

Numerical simulation of the manoeuvrability of an underwater vehicle

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Resumen

A six degree of freedom (6-DOF) nonlinear mathematical model for a submarine is proposed. Then, the problem of manoeuvrability control for this vehicle is formulated as an unconstrained optimal control problem which is solved by using a gradient descent method. Simulation results presented are rooted by two requirements which are of a major importance in naval industry, and they are compared to a classical linear model.

1. Introduction

In the development of a naval architecture tool for the guidance and autopilot of a submarine is important to choose both an accurate mathematical model for the equations of motion and a suitable control strategy. In spite of the practical importance of this matter, there are not so much works available in the literature, probably because of its applications in military technology. In fact, most of the results which are used by naval industry are based on simplified (meanly linear) versions of more accurate nonlinear models or on nonlinear models with lower degrees of freedom. Nowadays, the following two challenges deserve an special attention at the practical level:

- (i) optimality of sizing rudders (decision to be taken during the design state), and
- (ii) design of an automatic submarine autopilot to be able to guarantee smooth manoeuvres as physically admissible in order to reduce hydraulic oil consumption and so reduce noise generation.

Having this in mind, firstly, a vehicle dynamics model based on a combination of theory and empirical data is proposed in this work. Indeed, taking as a starting point the standard DTNSRDC (David Taylor Naval Ship Research and Development Center) nonlinear equations of motion for an underwater vehicle [5, 7, 10] and adapting these general equations to the particular characteristics of a prototype developed by the company Navantia S.A. Cartagena Shipyard (Spain), we propose a mathematical model composed of a highly coupled and nonlinear system of ordinary differential equations (ODEs) with six degrees of freedom.

The second aim of this work is to design numerically an appropriate nonlinear control system which may be used for simulation of the usual depth and course keeping manoeuvres (among others) as well as to provide some useful information to deal with the above two points (i)-(ii). To this end, we describe the controller design as an unconstrained optimal control problem and implement a gradient descent method to solve it. This is an iterative algorithm which requires a good initialization to get a rapid convergence. To achieve a good initialization and also for comparison purposes, we derive from the original nonlinear model a simplified (4-DOF) linear mathematical model and solve it by using classical results on controllability theory [8].

Finally, numerical results are given to show the resulting nonlinear manoeuvres comparing to the linear ones.

2. Vehicle modeling

The three-dimensional equations of motion for a marine vehicle are usually described by using two coordinate frames: the moving coordinate frame which is fixed to the vehicle and is called the *body-fixed* reference frame, and the earth-fixed reference frame which is called the *world* reference frame. The origin of body coordinates is at the half point along the symmetric longitudinal axis. Typically, this point is not so far from the center of buoyancy (CB) and from the center of gravity (CG). The body axes are longitudinal pointing in the nominal forward direction of the vehicle, lateral pointing through the right hand side of the level vehicle, and downward through the nominal bottom of the vehicle. The world coordinate system is defined by three orthogonal axes originating at an arbitrary local point at the ocean surface. North corresponds to x -axis, East corresponds to y -axis and increasing depth corresponds to z -axis. The position and orientation of the vehicle are described in the world system while the linear and angular velocities are expressed in the body-fixed coordinate system. The kinematic equations which relate the body-fixed reference frame to the world reference system can be found in [7], [9].

The dynamic equations of motion can be derived from a Newton-Euler formulation which is based on Newton's second law, analyzing the forces and movements acting in each of the submarine's axis one can obtain those equations. The development of the functional form of the hydrodynamic forces and moments is by now well-known [5, 7, 10]. Specific values of the particular hydrodynamic coefficients which appear in the equations depend on the specific vehicle and, therefore, would require modification if applied to other vehicles. The values of the coefficients used in this work have been experimentally obtained in deeply submerged conditions by using a scaling model. A detailed study of the mentioned model as well as several captive manoeuvring tests appear in the internal

report [2]. Those equations include the functional form of mechanical and hydrodynamic added mass, terms like:

$$\frac{\rho}{2}l^3[X'_u\dot{u} + X'_{vr}vr + X'_{wq}wq],$$

coriolis and centripetal forces, terms like:

$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})],$$

damping forces and moments due to skin friction and vortex shedding, terms like:

$$\frac{\rho}{2}l^4[X'_{qq}q^2 + X'_{rr}r^2 + X'_{rp}rp + X'_{q|q}|q|q|] + \frac{\rho}{2}l^2[X'_{uu}u^2 + X'_{vv}v^2 + X'_{ww}w^2 + X'_{w|w}|w|w|],$$

restoring (i.e., gravitation and buoyant) forces and moments, terms like:

$$(W - B) \sin(\theta),$$

propeller forces and moments, terms like:

$$\left(K_{T0} + K_{TJ} \frac{(1 - w_f)}{nD} u + K_{TJ^2} \frac{(1 - w_f)^2}{(nD)^2} u^2 + K_{TJ^3} \frac{(1 - w_f)^3}{(nD)^3} u^3 + K_{TJ^4} \frac{(1 - w_f)^4}{(nD)^4} u^4 \right) n^2 D^4,$$

and, rudders control forces, terms like:

$$\frac{\rho}{2}l^2[X'_{\delta_r \delta_r} u^2 \delta_r^2 + X'_{\delta_s \delta_s} u^2 \delta_s^2 + X'_{\delta_b \delta_b} u^2 \delta_b^2].$$

We refer to [7, 11] for a detailed description and analysis of these forces and moments, but for a better understanding of the problem we indicate here that the state variable is denoted by

$$\mathbf{x}(t) = [x(t), y(t), z(t), \phi(t), \theta(t), \psi(t), u(t), v(t), w(t), p(t), q(t), r(t)].$$

The model for a submarine proposed by us is a set of 12 first order ODEs, that is a six degree of freedom (6-DOF) model, which can be found in [9]. There the first 6 equations, called kinematic equations, relate the body fixed reference frame to the world reference system, where the second member or the ODEs is mainly composed for trigonometrical functions of the submarine position variables. The other 6 equations, called dynamical equations, are related to the dynamical analysis of the movement of the submarine, where we can find terms like the previous mentioned.

For the specific vehicle considered in this work, control inputs come in the form of thruster forces and moments and may be expressed as a vector

$$\mathbf{u}(t) = [\delta_b(t), \delta_s(t), \delta_r(t)], \tag{1}$$

where δ_b is deflection of bow plane, δ_s is deflection of stern plane, and δ_r is deflection of rudder.

From a pure mathematical point of view, it is important to point out the following facts in the proposed model:

- The high nonlinear character of the equations and the high order of the system (12 equations).

- The lack of differentiability of the system which is caused for several terms involving non-differentiable functions such as the absolute value and squared roots.
- The controls appear in nonlinear (quadratic) form.

Because of these three difficulties, it is certainly hard to study the model at the pure mathematical level (existence, uniqueness of solutions, etc.). Indeed, very few is known in the mathematical literature concerning to the control of non-differentiable systems with controls appearing in nonlinear form. We refer to [3] for a recent related paper. Since this work is mainly addressed to numerical simulations and application to a real-life engineering problem, we do not enter here in the above theoretical question which, however, will be analyzed in a future work.

Both kinematic equations and dynamic equations are the equations of motion for the submarine. In a compact form they can be written as:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)). \quad (2)$$

3. Nonlinear controller design

The control problem can be formulated as follows: given an initial state $\mathbf{x}(0) = \mathbf{x}^0 \in \mathbb{R}^{12}$ and a desired final target \mathbf{x}^T , the goal is to calculate the vector of control $\mathbf{u}(t)$ which is able to draw our system from the initial state \mathbf{x}^0 to (or near to) the final one \mathbf{x}^T in a given time T . In mathematical terms, this problem may be formulated as the unconstrained optimal control problem

$$\left\{ \begin{array}{l} \text{Minimize in } \mathbf{u} : \quad J(\mathbf{u}) = \Phi(\mathbf{x}(T), \mathbf{x}^T) + \int_0^T F(\mathbf{x}(t), \mathbf{u}(t)) dt \\ \text{subject to} \\ \quad \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \\ \quad \mathbf{x}(0) = \mathbf{x}^0 \end{array} \right. \quad (3)$$

Here $\Phi(\mathbf{x}(T), \mathbf{x}^T)$ and $F(\mathbf{x}(t), \mathbf{u}(t))$ are two generic functions which can be chosen as desired. At this point, a pair of remarks is in order:

- It is evident that the original problem includes some constraints on both the control inputs and the state variables. This is very important in order to choose an appropriate numerical control method but, at the practical point of view, all of these restrictions can be easily satisfied by taking the final time T large enough. For this reason, in this preliminary work we will not consider the above mentioned constraints.
- As for the cost function $J(\mathbf{u})$, we have written it in a general format because the method we plan to develop in the remaining can be applied in this general setting. Concerning to the particular problem of control of the manoeuvrability for an underwater vehicle, which is the main goal of this work, it is quite natural to take

$$\Phi(\mathbf{x}(T), \mathbf{x}^T) = \sum_{j=1}^{12} \alpha_j (\mathbf{x}_j(T) - \mathbf{x}_j^T)^2 \quad (4)$$

with $\alpha_j > 0$ penalty parameters, and

$$F(\mathbf{x}(t), \mathbf{u}(t)) = \sum_{j=1}^3 \beta_j (\mathbf{u}_j(t))^2 \quad (5)$$

with $\beta_j > 0$ another weight parameters. Therefore, the cost function $J(\mathbf{u})$ is a commitment between reaching the final target and a minimal expense of control on having done the corresponding manoeuvre.

There are several optimization methods which can be applied to solve (3). Due to the complexity of the state law and the large number of variables involved in the problem, it is quite reasonable to use a gradient descent method. We refer to [1, 4] for more details on this method.

4. Numerical simulations

In this section, we show some numerical results obtained by implementing the approach described in the preceding sections in Matlab[®] for a depth change. At each iteration the state and adjoint state equations have been solved by using the ODE45 Matlab function, which is a one-step solver based on an explicit Runge-Kutta (4,5) formula (see [13]). Our goal is to provide some useful numerical information to address challenges (i)-(ii) described in the introduction.

The depth change we are going to consider is defined through a change between an initial state and a final state, during a particular T and with the parameters λ and ε as follows:

$$\begin{cases} \lambda = 0,001 & \varepsilon = 10^{-6} \\ \mathbf{x}(0) = (0, 0, 400, 0, 0, 0, 2,5, 0, 0, 0, 0, 0) \\ \mathbf{x}^T = (0, 0, 50, 0, 0, 0, 0, 0, 0, 0, 0, 0), & T = 200. \end{cases} \quad (6)$$

We are going to compare four different cases:

Case 1. In order to check the algorithm with a typical manoeuvre, we take:

$$\begin{cases} \alpha_3 = 1, & \alpha_j = 0, \quad j \neq 3 \\ \beta_j = 0, & j = 1, 2, 3 \end{cases} \quad (7)$$

That is, we penalize only the final value for the depth state variable. We want the submarine goes from a depth of 400m to 50m, regardless the final value in the other state variables.

Case 2. In order to study *optimality of sizing rudders* (which is a typical example of a pre-contract navy requirement) we take:

$$\begin{cases} \alpha_2 = \alpha_3 = \alpha_5 = 1, & \alpha_j = 0, \quad j \neq 2, 3, 5 \\ \beta_j = 0, & j = 1, 2, 3 \end{cases} \quad (8)$$

In this case, we also penalize the final value for the state variables $y(t)$ and $\theta(t)$. We want the submarine goes from a depth of 400m to 50m and, in addition, the final pitch angle must be equal (or near) to zero in order to continue the movement with the final value on the control variable without changes in depth.

Case 3. In order to *reduce hydraulic oil consumption (and therefore noise generation)* we penalize, in addition, the L^2 -norm of the control variables. Hence, now we take:

$$\begin{cases} \alpha_2 = \alpha_3 = \alpha_5 = 1, & \alpha_j = 0, \quad j \neq 2, 3, 5, \\ \beta_1 = 100, & \beta_2 = \beta_3 = 0 \end{cases} \quad (9)$$

Case 4. The linear case which approaches this problem studied in (see [8]).

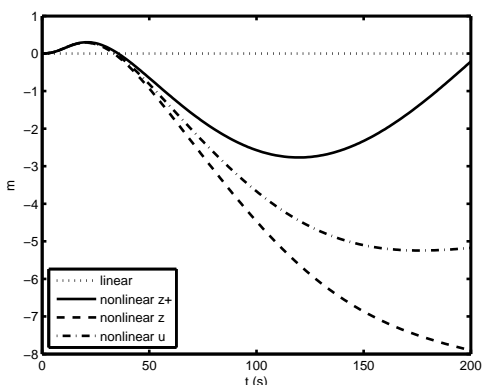


Figure 1. East movement($y(t)$)

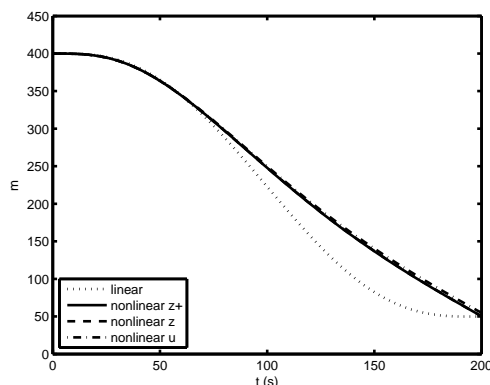


Figure 2. Depth movement($z(t)$)

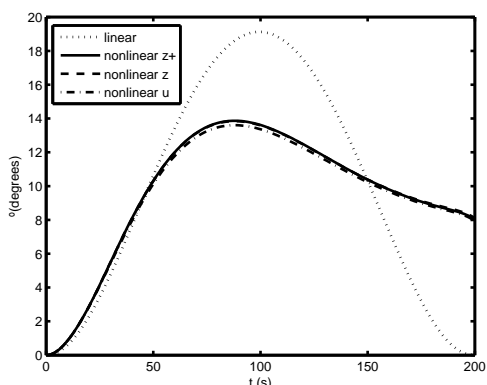


Figure 3. Pitch Angle ($\theta(t)$)

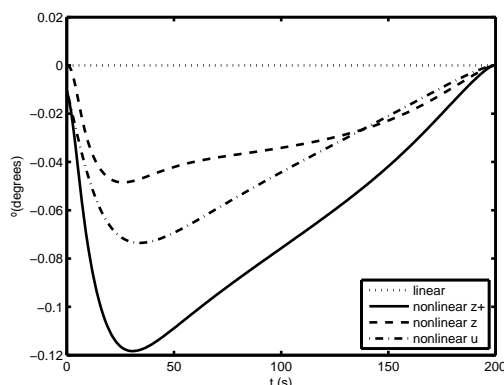


Figure 4. Deflection of rudder ($\delta_r(t)$)

The results are shown in Figures 1-7. Dashed lines (- - -) display results for the gradient method and for the set of parameters (7), we refer to this case as to *nonlinear z*. Continuous lines (—) display the results for (8), we refer to this case as to *nonlinear z+*. Dash/dot lines (-·-·-) show results for the gradient method and for the set of parameters as in (9) where the values for α_j are replaced by $\alpha_3 = 1$ and $\alpha_j = 0$ for $j \neq 3$, i.e., we keep the constrains in depth $z(t)$, east movement $y(t)$ and pitch angle $\theta(t)$, this last case is named *nonlinear u*. Finally, dotted lines (\cdots) correspond to results obtained with the *linear* model described in [8]. As shown in Figures 2, 3, 5 and 6 continuous, dash/dot and dashed lines are essentially the same. However, there is some differences between them in Figures 1 and 4. Obviously, the reason for these results is that in *nonlinear z+* and in *nonlinear u* we force the submarine to finish near to zero position on the y -axis, but in *nonlinear u* we also constrain the movement of the rudder δ_r which gives the movement on the y -axis. Therefore, δ_r needs to turn more in *nonlinear z+* than in *nonlinear u*, where

this movement is restrictive, or in *nonlinear z*, where the constraint on y -axis is inactive. Finally, Figure 7 shows the evolution of the cost function (with respect to the number of iterations) for the cases *nonlinear z*, *nonlinear z+* and *nonlinear u*. An exponential decay is observed in all cases. As we expect, the minimal cost is reached for the less restrictive problem *nonlinear z* in less number of iterations, and the largest cost is reached for the more restrictive problem *nonlinear u* in a larger number of iterations. We do not include pictures for the rest of state variables because they are not relevant in this manoeuvre.

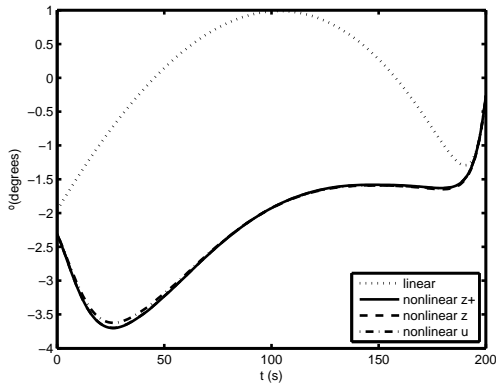


Figure 5. Deflection of stern plane ($\delta_s(t)$)

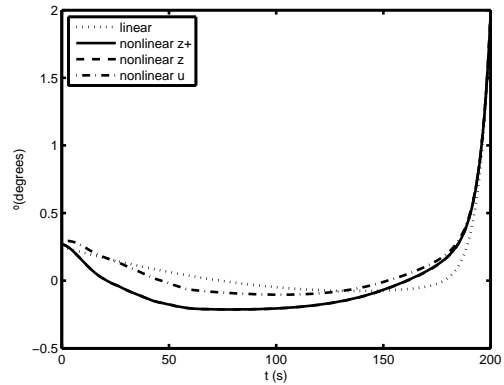


Figure 6. Deflection of bow plane ($\delta_b(t)$)

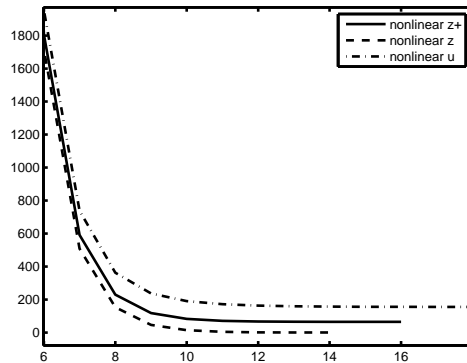


Figure 7. Cost Function

As is typical in a gradient algorithm, results depend on the initialization. For instance, when we initiate our algorithm with zero value for $\mathbf{u}(t)$, $0 \leq t \leq T$, after convergence, we obtain different results: the costs for the case *nonlinear z* are the same and very close to zero, but the optimal controls are slightly different; for the case *nonlinear z+*, the final optimal cost is higher than in the case tested before, and the optimal controls are also different. These results seem to indicate the existence of several local minima and/or non-uniqueness of solutions.

5. Conclusions

The problem of manoeuvrability control for a submarine is addressed. The vehicle dynamics is modelled as a simplified version of the classical DTNSRDC equations of motion that adapts to the specific type of submarine under consideration. The manoeuvrability

control problem is then formulated as a nonlinear unconstrained optimal control problem which is numerically solved by using a gradient descent method.

Numerical results show that, although the nonlinear model proposed in this work follows, in some sense, the same tendency as the more usual linear model, however, important changes in some state and control variables are present. As a conclusion, the nonlinearities which appear in the kinematic and dynamic equations of motion cannot be removed in order to have more accurate simulation results.

In what concerns the optimization method proposed, it allows us to obtain satisfactory simulation results in a model which presents serious mathematical difficulties.

As for the computational cost of the algorithm, for a depth change manoeuvre it takes about 60 seconds in a PC for the test cases 1, 2 and 3.

Finally, we emphasize that at the practical level it is interesting to include the propeller as a control variable and to impose some (a priori) constraints on the control variables. We plan to address these two issues as well as the more theoretical questions concerning existence and uniqueness of solutions in a future work.

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