Nonorthogonality Relations Between Complex Hybrid Modes: An Application for the Leaky-Wave Analysis of Laterally Shielded Top-Open Planar Transmission Lines

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***Abstract*—In this paper, the orthogonality relations between complex modes are investigated to demonstrate the coupling effect that exists along a transverse direction (z-axis) between TE^z and TM^z hybrid modes when power storage, loss, or leakage occur in the longitudinal direction (y-axis) of a transmission line. As an example, the parallel-plate waveguide modes are used to study a laterally shielded top-open microstrip line. The study of leaky waves is a very interesting application of the novel orthogonality relations derived in this paper. This study is carried out using the correct and incorrect orthogonality formulations, showing that physically valid results are obtained only when the orthogonality relations derived in this paper are properly introduced in the formulation. A physical explanation of this novel coupling effect between TE^z and TM^z hybrid modes with respect to the y-direction is given in terms of inhomogeneous plane waves. Comparisons with previous results for a laterally shielded slot-line antenna are presented to confirm the accuracy and usefulness of this novel proposed method.**

***Index Terms*—Coupled-mode analysis, electromagnetic coupling, leaky waves, planar waveguides.**

I. Introduction

THE orthogonality relations between propagation modes in guided structures are well known and are very important since, for instance, the completeness property of the set of normal modes of a guided structure allows to expand any electromagnetic field inside the geometry [1], and allows the analysis of discontinuities and feed models in many devices such as microwave filters or multiplexers [2], [3]. These relations have been studied by numerous authors, even for complex modes in planar transmission lines [4]–[6]. When the set of modes of a transmission line is used to describe its discrete spectrum, an inner product must be defined to establish their orthonormality relations. In these papers, the inner product is often described by the following power coupling equation:

$$P_{m,n}^{p,q} = \int_{x=0}^{a} \left[\vec{e}_m^{(p)}(x) \times \vec{h}_n^{(q)^*}(x)\right] \cdot \hat{z} \cdot \partial x \quad (1)$$

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where $e_m^{(p)}$ and $h_m^{(p)}$ are transverse vector modal functions with respect to the z-direction. The index m denotes the order of the modes, while p distinguishes between TE and TM polarizations with respect to the z-direction. It must be noticed that the z-axis is the "longitudinal" axis of propagation of the modes in the transverse-longitudinal formalism and, therefore, it is also the selected coupling direction in our formalism.

The above equation has been customarily applied for real modes (modes having a real propagation constant in the z-direction, k_{zm}). Many other investigations have been reported to check the orthogonality relations of complex modes (modes with a complex propagating factor in their longitudinal z-direction, k_{zm}), leading to many interesting results. Of particular relevance are the conclusions presented in [4], which confirmed that the following inner product can be used to maintain the orthogonality property between general complex modes:

$$Q_{m,n}^{p,q} = \int_{x=0}^{a} \left[\vec{e}_m^{(p)}(x) \times \vec{h}_n^{(q)}(x)\right] \cdot \hat{z} \cdot \partial x. \quad (2)$$

In this paper, we show that none of these inner products maintain the orthogonality between TE^z and TM^z parallel-plate waveguide (PPW) hybrid modes in the context of the analysis of leaky modes in laterally shielded top-open planar waveguides (see Fig. 1). This is due to the hybrid nature of these PPW modes with respect to the radiation losses direction, which is the longitudinal y-axis of the final structure, as shown in Fig. 1. In previous papers [4]–[6], the inner product (2) could maintain the orthogonality in the z-direction between complex modes since those modes were bounded in their transverse x–y-plane and, therefore, the only complex propagation factor was in their longitudinal z-direction (k_{zm}). However, a very different scenario appears in our study since PPW TE^z and TM^z leaky modes not only have a complex k_{zm} propagation factor, but also a complex k_y propagation factor, due to the unbounded z- and y-directions, as will be illustrated in Section II.

In the development of the space-domain Green's functions for this problem, the fields in the final structure are expanded

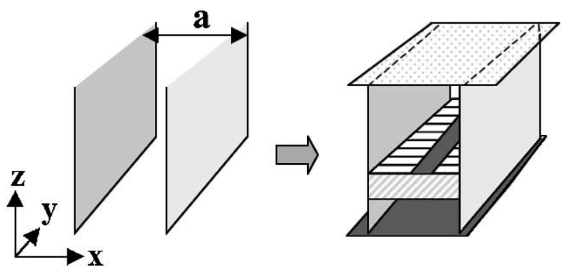

Fig. 1. Original PPW used to expand the fields of the final laterally shielded top-open suspended microstrip structure.

by means of the modes of the PPW that supports the planar waveguide. We choose the TE^z–TM^z set of modes (instead of TE^y–TM^y) in order to simplify the study of the air–dielectric interface. In Section II, it is demonstrated that these TE^z and TM^z PPW modes are coupled when power storage, loss, or leakage in the y-direction exists. Therefore, this coupling effect must be computed and introduced in the formulation of the fields' expansion for a correct analysis. In Section III, a simple but rigorous model is developed, which takes into account all these coupling effects in the relevant Green's functions.

In Section IV, results are presented for both "strong-leaky" and "weak-leaky" modes of a laterally shielded top-open suspended microstrip line (LShMSL). The "strong-leaky" mode has a much higher imaginary part in its complex axial wavenumber solution (k_y) than the "weak-leaky" LShMSL mode. Since this type of modes is complex, the influence of the coupling between TE^z and TM^z hybrid modes in the final result is shown to be important, especially for the "strong-leaky" mode. To check the importance of the coupling phenomenon, the induced currents on the strip are recovered using the magnetic fields. It is demonstrated that the right currents are only recovered when the new proposed model is used. If the proposed coupling is not taken into account, physically incoherent results between the electromagnetic fields and the induced currents occur. This coupling effect for complex modes can also be explained by analyzing the PPW TE^z and TM^z leaky modes as inhomogeneous plane waves, and studying their orthogonality relations. In Section V, a novel explanation of the coupling phenomenon is given by investigating the polarization properties of these inhomogeneous plane waves. To check the validity and accuracy of the novel method, together with its practical relevance, comparisons with previous results are presented for a laterally shielded top-open slot-line antenna in Section VI.

II. Complex PPW Modes

The PPW field modes are needed to analyze the propagation characteristics of laterally shielded top-open planar waveguides, as the LShMSL shown in Fig. 1. These are the customarily known PPW modes, but are modified to allow for a propagating factor in the y-axis. Their propagating constants can, in general, be of complex nature with phase and amplitude parts

$$k_y = \beta_y - j\alpha_y. \tag{3}$$

These PPW modal transverse fields (x–y-plane) can be analytically expressed by separately studying TE^z and TM^z plane-wave polarizations

$$\vec{e}_m^{\,\mathrm{TE}}(x,y,z) = \vec{e}_m^{\,\mathrm{TE}}(x)\cdot e^{-jk_y\cdot y}\cdot e^{-jk_{zm}\cdot z} \tag{4}$$

$$\vec{e}_m^{\,\mathrm{TM}}(x,y,z) = \vec{e}_m^{\,\mathrm{TM}}(x)\cdot e^{-jk_y\cdot y}\cdot e^{-jk_{zm}\cdot z} \tag{5}$$

$$\vec{e}_m^{\,\mathrm{TE}}(x) = \frac{-jk_y\cos(k_{xm}x)\hat{x} + k_{xm}\sin(k_{xm}x)\hat{y}}{N_m} \tag{6}$$

$$\vec{e}_m^{\,\mathrm{TM}}(x) = \frac{+k_{xm}\cos(k_{xm}x)\hat{x} - jk_y\sin(k_{xm}x)\hat{y}}{N_m} \tag{7}$$

$$\vec{h}_m^{(p)}(x) = \hat{z}\times\vec{e}_m^{\,(p)}(x) \tag{8}$$

where N_m are the normalization factors of the modes, and the propagation constant in the x-axis is determined by the lateral electric-wall standing-wave condition

$$k_{xm} = m\frac{\pi}{a}\ \text{(rad/m)}. \tag{9}$$

We choose the transverse-longitudinal notation with respect to the z-axis to easily obtain an equivalent modal transmission line in this multilayered stratification direction, and also easily model the top open boundary and bottom metal plate [7]–[9].

Each mode is characterized by the modal index m $(0,1,\ldots,\infty)$ and the polarization index p (TE^z or TM^z), and has a z-propagation factor in a medium with relative dielectric permittivity ε_r given by [10]

$$k_{zm} = \sqrt{k_o^2\cdot\varepsilon_r - k_{xm}^2 - k_y^2} = \beta_{zm} + j\alpha_{zm} \tag{10}$$

where k_y is the unknown propagation factor of the LShMSL complex mode introduced in (3). This LShMSL mode will be expanded by the PPW modes (as will be illustrated in Section III) and, therefore, k_y is shared by all the set of constituent PPW modes. However, it is first necessary to study the power orthogonality properties of these PPW modes in the selected propagating z-direction. By applying (1) to (6)–(8), it can be obtained that

$$P_{m,n}^{p,q} = \begin{cases} \mathbf{1}, & \text{if } m = n \text{ and } p = q \\ \mathbf{0}, & \text{if } m \neq n \text{ and } p \neq q \\ C_m, & \text{if } m = n \text{ and } p \neq q \end{cases} \tag{11}$$

where it is found that a coupling C_m coefficient appears for PPW modes with same space–harmonic indexes ($m = n$), but different polarizations ($p \neq q$). Straightforward derivations yield to

$$C_m = \frac{+k_{xm}\cdot 2\alpha_y}{|Nm|^2}\cdot\frac{a}{2}\delta_m \quad \delta_m = \begin{cases} 2, & m = 0 \\ 1, & m \neq 0. \end{cases} \tag{12}$$

This power coupling is nonzero only when k_y has an imaginary part, i.e., when power storage, loss, or leakage exists in the y-propagation direction for an LShMSL mode. These types of modes are important since they include evanescent (below cutoff and complex) modes, waves propagating in lossy materials, and leaky waves.

It might be thought that the inner product (2) described in [4] can decouple TE^z and TM^z polarizations for this type of LShMSL modes with an imaginary part in their propagation constant (k_y), but the following results are obtained when (2) is applied to the PPW transverse modes (6)–(8):

$$Q_{m,n}^{p,q} = \begin{cases} \mathbf{1}, & \text{if } m = n \text{ and } p = q \\ \mathbf{0}, & \text{if } m \neq n \text{ and } p \neq q \\ R_m, & \text{if } m = n \text{ and } p \neq q \end{cases} \tag{13}$$

where a new polarization-coupling coefficient is obtained as follows:

$$R_m = \frac{-j2k_{xm} \cdot (\beta_y - j\alpha_y)}{N_m^2} \cdot \frac{a}{2}\delta_m. \tag{14}$$

Not only the coupling phenomenon for complex modes still appears, but also it is extended for real modes (modes with a real k_y propagation constant) since now both the real and imaginary parts of k_y are involved in R_m. As mentioned in Section I, the z-axis is the selected propagation direction where the coupling between PPW modes is defined using (1) and (2). However, these PPW modes are hybrid with respect to the axial y-direction of the final waveguide. This situation leads to a transverse x–y-plane for the PPW modes, which is not bounded in its y-direction for the type of structures studied in this paper, as can be seen in Fig. 1. As a result, not only the leaky PPW modes have a complex k_{zm} wavenumber (10), but they also have a complex k_y constant (3). This scenario is quite different from that of the complex modes studied in waveguides with closed-transverse boundaries [4]–[6], where the only complex propagation constant occurred in the longitudinal z-direction (k_{zm}). This difference means that none of the inner products presented in (1) and (2) can maintain the orthogonality properties, and the coupling between PPW modes always appears for the case of modes with a complex k_y constant.

From (13) and (14) it is shown that the inner product (2) is not convenient for our purposes since it also extends the coupling formulation to real modes and the power coupling physical meaning of (1) is lost. For this reason, we choose the first inner product (1) to develop the orthonormality relations in our formalism. The relations derived in (11) must, therefore, be taken into account in our formulation, with special emphasis in the novel coupling coefficient between TE^z–TM^z PPW modes C_m, which is shown in (12).

III. Green's Functions for Laterally Shielded Top-Open Suspended Microstrip

The space-domain Green's functions for an electric source inside a multilayered-multiconductor structure have been developed in [7] and [8]. Following the same theory, the electric and magnetic transverse fields (x–y-plane) produced by an elementary electric current located inside the LShMSL at $x = x', z = 0$ can be expanded by the following PPW transverse-modal series:

$$\vec{E}_t(x'; x, y, z) = \sum_{p=1}^{2}\sum_{m=0}^{\infty} \vec{j}_m^{(p)}(x') \cdot V_m^{(p)}(z) \cdot \vec{e}_m^{(p)}(x) \cdot e^{-jk_y y} \tag{15}$$

$$\vec{H}_t(x'; x, y, z) = \sum_{p=1}^{2}\sum_{m=0}^{\infty} \vec{j}_m^{(p)}(x') \cdot I_m^{(p)}(z) \cdot \vec{h}_m^{(p)}(x) \cdot e^{-jk_y y}. \tag{16}$$

The z-dependent scalar functions—$V_m(z)$ and $I_m(z)$—of the above expressions can be obtained from Maxwell's transverse fields equations, leading to a set of modal equivalent transmission lines in the stratification z-axis (see Fig. 2). Following the same procedure as described in [7] and [8], but taking into account the novel coupling coefficient C_m between PPW modes (12), the following original equations are obtained:

$$\frac{\partial}{\partial z}\left[V_m^{\mathrm{TE}}(z) + C_m \cdot V_m^{\mathrm{TM}}(z)\right] = -j \cdot k_{zm}\left[Z_{0m}^{\mathrm{TE}} \cdot I_m^{\mathrm{TE}}(z) + C_m \cdot Z_{0m}^{\mathrm{TM}} \cdot I_m^{\mathrm{TM}}(z)\right] \tag{17}$$

$$\frac{\partial}{\partial z}\left[I_m^{\mathrm{TE}}(z) + C_m \cdot I_m^{\mathrm{TM}}(z)\right] = -j \cdot k_{zm}\left[\frac{V_m^{\mathrm{TE}}(z)}{Z_{0m}^{\mathrm{TE}}} + C_m \cdot \frac{V_m^{\mathrm{TM}}(z)}{Z_{0m}^{\mathrm{TM}}}\right] - \left[j_m^{\mathrm{TE}}(z) + C_m \cdot j_m^{\mathrm{TM}}(z)\right] \tag{18}$$

$$\frac{\partial}{\partial z}\left[V_m^{\mathrm{TM}}(z) + C_m \cdot V_m^{\mathrm{TE}}(z)\right] = -j \cdot k_{zm}\left[Z_{0m}^{\mathrm{TM}} \cdot I_m^{\mathrm{TM}}(z) + C_m \cdot Z_{0m}^{\mathrm{TE}} \cdot I_m^{\mathrm{TE}}(z)\right] \tag{19}$$

$$\frac{\partial}{\partial z}\left[I_m^{\mathrm{TM}}(z) + C_m \cdot I_m^{\mathrm{TE}}(z)\right] = -j \cdot k_{zm}\left[\frac{V_m^{\mathrm{TM}}(z)}{Z_{0m}^{\mathrm{TM}}} + C_m \cdot \frac{V_m^{\mathrm{TE}}(z)}{Z_{0m}^{\mathrm{TE}}}\right] - \left[j_m^{\mathrm{TM}}(z) + C_m \cdot j_m^{\mathrm{TE}}(z)\right] \tag{20}$$

where the characteristic impedances of each PPW mode in the z-direction are described by the following well-known equations for both TE^z and TM^z polarizations:

$$Z_{0m}^{\mathrm{TE}} = \frac{\omega\mu}{k_{zm}} \quad Z_{0m}^{\mathrm{TM}} = \frac{k_{zm}}{\omega\varepsilon}. \tag{21}$$

Equations (17)–(20) form a set of two coupled systems, each one corresponding to the coupled TE^z and TM^z equivalent transmission lines. In order to decouple them, we multiply (19) by C_m and subtract (17). Following a similar procedure with (18) and (20), the following system of differential equations is obtained for the TE^z case:

$$\frac{\partial}{\partial z}\left[V_m^{\mathrm{TE}}(z) - C_m^2 \cdot V_m^{\mathrm{TE}}(z)\right] = -j \cdot k_{zm} \cdot Z_{0m}^{\mathrm{TE}}\left[I_m^{\mathrm{TE}}(z) - C_m^2 \cdot I_m^{\mathrm{TE}}(z)\right] \tag{22}$$

$$\frac{\partial}{\partial z}\left[I_m^{\mathrm{TE}}(z) - C_m^2 \cdot I_m^{\mathrm{TE}}(z)\right] = -j \cdot \frac{k_{zm}}{Z_{0m}^{\mathrm{TE}}}\left[V_m^{\mathrm{TE}}(z) - C_m^2 \cdot V_m^{\mathrm{TE}}(z)\right] - \left[j_m^{\mathrm{TE}}(z) - C_m^2 \cdot j_m^{\mathrm{TE}}(z)\right]. \tag{23}$$

Proceeding in a similar way, an analogous system is obtained, but for the TM^z PPW modes. These two systems are decoupled in the sense that the voltage and current functions can be solved separately for each polarization as follows:

$$V_{m\mathrm{TOTAL}}^{(p)}(z) = V_m^{(p)}(z) - C_m^2 \cdot V_m^{(p)}(z) \tag{24}$$

$$I_{m\mathrm{TOTAL}}^{(p)}(z) = I_m^{(p)}(z) - C_m^2 \cdot I_m^{(p)}(z). \tag{25}$$

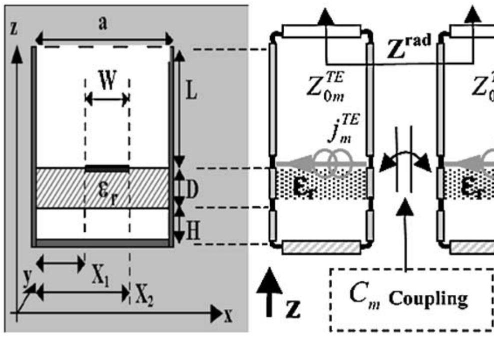

Fig. 2. Equivalent coupled transmission lines for LShMSL.

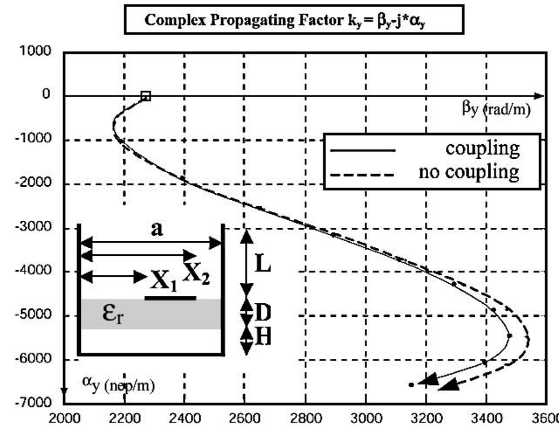

Fig. 3. Strong leaky-mode search with and without coupling effects.

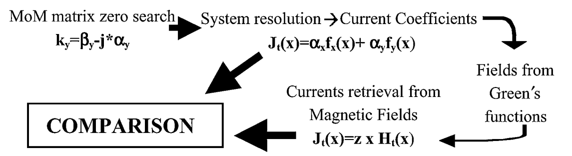

Fig. 4. Validity checking procedure using the currents induced on the strip.

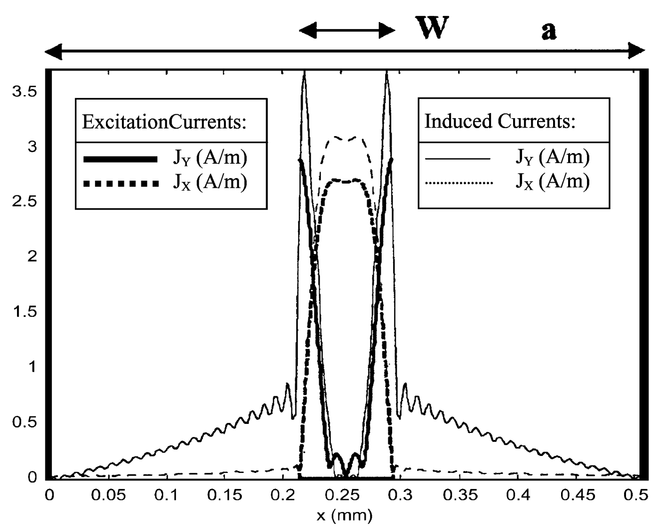

Fig. 5. Currents comparison neglecting coupling effects for the strong leaky mode.

However, the equivalent shunt current sources shown in Fig. 2 must also be computed. Following the procedure described in [7] and [8], we obtain

$$\vec{j}_m^{(p)}(x') = \frac{\vec{e}_m^{(p)^*}(x') - C_m \cdot \vec{e}_m^{(q)^*}(x')}{1 - C_m^2} \qquad \begin{cases} p = \mathrm{TE}^z \to q = \mathrm{TM}^z \\ q = \mathrm{TM}^z \to p = \mathrm{TE}^z \\ m = 0,1,2\ldots . \end{cases} \tag{26}$$

As can be seen, the source of the p-equivalent transmission line suffers the coupling of the q-polarization source given by C_m. This situation can be represented by Fig. 2, where the equivalent modal TE^z and TM^z transmission lines are shown for the LShMSL. From these equivalent modal-coupled networks, it is easy to find the equivalent voltage and current distribution along the z-axis by just applying the classical transmission-line theory. The electric- and magnetic-field amplitude functions—$V_m(z)$ and $I_m(z)$—involved in (15) and (16) can, thus, be obtained.

IV. Results for Leaky Waves

In order to investigate the coupling effect described in the previous sections, two leaky-wave modes of the top-open laterally shielded microstrip transmission line of Fig. 1 are studied, namely, a "strong-leaky" and "weak-leaky" mode. These types of modes have a complex propagation constant in the longitudinal axis of the open waveguide with a negative imaginary part due to the radiation losses, as shown in (3). The "strong-leaky" mode has a higher imaginary part α_y in its k_y solution due to either a strong leakage effect or to a reactive behavior (energy reflection) when the mode is below cutoff. The "weak-leaky" mode exhibits a much smaller attenuation constant α_y. Both modes were found at the frequency of 470 GHz for the LShMSL with the following geometrical parameters according to Fig. 2:

- $a = 0.508$ mm;
- $L = 0.254$ mm;
- $D = H = 0.127$ mm;
- $X_1 = 0.214$ mm;
- $X_2 = 0.294$ mm;
- $\varepsilon_r = 5$.

In Fig. 3, the "strong-leaky" wave is searched by following the procedure described in [9]. As can be seen, two different solutions are found by taking into account or neglecting the novel polarization coupling coefficient. In order to know which solution is the correct one, the "currents induced" on the metal strip are recovered with the use of the boundary condition for the tangential component of the magnetic field. The whole procedure is as follows: the currents on the metal strip are found by solving an electric field integral equation (EFIE) homogeneous system with the method of moments (MoM). Once the system is solved, what we call "excitation currents" can be expanded and, from them, the fields can be derived. Moreover, the "induced current" density in the strip can be computed from the magnetic fields boundary condition, and are then compared with the "excitation currents." Physically valid results are obtained only when both currents are equal. The whole proposed checking procedure leads to the flowchart shown in Fig. 4.

In Fig. 5, these two current densities are obtained for the case in which the coupling coefficients are neglected in the study of

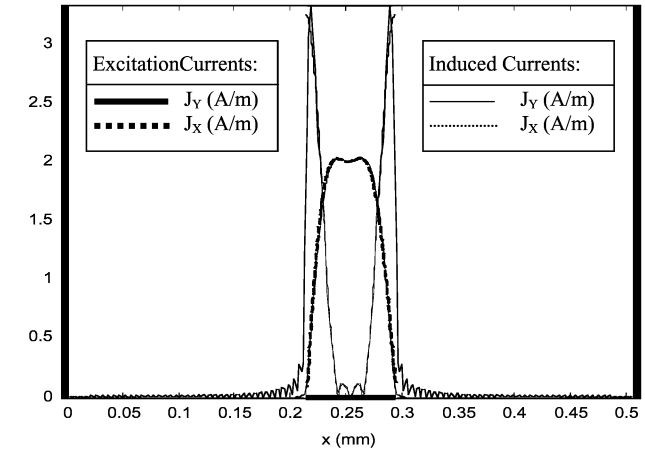

Fig. 6. Currents comparison with coupling effects for strong leaky mode.

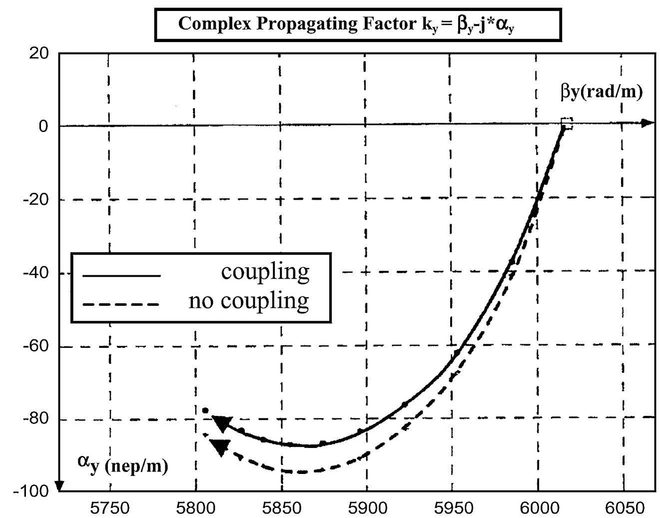

Fig. 7. Search of a weak leaky mode taking and not taking into account the coupling effects.

the "strong leaky" mode. As can be seen, both currents are very different and, in addition, a nonzero electric current appears outside the metal strip of width "a," which is physically impossible. Next, with the original coupling model just derived in this paper, the results obtained are shown in Fig. 6. As can be seen, the "excitation currents" computed through the MoM agree very well with the "induced currents" obtained with the magnetic-field boundary condition. In particular, we can observe that the magnetic transverse field is continuous across the dielectric interface, leading to a zero electric current outside the metal strip, which confirm the physical correctness of the method proposed.

In Fig. 7, complex propagation constant of a "weak leaky" mode is found. The value of α_y is relatively much smaller and the influence of the coupling phenomenon in the complex k_y solution is less significant. In fact, for this case, no difference in the final computed currents is observed when taking or not taking into account the coupling coefficients. In Fig. 8, the "induced" and "excitation" currents are represented without coupling effects, obtaining the same results when introducing the coupling coefficients in the transmission-line model. This is due to the fact that the leakage part of the solution is small, therefore, leading to small coupling C_m coefficients. However, the error introduced in the attenuation constant α_y can be significant when the coupling coefficient is not taken into account, as shown in Fig. 7.

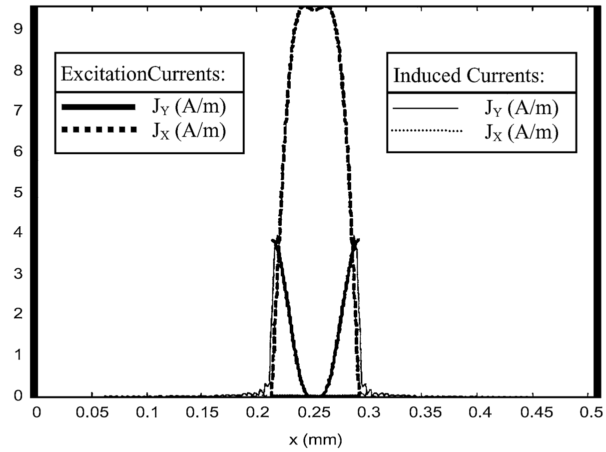

Fig. 8. Currents comparison without coupling effects for weak leaky mode (same results with coupling).

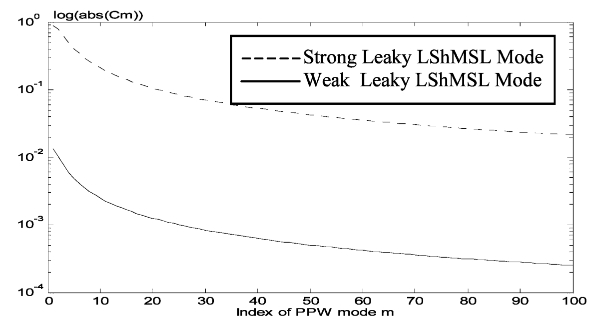

Fig. 9. Coupling modal coefficients C_m for the two leaky modes.

To illustrate this last conclusion, the modal coefficients C_m calculated from (12) are represented in Fig. 9 in logarithmic scale for both the "strong" and "weak" leaky LShMSL modes versus the index m of the PPW modes. It can be seen that the new coupling coefficient C_m is only important for the "strong leaky" mode, and it is much less important for the "weak leaky" mode. It can be also observed in this figure how the absolute value of C_m coefficients decreases as the order m of the PPW modes increases. This behavior will be clarified in Section V by studying the PPW leaky modes as inhomogeneous plane waves, together with an explanation of the nature of the coupling phenomenon.

V. Explanation of Coupling Effect

It is well known that evanescent and leaky modes are two different types of inhomogeneous waves [11], [12]. The properties of nonuniform plane waves have been widely studied [13], including their complex propagation vector, which, in our case, can be written from (3), (9), and (10) as

$$\begin{aligned}\vec{k} &= k_{xm}\hat{x} + k_y\hat{y} + k_{zm}\hat{z}\\ &= \vec{\beta} - j\vec{\alpha}\\ &= (k_{xm}\hat{x} + \beta_y\hat{y} + \beta_{zm}\hat{z}) - j(\alpha_y\hat{y} - \alpha_{zm}\hat{z}). \qquad (27)\end{aligned}$$

The PPW modes described in (4)–(10) can be decomposed into the sum of two generally inhomogeneous plane waves, as explained in [14]. The electromagnetic fields of each PPW TE^z and TM^z plane wave can, therefore, be expressed as follows:

$$\vec{E}^{(p)} = \vec{E}_0^{(p)} \cdot e^{-j\vec{k}\cdot\vec{r}} = \vec{E}_0^{(p)} \cdot e^{-\vec{\alpha}\cdot\vec{r}} \cdot e^{-j\vec{\beta}\cdot\vec{r}} \tag{28}$$

$$\vec{H}^{(p)} = \vec{H}_0^{(p)} \cdot e^{-j\vec{k}\cdot\vec{r}} = \vec{H}_0^{(p)} \cdot e^{-\vec{\alpha}\cdot\vec{r}} \cdot e^{-j\vec{\beta}\cdot\vec{r}} \tag{29}$$

$$\vec{E}_0^{\mathrm{TE}} = [-jk_y\hat{x} + jk_{xm}\hat{y}]\frac{\omega\mu}{k_{zm}} \tag{30}$$

$$\vec{H}_0^{\mathrm{TE}} = \left[-jk_{xm}\hat{x} + jk_y\hat{y} + j + \frac{k_{xm}^2 + k_y^2}{k_{zm}}\hat{z}\right] \tag{31}$$

$$\vec{E}_0^{\mathrm{TM}} = \left[+k_{xm}\hat{x} + k_y\hat{y} - \frac{k_{xm}^2 + k_y^2}{k_{zm}}\hat{z}\right] \tag{32}$$

$$\vec{H}_0^{\mathrm{TM}} = [-k_y\hat{x} + k_{xm}\hat{y}]\frac{\omega\varepsilon}{k_{zm}}. \tag{33}$$

It is of much interest to note that these plane waves present linear polarization in the case when they are not leaky (k_y real). In this situation, it can be seen how TE^z and TM^z plane waves with the same harmonic index m have orthogonal linearly polarized transverse fields (with respect to the z-axis), therefore, leading to a null coupling C_m coefficient. This orthogonality relation in the z-axis can also be derived from the following basic power-coupling calculation obtained from the cross-Poynting vector in the z-direction for two different PPW plane waves. It can be observed that only when k_y is real, the transverse fields are orthogonal, leading to a null cross-Poynting vector in z as follows:

$$\begin{aligned}\left(\vec{E}_0^{\mathrm{TE}} \times \vec{H}_0^{\mathrm{TM}^*}\right)\cdot\hat{z} &= \left(\vec{E}_0^{\mathrm{TM}} \times \vec{H}_0^{\mathrm{TE}^*}\right)\hat{z} \\ &= k_{xm}\cdot k_y^* - k_{xm}\cdot k_y \\ &\cdot \begin{cases} = 0, & k_y \text{ real} \\ \neq 0, & k_y \text{ complex.}\end{cases}\end{aligned} \tag{34}$$

For the case of inhomogeneous leaky PPW plane waves, (30)–(33) show that TE^z and TM^z polarizations change from cross-linear to elliptical as α_y becomes greater. These elliptical inhomogeneous plane waves are not orthogonal since (34) is not zero for k_y complex. This indicates that the two elliptical polarized waves do not have opposite rotation directions and equal axial ratio, and a coupling of power between them is created in the z-direction. This explains in an intuitive fashion the TE^z and TM^z modes coupling phenomenon for hybrid-complex modes derived in this paper: they change from linearly cross-polarized to nonorthogonal elliptic-polarized plane waves. This polarization change process is sketched in Fig. 10, where transverse electric fields for TE^z and TM^z plane waves are presented. It must be noticed again that the coupling between these waves is measured in the z-direction due to the mathematical formalism of our method, as explained in Sections I and II. Therefore, the transverse fields E_{0t} and H_{0t} of Fig. 10 are the x–y-plane components of (30)–(34).

It can also be checked from (30)–(34) how, as the order (m) of the PPW modes increases, TE^z and TM^z inhomogeneous plane waves tend to be linearly cross-polarized again, leading to a decreasing C_m coupling coefficient, as can be seen in Fig. 9. This phenomenon can also be explained since higher order PPW modes tend to propagate axially along the waveguide, leading to none radiation, as demonstrated in [14]. Therefore, as the order m of PPW modes increases they become "less leaky," and the

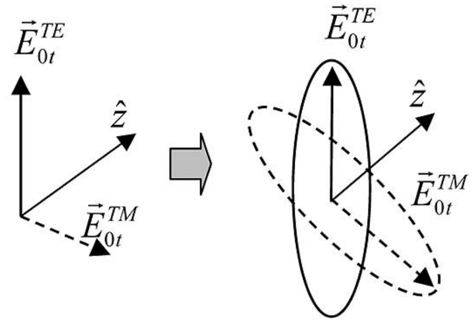

Fig. 10. PPW TE^z–TM^z transverse electric-field polarization change.

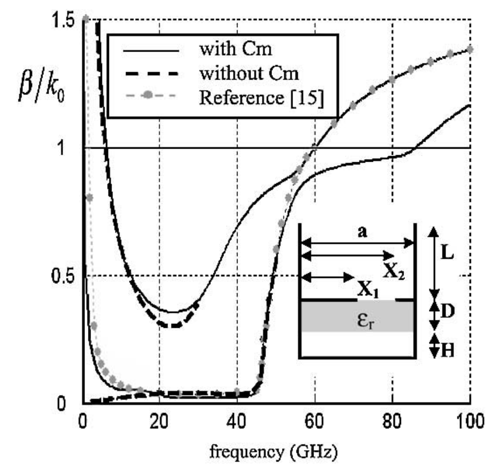

Fig. 11. Comparison with previous results for a slot-line antenna. Our method detects two modes (desired and channel-guide leaky modes), while the method in [15] only detects one mode.

coupling C_m coefficient between TE^z and TM^z polarizations tends to zero (see Fig. 9). It is also interesting to note that the coupling coefficient C_m defined in (12) is zero for $m = 0$. This is an evident result since the TM^z PPW mode for $m = 0$ does not exist. This is also why other analysis techniques based on single-mode equivalent networks ($m = 0$), as derived in [15], do not need to take into account this coupling phenomenon.

VI. Validation Results

In the last sections, the necessity of introducing the novel coupling coefficient to obtain physically valid results has been established. Here, we are going to compare our results with previous ones to check the accuracy and usefulness of this new proposed method in the analysis of complex modes in practical laterally shielded open planar transmission lines. In Fig. 11, we present the results for the millimeter-wave laterally shielded slot-line leaky-wave antenna studied by Lampariello *et al.* in [15]. The geometrical parameters according to our nomenclature (see inset of Fig. 11) are the following:

- $a = 2.2$ mm;
- $X_1 = 1.1$ mm;
- $X_2 = 2.1$ mm;
- $\varepsilon_r = 2.56$ mm;
- $D = 1.59$ mm;
- $H = 0$ mm;
- $L = 2$ mm.

The authors of [15] used a single-mode transverse equivalent tee network to model the slit discontinuity in a simple closed form. In this way, a transverse resonance procedure was applied for only the fundamental mode (a TEM wave traveling in our x–y-plane, which is the TE^z $m = 0$ PPW mode in our formulation). No full-wave analysis was performed and, therefore, no modal-coupling computation was needed. However, approximate equivalent circuits should be derived to model the discontinuities, and higher order modes effects cannot be accurately taken into account in all frequency or structural ranges. The full-wave multimode technique proposed in this paper can theoretically deal with any LShMSL leaky-wave mode, including higher order mode interaction, no matter the geometrical parameters or the frequency values involved.

A leaky-wave mode with practical interest for the design of an antenna (since the phase and amplitude parts of its complex propagation constant could be controlled separately) was studied. This desired leaky mode becomes purely bounded above a given frequency near 60 GHz, and it is below cutoff for frequencies under 45 GHz. In Fig. 11, we can see the dispersion curves obtained with our method, taking into account the TE^z–TM^z coupling phenomenon (continuous line) and neglecting it (dotted line). As expected, the effects of the coupling coefficient C_m are important only below cutoff, where the leakage constant α_y is large enough. Our results are in very good agreement with those obtained in [15] for this desired leaky mode at all frequency ranges, which confirm the accuracy of the proposed method. Besides, below cutoff it can be seen how it is necessary in our method to compute the coefficients C_m to obtain accurate results. In fact, we observe in Fig. 11 that the results without C_m (dotted line) are wrong for frequencies below 15 GHz for the desired leaky mode.

Moreover, another leaky mode is present in this structure, namely, a "channel-guide" leaky mode. This type of leaky mode appears between the parallel plates and has been studied for a large variety of laterally shielded leaky-wave antennas [16]. Although the "channel-guide" modes are not desired, in practice, their propagation properties must be studied since they can couple with the wanted leaky mode, as can be seen in Fig. 11 between 50–60 GHz. In [15], the length of the parallel-plate stub (L) was supposed to be infinite and, therefore, no channel mode was found. Our full-wave method allows the study of these "channel-guide" modes in an accurate and simple way. However, these modes usually have a strong leakage constant α_y. Consequently, in our formulation, the C_m coefficients play an important role, as can be seen in the results of Fig. 11 in the range from 15 to 30 GHz for this higher order undesired leaky mode.

We can conclude that the study of complex modes in open structures can be easily treated with the full-wave method proposed. With the introduction of a simple coupling coefficient between TE^z and TM^z modes, complex solutions with a high imaginary part in their propagation constant can be accurately computed. Among these types of modes, we can find "channel-guide" leaky modes and evanescent modes below cutoff. It is very important to precisely study their propagation features since "channel-guide" leaky modes can couple to desired leaky modes [16], and modes below cutoff are essential to characterize discontinuities [2], [3].

VII. Conclusion

In this paper, it has been demonstrated that complex hybrid polarizations TE^z and TM^z are not orthogonal in the z-direction when storage, losses, or power leakage exist in the axial y-direction. A suitable formulation has been developed in order to take into account for this polarization coupling effect in the Green's functions of multilayered shielded structure, leading to an original and precise transmission-line equivalent model. This correction has been checked by studying two leaky-wave modes in a laterally shielded top-open suspended microstrip waveguide with different leakage properties. A more intuitive, but rigorous and novel explanation of this coupling phenomenon has been given by analyzing the polarization properties of the inhomogeneous TE^z and TM^z plane leaky waves, in which any PPW can be decomposed. It has been identified that the coupling occurs due to the elliptic polarization nature of the inhomogeneous plane waves associated with leakage. Finally, the necessity of taking into account the influence of the coupling between modes has been shown if accurate results are to be obtained when calculating the propagation characteristics of complex waves with a large imaginary part in their propagating constants.

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