Manuscript Number: HMT-D-13-01412

Title: Heat transfer enhancement of laminar and transitional Newtonian and non-Newtonian flows in tubes with wire coil inserts

Article Type: Full Length Article

Keywords: heat transfer enhancement; wire-coil inserts; non Newtonian flow; transitional flow

Corresponding Author: Mr. David S Martínez, Ph.D. Student

Corresponding Author’s Institution: Universidad Politécnica de Cartagena

First Author: David S Martínez, Ph.D. Student

Order of Authors: David S Martínez, Ph.D. Student; Alberto García; Juan P Solano; Antonio Viedma

Abstract: This work presents an experimental study on two different wire coils inserted in a smooth tube using both Newtonian and non-Newtonian fluids to characterize their thermohydraulic behaviour in laminar and transition flow. Dimensionless pitches of the wire coils (based on the empty tube inner diameter) were chosen as $p/D = 1$ and 2, whereas dimensionless wire diameter was $e/D = 0.09$ for both wire coils. Non-Newtonian tests considered different viscosity types with concentration of 1% of CMC (Carboxyl-methyl-cellulose) solution in water at several temperatures; a wide range of flow conditions has been covered: Reynolds number from 10 to 1200 and Prandtl number from 150 to 1900. Newtonian test were carried out with propylene glycol as working fluid, covering a similar range of Prandtl and Reynolds number as the previous indicated for non-Newtonian fluids. This range of flow conditions was previously measured on the empty smooth tube, and compared with the well-known solutions. Isothermal pressure drop tests and heat transfer experiments under uniform heat flux conditions were done. At low Reynolds numbers, both wire coils behave as a smooth tube but accelerate transition to critical Reynolds numbers down to 400. Maximum augmentations of Fanning friction factor of 3.5 times and of 4.5 times of Nusselt number have been found respect to the smooth tube.
Abstract

This work presents an experimental study on two different wire coils inserted in a smooth tube using both Newtonian and non-Newtonian fluids to characterize their thermohydraulic behaviour in laminar and transitional flow. Dimensionless pitches of the wire coils were chosen as \( p/D = 1 \) and \( 2 \), whereas dimensionless wire diameter was \( e/D = 0.09 \) for both wire coils. Non-Newtonian tests considered different viscosity types with concentration of 1% of CMC (Carboxyl-methyl-cellulose) solution in water at several temperatures; a wide range of flow conditions has been covered: Reynolds number from 10 to 1300 and Prandtl number from 150 to 1900. Newtonian tests were carried out with propylene glycol as working fluid, covering a similar range of Prandtl and Reynolds number as the previously indicated for non-Newtonian fluids. Isothermal pressure drop tests and heat transfer experiments under uniform heat flux conditions were performed, and results were contrasted with own experimental data for the smooth tube and with well-known analytical solutions. At low Reynolds numbers, both wire coils behave as a smooth tube but accelerate transition to critical Reynolds numbers down to 500. Maximum augmentations of Fanning friction factor of 3.5 times and of 4.5 times of Nusselt number have been found with respect to the smooth tube.

Keywords:
heat transfer enhancement, wire coil inserts, non Newtonian flow, transitional flow
1. Introduction

Heat transfer process of high-viscous fluids is commonly encountered in the chemical, food, pharmaceutical or petroleum industries. Food-related products like tomato paste and carrot pure, polymer melts like polystyrene and nylon, as well as many personal care products are typical examples. Due to its nature, these kind of fluids involve elevated Prandtl number values ($Pr>1000$) together with low Reynolds values [1]. As a consequence, most of them are processed in the laminar or transitional regimes, where heat transfer rates are particularly low. Here, the various forms of insert devices are effective to enhance the heat transfer performance of tubular heat exchangers.

The dominant literature (Bergles [2], Webb and Kim [3]) usually mentions five devices: wire coils, twisted tapes, extended surface devices, mesh inserts and displaced elements. The main advantage of inserts, with respect to other enhancement techniques, is that they allow an easy installation in an existing exchanger of smooth tubes. Due mainly to its low cost, the insert devices which are most frequently used in engineering applications are wire coils and twisted tapes. The difficulties for carrying out experimental studies on heat transfer in laminar flow is well known (Bergles [4]), as this flow is sensitive to entry length effects, the thermal boundary condition and the buoyancy forces effect. Frequently, highly-viscous fluids exhibit a non-Newtonian behaviour; which implies that an appropriate rheological characterization needs to be done prior to the experimentation. Another problem that can arise when dealing with non-Newtonian fluids is thixotropy, where their apparent viscosity changes along the course of the tests. Consequently, few experimental studies have been reported on the enhancement of non-Newtonian laminar heat transfer. Nazmeev [5] studied the enhancement of pseudoplastic fluid flows using twisted-tape
inserts. He found a substantial increase in heat transfer (50-300%), similar to that observed in the studies with Newtonian fluids carried out by Hong and Bergles [6] and Marner and Bergles [7]. Manglik et al. [8] extended the available experimental data by performing new experiments with different tapes, working with aqueous solutions of cellulose ether powder. They modified the Hong and Bergles [6] Newtonian correlation for Nusselt number, in order to account for the non-Newtonian and variable consistency effects. They also proposed a correlation for the friction factor, and concluded that additional experimental data would help confirm the validity of their correlations. More recently, Patil [9] presented an experimental investigation of heat transfer and flow friction of a power-law fluid in tubes with twisted tape inserts. Correlations were presented for isothermal and heating friction factors and Nusselt numbers under uniform wall temperature condition. They observed heat transfer enhancement ratios of up to 2.4, in the basis of fixed geometry and pumping power, at Reynolds numbers around 200. The abundance of design correlations for twisted tapes does not mean, however, that they are the best option to enhance heat transfer in laminar flow, such as Webb and Kim [3] have pointed out. Wire coil inserts are an alternative to twisted tapes and other insert devices for heat transfer enhancement at moderate Reynolds numbers. However, very few authors have studied wire coils in non-Newtonian flow so far. An early work from Igumentsev and Nazmeev [10] studied the effect of wire coils in the intensification of convective heat exchange, for anomalously viscous liquids (aqueous solutions of sodium carboxymethyl cellulose). The dimensionless geometrical range was $p/D=0.3-3.0$ and $e/D=0.072-0.109$. However, the authors only informed about the pitch length of the wire coils studied, and not about the values of wire diameters. The authors performed heat transfer and pressure drop experiments under uniform heat flux conditions; they did not isolate the test section and placed an additional electric heater above the tube, in order to
compensate for the heat losses to the surrounding medium. The experimental data was shown in terms of relative increases of Nusselt number and friction factor with respect to the smooth tube solutions. Friction factor and Nusselt number results were absent. Thus, the accuracy of Igumentsev and Nazmeev’s [10] measurements cannot be estimated. At least the smooth tube results should have been presented and compared with contrasted correlations for non-Newtonian flow. Another concern about Igumentsev and Nazmeev’s [10] experiments is that the flow was not hydrodynamically developed at the test section inlet. Oliver and Shoji [11] tested three types of insert devices: twisted tapes, Cal Gavin patent wire meshes and wire coils. Three geometries of each type were characterized, and aqueous solutions of sodium carboxymethyl cellulose (SCMC) were used as test fluids. Measurements of both isothermal pressure drop and heat transfer at constant wall temperature were made, in a range of Prandtl number Pr= 30-90 and Reynolds number Re=20-2000.

The mesh inserts performed better than wire coils and twisted tapes at Reynolds numbers below 200. However, wire coils rapidly became more effective as the flow became turbulent, and a poor performance of twisted tape inserts was observed. At Reynolds numbers higher than 300, with a 0.2% solution of SCMC, a wire coil showed to achieve a better performance than mesh inserts, with a higher heat transfer rate and a much lower pressure drop level. Oliver and Shoji [11] gave attention to the degradation with time (thixotropy) of the pseudoplastic test fluid: rheological properties were checked for each heat transfer run made. Oliver and Shoji [11] considered the contribution of natural convection to their heat transfer results, and also the effect of radial viscosity variation. Moreover, they took into account the axial variation of effective viscosity along the test section. They did not inform about their experimental uncertainty. However, their smooth tube heat transfer results did not match the theoretical predictions.
This work presents an experimental study on two wire coils inserted in a smooth tube using non-Newtonian and Newtonian fluids. A wide range was covered in order to characterize its thermohydraulic behaviour in laminar and transition flow. The employed non-Newtonian fluid was made of different types (high and medium viscosity) of a carboxyl-methyl-cellulose (CMC) solution in water at several temperatures (taking into account the thixotropic effects). The Newtonian fluid employed was propylene glycol, and it was tested under the same flow conditions as the non-Newtonian fluid. This working range was previously analyzed for the smooth tube and compared to the solutions of Bird [12] and Mahalingam [13]. The ranges of the investigated experimental variables are shown in Table 1.

1.1. Wire coil inserts

The wire coil of the present work was made of steel and covered with an insulating coating to prevent electrical conduction. Fig. 1 shows a sketch of a wire coil inserted in a smooth tube, where \( p \) stands for the helical pitch and \( e \) for the wire diameter. These parameters can be arranged to define the wire geometry in a non-dimensional form: dimensionless pitch \( p/D \), dimensionless wire-diameter \( e/D \) and pitch to wire-diameter ratio \( p/e \). The wire coil pitch revealed to be a decisive parameter of the inserts with a clear effect on the R3 criterion. (see Sec. 2.3). Whereas low pitch values increase the heat transfer but also the pressure drop in an excessive way, too low pitches lead to almost negligible heat transfer augmentation values. Two wire coils with different pitches were employed in order to test the pitch effect on the non-Newtonian thermal-hydraulic performance. The dimensions of the wire coils are shown in Table 2.
1.2. Experimental set-up

The schematic diagram of the experimental setup is shown in Fig. 2. The experimental facility consists of two independent circuits. The primary circuit, which contains the test fluid, is in turn divided in two sub-loops. The test section is placed in the main one, including a gear pump (2) driven by a frequency controller (3). The test fluid in the supply tank (1) is continuously cooled in the second sub-loop through a plate heat exchanger (13) with a coolant flow rate settled by a three-way valve (15). The coolant liquid is stored in a 1000 l tank (16) from where it flows to a cooling machine. The thermal inertia of this tank, with a capacity of 1000 l, together with the operation of the PID-controlled three-way valve provides stability to the temperature of the test fluid in the supply tank, which can be accurately fixed to a desired value.

The test section was placed in the main circuit and consisted of a thin-walled, 4 m long, 316L stainless steel tube with a wire coil insert. The inner and outer diameters of the tube were 18 mm and 20 mm, respectively. Two oversized, low-velocity gear pumps (one on each circuit) were used for circulating the working fluid, in order to minimize the degradation of CMC solutions during the tests. Mass flow rate was measured by a Coriolis flow meter.

Pressure drop experiments were carried out in the hydro-dynamically developed region under isothermal conditions. Four pressure taps separated by 90° were coupled to each end of the pressure test section ($l_p=1.85$ m). Pressure drop $\Delta P$ was measured by means of a highly accurate pressure transmitter. The test section was preceded by 2 m of smooth tube with wire coil insert, in order to ensure fully developed flow conditions. The fanning friction factor $f$ was calculated from measurements of mean...
pressure drop and fluid mass flow rate as:

$$f = \frac{\Delta P}{\frac{D}{l_p} \frac{D}{2\rho \bar{u}^2}} = \frac{\Delta P}{\frac{\rho \pi^2 D^5}{32 \dot{m}^2}}$$

(1)

Heat transfer experiments were carried out under uniform heat flux (UHF) conditions, with hydrodynamically developed flow at the test section entry. Energy was added to the working fluid by Joule effect heating. A 6 kVA transformer was connected to the smooth tube by copper electrodes and the power supply was regulated by means of an auto-transformer. The length between electrodes defined the heat transfer test section ($l_h = 1.49$ m). The test section was insulated by an elastomeric thermal insulation material of 20 mm thickness and thermal conductivity 0.04 W/(m·K) to minimize heat losses. The power input added to the heating section was calculated by measuring the voltage between electrodes (0-15 V) and the electrical current (0-600 A). The heat input added to the test fluid, $Q$, was estimated after correcting the electrical power for heat losses through the outer wall. Fluid inlet and outlet temperatures $T_{in}$ and $T_{out}$ were measured by immersion RTD sensors. The axial position of the measuring point (element 12 on Fig. 6) was defined from the upstream electrode. In the present work it was fixed at a distance $l_x = 1.02$ m. Since heat was added uniformly along the tube length, the bulk temperature of the fluid at the measuring section, $T_b(l_x)$, was calculated by assuming a linear variation of mean fluid temperature with axial direction. The outside wall temperature at the measuring section, $T_{wo}$, was measured by eight surface thermocouples placed 45° apart circumferentially, and electrically insulated from the tube. $T_{wo}$ was estimated by averaging the eight wall-temperature readings. The local Nusselt number was calculated as:

$$Nu_x = \frac{D}{k} \frac{q''}{T_{wi} - T_x},$$

(2)
where $q''$ stands for the heat flux at the inner wall and $T_{wi}$ is the inner wall temperature, which was obtained from the numerical solution of the radial, 1D heat conduction across the insulated tube with internal heat generation.

The rheological characterization of the non-Newtonian test fluids ($n$ and $K$ values) as well as the viscosity measurement $\mu$ for the Newtonian one were obtained by employing an in-line viscometer, parallel to the testing tube. It consists of a smooth tube in which the mass flow rate and pressure drop are measured, retrieving the values of $n$, $K$ and $\mu$ (see Sec. 1.4). In that way, measurements of the rheological properties could be done at the beginning and at the end of each set of experiments, minimizing the thixotropy effect. Moreover, the measurement technique is based in the same principle (pressure drop on a tube) as the experimental tests, giving more accurate values of the properties.

Further details of the working apparatus and the calibration procedure are given in García et al. [14, 15]. The experimental uncertainty was calculated by following the "Guide to the expression of uncertainty in measurement", published by ISO [16]. Details of the uncertainty assignation to the experimental data are given by the authors in Vicente et al [17]. Uncertainty calculations based on a 95% confidence level showed maximum values of 3% for Reynolds number, 3% for Graetz number, 3% for Nusselt number and 4% for friction factor.

1.3. Test fluid characteristics

Two different types of test fluids were employed in the experiments; a Newtonian and a non-Newtonian fluid. The Newtonian test fluid was propylene-glycol whereas the non-Newtonian test fluids were 1% wt aqueous solutions of carboxymethyl cellulose (CMC), supplied by Sigma-Aldrich Co. Two different non-Newtonian test fluids were obtained by using two types of CMC: medium-viscosity (mv) and high-viscosity
(hv) grade. The solutions were prepared by dissolving the polymer powder in distilled water and then raising the pH values of the solution to increase viscosity. All propylene-glycol thermophysical properties were obtained from tables except viscosity, which was obtained by using the in-line viscometer (see Sec. 1.4). On the other hand, all CMC thermophysical properties except the rheological parameters were assumed to be the same as pure water. Solutions of CMC in water at low concentrations are pseudoplastic in nature, and their constitutive relationship can be expressed as (Chabria et al [18]):

\[
\tau_w = K \left[ \frac{8 \bar{u}}{D} \left( \frac{3n + 1}{4n} \right) \right]^n
\]

where \( \tau_w \) is the wall shear stress, \( K \) the flow consistency index, \( n \) the flow behaviour index and \( (8\bar{u}/D) \) is the velocity gradient at the wall for Newtonian fluids in fully developed laminar flow. The values of \( n \) and \( K \) for the test fluids were obtained by using the in-line smooth tube as a viscometer. The parameter \( (8\bar{u}/D) \) was calculated from the fluid flow rate, and \( \tau_w \) was computed from the isothermal pressure drop measurements at 25°C, 45°C and 60°C by means of:

\[
\tau_w = \frac{\Delta P D}{l_p} \frac{D}{4}.
\]

The procedure to obtain the values of the Newtonian viscosity \( \mu \) for propylene-glycol was analogue. Replacing \( n \) by 1 in Eq. 3 the flow consistency index \( K \) becomes the viscosity \( \mu \), and the Newtonian expression appears:

\[
\tau_w = \mu \left( \frac{8\bar{u}}{D} \right).
\]

Therefore, following the same steps as with CMC, viscosity can be calculated. Fig. 3 shows the flow curves (\( \tau_w \ vs. \ 8\bar{u}/D \)) for the non-Newtonian fluids used in the
smooth-tube experiments. Fresh CMC solutions were prepared to be used in the wire coil experiments. The values of $n$ and $K$ for the wire coil and the smooth-tube experiments are listed in Table 3, whereas the values of propylene-glycol viscosity $\mu$ are listed in Table 5. The uncertainty calculations based on a 95% confidence level shown maximum values of 0.2% for $n$ and 3% for $K$ and $\mu$.

1.4. Experimental details

The aqueous solutions of CMC degrade with shear and temperature because of the breakage of polymer chains (thixotropy). In this work, the experimental program was designed to shorten the time of testing. The tests for a given geometry (plain tube or tube with wire coil) with one type of aqueous solution of CMC (CMC high viscosity or medium viscosity grade) took about 450 minutes. A typical measurement cycle is schematically represented in Fig. 4, consisting of: 1-Pressure drop and flow-rate measurements at $T_b=25^\circ C$ (in-line viscometer), 2-Pressure drop test at $T_b=25^\circ C$ (wire coil tube), 3-Heat transfer test at $T_b(l_x)=25^\circ C$ (wire coil tube), 4-Pressure drop and flow-rate measurements at $T_b=45^\circ C$ (in-line viscometer), 5-Pressure drop test at $T_b=45^\circ C$ (wire coil tube), 6-Heat transfer test at $T_b(l_x)=45^\circ C$ (wire coil tube), 7-Pressure drop and flow-rate measurements at $T_b=60^\circ C$ (in-line viscometer), 8-Pressure drop and flow-rate measurements at $T_b=25^\circ C$ (in-line viscometer). For each complete measurement cycle performed (1–8), the test fluid was replaced with a fresh one.

The in-line viscometer results (tests 1, 4, and 7) were processed to obtain the rheological fluid properties, $K$ and $n$, as described in the preceding section. These properties were used in the data reduction routines. During the heat transfer experiments, the power input was adjusted to control the wall temperature at the measuring section, $T_w(l_x)$. In the tests at $T_b(l_x)=25^\circ C$, $T_w(l_x)$ was fixed at $45^\circ C$. 

10
and at $T_b(l_x)=45^\circ\text{C}$, $T_w(l_x)$ was $60^\circ\text{C}$ (see Fig. 4). Thus, in the data processing, the values of $n$, $K$ (for bulk temperature) and $K_w$ (for wall temperature) were completely known (see Sec. 2.2). The difference between the values of $K$ and $n$ obtained from tests 1 and 8 (performed at a same temperature in an interval of about 450 minutes) gives insight into the degradation rate of the fluid. Table 4 shows the values of $K$ and $n$ at the beginning and the end of each measurement cycle. A maximum fall of 38.9% in the value of $K$ and 6.9% in the value of $n$ is observed for the worst case (wire coil with CMC-mv). The arithmetic averaging procedure proposed by Joshi and Bergles [19] was employed, in order to estimate the corresponding values of $K$ and $n$ for a given time.

2. Results and discussion

2.1. Friction factor results

Fig. 5 shows the friction factor results for the plain tube, including both CMC and propylene-glycol (PG) fluids. The Reynolds number proposed by Metzner and Reed [20] for non-Newtonian power-law fluids has been used:

$$Re_{MR} = \frac{8^{1-n}D^n u^2 \rho}{K \left[\frac{3n+1}{4n}\right]^n},$$

which for laminar flow in a smooth tube is related to the friction factor in the same way as is for Newtonian fluids ($f = 16/Re_{MR}$). According to that, $Re_{MR}$ number is also valid for Newtonian fluids and particularly for propylene-glycol, in which $n=1$ and $K$ becomes the Newtonian viscosity $\mu$. This property of $Re_{MR}$ number allows comparisons between Newtonian and non-Newtonian fluids. The smooth tube results of Fig. 5 are in excellent agreement with the analytical solution, with a maximum deviation of $\pm1.6\%$. 

11
Fig. 6 depicts the friction factor results for the two tested wire coils inserts, including both CMC and propylene-glycol. These results show a different friction factor tendency for each wire coil. Although the transition starts for both wire coils at $Re_{MR} \approx 500$, the shorter pitch WC1 brings the transition forward slightly compared with WC2, which leads to higher $f$ values for WC1 when equal $Re_{MR}$ numbers are compared. Regarding the fluid type, a similar behavior between Newtonian and non-Newtonian fluids is observed. This similarity agrees with the exposed above: the use of $Re_{MR}$ yields the same relationship for Newtonian and non-Newtonian fluids in the laminar region, showing also equal values of $f$ for the transition region.

Fig. 7 shows the increase in friction factor $f_{wc}/f_s$ vs. Reynolds number $Re_{MR}$. Both wire coils inserts produced a moderate increase (50%) in pressure drop at Reynolds numbers below 500. WC1 shows a greater turbulence transition promoting effect respect to WC2. At $Re_{MR} \approx 1000$ the friction factor increment respect to the plain tube for the smaller pitch wire coil is about 3 whereas for the bigger pitch wire coil is around 2. For higher values of $Re_{MR}$ ($\approx 1300$) the friction factor increment for WC1 is equal to 8 whereas for WC2 is 6. On the other hand and in the same way as in 6, the type of fluid has no effect when $Re_{MR}$ is used; equal $Re_{MR}$ values lead to the same friction factor increments respect to the plain tube.

2.2. Nusselt number results

The heat transfer results for the smooth tube have been compared with the correlation of Bird et al. [12] for forced convection heat transfer in laminar flow:

$$Nu = 1.41 \Delta^{1/3} Gz^{1/3},$$  
(7)

$$Gz = \frac{\dot{m}c_p}{k l_x},$$  
(8)
and the term $\Delta$ is expressed as:

$$\Delta = \frac{3n + 1}{4n},$$  

(9)

Both the heat transfer measurements in the plain tube and the tube with wire coil have been corrected by the factor $\Delta^{1/3}$, in order to show the results free of non-Newtonian effects. A consistency index correction has also been applied to account for radial temperature variation, according to Joshi and Bergles [19]:

$$\left(\frac{K_b}{K_w}\right)^{0.58-0.44n}$$

(10)

where $K_b$ and $K_w$ are the fluid consistency index evaluated at bulk and wall temperatures, respectively. In the same way as in the $Re_{MR}$ number, these expressions can be applied to a Newtonian fluid (i.e. propylene-glycol), in which $\Delta=1$ and Eqs. 7 and 10 become the well-known Newtonian expressions. As mentioned in section 1.2, the outside wall temperature at the measuring point, $T_{wo}$, was measured with eight surface thermocouples placed $45^\circ$ apart. This arrangement allowed to determine if a circumferential temperature variation existed, which would suggest the presence of mixed convection heat transfer phenomena. Within the experimental range covered, no effect of buoyancy forces on heat transfer has been observed. The heat transfer experimental data for the plain tube including both propylene-glycol and CMC is presented in Fig. 8, in terms of local Nusselt number $vs$ Graetz number.

As it is shown in Fig. 8, the Nusselt number results for the plain tube are in good agreement with Bird correlation (Eq 7), with a deviation of $\pm 6.5\%$ for 95$\%$ of data. Fig. 9 shows Nusselt number $vs$. Reynolds number for the plain tube and for the wire coil inserts, including again propylene-glycol, CMC-hv and CMC-mv. The three reference lines shown in Fig. 9 correspond to the solution of Eq. 7 for each working fluid. At low Reynolds numbers, wire coils have no effect in heat transfer.
However, both wire coils become more effective as turbulence is established. In terms of relative effectiveness between them, results seem to show a higher increase of heat transfer for wire coil WC1 (lower pitch), as it was expected from the early transition to turbulence observed in Figs. 6 and 7. On the other hand, the performance of the wire coil regarding the type of fluid seems to be lower for wire coil WC1, when it is compared with the Newtonian results. The increment in heat transfer due to transition is shifted from \( Re_{MR} \approx 300 \) to \( Re_{MR} \approx 500 \). For the case of wire coil WC2, the difference between Newtonian and non-Newtonian fluid is much lower, while an early transition for the non-Newtonian fluid can be observed respect to the Newtonian one.

All these facts are more clearly noticeable if the heat transfer results are processed in terms of Prandtl number and \( Nu_{wc}/Nu_s \) ratio, which relates the heat transfer coefficient in the tube with wire coil insert with the one in the plain tube, at the same Reynolds number. Figs. 10 and 11 show respectively the wire coils WC1 and WC2 for the \( Nu_{wc}/Nu_s \) ratio vs. Reynolds number, including also the Prandtl number for each \( Re_{MR} \) value. Propylene-glycol (Newtonian fluid) \( Nu \) values from \( Re_{MR} > 2000 \) onwards have been divided by the turbulent regime correlation for smooth tube (García et al. [14]):

\[
Nu_{s,t} = 0.0147 \left[ (Re - 1000)^{0.86} Pr^{0.39} \right]
\]  

Prandtl number is obtained from the apparent viscosity \( \mu_{eff} \),

\[
\mu_{eff} = K \dot{\gamma}_w^{n-1}
\]

where \( \dot{\gamma}_w \) is the wall shear rate, defined as,

\[
\dot{\gamma}_w = \left[ \frac{3n + 1}{4n} \left( \frac{8\bar{u}}{D} \right) \right]
\]
Thus, Prandtl number for non-Newtonian fluids depends on the fluid velocity and therefore on the Reynolds number. This fact is clearly noticeable in Figs. 10 and 11. Comparing both figures, an early transition to turbulent regime is observed for wire coil WC1 (Fig. 10). Above \( Re_{MR} \approx 500 \), the Nusselt number results for this wire coil are significantly higher than the plain tube results, while in the wire coil WC2 the same situation is reached for \( Re_{MR} \approx 600 \). This different tendency disappears after reaching the turbulent regime, and both wire coils tend to the same values. The \( Nu_{wc}/Nu_s \) ratio increase with Reynolds number; a maximum value of \( Nu_{wc}/Nu_s = 7.5 \) is observed at \( Re_{MR} = 1900 \), for both wire coil WC1 and WC2. In terms of Newtonian and non-Newtonian behavior, an early transition of propylene-glycol with respect to CMC have been found for wire coil WC1 (\( p/D = 1 \)) while the opposite tendency is found in the wire coil WC2 (\( p/D = 2 \)). This different tendency disappears as long as the transition to turbulent regime results in a completely turbulent regime, in the same way that occurs for the two wire coils results (mentioned above). Moreover, the results of CMC and propylene-glycol with the same Prandtl value present a similar ratio \( Nu_{wc}/Nu_s \).

2.3. Performance evaluation

The \( R3 \) criterion outlined by Bergles et al. [21] has been calculated to quantify the performance of the wire coil inserts. This criterion yields the heat transfer augmentation \( (Nu_{wc}/Nu_s) \) when the wire coil is inserted in a smooth tube, for equal pumping power and heat exchange surface area. To satisfy the constraint of equal pumping power, \( Nu_s \) is evaluated at the equivalent smooth tube Reynolds number \( Re_s \), which matches:

\[
R3^3 = \frac{f_{wc}}{f_s} Re_{MR}
\]  \( (14) \)
Fig. 12 shows the results of $R3$ vs. the equivalent smooth tube Reynolds number, $Re_s$. It includes both the two wire coils as well as the tested fluids (propylene-glycol, CMC-hv and CMC-mv). The wire coil inserts have a poor performance at low Reynolds numbers ($Re_s < 500$), where they even adversely affect the heat transfer ($R3 \approx 1$). The $R3$ values make evident that there are no significant differences on the thermal-hydraulic performance for the two wire coils investigated: the same type of fluid shows similar $R3$ values for different pitches. However, the results exhibit higher $R3$ values for the Newtonian fluid compared with the non-Newtonian one. Thereby, the transition of the propylene-glycol starts at $Re_s \approx 300$, whereas for the CMC solutions it does at $Re_s \approx 500$ (both independently of the wire coil pitch).

At $Re_s \approx 700$ $R3$ reaches a value of 4 for propylene-glycol and around 3 for the CMC solutions. However, once the flow becomes more and more turbulent the difference is smaller, leading to a value of $R3 \approx 7$ for $Re_s \approx 1300$ in both cases, which means that at $Re_s \approx 1300$ the heat transfer rate will be increased by 300% if the wire coil is inserted in a smooth tube.

Therefore, the higher heat transfer augmentation reported for the wire coil WC1 with respect to the wire coil WC2 in Fig. 9 is outweighed here by the higher values of friction factor (Fig. 6). Hence the $R3$ values result in a similar performance for both wire coils if the same type of fluid is compared. However the presence of the wire coil shows a more positive effect on Newtonian fluid than in non-Newtonian ones, advancing the transition to turbulence.

3. Conclusions

1. Isothermal pressure drop and heat transfer experiments under UHF for Newtonian and pseudoplastic non-Newtonian flow have been performed on two wire coils inserted in a smooth tube, covering the laminar and transition regimes:
$Re_{MR} = 10-1200$ and $Pr = 150-1900$. Rheological properties were measured with an in-line viscometer.

2. In laminar regime, results show a negligible effect of the wire coils whereas the transition to turbulent flow is brought forward to $Re_{MR} \approx 500$ smoothly. Higher pitch-diameter ratio leads to a greater effect in the promotion to turbulence, whereas no difference was found between non-Newtonian and Newtonian fluids.

3. The wire coil inserts have no effect in heat transfer for low Reynolds numbers (below $Re_{MR} \approx 500$), becoming more effective as turbulence is established and tending both wire coils to the same values: maximum Nusselt number augmentations of 7.5 were found at $Re_{MR} \approx 1900$.

4. According to the criterion $R3$ both wire coil inserts have a poor performance at low Reynolds numbers, becoming effective for Reynolds numbers above 500. Both wire coils have a similar performance for equal type of fluids being the wire-coil enhancement more noticeable for the Newtonian flow: at $Re_{MR} \approx 700$ $R3$ reaches a value of 4 for propylene-glycol and around 3 for the CMC solutions, whereas at $Re_{MR} \approx 1300$ the $R3$ value is $\approx 7$ for both fluids.

5. The prediction of the friction factor values in plain tubes with wire coil inserts for the tested non-Newtonian fluids can be accurately retrieved from the existent correlations for Newtonian fluids by simply using the $Re_{MR}$. The Nusselt number values can be approached by the existent correlations for Newtonian fluids applying the factor $\Delta^{1/3}$.

References


Figure captions

Figure 1: Sketch of a wire coil fitted inside a smooth tube.

Figure 2: Schematic diagram of the experimental setup: (1) supply tank, (2) gear pumps, (3) frequency controller, (4) immersion resistances, (5) Coriolis flowmeter, (7, 8) inlet and outlet immersion RTDs, (9) pressure transmitter, (10) electrical transformer, (11) autotransformer, (12) surface thermocouples (13) plate heat exchanger, (14) PID, (15) three-way valve, (16) cooling tank, (17) cooling machine, (18) in-line viscometer.

Figure 3: Experimental relation shear stress-shear rate for the CMC solutions obtained in smooth tube tests.

Figure 4: Typical measurement cycle (clockwise direction): in-line viscometer rheological properties $n$ and $K$ and their application in each pressure drop ($\Delta P$) and heat transfer ($Q$) wire-coil test.
Figure 5: Fanning friction factor results for the smooth tube.

Figure 6: Fanning friction factor results for the two wire-coils inserts.

Figure 7: Fanning friction increase $f_w/f_s$ vs. Reynolds number.

Figure 8: Local Nusselt number as a function of Graetz number for the smooth tube in forced convection.

Figure 9: Fully developed Nusselt number results for the smooth tube and wire coils as a function of Reynolds number.

Figure 10: Nusselt number augmentation, $Nu_w/Nu_s$, and $Pr$ number vs. Reynolds number for wire coil WC1.
Figure 11: Nusselt number augmentation, $Nu_w/Nu_s$, and $Pr$ number vs. Reynolds number for wire coil WC2.

Figure 12: $R3$ performance evaluation vs equivalent smooth tube Reynolds number.
<table>
<thead>
<tr>
<th></th>
<th>CMC-mv 1%wt</th>
<th>CMC-hv 1%wt</th>
<th>Propylene glycol</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q'' (\text{W m}^{-2}) )</td>
<td>7,320-37,541</td>
<td>7,936-66,463</td>
<td>4,563-49,114</td>
</tr>
<tr>
<td>( Re_{MR} )</td>
<td>8-400</td>
<td>58-1819</td>
<td>221-3160</td>
</tr>
<tr>
<td>( Pr )</td>
<td>490-1900</td>
<td>150-380</td>
<td>142-314</td>
</tr>
<tr>
<td>( n )</td>
<td>0.41-0.43</td>
<td>0.86-0.94</td>
<td>0.027-0.011</td>
</tr>
<tr>
<td>( \dot{\gamma} (\text{s}^{-1}) )</td>
<td>87-778</td>
<td>74-700</td>
<td>-</td>
</tr>
<tr>
<td>( \mu (\text{kg m}^{-1}\text{s}^{-1}) )</td>
<td>-</td>
<td>-</td>
<td>0.027-0.011</td>
</tr>
<tr>
<td>( \dot{m} (\text{kg h}^{-1}) )</td>
<td>150-1400</td>
<td>150-1400</td>
<td>160-1800</td>
</tr>
</tbody>
</table>

Table 1: Ranges of investigated experimental variables.
<table>
<thead>
<tr>
<th></th>
<th>D(mm)</th>
<th>p/D</th>
<th>e/D</th>
<th>p/e</th>
<th>α(°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire Coil WC1</td>
<td>18</td>
<td>1</td>
<td>0.088</td>
<td>11.3</td>
<td>63.4</td>
</tr>
<tr>
<td>Wire Coil WC2</td>
<td>18</td>
<td>2</td>
<td>0.088</td>
<td>22.5</td>
<td>45.0</td>
</tr>
</tbody>
</table>

Table 2: Geometry of the wire coils tested.
<table>
<thead>
<tr>
<th>Test</th>
<th>T (°C)</th>
<th>n</th>
<th>K</th>
<th>n</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth tube</td>
<td>25</td>
<td>0.86</td>
<td>0.10</td>
<td>0.39</td>
<td>4.82</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>0.94</td>
<td>0.04</td>
<td>0.41</td>
<td>2.91</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1.01</td>
<td>0.01</td>
<td>0.44</td>
<td>2.72</td>
</tr>
<tr>
<td>WC1</td>
<td>25</td>
<td>0.85</td>
<td>0.11</td>
<td>0.38</td>
<td>4.83</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>0.96</td>
<td>0.05</td>
<td>0.42</td>
<td>2.93</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1.02</td>
<td>0.02</td>
<td>0.45</td>
<td>2.71</td>
</tr>
<tr>
<td>WC2</td>
<td>25</td>
<td>0.87</td>
<td>0.09</td>
<td>0.40</td>
<td>4.81</td>
</tr>
<tr>
<td></td>
<td>45</td>
<td>0.95</td>
<td>0.06</td>
<td>0.40</td>
<td>2.92</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>1.03</td>
<td>0.01</td>
<td>0.43</td>
<td>2.73</td>
</tr>
</tbody>
</table>

Table 3: Values of $n$ and $K$ (kg·m$^{-1}$·s$^{-1}$) for CMC used for wire coil tests and smooth tube tests.
<table>
<thead>
<tr>
<th>Test</th>
<th>Measure</th>
<th>Initial</th>
<th>Final</th>
<th>Variation (%)</th>
<th>Initial</th>
<th>Final</th>
<th>Variation (%)</th>
<th>Initial</th>
<th>Final</th>
<th>Variation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth tube</td>
<td>Initial</td>
<td>0.86</td>
<td>0.92</td>
<td>6.98</td>
<td>0.85</td>
<td>0.93</td>
<td>9.41</td>
<td>0.87</td>
<td>0.91</td>
<td>8.33</td>
</tr>
<tr>
<td></td>
<td>Final</td>
<td>0.10</td>
<td>0.06</td>
<td>40.00</td>
<td>0.11</td>
<td>0.07</td>
<td>36.36</td>
<td>0.10</td>
<td>0.05</td>
<td>50.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.39</td>
<td>0.43</td>
<td>10.26</td>
<td>0.38</td>
<td>0.41</td>
<td>7.89</td>
<td>0.40</td>
<td>0.42</td>
<td>7.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.82</td>
<td>3.89</td>
<td>19.29</td>
<td>4.83</td>
<td>3.88</td>
<td>19.67</td>
<td>4.81</td>
<td>3.87</td>
<td>19.54</td>
</tr>
</tbody>
</table>

Table 4: Values of $n$ and $K$ (kg·m$^{-1}$·s$^{-1}$) for the test fluids, at the beginning and the end of 25°C experiments.
Table 5: Experimental data for dynamic viscosity $\mu$ (kg·m$^{-1}$·s$^{-1}$) for propylene-glycol used for wire coil tests and smooth tube tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>$T$ (°C)</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth tube and wire coils</td>
<td>25</td>
<td>0.027</td>
</tr>
<tr>
<td>Smooth tube and wire coils</td>
<td>45</td>
<td>0.011</td>
</tr>
<tr>
<td>Smooth tube and wire coils</td>
<td>45</td>
<td>0.009</td>
</tr>
</tbody>
</table>
$8 \cdot u/D$

$\tau_w$

CMC 1%hv 25°C
CMC 1%hv 45°C
CMC 1%hv 60°C
CMC 1%mv 45°C
CMC 1%mv 60°C
CMC 1%mv 25°C
$f = \frac{16}{Re_{MR}}$

- PG
- CMC hv
- CMC mv
\( f = \frac{16}{Re_{MR}} \)

- PG WC−1
- PG WC−2
- CMC hv WC−1
- CMC mv WC−1
- CMC hv WC−2
- CMC mv WC−2
Wire coil

- PG WC–1
- PG WC–2
- CMC hv WC–1
- CMC mv WC–1
- CMC hv WC–2
- CMC mv WC–2

Analytic smooth

$\frac{Nu}{Pr^{1/3}}$ vs $Re_{\text{eff}}$
The diagram shows the relationship between $Re_{str}$ and $Nu_{str}/Nu_{n}$ on the top axis and $Pr$ on the bottom axis. The symbols represent different data sets: PG, CMC hv, and CMC mv. The x-axis represents $Re_{str}$ with logarithmic scale, while the y-axis represents $Nu_{str}/Nu_{n}$ and $Pr$ with linear scale. The data points are spread across the graph, indicating varying trends for different conditions.