars fainter than those required for the natural guide star AO technique.

The main handicap when using LI is related to the

temporal evolution of atmospheric turbulence. The decorrelation timescale of the atmosphere in the case of Lucky Imaging is about 30 milliseconds (atmospheric coherence time). Hence, exposure times employed with LI technique must be shorter than this coherence time to freeze the atmospheric evolution.

Under this conditions we obtain a distorted PSF whose shape depends on D/r_0 , the ratio between the telescope diameter (D) and the Fried parameter r_0 , which is the atmospheric coherence length. The number of speckles appearing over the PSF is roughly given by $(D/r_0)^2$ and they are randomly distributed over a circular region of the image with angular diameter λ/r_0 .

It must be taken into account that r_0 depends on the detection wavelength (or band) and consequently the number of speckles and the area covered by them are strongly dependent on the wavelength as well. In general, a good balance for high resolution observations is found observing at I-band (700-800 nm wavelength) with a 2.5 m diameter telescope.

In this paper we propose a new algorithm which takes advantage of temporal atmospheric fluctuations to uncover possible companions surrounding main stars. As we will see later, the intensity of all the pixels where a faint companion is placed will fluctuate in phase with the main star intensity along the image series. However, the pixels containing inco-

herent speckles will fluctuate in counter-phase. Hence, the finding of pixels in the image series which are fluctuating in phase with pixels gathering light from the main star is a method for a robust detection of hidden objects. This goal is accomplished by evaluating the normalized covariance (also known as the correlation function) between the main star and the rest of the image pixels along the selected LI series. The result is a kind of bi-dimensional covariance map so that the pixel intensity is the normalized covariance value. The resulting map is, obviously, normalized to unity. This technique can be applied either for extracting undetected faint companions from the background or to improve spatial resolution of images with detected companions to a main star. In this paper we define the principles of the COELI algorithm and perform an estimation of the expected contrast of one object placed in the proximities of a main star.

We test the COELI technique with a set of LI images of GJ822 taken at the I-band by the 2.2 m diameter Calar Alto telescope. Starting from the experimental LI series, we simulate a double star with different relative intensities and distances. By applying COELI we are able to detect the presence of point-like sources in regions where the primary halo dominates. In some cases, two stars as close as $1.22\lambda/D$, the telescope diffraction limit, can be resolved.

2 THE COELI ALGORITHM

The image of a point source obtained by a perfect optical system can be described by the Airy pattern. However, in ground-based telescopes where the atmosphere refractive index inhomogeneities distort the incoming wavefront, this image consists of a central peak surrounded by a number of speckles whose temporal average is commonly known as halo. The central peak is formed by the coherent part of the energy at the incoming wavefront added to an incoherent halo whilst the surrounding speckle is only due to the incoherent wavefront energy. The amount of coherent energy depends on the D/r_0 value. Hence, to increase the central peak energy it is enough to have a less aberrated incoming wavefront or to compensate it by an AO system. In this analysis, we will not consider the use of any AO system, since in that case the intensity statistics will depend on the position at the final image plane (Cagigal 2004). Hence, when imaging a distant star, the total star peak intensity would be $i_{sp} = i_{cp} + i_h$, where i_{cp} is the coherent part of the star peak intensity and i_h is the halo peak intensity. The coherent part will increase as the phase variance of the incoming wavefront σ^2 decreases ((Hardy 1998), (Cagigal 2000) and (Canales 1999)).

$$i_{cp} = e^{-\sigma^2} \quad (1)$$

whilst the halo peak intensity i_h evolves as:

$$i_h \simeq \frac{1 - e^{-\sigma^2}}{(D/r_0)^2} \quad (2)$$

The total intensity has been normalized to unity. We can see that the halo intensity behaves anticorrelated with intensity of the reference star peak.

Combining equations (1) and (2) we obtain that the height of the coherent peak is the same as that of the surrounding speckles for about $D/r_0 = 8$. The technique we

Figure 1. Example of experimental correlations in GJ822. Host star peak intensity (blue line), companion intensity (green line), the inverse of the averaged intensity in an area surrounding the central peak (red line) and the background (black line).

propose here is limited to D/r_0 values ranging under that limit. In a good observing site, a standard r_0 would be around 20 cm. If the technique of LI is applied for selecting the best frames, effective r_0 values of around 30 cm can be reached. This means that, keeping the restriction $D/r_0 = 8$, the suitable telescope diameter would have around 2.4 m of diameter.

If we apply the LI technique for obtaining a short exposure frame series, the central peak intensity of a star will evolve along the frame series in counter phase with respect to the surrounding halo intensity. This behaviour will be the same for any other object contained in the scientific image. Hence, the intensity value of those pixels containing the central peak of an astronomical object will oscillate in phase, whilst the pixels containing the speckled halo will oscillate in counter phase with respect to the peak intensities. As an example, Fig. 1 shows experimental curves for the object GJ822 corresponding to the LI experiment that is described in detail later. The host star peak intensity (blue line), the companion intensity (green line), the inverse of the averaged intensity in an area surrounding the central peak (red line) and the background (black line) are plotted once normalized for a series of LI frames. It can be seen that there exists a strong correlation between the host star and companion intensities. The correlation between the host star intensity and the inverse of the average halo intensity is also evident. The noise background remains basically constant along the frame series. It is evident that whilst all the objects fluctuate in phase along the frame series, their corresponding halo fluctuate in counter phase.

COELI basically consists of the calculation of the covariance between the main star peak intensity and the intensity of the rest of the pixels forming the image. This covariance is estimated using a series of short exposure Lucky Images. The result is a covariance map where each pixel of the map contains the value of its covariance with respect to that of the main star. The normalized covariance values will range from 1, corresponding to the reference star, to -1 for those pixels fluctuating in counter phase with respect the main star.

The algorithm is composed by the following steps:

1. To obtain accurate covariance estimate we have to re-

center the image series. For an efficient centering, we have choose the superimposition of the most intense pixel of every frame.

2. The second step is to eliminate intensity background pixels with slow spatial dependence. To accomplish that we convolved the frame series with a one-pixel radius Laplacian filter (Gonzalez 2002). This kind of filters are commonly known as point detectors.

3. After this simple preprocessing it is possible to estimate the normalized covariance (Pearson correlation) between the most intense peak of the reference star and the rest of the frame pixels along the frame series.

The procedure we followed was to calculate the normalized covariance given by the expression:

$$C[i_{sp}, i(r)] = \frac{Conv[i_{sp}, i(r)]}{\sigma_{sp} \sigma_{ir}} \quad (3)$$

and the convolution, given by:

$$Conv[i_{sp}, i(r)] = \langle i_{sp} i(r) \rangle - \langle i_{sp} \rangle \langle i(r) \rangle \quad (4)$$

where i_{sp} stands for the star peak intensity, $i(r)$ is the intensity detected at a position r from the star peak [$r = (i, j)$], σ is the standard deviation and $\langle \rangle$ is the ensemble average (frame series average).

In general, the intensity $i(r)$ is the addition of the star halo background plus noise, $i(r) = i_h(r) + i_n$. However, in those pixels where there is an object it would be necessary to add the object intensity, $i(r) = i_h(r) + i_n + i_o$. The set of values obtained by applying Eqs. (3) and (4) are saved at the corresponding (i, j) pixel position thus forming a normalized covariance map. All the preceding steps have been included in an ImageJ (<http://imagej.nih.gov/ij>) plugin named COELI.

3 COVARIANCE CONTRAST

To evaluate the capability of this tool for detecting objects it is necessary to define the contrast of the object against the background at the covariance map estimated using Eq. (3).

To accomplish this task we define the object peak intensity i_o , which is proportional to i_{sp} :

$$i_o = k_o i_{sp} \quad (5)$$

The star peak intensity (i_{sp}) can be obtained as the addition of the coherent peak intensity (i_{cp}) plus the intensity star halo at the center (i_h):

$$i_{sp} = i_{cp} + i_h \quad (6)$$

The intensity star halo is a function of the distance to the main star and it is related to i_{cp} through the expression:

$$i_h(r) = k(r)(1 - i_{cp}) \quad (7)$$

Where $k(r)$ is a function which states the halo intensity spatial dependence. Finally, the readout noise intensity is given by i_n . The normalized covariance between the central star peak, i_{sp} , and pixels inside the halo is given by:

$$C(i_{sp}, i_h + i_n) = \frac{-k\sigma_{cp}^2 + \sigma_h^2}{\sigma_{sp}\sqrt{\sigma_n^2 + \sigma_h^2}} \quad (8)$$

Where we have used Eq. (7) for obtaining an approximated expression of the covariance between i_{sp} and i_h (the explicit dependence with position r has been omitted). The covariance between the central star peak and those halo pixels containing an object will be:

$$C(i_{sp}, i_h + i_n + i_o) = \frac{(-k + k_o - k k_o)\sigma_{cp}^2 + \sigma_h^2}{\sigma_{sp}\sqrt{\sigma_o^2 + \sigma_n^2 + \sigma_h^2}} \quad (9)$$

It is interesting to note that we have considered the reading noise intensity at the star peak position negligible compared to the other noises affecting the measurement. Hence the covariance contrast between pixels containing an object and those without object for pixels inside the halo can be defined by the quotient of Eqs. (9) and (8):

$$Contrast(r) = \frac{[(-k + k_o - k k_o)\sigma_{cp}^2 + \sigma_h^2] \sqrt{\sigma_n^2 + \sigma_h^2}}{(-k\sigma_{cp}^2 + \sigma_h^2) \sqrt{\sigma_o^2 + \sigma_n^2 + \sigma_h^2}} \quad (10)$$

where σ_x^2 is the variance corresponding to the intensity i_x . We build up a covariance contrast map evaluating this expression for all the pixels of the image. As it can be seen, the contrast of one object at the halo estimated from the covariance map will depend on a series of parameters (k_o , and k) and variances (σ_{sp}^2 , σ_h^2 , σ_o^2 and σ_n^2). To estimate the value of the expected contrast we will use some approximated expressions for the different variances.

An approximated expression for the variance of the peak intensity i_{sp} has already been evaluated for astronomical images ((Yaitskova 2012) and (Gladysz 2009)) as:

$$\sigma_{sp}^2 = \frac{2}{N} \langle i_{sp} \rangle [1 - \langle i_{sp} \rangle]^2 \quad (11)$$

where N is the number of homogeneous areas in the telescope pupil and can be approximated by:

$$N = \left(\frac{D}{r_0}\right)^2 \quad (12)$$

In low light level it is necessary to include the variance due to the Poissonian detection process which is equal to the intensity mean value:

$$\sigma_{sp}^2 = \frac{2}{N} \langle i_{sp} \rangle [1 - \langle i_{sp} \rangle]^2 + \langle i_{sp} \rangle \quad (13)$$

On the other hand, we can consider that the halo variance comes from the speckle statistics as Aime et al. suggested ((Aime 2004a) and (Aime 2004b)):

$$\sigma_h^2 = i_h^2 + 2i_{cp}i_h + i_{cp} + i_h \quad (14)$$

Where we have not considered the radial dependence of the halo intensity but the variance due to the Poissonian detection process ($\sigma_p^2 = i_{cp} + i_h$) has been included.

Hence, the coherent peak variance can be obtained from the difference of the previous ones:

$$\sigma_{cp}^2 = \sigma_{sp}^2 - \sigma_h^2 \quad (15)$$

Finally, we shall assume that the object adds a constant value in one pixel and its variance is only due to the Poissonian detection process (Aime 2004a). Therefore, the variance of a companion of intensity i_o will be:

$$\sigma_o^2 = \langle i_o \rangle \quad (16)$$

The detection noise variance σ_n^2 is not estimated since the analysis will be performed as a function of its possible values.

Figure 2. Contrast (red dot-dash line), background covariance (blue solid line) and background plus object covariance (green dashed line) as a function of the detection noise variance. Black line indicates a reference contrast value of 3.

4 CONTRAST ANALYSIS

The purpose of the COELI algorithm is to increase the visibility of the objects whose light suffers a temporal oscillation which is in phase with that of the reference star. Hence, it can be applied for extracting a faint companion from a noisy background. The only limitation relies on the noise level affecting the companion intensity measurement. If the light intensity of the pixel where a companion is located is clearly dominated by the noise, the COELI algorithm will consider that in the pixel there is not any object. Hence, the detection noise reduction (cameras with small electronic noise, camera cooling, etc.) will allow faint objects to appear.

As an example, let us consider a main star with a Strehl of 0.1 corresponding to $D/r_0 = 5.5$. We estimate the contrast corresponding to a companion with an intensity equal to a half of the main star intensity ($k_o = 0.5$) placed near to the peak star, so that we can use the following approximated expression for evaluating the halo height, $i_h \propto (r_0/D)^2$ ((Hardy 1998) and (Cagigal 2000)). Figure 2 shows the covariance curves corresponding to Eqs. (8) and (9) along with that of the contrast, Eq (10) as a function of the noise variance. To evaluate the curves the approximated variances of Eqs. (12) to (16) have been used. It can be seen that the contrast (red line) tends very quickly to a value of 5 as the reading noise increases, since the covariance drops significantly in those pixels where there is not any object (blue line) whilst it keeps almost a constant value in the pixel where the object is present (green line).

5 EXPERIMENTAL CHECKING

To check the COELI technique a series of experimental measurements were completed. The observations were carried out during September of 2013 using Astralux at the 2.2 m Telescope at CAHA (Almería, Spain). This instrument incorporates a fast readout electron multiplying CCD chip (EMCCD) which is able to acquire images with a very low readout noise thanks to internal charge amplification before conversion to voltage by an output amplifier. Astralux allows the acquisition of a large number of images, typically several thousand for each target, with exposure times about

a few tens of milliseconds. Images that clearly show frozen atmospheric speckles. Conventional large integration times average all of these speckles, which yields to the usual seeing-limited point spread functions with a seeing dependent on atmospheric perturbations.

The observations were done in SDSS I band with a pixel scale of 47 mas/pixel and 7000 images were acquired, each with exposure time of 30 ms. The internal electron multiplying gain was adopted to work in the EMCCD linear regime and therefore determined by the luminosity of the target. To carry out a precise calibration of the pixel scale and camera rotation we observed the core of the globular clusters M15, and correlated the astrometric data with catalogues from Hubble Space Telescope. This provided an accurate astrometric calibration for each observing night with plate scale precision as good as 0.01 mas. The main source of astrometric error for a given star results from the uncertainty in the measurement of the barycenter which is in turn mainly determined by the signal to noise ratio. In our data this uncertainty is typically 0.1 pixel and reaches 0.2 in the faintest stars. This leads to typical errors in separation of the order of 10 mas.

To check if our algorithm can reduce the main star halo without affecting the detectability of other fainter objects we selected the 100 frames of to the object GJ822 with highest Strehl. Fig. 3 (a) shows the result of applying a Shift-And-Add algorithm (SAA) to the stack. In this figure the chosen scale allows the 30 times fainter companion to appear. When the image is scaled to normalize the peak high, the companion disappears (Fig. 3 (b)). We have already applied COELI to the same 100 frames stack obtaining the image shown in Fig. 3 (c). It can be seen that a drastic halo reduction has happened whilst the secondary object is maintained. We can also see that the photometry has been complete lost since what Fig. 3 (c) is showing is only the covariance map of the image stack with respect to the central star.

We have seen that COELI is able to improve the visibility of faint objects but, at the same time, it is very effective suppressing the speckle halo surrounding the star coherent peak, since the halo oscillates in counter phase with respect to the peak. This result shows the feasibility of the algorithm to resolve companions to main stars with angular separations close to the telescope diffraction limit.

6 RESOLUTION ANALYSIS

Our aim in this section to measure the ability of the algorithm to resolve objects with small angular separations. Let us consider two punctual sources with an angular size given by the diffraction theory. The central peak angular radius is given by $1.22\lambda/D$, where λ is the detection wavelength and D the telescope pupil diameter. The Rayleigh criterion establishes that two punctual sources are considered as resolved when the principal diffraction maximum of one image coincides with the first minimum of the other. Only perfect optical systems are able to meet the Rayleigh criterion but when an aberrating medium is introduced it may be impossible. In particular, to reach diffraction limited images in ground-based telescopes, where the light coming from the stars has to go across the atmosphere, the telescope size has to be similar to the Fried parameter. Recently,

Figure 3. (a) Plot obtained after applying SAA to the 100 best frames of the object GJ822, the grey level scale has been chosen to allow the image of the companion to be visible. (b) Same SAA image but normalized to its peak value. (c) Plot after applying COELI to the same stack.

some successful results have been reported applying AO to a medium size (1.5 m) telescope (Serabyn 2010). Our goal is to reach diffraction limited images using only a post processing technique to Lucky Images detected in a 2.2 m telescope.

We have already shown that before applying the correlation algorithm given by Eq. (3) it is necessary to improve the object contrast by passing a Laplacian filter. However, this raises a number of questions. For example, the dependence of the contrast on the radius of the applied mask or the number of mask iterations required. To answer these questions we have carried out a simulation using an experimental stack containing the 100 best frames obtained from the previously described experiment. We have duplicated it, translated it a number of pixels and multiplied it by a reducing coefficient. This modified stack is added to the unmodified one to create a double object. We have repeated the same process for different displacements and different reducing coefficient values. This simulation technique has been widely used for simulating binary stars ((Bagnuolo 1982) and (Lee 2003)).

Figure 4 shows the contrast (ratio between the object covariance and the covariance average value of the surround-

Figure 4. Contrast at the covariance map as a function of the distance between companion and main star. The companion intensity is a half of that of the main star. Contrast reached using only once the Laplacian filter with radius one (green dashed line) and two pixels (red dotted line) and that reached using twice the Laplacian filter with radius one (blue dot-dash line) and two (black solid line) pixels.

ing area) as a function of the distance in pixels between the two objects when the companion intensity is a half of that of the main star. It can be seen that when the Laplacian filter is convolved with the image stack only once before applying the covariance estimating algorithm the result is almost identical for a mask radius of one and two pixels (green and red curves, respectively). When a Laplacian filter is convolved twice with the stack the result is independent of the filter radius too (blue and yellow curves, respectively) and clearly improves the contrast obtained with only one convolution. We have already checked that a third convolution with the Laplacian mask does not improve the result obtained with only two convolutions.

As an example we have compared the covariance map obtained for the objects placed two pixels apart and with relative intensity of 0.7. The result clearly depends on the number of times the one-pixel radius Laplacian mask has been convolved before applying the covariance calculation. In Fig. 5(a), where we have convolved the Laplacian mask only once, we can see a broad object that suggests a double star (marked with a white arrow). However, the image is noisy and it is difficult to make a decision. Figure 5(b) is the same case but now the Laplacian mask has been convolved twice. The noise has been drastically reduced and we can see that the broad object we had in Fig. 5(a) is now split into two different ones.

Another interesting point is the contrast dependence on the relative intensity of the object. To check this, we have used the same stack as before for evaluating the attainable contrast for different intensity ratios. As a result of Fig. 4, to evaluate the dependence of the contrast on the relative intensity we have convolved the image stack twice with a one-pixel radius Laplacian filter before applying the covariance algorithm.

Figure 6 shows that there is a general behavior; the contrast increases when the companion intensity or the distance between objects increases. In particular, the curve shows that objects as close as two pixels, which is the diffraction limit of our telescope according to the Rayleigh criterion,

Figure 5. Covariance map for two objects placed at a distance of two pixels and with a relative intensity of 0.7. (a) The one-pixel Laplacian mask has been applied once, (b) the one-pixel Laplacian has been convolved twice

Figure 6. Contrast as a function of the companion intensity for a distance between companion and main star of two (red solid line), five (blue dashed line) and twenty pixels (green dot-dash line).

can be resolved. However, for this particular case, only when the companion intensity is larger than 0.1 times that of the main star the contrast is above the 5σ value required for detection.

Fig. 7(a) shows SAA result for a series of images containing two objects with a relative intensity of 0.6 and a relative distance of two pixels. Fig. 7(b) shows COELI result for the same image series. Fig. 7(c) shows SAA result for two objects with a relative intensity of 0.8 and a relative distance of two pixels and Fig. 7(d) shows COELI re-

Figure 7. Images corresponding to two objects placed at a distance of two pixels. The relative intensity is 0.6 for (a) (obtained by SAA) and (b) (obtained by COELI). The relative intensity is 0.8 for (c) (obtained by SAA) and (d) (obtained by COELI).

sult for the same image series. By comparing Fig. 7(a) and 7(b), we see that the companion clearly appears when using COELI whilst SAA provides a single peak. The same result is reached by comparing Fig. 7(c) and 7(d). This comparison states the advantage of using COELI for detecting objects inside the speckled halo. Relative intensity between main star and companion is a key factor for companion detection. For a relative intensity of 0.6 and a relative distance of two pixels we can clearly distinguish between the two objects (Fig. 7(b)). However, when the relative intensity is 0.8 it is difficult to distinguish them as Figure 7(d) shows.

7 CONCLUSIONS

We have introduced a new technique based on the estimation of the covariance of the intensity applied over a series of Lucky Images. This technique takes advantage of the fact that the two components of the image of an astronomical object, coherent central peak and speckled halo, have intensities that oscillate in counterphase. We have shown how to evaluate the covariance map and how different noises involved in the image detection may affect the covariance map estimate. We have checked the COELI algorithm using actual Lucky Images taken at the 2.2m CAHA telescope.

We have seen that the application of our technique allows the speckled halo to be extremely reduced, which allows very close companions to be detected. In fact, we show that the diffraction limit of the telescope has been achieved under certain conditions of relative intensity between objects.

Figure 8. The area of applicability is found between the first Airy ring ($1.22\lambda/D$) and the outer radius $1.22\lambda/r_0$ corresponding to the speckled halo limit.

Since the COELI technique cancel out the speckled halo surrounding the coherent peak of the main star, the technique is particularly effective in the area covered by the halo. Figure 8 shows a plot of the area of interest with an outer radius of $1.22\lambda/r_0$, which is equivalent to D/r_0 times the Airy ring radius. As we stated previously, the limiting D/r_0 value for applying COELI is about 8. Hence, the detection area is an annulus with inner radius of $1.22\lambda/D$ and outer radius about 8 times the inner one.

A clear limiting factor for applying this technique is the detection noise affecting the captured images. Theoretical analysis shows that the lower camera noise the better achievable contrast, as it could be expected.

We have experimentally checked that COELI detects all the successive images of the main star caused by misalignment of the optical set up. Hence, this technique could also be an effective tool for detecting set up misalignment prior to use it for capturing scientific images.

A drawback of the technique is that it does not maintain the photometry since what we obtain is not an image any more, but a map of covariance values.

Nevertheless, we consider that this technique may be considered as an interesting tool for reaching telescope diffraction limit from ground based telescopes with sizes under 2.5 m. Besides, it has the additional advantage of a much reduced cost, in particular when compared with adaptive optics.

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Chapter 6

Conclusions

The present dissertation has presented the three main contributions that have been made by the author during his doctoral period in the field of engineering applied to observational astronomy. Each contribution was associated to a different project and also to a different point in the engineering process where research can be conducted. In all the cases, the performed research has advanced the state of art of the respective fields, summing three publications in specialized scientific journals with high impact factors.

On the field of space-based astronomy, the participation in the payload of the Euclid satellite motivated the research of new methods to improve the resilience of digital circuits to the high levels of harmful radiation that are always present in the space environment. Among all the possible ways to address this problem, the strategy that was followed in this thesis was to develop new design tools capable of improving the reliability of the circuits that are most commonly used to protect the volatile memories in a spacecraft, that is, the EDAC codecs. When implemented in SRAM-based FPGA devices, it was estimated that the proposed methods can increase the reliability of the EDAC codecs in a 10% when compared to the currently available ones, thus improving the overall reliability of the systems that benefit from such optimized codecs. That way, it will be possible in future space missions to reduce the probability of temporary or permanent failures that may have an impact on the established scientific objectives or even on human safety.

The increase of reliability is perhaps the most relevant feature of the proposed algorithms, but it must not be forgotten that these algorithms also improve the rest of figures of merit of the generated circuits, namely: area utilization, maximum path delay and power consumption. The amount of improvement in these parameters can be as high as 34.48%, 17.72% and 34.37%, respectively. In any application, it is always beneficial to obtain the best possible numbers for these parameters, but in the case of space missions the need of optimization is of paramount importance because of the limited amount of available resources (mass, volume,

energy supplies, money). With recurring elements such as the EDAC codecs getting smaller, faster and more efficient, the benefits at system level become more and more remarkable. For example, the simple fact of occupying less hardware resources makes it possible to integrate more functions in a single chip, which in turn has multiple benefits like the reduction of power consumption, the smaller physical size, the simplification of electronic interfaces, etc.

Regarding the contributions in ground-based astronomy, the tasks performed during the doctoral period were linked to the AOLI instrument and the COELI algorithm. The participation in AOLI led to a fully operational adaptive optics system based on a novel wavefront sensor that will potentially increase the sky coverage of AO-enabled instruments using this newer sensor instead of the ones currently available. The specific contributions to the AOLI project made by the author may be enumerated as follows: the development of the real-time control software of the AO subsystem, the coordination of the software interfaces that are present in the AO processing pipeline, the full characterization of the AO control plant both in the laboratory and in the telescope, the operation of the instrument at the WHT in two short commissioning runs held throughout 2016 and the off-line analysis of the acquired AO-related data. In addition, the author has actively participated in the integration and assembly of the hardware side of the instrument and has later performed a preliminary analysis of the science data acquired at the WHT. All of these contributions are properly documented in the corresponding research paper that is part of the present thesis, and will be of great aid to other teams willing to replicate the technology developed for AOLI, with the TP3-WFS deserving a special mention.

Further telescope tests with AOLI will characterize the limiting magnitude of the reference AO star, which is actually the main parameter that was meant to be improved with the development of the novel TP3-WFS. Then, the performance of the complete system (including LI post-processing) will be evaluated so as to check whether the diffraction limit of the 4.2m WHT is reached as expected. The preliminary analyses performed on this matter, also included as part of this thesis, indeed indicate that the closure of the control loop has a positive impact from the point of view of the LI algorithm. Finally, as a long-term objective, AOLI is expected to be adapted to even larger telescopes such as the 10.4m Gran Telescopio Canarias (GTC), which would theoretically deliver a higher resolving power than the 6.5m James Webb Space Telescope (JWST), to be launched by the end of 2018.

On the other hand, the tests executed over the newly developed COELI algorithm have demonstrated excellent performance with actual astronomical images acquired at the 2.2m telescope in Calar Alto Observatory. It was demonstrated that COELI is capable of achieving resolutions as close as the diffraction limit in cases where it is not possible to do so with conventional LI because of the high contrast between the objects to be resolved. The contrast

that can be achieved with COELI is only limited by the number of acquired lucky images and the detection noise, which means that further improvements will be possible with the advance of image sensing technologies. It is important to highlight that, in spite of all its advantages, COELI does not completely replace LI because the produced covariance maps do not retain photometric information about the targets.

It remains to be confirmed whether the adaptive optics of AOLI would indeed be capable of providing the necessary aid to apply COELI at the larger WHT, whose aperture size makes it extremely unlikely to obtain a lucky image unless low-order AO corrections are applied. The same lucky images that would be produced as the nominal output of AOLI could be effectively processed by COELI without the need of developing any additional piece of hardware. The only reason why COELI may be incompatible with AOLI is that the closure AO control loop alters the statistical properties of the pixels that are part of the speckle patterns, more specifically their covariance along the aligned image stacks. If research in this topic finally confirms the compatibility of AOLI with COELI, such combination will deliver unprecedented performance in the search of faint companions located at very small angular separations from their host stars.

Overall, it is the author's opinion that the work carried out in this thesis represents steps forward in the field of astronomical technology, thus providing a clear contribution in the search of answers to (probably) the most important questions of science today.

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Appendix A

Journal Impact Factors

The following pages provide information about the impact factor of the two scientific journals where the three papers that support the present thesis were published.

Rank in Category: IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS

Journal Ranking

For **2015**, the journal **IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS** has an Impact Factor of **1.672**.

This table shows the ranking of this journal in its subject categories based on Impact Factor.

Category Name	Total Journals in Category	Journal Rank in Category	Quartile in Category
ENGINEERING, AEROSPACE	30	3	Q1
ENGINEERING, ELECTRICAL & ELECTRONIC	257	94	Q2
TELECOMMUNICATIONS	82	24	Q2

Category Box Plot

For **2015**, the journal **IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS** has an Impact Factor of **1.672**.

This is a box plot of the subject category or categories to which the journal has been assigned. It provides information about the distribution of journals based on Impact Factor values. It shows median, 25th and 75th percentiles, and the extreme values of the distribution.

Key

- A - ENGINEERING, AEROSPACE
- B - ENGINEERING, ELECTRICAL & ELECTRONIC
- C - TELECOMMUNICATIONS

Rank in Category: MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY

Journal Ranking

For **2015**, the journal **MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY** has an Impact Factor of **4.952**.

This table shows the ranking of this journal in its subject categories based on Impact Factor.

Category Name	Total Journals in Category	Journal Rank in Category	Quartile in Category
ASTRONOMY & ASTROPHYSICS	62	13	Q1

Category Box Plot

For **2015**, the journal **MONTHLY NOTICES OF THE ROYAL ASTRONOMICAL SOCIETY** has an Impact Factor of **4.952**.

This is a box plot of the subject category or categories to which the journal has been assigned. It provides information about the distribution of journals based on Impact Factor values. It shows median, 25th and 75th percentiles, and the extreme values of the distribution.

Key

A - ASTRONOMY & ASTROPHYSICS