**Abstract**—This paper presents a new theory to analyze and design a novel leaky-wave coupler based on a hybrid dielectric-waveguide printed-circuit technology. The analysis theory is based on a combined method of moments (MoM) geometrical optics (GO) technique, which makes use of the leaky mode complex propagation constant. This MoM–GO approach allows to easily obtain the response of the coupler, and to illustratively understand the working mechanism of the proposed structure. A coupler is designed to work at 5.5 GHz. Leaky mode results are compared with three-dimensional finite-element method simulations, showing the validity and accuracy of the new theory developed. This novel directional coupler presents interesting advantages when compared to previous wave couplers.

**Index Terms**—Directional coupler, geometrical optics (GO), leaky waves, method of moments (MoM).

**I. INTRODUCTION**

A **N ORIGINAL** directional coupler, based on hybrid dielectric-waveguide printed-circuit technology, was presented in [1]. The design procedure was illustrated using three-dimensional finite-element method (FEM) simulations, and the frequency response was obtained, showing the ability to control the coupling level. In this paper, an original theory based on the leaky mode complex propagation constant is described. With this theory, the response of the coupler can be obtained in a much faster way since the formulation is totally analytical and based on the leaky mode expressions. In this way, a straightforward design can be performed following the same procedure described in [1], but avoiding three-dimensional FEM simulations (which are very time consuming for large structures, as with the coupler studied in this paper). Moreover, the proposed theory helps to better understand the coupling working mechanism inherent to this coupler, illustrating the importance of higher order effects.

The structure of the proposed leaky-wave coupler is shown in Fig. 1. Two radiating dielectric waveguides, laterally shielded by parallel plates, are faced toward each other in order to transfer energy from one to the other. Planar slot circuits are printed at the broad sides of the dielectric guides in order to excite a leaky-wave mode and to control its propagation and radiation properties. From a technological point-of-view, the proposed coupler presents a more flexible and simpler design than conventional grating couplers since the dielectric guide structure must not be mechanized to obtain a maximum efficiency design [1]. Only the printed circuit must be tapered [1]–[3]. In addition, the new coupler is based on radiation from the main space harmonic. In this way, the structure dimensions are minimized as compared to grating couplers, which are based on higher order radiating harmonics [4], [5].

**II. THEORY**

Here, the theory to analyze and design the proposed coupler is described for the first time. The working mechanism of the proposed coupler will be explained, describing the theory to analyze and design each part of the circuit in order to obtain a successful response. The theory presented in this paper is novel, and it is based on a combination of the method of moments (MoM) and geometrical optics (GO) approaches.

**A. MoM Analysis and Design of Dielectric Waveguides and Slot Printed Circuits**

Rectangular dielectric waveguides of width $a$, height $D$, and relative permittivity $\varepsilon_r$ are used to propagate the main $\text{TE}_{20}$ mode. These dimensions control the $\text{TE}_{20}$ mode cutoff and, therefore, determine the fast-wave frequency region where this mode can become leaky. The width of the slot, printed at the dielectric–air interface, also controls the leaky-wave region. A
MoM technique, especially conceived for the analysis of hybrid waveguide-planar transmission lines [2], is used to obtain the TE$_{10}$ leaky mode phase and leakage constant. Fig. 2 shows the normalized phase constant ($\beta_{y}/\omega_0$) of the TE$_{10}$ mode for three different slot widths $W$ (see the inset of Fig. 2).

As it can be seen in Fig. 2, the fast wave region of the TE$_{10}$ mode (dark area in Fig. 2) extends from 4.8 GHz (cutoff region) to 6.5 GHz (bound wave region) for a dielectric guide of dimensions $a = 22.86$ mm, $D = 10.16$ mm, and $\epsilon_r = 2.2$, and a centered slot of width $W = 10$ mm. In addition, the slot must be centered with respect to the parallel plates (see Fig. 3) in order to launch the leaky wave. This asymmetry radiation mechanism is explained in detail in [3], where interesting leaky-wave antennas (LWAs) were designed using this technology. In this way, no coupling will occur between the two dielectric guides if the separation $L$ between them is large enough, and the printed slot-circuit is centered, as sketched in Fig. 3(a) and (b). However, if the slot is uncentered, there will be radiation and coupling no matter the value of $L$, as illustrated by Fig. 3(c).

To compute the coupling between the input waveguide [transmitter (TX)] and the output waveguide [receiver (RX)], an “overlapping integral” along the whole structure length can be computed, as proposed in [4] and [5]. Note that the total length is divided into $(L_A + L_D)$ according to Fig. 1, where $L_A$ is the radiating slot length and $L_D$ is the spatial offset between the TX and the RX shown in Fig. 1.

This integral computes the overlapping between the beams created by each waveguide at the reference plane located in the middle height. As sketched in Fig. 4, the emitted beams must be shaped to obtain maximum overlapping and, therefore, maximum coupling [5]. For this purpose, the printed slot circuit must be tapered along the coupler length, as explained in [3], in order to obtain a cosine shaped beam [see Fig. 4(b)]. The MoM analysis approach allows to obtain the variation of the TE$_{10}$ leaky mode complex longitudinal propagation constant ($k_y$) along the waveguides length ($y$-axis, see Fig. 1) for both the TX and RX couplers

$$k_{y}^{\text{TX}}(y) = i\beta_{y}^{\text{TX}}(y) - j\alpha_{y}^{\text{TX}}(y)$$

$$k_{y}^{\text{RX}}(y) = i\beta_{y}^{\text{RX}}(y) - j\alpha_{y}^{\text{RX}}(y).$$

The normalized illuminations at the TX and RX antenna apertures $M^{\text{TX}}$ and $M^{\text{RX}}$ can be computed from the following well-known equations [6]:

$$M^{\text{TX}}(y, z = -L/2) = \max \left\{ \frac{\partial F^{\text{TX}}(y)}{\partial y} \right\} \cdot e^{-j\beta_{y}^{\text{TX}}(y)}$$

$$M^{\text{RX}}(y, z = +L/2) = \max \left\{ \frac{\partial F^{\text{RX}}(y)}{\partial y} \right\} \cdot e^{+j\beta_{y}^{\text{RX}}(y)}.$$
and an offset, as shown in Fig. 4. The transverse plane located at \( z = 0 \) will be used as the reference plane to compute the coupling coefficients. For this purpose, a simple GO model [7] is applied. Due to the fact that a uniform propagation direction is obtained for all the length of the synthesized beams \( \{ \theta(y) \} \) is almost constant in Fig. 5(b), a simple displacement can be used to obtain the illuminations created by the TX and RX antennas at the reference plane \( (z = 0) \)

\[
\delta_{\text{TX}} = \frac{L}{2} \tan(\overline{\theta}_{\text{TX}}) \delta_{\text{RX}} = \frac{L}{2} \tan(\overline{\theta}_{\text{RX}})
\]

(9)

\[
M_{\text{TX}}(y, z = 0) = M_{\text{TX}} \left( y - \delta_{\text{TX}}, z = -\frac{L}{2} \right)
\]

(10)

\[
M_{\text{RX}}(y, z = 0) = M_{\text{RX}} \left( y + \delta_{\text{RX}}, z = +\frac{L}{2} \right)
\]

(11)

where \( \overline{\theta}_{\text{TX}} \) and \( \overline{\theta}_{\text{RX}} \) are the mean values for the beam pointing direction of the TX and RX LWAs, respectively [see Fig. 5(b)]. Once the beams complex illuminations have been computed at the reference plane \( (z = 0) \), a normalized overlapping integral can be used to compute the coupling between these two beams along all the coupler length \( (L_A + L_D) \) [5], shown in (12) at the bottom of this page.

In order to have high coupling coefficient \( C_1 \), the beams must match in the same position with the same amplitude function and with conjugated phases. This will depend on the amplitude and phases of the emitted beams \( (M_{\text{TX}} \) and \( M_{\text{RX}} \)), but it will also depend on the height of the coupler \( (L) \), the length of the radiating aperture \( (L_A) \), and the offset between the LWAs \( (L_D) \). Fig. 6 illustrates different scenarios. The first-order coupling computed by \( C_1 \) occurs when the TX and RX beams overlap, as sketched in Fig. 6(c). A wrong shift of the beams will decrease the overlapping, as shown in Fig. 6(b) and (d).

The energy of the TX, which is not coupled to the RX, will be reflected back from the RX waveguide to the TX waveguide. In Fig. 6(a), a higher order coupling scenario is sketched. The reflected beam will be retransmitted up to the RX LWA, as is shown in Fig. 6(a). In this way, a second-order coupling coefficient can be computed from the new incident beam, shown in (13) at the bottom of the following page, where the illumination

\[
C_1 = \frac{\int_{y=0}^{y=L_A+L_D} \left\{ M_{\text{TX}}(y, z = 0) \right\} \cdot \left\{ M_{\text{RX}}(y, z = 0) \right\}^* \cdot dy}{\sqrt{\int_{y=0}^{y=L_A+L_D} \left| M_{\text{TX}}(y, z = 0) \right|^2 dy \cdot \int_{y=0}^{y=L_A+L_D} \left| M_{\text{RX}}(y, z = 0) \right|^2 dy}}
\]

(12)

TABLE I

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</table>

Fig. 5. Complex illumination functions for the beams created by the TX and RX LWAs at their apertures.
created by the TX beam after the multiple reflections is computed using the displacement $\delta_{TX}$

$$M_2^{TX}(y, z = 0) = M_2^{TX}(y - 4\delta_{TX}, z = 0).$$

This process can be recursively repeated so that the total amount of energy coupled from the TX to the RX can be computed, after multiple reflections, as

$$C = \eta \cdot \left\{ |C_1| + \sum_{n=2}^{N} \prod_{m=1}^{n-1} (1 - |C_m|) \right\} |C_n|.$$  \hspace{1cm} (15)

In (15), $N$ represents the total number of reflections considered, and $\eta$ is the maximum coupling coefficient, computed from the TX and RX antennas radiation efficiencies $\eta^{TX}$ and $\eta^{RX}$. They are obtained from their respective leakage functions ($\alpha^{TX}(y)$ and $\alpha^{RX}(y)$), computed in a lossless scenario and in the absence of the other antenna. The procedure for the calculation of $\alpha^{TX}(y)$ and $\alpha^{RX}(y)$ is detailed in [6] as follows:

$$\eta = \eta^{TX} \cdot \eta^{RX}$$

$$\eta^{TX} = 1 - e^{-2\int_{\tau=0}^{\tau=L_A} \alpha^{TX}(\tau)d\tau}$$
$$= 1 - f_{TX}(L_A + L_D)$$

$$\eta^{RX} = 1 - e^{-2\int_{\tau=L_A+L_D}^{\tau=L_A+L_D} \alpha^{RX}(\tau)d\tau}$$
$$= 1 - f_{RX}(0).$$

To obtain the highest coupling, the radiation efficiency of the two LWAs, $\eta^{TX}$ and $\eta^{RX}$ [see (16)–(18)] must be maximized. Once the leakage rate of the LWA is known (in our case, it is typically $\alpha/\lambda_0 = 0.1$), the radiating slot length $L_A$ must be chosen to fulfill the high-efficiency requirements of a particular application [the higher $L_A$ is, the higher $\eta$ is obtained, see (17)] and (18)]. In our case, we assure good radiation efficiency for all the designs with $L_A = 10\lambda_0$ ($\eta^{TX} = \eta^{RX} = 98\%$).

In a practical LWA coupler, the TX and RX antennas are arranged to have direct coupling (high $C_2$) at the design frequency, as shown in Fig. 6(c). Due to the frequency beam scanning response of LWA [6], the coupling decreases for frequencies below [see Fig. 6(b)] and above [see Fig. 6(d)] the center frequency. For quite low frequencies, higher order couplings can be very important, as shown in Fig. 6(a). In any case, the energy that has not been coupled after $N$ higher order couplings reaches the structure far end at $y = L_A+L_D$, where the perfect absorber must be located to avoid unwanted standing-wave phenomena. In Section III, a practical design of an LWA coupler in hybrid waveguide printed-circuit technology is performed using the previously described theory. Results obtained with a three-dimensional finite-element electromagnetic solver (FEM) will be presented to validate the theory.

III. RESULTS

Once the LWAs have been designed to synthesize $L_A = 10\lambda_0$ long cosine-shaped constant pointing beams (see Fig. 5), the total coupler can be analyzed and designed using the combined MoM–GO theory. As demonstrated in [1], the height of the parallel plates ($L$) must be high enough to avoid direct coupling due to the evanescent bound wave (with $L > 80$ mm, the direct coupling is below $-50$ dB). In this way, the coupling mechanism can only be due to the emitted leaky-wave beams.

For a given central frequency, which, in our case, is 5.5 GHz, the TX and RX slot circuits of length $L_A$ must be displaced at an offset $L_D$ [see Figs. 1 and 4(a)] in order to make the emitted beam to overlap with the trace of the receiver [5]. The theoretical value of $L_D$, which provides this overlapping, is given by

$$L_D = L \cdot \tan \theta_{RAD}$$

where $\theta_{RAD} = \theta_{TX} = \theta_{RX}$ [if the design is correct, as shown in Fig. 5(b)]. The radiating angle obtained from the $TE_{30}$ leaky mode dispersion curve for the design frequency (5.5 GHz) is $\theta_{RAD} = 40^\circ$ [see Fig. 5(b)] and $L$ is chosen to be $L = 90$ mm. This gives rise to a theoretical offset (19) $L_D = 75,51$ mm. Fig. 7 shows the variation of the coupling as a function of $L_D$ for the designed 10$\lambda_0$-long cosine-shaped slot-circuits, pointing at $\theta_n = 40^\circ$ at 5.5 GHz. The results obtained with the leaky mode theory described in this paper are compared with a commercial FEM solver. It is checked that the maximum coupling takes place between $L_D = 70$ mm and 80 mm. For other values of $L_D$, the traces are shifted, obtaining a smaller overlapping integral, which is translated into a lower coupling, as can be seen

$$C_2 = \frac{\int_{y=0}^{y=L_A+L_D} \left\{ M_2^{TX}(y, z = 0) \right\}^* \cdot \left\{ M_2^{RX}(y, z = 0) \right\}^* dy}{\sqrt{\int_{y=0}^{y=L_A+L_D} \left[ M_2^{TX}(y, z = 0) \right]^2 dy \cdot \int_{y=0}^{y=L_A+L_D} \left[ M_2^{RX}(y, z = 0) \right]^2 dy}}$$

\hspace{1cm} (13)
in Fig. 7. Very good agreement is observed between the two different analysis approaches (leaky mode and FEM).

When frequency is varied around the design point (5.5 GHz), a beam-shifting phenomenon occurs. Due to the frequency-scanning response of LWAs [6], the beams are displaced and the overlapping is reduced. This phenomenon is illustrated in Fig. 8, where the traces created by the TX and RX LWAs at the reference level \( z = 0 \) are plotted for different frequencies. The offset between the two antennas has been chosen to be \( L_D = 70 \text{ mm} \). The beams obtained with the combined MoM–GO theory [see (9)–(11)] are compared to the plots obtained with FEM analysis. Very good agreement is observed for the beam shape and frequency shift.

Fig. 9 shows the dependence with frequency of the pointing direction \( \theta_{\text{RAD}} \) and of the radiation efficiency \( \eta_{\text{RAD}} \) for the designed tapered LWAs. The pointing direction \( \theta_{\text{RAD}} \) is derived from the mean value applied to (7) and (8), while the radiation efficiency \( \eta_{\text{RAD}} \) is computed with (17) and (18). The leaky mode results are compared with FEM analysis, showing excellent agreement. As can be seen in Fig. 9, the antennas are designed to point at \( \theta_{\text{RAD}} = 40^\circ \) for \( f = 5.5 \text{ GHz} \) with nearly \( \eta_{\text{RAD}} = 100\% \). For higher frequencies, the radiation tends to the endfire direction \( [\theta_{\text{RAD}} = 90^\circ, \text{see Fig. 6(c)}] \), while the radiation efficiency decreases [6]. For lower frequencies, the beam points closer to the broadside radiation \( (\theta_{\text{RAD}} = 0^\circ) \), keeping the nearly 100% radiation efficiency. Below 4.8 GHz, the \( \text{TE}_{10} \) mode is at cutoff, and above 6.8 GHz, the \( \text{TE}_{10} \) leaky mode becomes a bound wave [6].

These three regimes can also be identified in the frequency response of the coupler shown in Fig. 10, where both the coupling and direct output responses are shown. Coupling and direct output are very low below 4.8 GHz due to the cutoff behavior of the \( \text{TE}_{10} \) mode. The leaky-wave region corresponds to the range between 4.8–6.8 GHz, where it can be seen in Fig. 10 that the maximum coupling occurs. Above 6.5 GHz, the \( \text{TE}_{10} \) mode becomes a bound wave, making most of the injected energy to travel to the OUT port (see Fig. 1 for the definition of the ports), therefore decreasing the coupling. In the leaky-wave band, it is observed that the maximum coupling is obtained at the design frequency (5.5 GHz). Around this frequency, the coupling decreases due to the frequency beam-shifting phenomenon illustrated in Fig. 8. However, another maximum appears at 5.1 GHz. This maximum is created by a second-order coupling, as was described and illustrated in Fig. 6(a). For frequencies above 5.5 GHz, the emitted beams diverge [see Fig. 6(b)] and coupling decreases as frequency increases, as it is checked
in Fig. 10. Very good agreement is observed between the FEM analysis and the results obtained with the MoM–GO model. The total coupling was obtained from (15) using three higher order integrals. Moreover, the couplings to the OUT port were obtained assuming perfect matching and perfect isolation to the ISOLATED port (it was checked with three-dimensional FEM analysis that the return losses and isolation are below −15 dB in the entire coupling band)

\[
\text{COUPLING(dB)} = 10 \log_{10}(|C|),
\]

\[
\text{OUT(dB)} = 10 \log_{10}(1 - |C|),
\]

Fig. 11 shows the absolute values of the first three overlapping factors \(C_1, C_2, \) and \(C_3\), computed with the MoM–GO model using (9)–(14). The first-order overlapping \(C_1\) dominates at the design frequency \(C_1 = 1\) at 5.5 GHz for which there is direct sight between the TX and RX LWAs. For lower frequencies, the TX and RX beams overlap after higher order reflections due to the nearly normal angle of emission [see Fig. 6(a)]. This phenomenon is checked by the high values of \(C_2\) and \(C_3\) for the frequency of 5.1 GHz. As a result, another maximum of total coupling \(C\) is obtained at 5.1 GHz, as can be seen in Fig. 10. It is necessary to take into account these higher order overlapping factors to accurately model the response of the designed LWA directional coupler.

Moreover, Fig. 11 helps to understand the election of the index \(N\) in (15), which stands for the total number of reflections considered. As can be seen in Fig. 11, there are two significant contributions to the total coupling: the one from the direct beam \(C_1\) at 5.5 GHz and the higher order reflections \(C_2\) and \(C_3\) at 5.1 GHz, just after the cutoff frequency. Although the value for \(C_3\) is quite high at 5.1 GHz \(|C_3| \approx 1\), one can readily see that, according to (15), \(C_3\) does not contribute much to the total response. This is due to the fact that most of the energy is already coupled to the second-order coefficient \(|C_2|\), leaving very little energy left \((1 - |C_2|)\) in (15) to be coupled to the third-order reflection. This phenomenon explains the results shown in Fig. 12, where the convergence of the total coupling \(C\) frequency response is shown as a function of the index \(N\). It can be seen how with \(N = 2\) coefficients, the total response has almost converged. All the results shown in this paper are for \(N = 3\) to assure convergence.

All the previous results correspond to a directional coupler designed to obtain a coupling level of \(C = -0.02\) dB at the design frequency of 5.5 GHz. The maximum coupling level is controlled by the radiation efficiency of the TX and RX LWAs, \(\eta_{\text{TX}}\) and \(\eta_{\text{RX}}\) [see (17) and (18)]. Once the first-order overlapping integral (12) is maximized at the design frequency \(|C_1| = 1\), the coupling level (15) can be computed as

\[
C = \eta = \eta_{\text{TX}} \cdot \eta_{\text{RX}}.
\]

If the RX LWA is designed with maximum radiation efficiency \((\eta_{\text{RX}} \approx 1)\), the coupling level can be determined by the TX LWA efficiency \(\eta_{\text{TX}}\). In this way, the input guide printed circuit will be responsible for controlling the coupling level of the entire directional coupler. Fig. 13 shows the results obtained for three couplers with maximum overlapping \(C_1\) designed at 5.5 GHz. One coupler is designed for a 95% coupling level \((\eta_{\text{TX}} = 0.97, C = 0.95 = -0.2\) dB), the other for 52% of coupling \((\eta_{\text{RX}} = 0.97, \eta_{\text{TX}} = 0.53, C = 0.52 = -2.8\) dB), and the last one for a coupling level of 22% \((\eta_{\text{RX}} = 0.97, \eta_{\text{TX}} = 0.23, C = 0.22 = -6.6\) dB). On the right-hand side of Fig. 13, the layout of the three TX LWAs printed circuits is sketched, which is designed using the asymmetry radiation
principle described in [3]. To decrease the radiation efficiency of the TX LWA $\eta^{TX}$, the asymmetry of the tapered slot-circuit must be decreased, as can be seen in the layouts of Fig. 13. The results obtained with the proposed MoM–GO theory are compared with FEM analysis, observing very good agreement. For all cases, it is observed the aforementioned higher order coupling at low frequencies (another maximum of coupling around 5 GHz).

Finally, a study on the influence of the slot profiles on the coupling characteristics was performed. The authors have obtained similar coupling frequency responses by using three different symmetrical beam shapes, namely, cosine-beam, triangular-beam, and square-cosine beam. Each shape is synthesized using a different tapered slot profile, as explained in [3]. The maximum coupling level (obtained at the frequency of design) does not depend on the beam shape, but on the radiation efficiency of the TX and RX LWAs [see (22)], provided the correct alignment between the two radiating waveguides is assured [see (19)].

Fig. 14 shows the beams created by the TX and RX LWAs at the reference plane ($z = 0$, where the overlapping integrals are computed) for the aforementioned beam shapes and for four different frequencies of the coupling band (5.1, 5.5, 6, and 6.5 GHz). At the design frequency (5.5 GHz), the desired beam shape is synthesized, and perfect overlapping is achieved by the correct alignment of the couplers [due to $L_D$, see (19)]. For other frequencies, the overlap decreases due to the beam-frequency-shifting effect, and also due to the distortion of the emitted beams. The graphs shown in Fig. 14 are obtained with the MoM–GO approach, and have been compared to FEM results, showing very good agreement.

The frequency response does not change much for these three different slot profiles, as illustrated by Fig. 15, where 95% coupling efficiency couplers were designed using the three aforementioned beam shapes. A slight difference in the coupling frequency response is observed. This is due to the fact that the evolution of the overlapping factor with frequency varies from one beam shape to another, as can be depicted from Fig. 14. Fig. 15 shows FEM simulations, together with results from MoM–GO model for the cosine case (the results for the triangle and square cosine shapes are similar, and they are not shown for the sake of clarity).

It must be emphasized that the proposed theory provides fast and accurate results for complex three-dimensional structures due to its simplicity and analytical nature. This simplicity has been achieved by splitting the analysis of the whole three-dimensional structure into several steps. First, the evolution along the $y$-axis of the leaky-wave complex propagation constants is obtained for the TX and RX radiating waveguides by means of a two-dimensional leaky mode analysis ($x$–$z$-plane, see Fig. 1). The complex illumination functions along the longitudinal axis are then straightforwardly derived for both the emitted and received beams. Overlapping integrals only in the $y$-direction (one-dimensional analysis) can be applied without losing accuracy. The beam coupling does not depend on the $x$-axis due to the fact that the field distribution along the $x$-direction is uniform (the radiated beam is basically a horizontally polarized oblique plane wave, as explained in [3] and [10]). A simple GO model in one dimension ($y$-axis) can also be applied without losing accuracy due to the uniform pointing direction of the synthesized beams. Moreover, the mutual influence between the TX and RX waveguides is negligible, provided that the parallel...
plates’ height is higher than \( L/\lambda_0 = 1 \). Under this condition, it is a good approximation to compute the normalized illumination functions considering that TX and RX are isolated.

The analysis and design of these types of couplers is much more time consuming if generic three-dimensional numerical electromagnetic solvers are used as FEM analysis. The development of a specific and simpler theory is very convenient due to the large electrical dimensions of this type of traveling-wave couplers. This is also the case for other couplers based on non-radiative dielectric (NRD) technology \([8, 9]\). For instance, the coupler designed in this paper has a total length \( L_A + L_D \approx 11.5\lambda_0 \), a width \( a \approx 0.5\lambda_0 \), and a total height \( L + 2D \approx 2\lambda_0 \) (see Fig. 1).

The simple combination of the MoM and GO gives accurate results due to the particular configuration of the studied leaky-wave coupler, which is laterally shielded by metallic walls. Other authors have used other approaches to analyze other type of couplers such as the perturbation theory \([4]\), Marcuse’s coupling formula \([8]\), or the coupled mode theory \([9]\). In \([5]\), a similar overlapping integral approach was used to obtain a scattering matrix representation of a dielectric grating coupler. However, in \([5]\), only the conditions to obtain maximum coupling at the design frequency were studied. With the proposed theory, the response of a leaky-wave coupler can be accurately obtained, for the first time, for any frequency in the leaky-wave region, using a simple GO analysis based on the dispersion of the synthesized beams.

IV. CONCLUSION

A new theory to analyze and design a new leaky-wave directional coupler in hybrid dielectric-waveguide printed-circuit technology has been presented. A MoM approach has been used to obtain the rectangular dielectric guide dimensions and the tapered geometry of the printed slot circuit in order to control the complex propagation constant of the \( TE_{30} \) leaky mode. In this way, a cosine shaped beam can be synthesized to obtain maximum coupling efficiency between the input and output couplers. Using GO, the displacement of the transmitted beams as they propagate through the coupler can be computed. In this way, total coupling is obtained from the computation of the overlapping integrals in the reference plane. It has been demonstrated that the offset between the transmitter and receiver leaky-wave radiating slots must be properly chosen to maximize the overlapping at the design frequency. Results obtained using a three-dimensional electromagnetic solver based on the FEM have been compared to predictions based on this leaky-wave MoM–GO approach, showing very good agreement. It has been confirmed that it is important to take into account higher order coupling effects due to reflections in the structure. Finally, it has been shown how the printed circuit can also control the radiation efficiency of the transmitter slot, allowing for the adjustment of the coupling level. The influence of the slot profile has also been studied by comparing the coupling for three different radiated beam shapes (cosine, triangular, and square cosine). The theory developed in this paper allows to easily design these types of couplers using the information of the complex propagation constant of the radiating leaky mode. This modal approach has been validated by comparisons with full-wave three-dimensional FEM simulations.

REFERENCES


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