Performance Evaluation of MAC Transmission Power Control in Wireless Sensor Networks

Javier Vales-Alonso‡, Esteban Egea-López, Alejandro Martínez-Sala, Pablo Pavón-Mariño,
M. Victoria Bueno-Delgado, Joan García-Haro
Department of Information Technologies and Communications
Polytechnic University of Cartagena, Spain
e-mail: {javier.vales, esteban.egea, alejandros.martinez, pablo.pavon, mvictoria.bueno, joang.haro}@upct.es
‡ Corresponding author. Address: Campus Muralla del Mar, 30202, Cartagena, Spain
phone: +34 968 326588, fax: +34 968 325973, e-mail: javier.vales@upct.es

Abstract*— In this paper we provide a method to analytically compute the energy saving provided by the use of Transmission Power Control (TPC) at the MAC layer in Wireless Sensor Networks (WSN). We consider a classical TPC mechanism: data packets are transmitted with the minimum required power to achieve a given packet error probability, whereas the additional MAC control packets are transmitted with the nominal (maximum) power. This scheme has been chosen because it does not modify the network topology, since control packet transmission range does not change. This property also allow us to compute analytically the expected energy savings. Besides, this type of TPC can be implemented in the current sensor hardware, and can be applied directly to several MAC protocols already proposed for WSN. The foundation of our analysis is the evaluation of L ratio, defined as the total energy consumed by the network using the original MAC protocol divided by the total energy consumed if the TPC mechanism is employed. In the L computation we emphasize the basic properties of sensor networks. Namely, the savings are calculated for a network that is active a very long time, and where the number of sensors is supposed to be very large. The nodes position is assumed to be random -for the sake of example a normal bivariate distribution is assumed in the paper- and no node mobility is considered. In the analysis we stress the radio propagation and the distribution of the nodes in the network, that will ultimately determine the performance of the TPC. Under these conditions we compute the mean value of L. Finally, we have applied the method to evaluate the benefits of TPC for TDMA and CSMA with two representative protocols, L-MAC and S-MAC using their implementation reference parameters. The conclusion is that, while S-MAC does not achieve a significant improvement, L-MAC may reach energy savings up to 10-20%.

***Index Terms*— Energy saving, MAC, network lifetime, transmission power control, WSN**

I. Introduction

Wireless Sensor Networks (WSNs) consist of a large set of autonomous wireless sensing nodes [1]. They are designed and deployed to accomplish specific tasks, e.g. environmental monitoring, industrial sensing or space exploration. This fact yields to specific traffic patterns and network topologies, that are strongly application dependent. WSNs also inherit key properties from ad-hoc networks: decentralized control, common transmission channel, broadcast nature, multi-hop routing, and ephemeral topologies among others. Besides, there are two WSN basic constraints. First, the expected hardware, program and data memory resources are scarce, and consequently, impose limitations on the protocol complexity. Second, the amount of energy available per node is finite if they are battery powered. Therefore, the development of energy-efficient protocols and applications is a major challenge in current WSN research, in order to develop systems that run unattended (without battery replacement) for an arbitrary long time (e.g. years).

Actually, the major sources of energy "waste" are related to radio communication issues [2], [3]. For instance, communicating one bit of information consumes as much energy as executing hundreds of instructions in typical sensor nodes like Mica2 motes [4]. The radio interface consumption depends on its state, which can be one of the following:

- Transmission state (tx). Packet transmission. Power consumption is proportional to the the radiated output power, which can be selected from a set of discrete values (e.g. from -20 to 5 dBm in steps of 1 dB in the Mica2 motes, see Table I). However, in practice the output power is kept fixed at a *nominal* value (usually, the maximum possible output power).
- Reception -and listening- state (rx). Packet reception and channel listening (carrier sense). It also consumes a significant amount of power independently of receiving actual data or just listening (e.g. 35.4 mW in the Mica2 motes).
- Sleep state (sl). Radio off. Negligible power consumption (e.g. 3 μW in the Mica2 motes).

From the MAC (Medium Access Control) layer perspective consumption may be minimized if the nodes sleep during inactivity instead of being in the reception state. Thereby the average consumption is significantly reduced. This strategy requires coordination among nodes (all neighbors must sleep and awake simultaneously) and a tradeoff between the sleeping time and the achievable throughput (since nodes cannot send or receive data in the sleep state). Consumption may also be reduced using only the power needed for each data transmission. Figure 1 illustrates this idea. Node 1 reaches nodes 2 and 3 with power Q_1, however nodes 4, 5 and 6 are only reached at power Q_2. In this case Q_2 is the nominal power that guarantees full conectivity. But, if power is set

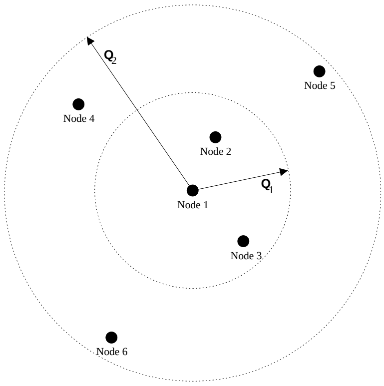

Fig. 1. Discrete output powers

fixed, there is a waste when data packets are delivered to nodes 2 and 3 at power Q_2. Selecting the *best* output power in each transmission may reduce considerably the output power, and thus the total consumption. This type of strategy is called Transmission Power Control (TPC). Currently, only the former scheme -coordinated sleeping- has been exhaustively studied as a technique for energy saving in WSNs, since idle listening avoidance has been widely considered the dominant factor to reduce power consumption. This has been the major design objective of a large set of specific WSN MAC protocol proposals [3], [5]–[8].

In this paper, we are interested in the additional benefits, in terms of energy saving, that the use of TPC may provide to WSN MAC protocols. First, we have to select a TPC mechanism suitable (or at least adaptable) for the vast majority of the WSN MAC approaches, in order to evaluate the performance of TPC. We have selected a well-known approach for TPC [9]: the considered TPC mechanism uses the minimum necessary power to transmit the network layer packet data units with a bounded packet error probability, and the nominal (maximum) power to transmit the additional MAC control packets. For instance, in the example depicted in Fig. 1, considering the IEEE 802.11 protocol [10]: RTS/CTS/Data/ACK[1] sequence. If node 1 has to send a data packet to node 2, then node 1 would start transmiting the RTS at the maximum (nominal) power Q_2. Afterwards, node 2 would answer with the CTS at the same nominal power. Then Data is exchanged, but at the reduced power Q_1, and finally the ACK is transmitted with the nominal power Q_2.

This approximation does not modify network topology since control packet transmission range is not modified. Thereby, upper layer behavior is not affected. This property allow us to compare fairly the energy consumption between TPC and non-TPC protocols.

As a drawback, this type of TPC requires that nodes have an exact knowledge about the output power needed for every packet exchange. Nevertheless, since we are interested in the maximum energy saving achieved by an ideal TPC, we will assume that this information is already known by the nodes.

To evaluate such a mechanism, we compute the ratio of the energy without TPC to the energy using the TPC algorithm after a very long running time. We denote this ratio as L. We find that, for a given node distribution, L depends on two factors: (i) one characterizes the geometrical distribution of the nodes and the other (ii) is mainly influenced by the MAC protocol and the traffic of the network. Both parameters are evaluated under the most widely accepted WSN assumptions (see Section IV), stressing the geometrical distribution and the transmission properties of the network. It is shown that L converges if the number of nodes is large. Once developed the theoretical framework, it can be applied to a wide range of MAC proposals, just by adjusting the parameters of the model according to a given MAC operation. Therefore, we describe how to put our results in practice: the theoretical result is applied to a set of traffic loads and nodes densities for two MAC protocols, a TDMA one (L-MAC) [8] and a contention one (S-MAC) [3]. The results reveal that energy savings are considerable for L-MAC (from 10% to 20%), while rather average for S-MAC (< 10%).

The rest of the paper is organized as follows. Section II presents the related work in the field. In section III the network lifetime ratio L is defined and computed as a function of two factors (ξ and s). In section IV a representative WSN scenario is discussed and studied. Sections V and VI computes the mean of ξ and s, respectively, under the assumptions considered in the scenario of Section IV. Then, Section VII applies these results to compute the mean of L for the L-MAC and S-MAC protocols. Finally, section VIII concludes.

Note: From then on, the following notation and conventions are used:

- Probabilities are denoted as Pr[Event].
- Random variables (rv) are denoted as $\boldsymbol{x}$.
- Average values are denoted as $\overline{\boldsymbol{x}}$ or $E\{x\}$.
- Stochastic processes are denoted as $x(t)$.
- Discrete output transmission powers are denoted with the (quantum) letter Q.
- Power consumptions are denoted with the letter P.

II. Discussion and related work

The goal of the TPC mechanisms is to select the *optimum* power transmission level to be employed in each packet delivery. The precise meaning of optimum depends, indeed, on the scope and objective. TPC protocols for Mobile Ad-hoc NETworks (MANETs) have been commonly designed with the aim of increasing the capacity in wireless media by means of channel reuse or to ensure network connectivity. Two types of strategies have been already considered: (i) In network layer TPCs transmission power control is used to select the *best* subset of the actual neighbors to be reached, that is, for topology control purposes (e.g. COMPOW [12] and PSP [13]). And (ii), the MAC layer TPCs, where power is selected for each packet to improve channel reuse or reduce packet collision probability (e.g. PCM [14], PCDC [15], PCMA [16] and DCAPC [17]). The three latter proposals are based on multichannel devices, using the additional channels to signal incoming transmissions and to compute the best output power to use. Besides, the PCM protocol must run on specialized radio hardware that allows very fast output power variation.

[1]Request To Send/Clear To Send/Data/ACKnowledgement.

These solutions effectively reduce collisions and improve capacity, which is the major issue in MANETs. However, in WSN the primary concern is *energy consumption efficiency*, rather than high channel throughput and reusability. Moreover, WSN protocols are constrained by the scarce memory, CPU and radio resources available. Sensor nodes are too limited to support most of the previously mentioned approaches proposed for MANETs. For instance, multichannel proposals cannot be implemented since current sensors are mainly monochannel.

There is a number of TPC algorithms designed specifically for sensor networks. In [18] a set of distributed TPC algorithms is proposed. These mechanisms select a single transmission power level for each node. The different proposals are compared with each other, using network connectivity and lifetime as performance metrics, but no comparison with non-TPC protocols is provided. In this paper, on the contrary, we evaluate the expected energy savings that can be achieved by using a generic TPC protocol, without describing a particular way of finding the power level necessary to reach a specific neighbor. Thus, our analyisis provides an ideal upper bound on energy saving.

Analytical studies on several aspects of TPC can also be found in the scientific literature. In [19] the auhors look for the optimal transmission range that maximizes a parameter called “expected one-hop progress in the desired direction”, but energy consumption is not considered. In [20], an analytical comparison between common range and variable-range TPC for MANETs is provided. The study is focused on the impact of TPC on network connectivity, capacity and routing protocols. An energy consumption model, as a function of the packet size, MAC protocol and radio characteristics, is used in [21] to derive an optimal transmission power in terms of end-to-end energy consumption. Our approach focuses on the improvement that TPC may provide. In addition, the TPC mechanism used in our evaluation can be combined with proposals focused on network layer operation. It should be noticed that the network topology is determined by the range of control packets, which here is set to the maximum (nominal) transmission power. Therefore, this nominal transmission power can be selected in order to achieve a desired network property, such as full connectivity, and still TPC can be used for data transmission.

III. Performance analysis

In this section, we introduce the calculation of the energy saving which can be obtained from the TPC mechanism selected. Let n be the number of nodes in the network. Let $T^i_{tx}(t)$, $T^i_{rx}(t)$ and $T^i_{sl}(t)$ be stochastic proccesses representing the accumulated time that node i, for i=1..n, is in each state -transmission (*tx*), reception (*rx*) and sleep (*sl*)- during the interval [0,t). Then, we can define the total network accumulated time in each state, summing all nodes contributions, $T_{tx}(t)$, $T_{rx}(t)$ and $T_{sl}(t)$, as:

$$\begin{aligned} T_{tx}(t) &= \sum_{i=1}^{n} T^i_{tx}(t) \\ T_{rx}(t) &= \sum_{i=1}^{n} T^i_{rx}(t) \\ T_{sl}(t) &= \sum_{i=1}^{n} T^i_{sl}(t) \end{aligned} \tag{1}$$

Accordingly, let P_{tx}, P_{rx} and P_{sl} be the power consumptions associated to each state. Then, the total energy consumed until an arbitrary instant t, $E(t)$, is given by eq. (2).

$$E(t) = P_{tx}T_{tx}(t) + P_{rx}T_{rx}(t) + P_{sl}T_{sl}(t) \tag{2}$$

We should specify a way to measure the energetic efficiency improvement due to the application of the TPC mechanism to a particular MAC protocol. One metric may be the increase of the *lifetime* of the network. However, this is difficult to define in a general way, since it is not clear when the network (as a whole) cannot continue its correct operation, which usually depends on the application. Instead, we can compute a metric of the efficiency (we name it L) of TPC based on the assimptotical ratio for a large t of the energy consumption of the network in both cases (with and without TPC) through equation (3).

$$L = \lim_{t\to\infty} \frac{E(t)|_{\text{no-TPC}}}{E(t)|_{\text{TPC}}} \tag{3}$$

This expression can be further developed taking into consideration the assumption that control packets are always sent at the maximum power. Therefore, neither network topology, nor $T^i_{tx}(t)$, $T^i_{rx}(t)$ and $T^i_{sl}(t)$, changes when TPC is used.

Let us assume that the network is composed of homogeneous sensors with p possible transmission output powers (Q_j, $j = 1, \ldots, p$). Let us denote P_{tx_i} as the power consumption associated to output transmission power Q_i. Let Q_p be the nominal (maximum) transmission output power, and hence P_{tx_p} is the nominal consumption. Now, let us note that the transmission time of each node can be decomposed into two contributions: data and signaling. Let us define the stochastic processes $T^i_{data}(t)$, $T^i_{sign}(t)$ for $i = 1, \ldots, n$ as the accumulated transmission time dedicated to data and signaling respectively for each node. Let $T_{data}(t) = \sum_{i=1}^n T^i_{data}(t)$, $T_{sign}(t) = \sum_{i=1}^n T^i_{sign}(t)$. In addition, $T^i_{data}(t)$ can be further subdivided in the time spent transmitting at each output power j: $T^i_{data_j}(t)$. Let us define $T_{data_j}(t) = \sum_{i=1}^n T^i_{data_j}(t)$ for $j = 1, \ldots, p$.

Then, assuming that the power consumption during the sleep periods is negligible, the energy consumption without and with TPC is given by equations (4) and (5), respectively.

$$E(t)|_{\text{no-TPC}} = T_{data}(t)P_{tx_p} + T_{sign}(t)P_{tx_p} + T_{rx}(t)P_{rx} \tag{4}$$

$$E(t)|_{\text{TPC}} = \sum_{j=1}^{p} T_{data_j}(t)P_{tx_j} + T_{sign}(t)P_{tx_p} + T_{rx}(t)P_{rx} \tag{5}$$

Therefore, from eq. (3), (4) and (5):

$$L = \lim_{t\to\infty} \frac{T_{data}(t)P_{tx_p} + T_{sign}(t)P_{tx_p} + T_{rx}(t)P_{rx}}{\sum_{j=1}^{p} T_{data_j}(t)P_{tx_j} + T_{sign}(t)P_{tx_p} + T_{rx}(t)P_{rx}}$$

L can be rewritten dividing both numerator and denominator by $P_{tx_p}T_{data}(t)$; we obtain:

$$L = \lim_{t\to\infty} \frac{1 + \frac{T_{sign}(t)}{T_{data}(t)} + \frac{P_{rx}T_{rx}(t)}{P_{tx_p}T_{data}(t)}}{\sum_{j=1}^{p} \frac{P_{tx_j}T_{data_j}(t)}{P_{tx_p}T_{data}(t)} + \frac{T_{sign}(t)}{T_{data}(t)} + \frac{P_{rx}T_{rx}(t)}{P_{tx_p}T_{data}(t)}} \quad (6)$$

Now, let us define the variable ξ as,

$$\xi = \lim_{t\to\infty} [\frac{T_{sign}(t)}{T_{data}(t)} + \frac{P_{rx}}{P_{tx_p}} \frac{T_{rx}(t)}{T_{data}(t)}] \quad (7)$$

Besides, let $\boldsymbol{Q}$ be the “output transmission power” random variable that assigns to each transmission output power Q_j its corresponding probability $\Pr[\boldsymbol{Q} = Q_j]$ (i.e., the probability that a data transmission occurs in the j-th power quantum). Let us note that,

$$\Pr[\boldsymbol{Q} = Q_j] = \lim_{t\to\infty} \frac{T_{data_j}(t)}{T_{data}(t)} \quad (8)$$

Hence,

$$L = \frac{1 + \xi}{\sum_{j=1}^{p} \frac{P_{tx_j}}{P_{tx_p}} \Pr[Q{=}Q_j] + \xi} \quad (9)$$

Finally, calling $s = \sum_{j=1}^{p} \frac{P_{tx_j}}{P_{tx_p}} \Pr[Q{=}Q_j]$, we arrive to:

$$L = \frac{1 + \xi}{s + \xi} \quad (10)$$

This expression says that the energy ratio of the two approaches converges after a long running time to a value that is a function of two variables: s and ξ. The s-coefficient characterizes the geometrical distribution of the network since it depends on the relative distance between neighbors, which determines the output power required. Moreover, s values are always within the interval (0,1]. Low values indicate that nodes are closer, and thus savings are more noticeable (larger values of L). High values indicates that it is unlikely that the output quantum can be reduced. In fact, $s = 1$ means that no saving is possible at all. Thus, in this case TPC lacks interest.

On the other hand, the ξ-coefficient measures the balance between the time devoted to data transmission and the time used for signaling and reception. Note that $\partial L/\partial\xi = (s - 1)/(s+\xi)^2 < 0$ for all $s \in (0, 1)$. Therefore, L decreases as ξ increases. Protocols with a good balance have a low ξ value, yielding to a higher L ratio, that is, larger savings.

Finally, let us note that both coefficients are influenced by the traffic properties, and that if either the position of nodes or the number of sensors change, these variables (s and ξ) will also change, and so the L factor. That is, L is a function of the number of nodes and their positions. Hence, if nodes position is random (a likely case in WSN), the L factor is also random.

We will compute the mean of L, $\overline{\boldsymbol{L}}$, in the next sections. We will show that for large values of n, in a representative WSN deployment, $\overline{\boldsymbol{L}}$ is given by the following expression:

$$\overline{\boldsymbol{L}} = \frac{1 + \overline{\boldsymbol{\xi}}}{\overline{\boldsymbol{s}} + \overline{\boldsymbol{\xi}}} \quad (11)$$

Both $\overline{\boldsymbol{s}}$ and $\overline{\boldsymbol{\xi}}$ (and, therefore, $\overline{\boldsymbol{L}}$) are evaluated in the following sections applying the realistic assumptions for WSN discussed in Section IV.

IV. WSN MODEL

At this point, we aim to compute the general L function derived in the previous section for a representative WSN model. Such scenario is specified by: (i) a suitable propagation model, (ii) a node distribution, (iii) a traffic pattern and (iv) a multiple access scheme.

A. Propagation model

WSN media can be considered a time-invariant narrow-band channel, which can be modeled using a path-loss approximation [22]. In this case, the transmission power required ($\widehat{P_{tx}}$) to achieve a target probability error ($\widehat{p_e}$) for nodes at a given distance d is:

$$\widehat{P_{tx}} = d^{\alpha}\Omega(\widehat{p_e}) \quad (12)$$

being α the propagation coefficient of the path-loss model and Ω a function that depends on $\widehat{p_e}$ and the communication environment. Appendix 1 describes how this expression is found.

For instance, for the Mica2 motes hardware, a packet size of 100 bytes, a target error probability $\widehat{p_e} = 10^{-3}$, and $\alpha = 3.95$ (value experimentally obtained in our test-beds [11]), and a bit rate of 30 Kbps we have $\Omega \approx$ -97.5 dB[2]. For the discrete transmission output powers of the Mica2 motes, the associated distances using the previous parameters are summarized in Table I.

Maximum range. Additionally, receivers have a sensitivity ($P_S \approx -102$ dBm for the Mica2 motes) that establishes the maximum distance between nodes (d_S). For instance, for the path-loss model with the previous propagation coefficient ($\alpha = 3.95$) the associated maximum distance d_S is 89.92 m.

B. Nodes distribution

In a general WSN deployment the position of nodes may not be controlled, and is *a-priori* unknown. In this paper we consider a reasonable type of node position pattern, where nodes are concentrated around a *point of interest*. This pattern is likely in several cases, like natural disaster zones, where sensing nodes are thrown close to a target area. A simple way

[2]This value has been calculated for Manchester codification, which doubles the baud rate.

Output power	Consumption	Range
-20 dBm (0.0100 mW)	25.8 mW	19.30m
-19 dBm (0.0126 mW)	26.4 mW	20.46 m
-18 dBm (0.0158 mW)	27.0 mW	21.69 m
-17 dBm (0.0200 mW)	27.0 mW	22.99 m
-16 dBm (0.0251 mW)	27.3 mW	24.38 m
-15 dBm (0.0316 mW)	27.9 mW	25.84 m
-14 dBm (0.0398 mW)	27.9 mW	27.39 m
-13 dBm (0.0501 mW)	28.5 mW	29.03 m
-12 dBm (0.0631 mW)	29.1 mW	30.78 m
-11 dBm (0.0794 mW)	29.7 mW	32.62 m
-10 dBm (0.1000 mW)	30.3 mW	34.58 m
-9 dBm (0.1259 mW)	31.2 mW	36.66 m
-8 dBm (0.1585 mW)	31.8 mW	38.86 m
-7 dBm (0.1995 mW)	32.4 mW	41.19 m
-6 dBm (0.2512 mW)	33.3 mW	43.67 m
-5 dBm 0.3162 mW)	41.4 mW	46.29 m
-4 dBm (0.3981 mW)	43.5 mW	49.07 m
-3 dBm (0.5012 mW)	43.5 mW	52.01 m
-2 dBm (0.6310 mW)	45.3 mW	55.13 m
-1 dBm (0.7943 mW)	47.4 mW	58.44 m
+0 dBm (1.0000 mW)	50.4 mW	61.95 m
+1 dBm (1.2589 mW)	51.6 mW	65.67 m
+2 dBm (1.5849 mW)	55.5 mW	69.61 m
+3 dBm (1.9953 mW)	57.6 mW	73.79 m
+4 dBm (2.5119 mW)	63.9 mW	78.22 m
+5 dBm (3.1623 mW)	76.2 mW	82.92 m

TABLE I

PATH-LOSS MODEL (α = 3.95) RANGES AND CONSUMPTIONS FOR THE MICA2 OUTPUT POWERS

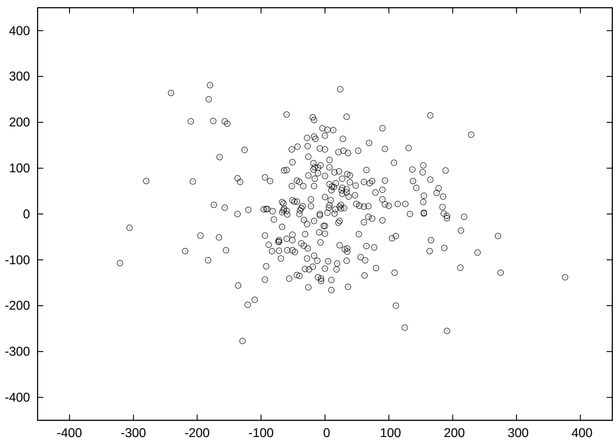

Fig. 2. 250 nodes positioned with a normal bivariate distribution of parameter σ = 100

to model this scenario is to select nodes with a random normal bivariate distribution around the coordinates of the "focus" point. The typical deviation parameter σ will control the node dispersion. An example of this type of distribution is plotted in Fig. 2, for n = 250 nodes and σ = 100.

Let us assume, without lost of generality, that the "focus" point is situated in the center of the real plane. Then, the position of the i-th node is ($\boldsymbol{X_i}, \boldsymbol{Y_i}$), being $\boldsymbol{X_i}, \boldsymbol{Y_i}$ independent and identically distributed random variables $N(0, \sigma)$. The quadratic distance between two nodes i, j is $\boldsymbol{d^2} = \boldsymbol{\Delta X}^2 + \boldsymbol{\Delta Y}^2$, being $\boldsymbol{\Delta X} = \boldsymbol{X_i} - \boldsymbol{X_j}$, $\boldsymbol{\Delta Y} = \boldsymbol{Y_i} - \boldsymbol{Y_j}$. The probability density function (pdf) of $\boldsymbol{d^2}$ is (see Appendix 2):

$$f_{d^2}(x) = \frac{1}{4\sigma^2} \exp(-\frac{x}{4\sigma^2}) \quad (13)$$

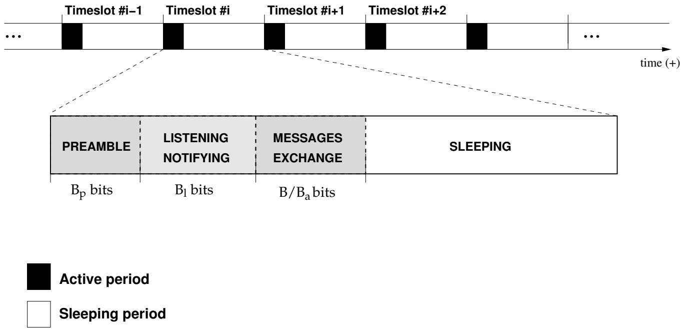

Fig. 3. General timeslot distribution

C. Traffic pattern

Usually, WSN nodes are expected to carry unicast traffic flows from the sensors to the *sinks*. Sink nodes are special nodes that receive and process the data from all the sensors in the network. Therefore, nodes closer to the sinks transmit more traffic, since they must relay the information of other nodes. The average load in the network is assumed to be low, with a small data packet size (around 100 bytes long). According to these premises, and for the sake of simplicity, we will make the following considerations in our analytical model:

- The number of nodes is very large.
- The sink nodes are situated close to the center of the plane. Therefore, traffic flows go from the *edges* to the center.
- The link utilization is uniform in all the links of the network, and equal to ρ. This simplification is sound with the idea that there is more traffic in central nodes than in the edge ones, since central nodes will have more links. This simplification will allow us to anallyticaly determine the L factor.

D. Multiple access scheme

We consider a generic model in which time is divided into timeslots, composed by an active period for transmission/reception and a passive period to sleep. This is the most common solution in WSN, it is employed in a vast range of protocols both deterministic (TDMA) [7], [8] and contention ones (CSMA variations) [3], [5], [6]. In our analysis we will assume that no collisions are possible. Such assumption is justified since in TDMA access schemes nodes exclusively own timeslots for transmission, and *no collisions are possible*, whereas in contention ones the low load premise discussed in the previous section allow us to consider collisions an unlikely event. Indeed, if such premise cannot be applied to CSMA proposals, and collisions may occur frequently, the behavior of TPC will degrade, but the value obtained for L still will be an *upper bound* of the efficiency of the TPC algorithm.

In our model lengths are expressed in bits, and each timeslot consists of (see Fig. 3):

- Preamble (B_p bits). The preamble is usually intended to recover from clock drifts, discovery of new neighbors and maintenance functions. All nodes in range must listen during the B_p bits to any other node preamble transmission, which is cyclically sent. That is, each node selects

a timeslot and retransmits its preamble each C timeslots (the preamble period). Actually, it is possible that the preamble packet lasts less than B_p bits (for instance, if contention is employed, there must be additional time to accomodate the contention window). Thus, let us denote B'_p the length of the preamble that is really transmitted. The preamble is always sent at the nominal power.
- Listening/notifying (B_l bits). It is intended to notify nodes about an incoming transmission by other node (for instance, with a RTS/CTS exchange). So, all nodes keep listening during this period. Nodes sleep until next timeslot if there is no tranmission or if they are not the intended receiver. Similarly to the Preamble, let denote B'_l the actual length in bits of packets transmitted (see Fig. 3). If a node has to transmit in this timeslot it notifies the transmission during this phase using the nominal output power.
- Messages exchange (B data bits plus B_a auxiliary bits). During this period, Data and MAC auxiliary packets are exchanged (e.g. Data/ACK). If TPC is employed Data messages are transmitted using the minimum power needed to achieve a target bit error probability (see section IV-A), while auxiliary (control, signalling) messages are sent at the nominal transmission output power.
- Sleeping period. After packet transmission, receiver and transmitter go to sleep until next timeslot. In the sleeping phase the power consumption is negligible.

Let us notice that for each stage, in TDMA-like protocols only one node transmits at a time in a spatial region of the network[3] (and, therefore, there are no collisions), whereas in CSMA-like proposals all nodes with information to transmit contend to access the channel.

V. $\overline{\xi}$ EVALUATION

Let $c(t)$ be the number of timeslots until instant t. Notice, that $c(t)$ is the same for all the nodes in the network. Let us denote K as the number of links that exists in the network. Let $m(t)$ be the stochastic process representing the number of packets sent in each one of the K network links until time t. Then, $m(t) = c(t)\rho$. In addition, let $v^i(n)$ be the random variable representing the number of neighbors of the i-th node, for $i = 1, \ldots, n$. Obviously, it depends on the total number of nodes n. Let us define $v(n) = \sum_{i=1}^{n} v^i(n)$, the sum of the neighbors that each node has. Note that $v(n) = 2K$, as each neighbor node corresponds to a link in the network, which is counted twice in the sum. Then, the following relationship holds,

$$c(t) = 2\frac{Km(t)}{v(n)\rho} \tag{14}$$

Besides, it is now possible to compute the amount of time that nodes are in each state. Let ν be the bit transmission time of the radio transceiver. Until time t, $m(t)$ packets have been sent in each link, and $c(t)$ timeslots have passed in each node. Then, the contributions to each time are the following,

Data transmission: B bits for each packet sent. $Km(t)$ packets altogether.

$$T_{data}(t) = B\nu Km(t) \tag{15}$$

Signaling transmission: B'_p bits out of every C timeslots per node ($nc(t)$ altogether). $B'_l + B_a$ bits for each packet sent.

$$T_{sign}(t) = \frac{B'_p\nu}{C}nc(t) + (B'_l + B_a)\nu Km(t) \tag{16}$$

Reception: B_p bits during $C-1$ out of every C timeslots (in the remaining timeslot, the node transmits its own preamble, and only listens to the channel $B_p - B'_p$ bits). B_l bits for each timeslot in which the node itself does not transmit (if the node transmits it listens only $B_l - B'_l$ bits). $B + B_a$ bits for each packet received.

$$\begin{aligned} T_{rx}(t) &= \nu\frac{B_pC - B'_p}{C}nc(t) + B_l\nu nc(t) + \\ &- B'_l\nu Km(t) + (B + B_a)\nu Km(t) \\ &= nc(t)\nu(B_p\frac{C-1}{C} + B_l) + Km(t)\nu(B + B_a - B'_l) \end{aligned} \tag{17}$$

Using these equations and equation (14), we have that (notice that ν dissapears in the fractions since it was both in denominator and numerator):

$$\begin{aligned} \frac{T_{sign}(t)}{T_{data}(t)} &\approx \frac{B'_p\frac{n}{C}c(t) + (B'_l + B_a)Km(t)}{BKm(t)} = \\ &= \frac{2B'_p}{BC\rho}\frac{n}{v(n)} + \frac{B'_l + B_a}{B} \end{aligned} \tag{18}$$

and,

$$\begin{aligned} \frac{T_{rx}(t)}{T_{data}(t)} &\approx \frac{nc(t)(\frac{B_pC - B'_p}{C} + B_l) + Km(t)(B + B_a - B'_l)}{BKm(t)} = \\ &= 2\frac{B_p\frac{C-1}{C} + B_l}{B\rho}\frac{n}{v(n)} + \frac{B + B_a - B'_l}{B} \end{aligned} \tag{19}$$

These equations are approximations, since packets can be partially transmitted at instant t. However, this approximation will be fine for large values of t. Thus, as t tends to infinity, ξ is equal to

$$\begin{aligned} \xi =& \frac{2B'_p}{BC\rho}\frac{n}{v(n)} + \frac{B'_l + B_a}{B} + \\ &+ \frac{P_{rx}}{P_{tx_p}}[2\frac{\frac{B_pC - B'_p}{C} + B_l}{B\rho}\frac{n}{v(n)} + \frac{B + B_a - B'_l}{B}] \end{aligned} \tag{20}$$

To sum up, notice that the ξ coeffient depends on the power ratios (a hardware parameter) and on the traffic load and message lengths (a protocol parameter). Also, ξ is a function of

[3] Obviously, timeslots may be reused spatially in the network without causing collisions.

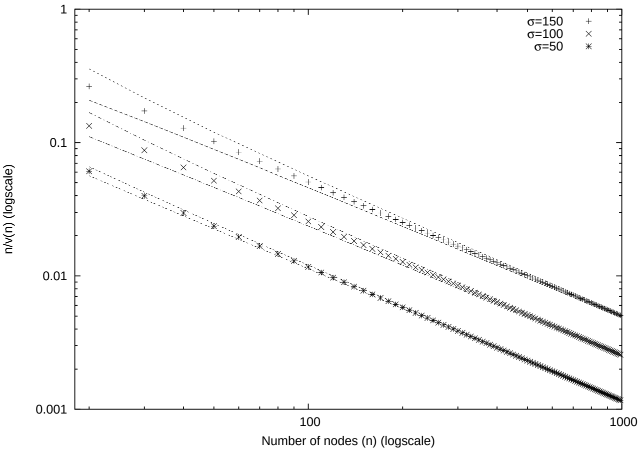

Fig. 4. $n/v(n)$ evaluated versus n for different σ

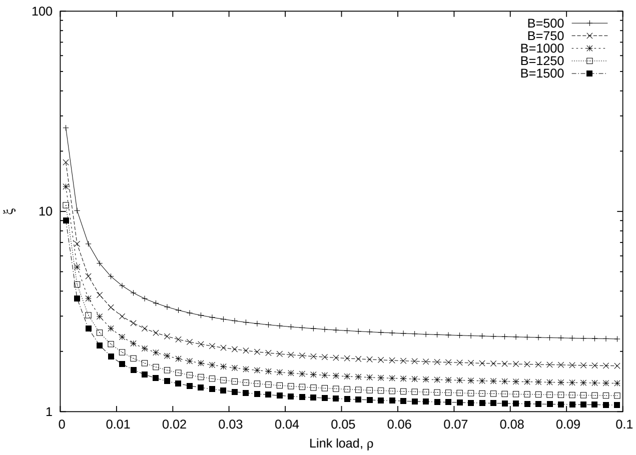

Fig. 5. $\overline{\boldsymbol{\xi}}$ evaluated versus ρ for different data packet sizes (B)

$n/v(n)$, which, for random topologies like the one considered, is a random variable.

Proposition 1 *The mean of the random variable $n/v(n)$ is*

$$E\{\frac{n}{v(n)}\} = \frac{1}{\overline{\boldsymbol{v}}} = \frac{1}{(n-1)[1-\exp{(-\frac{d_S^2}{4\sigma^2})}]} \tag{21}$$

where $\overline{\boldsymbol{v}}$ denotes the average number of neighbors of a node in a network of n nodes .

The expression for $\overline{\boldsymbol{v}}$ is obtained in Appendix 3. We have no direct proof for the former expression. Instead, we have analized its correctness using intensive computing. Figure 4 shows some of the results. For σ values of 50, 100 and 150 we have dropped random networks of n nodes and calculated the mean and typical deviation of $n/v(n)$ using 10000 samples. The lines in the figure 4 are the mean $\pm$ the typical deviation obtained in the experiments, while the points represent the analitycal computation of $1/\overline{\boldsymbol{v}}$. As can be clearly observed the probability concentrates around the expected value as n grows.

Using the previous expression, the mean of ξ is given by eq. (22).

$$\begin{aligned}\overline{\boldsymbol{\xi}} = & \frac{2B'_p}{BC\rho}\frac{1}{\overline{\boldsymbol{v}}} + \frac{B'_l+B_a}{B} + \\ & + \frac{P_{rx}}{P_{tx_p}}[2\frac{\frac{B_pC-B'_p}{C}+B_l}{B\rho}\frac{1}{\overline{\boldsymbol{v}}} + \frac{B+B_a-B'_l}{B}]\end{aligned} \tag{22}$$

Figure 5 represents $\overline{\boldsymbol{\xi}}$ evaluated versus ρ for different Data packet sizes (B) using n = 100 nodes. The parameters used to compute these plots are B_p = B'_p = 100 bits, B_l = B'_l = 400 bits, B_a = 400 bits, $C = 20$ and P_{rx} = 35.2 mW, P_{tx_p} = 76.2 mW. Packet lengths have been chosen as representative values of a WSN configuration, and the consumption powers correspond with actual Mica2 motes consumptions. The curves show that the expected value of ξ decreases as ρ increases, and that the value is greater for lower values of B.

VI. $\overline{\boldsymbol{s}}$ EVALUATION

As shown in the Section III, the energy saving obtained depends on the value of the s coefficient. This coefficient is a function of the mass probability function of the random variable $\boldsymbol{Q}$ found in Section III and given by eq. (23).

$$s = \sum_{j=1}^{p} \frac{P_{tx_j}}{P_{tx_p}} \Pr[Q{=}Q_j] \tag{23}$$

Indeed, s is actually a function of the position of the nodes and the number of nodes. Since position is random, s is also random.

Proposition 2 *The mean of the random variable s is*

$$\begin{aligned}\overline{\boldsymbol{s}} = & \frac{\sum_{j=1}^{j=p-1} P_{tx_j}[\exp{(-\frac{(\frac{Q_{j-1}}{\Omega})^{2/\alpha}}{4\sigma^2})} - \exp{(-\frac{(\frac{Q_j}{\Omega})^{2/\alpha}}{4\sigma^2})}]}{P_{tx_p}[1-\exp{(-\frac{d_S^2}{4\sigma^2})}]} + \\ & + \frac{\exp{(-\frac{(\frac{Q_j}{\Omega})^{2/\alpha}}{4\sigma^2})} - \exp{(-\frac{d_S^2}{4\sigma^2})}}{[1-\exp{(-\frac{d_S^2}{4\sigma^2})}]}\end{aligned} \tag{24}$$

The aim of this section is to prove Proposition 2.

A data transmission between two nodes uses transmission output power Q_j if the distance between them is within an interval $D_j = [d_{min_j}, d_{max_j})$. If the link usage is equiprobable, then Pr[$\boldsymbol{Q} = Q_j$] is just the probability that the distance between nodes is within D_j. TPC MAC algorithms must select a value of transmission power that guarantees a given error probability $\widehat{p_e}$ at the receiver (or, equivalently that the SNR be greater than a given threshold). Let Pr[Nodes in range] denote the probability that the nodes are in range. For consistency, let Q_0 = 0. Then, from eqs. (12) and (13) we obtain:

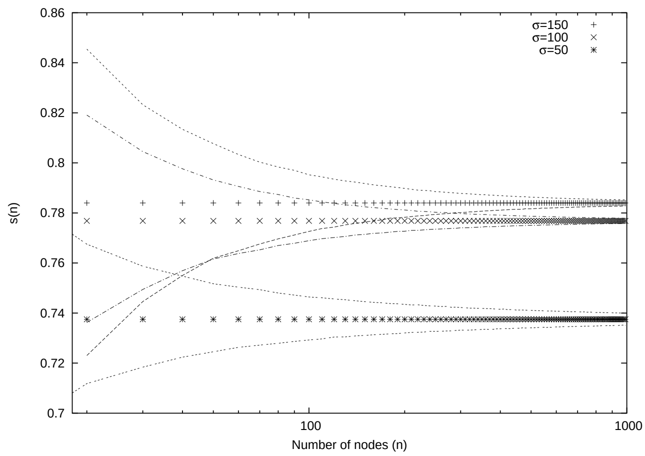

Fig. 6. s evaluated versus n for different σ

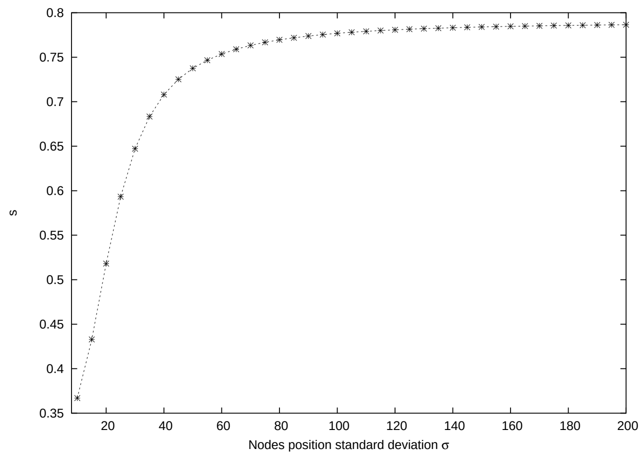

Fig. 7. $\overline{\boldsymbol{s}}$ evaluated versus σ

$$
\begin{aligned}
&\Pr[\boldsymbol{Q}=Q_j]\Pr[\text{Nodes in range}] = \\
&=\Pr[Q_{j-1}\leq \widehat{\boldsymbol{P_{tx}}}, Q_j \geq \widehat{\boldsymbol{P_{tx}}}] = \\
&=\Pr[Q_{j-1}\leq \widehat{\boldsymbol{P_{tx}}} \leq Q_j] = \\
&=\Pr[Q_{j-1}\leq \boldsymbol{d}^{\alpha}\Omega \leq Q_j] = \\
&=\Pr[\tfrac{Q_{j-1}}{\Omega}\leq \boldsymbol{d}^{\alpha} \leq \tfrac{Q_j}{\Omega}] = \\
&=\Pr[(\tfrac{Q_{j-1}}{\Omega})^2\leq (\boldsymbol{d}^{\alpha})^2 \leq (\tfrac{Q_j}{\Omega})^2] = \\
&=\Pr[(\tfrac{Q_{j-1}}{\Omega})^{\frac{2}{\alpha}}\leq \boldsymbol{d}^{2} \leq (\tfrac{Q_j}{\Omega})^{\frac{2}{\alpha}}] = \\
&=\exp\big(-\frac{(\frac{Q_{j-1}}{\Omega})^{2/\alpha}}{4\sigma^2}\big)-\exp\big(-\frac{(\frac{Q_{j}}{\Omega})^{2/\alpha}}{4\sigma^2}\big)
\end{aligned}
\tag{25}
$$

for $j=1,\ldots,p-1$. For the p-quantum there is an expanded region between $(\frac{Q_j}{\Omega})^{\frac{2}{\alpha}}$ and d_S^2 where reception is still possible, but with a higher error rate. In this case, the probability associated to the last quantum (nominal power) is:

$$
\begin{aligned}
&\Pr[\boldsymbol{Q}=Q_j]\Pr[\text{Nodes in range}] = \\
&=\Pr[(\tfrac{Q_{p-1}}{\Omega})^{\frac{2}{\alpha}}\leq \boldsymbol{d}^{2} \leq d_S^2] = \\
&=\exp\big(-\frac{(\frac{Q_{p-1}}{\Omega})^{2/\alpha}}{4\sigma^2}\big)-\exp\big(-\frac{d_S^2}{4\sigma^2}\big)
\end{aligned}
\tag{26}
$$

The probability that nodes are in range is just $\Pr[\text{Nodes in range}] = \Pr[\boldsymbol{d^2}\leq d_S^2]$. Therefore, from eq. (23), the average s coefficient ($\overline{\boldsymbol{s}}$) for a normal distribution is:

$$
\begin{aligned}
\overline{\boldsymbol{s}} &= \frac{\sum_{j=1}^{j=p} P_{tx_j}\Pr[\boldsymbol{Q}=Q_j]}{P_{tx_p}} = \\
&= \frac{\sum_{j=1}^{j=p-1} P_{tx_j}[\exp(-\frac{(\frac{Q_{j-1}}{\Omega})^{2/\alpha}}{4\sigma^2})-\exp(-\frac{(\frac{Q_{j}}{\Omega})^{2/\alpha}}{4\sigma^2})]}{P_{tx_p}[1-\exp(-\frac{d_S^2}{4\sigma^2})]} + \\
&+ \frac{\exp(-\frac{(\frac{Q_{j}}{\Omega})^{2/\alpha}}{4\sigma^2})-\exp(-\frac{d_S^2}{4\sigma^2})}{[1-\exp(-\frac{d_S^2}{4\sigma^2})]}
\end{aligned}
\tag{27}
$$

We additionally verified Proposition 2 using numerical computation. Figure 6 shows the results. For σ values of 50, 100 and 150 we have dropped random networks of n nodes and calculated the mean and typical deviation of s using 10000 samples for each point. The lines in the figure 6 are the mean $\pm$ typical deviation obtained in the experiments, while the points represent the analytical computation of $\overline{\boldsymbol{s}}$ (eq. 2). It can also be observed that the probability concentrates around the expected value as n grows.

Figure 7 shows an evaluation of this expression, for different values of parameter σ, and assuming Ω = -97.5 dB, and α = 3.95 (same values used in the previous numerical examples). Naturally, lower values of parameter s (which imply higher values of L) are obtained for lower σ distributions, since the distance between nodes is also shorter. It should be noted that the resulting function has an interval of fast growing for $\sigma \in (10, 50)$. For the values $\sigma > 100$ the function asymptotically increases toward a limit value of $\overline{\boldsymbol{s}}$ = 0.78.

VII. $\overline{\boldsymbol{L}}$ EVALUATION

Based on the previous results, we can enunciate the following proposition:

Proposition 3 *For large values of n, the mean of the random variable L converges to*

$$
\overline{\boldsymbol{L}}|_{n\gg 1} \approx \frac{1+\overline{\boldsymbol{\xi}}}{\overline{\boldsymbol{s}}+\overline{\boldsymbol{\xi}}} \tag{28}
$$

The reason behind this assimptotical behavior is the concentration of the pdf around the mean of ξ and s as n grows (as can be seen in Figs. 4 and 6, respectively). Thus, for large values of n the uncertainty on the value of ξ and s is very small and, therefore, so the uncertainty of L. That is the reason that allow us to substitute the random variables for its expected value in equation (28). Nevertheless, we have no direct proof for Proposition 3. Instead, we have performed computational tests to verify this statement. Fig. 8 depicts the evaluation of L for growing values of n. The experiments are similar to those shown in Figs. 4 and 6. For each point, 10000 random networks are thrown and L is averaged. The mean $\pm$ typical deviation obtained are represented by the lines depicted in Fig. 8. This figure also shows the analytical results computed via eq. (28) (points). The configuration used is similar to that of

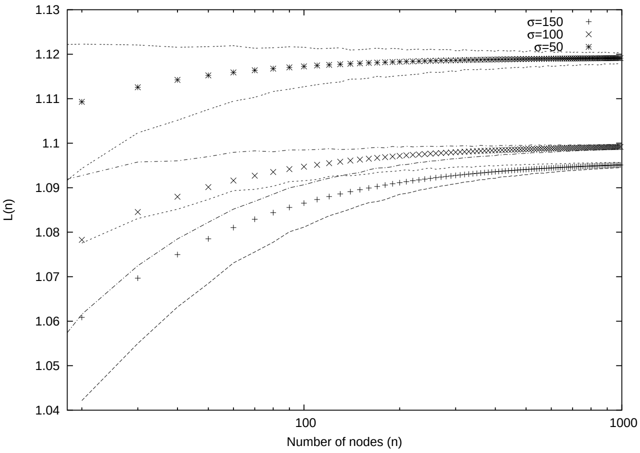

Fig. 8. L evaluated versus n for different σ

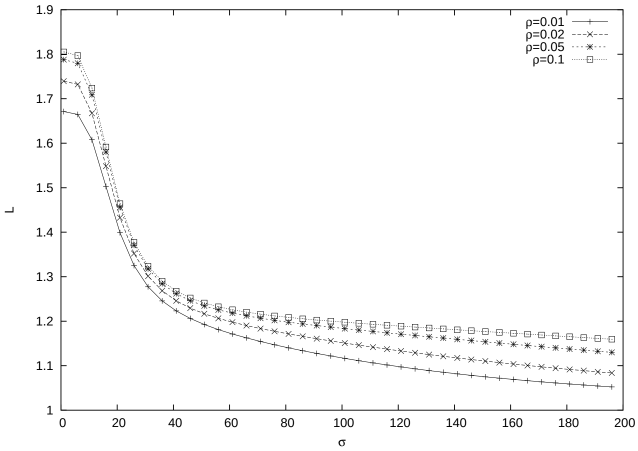

Fig. 9. L vs. σ for different Link loads (ρ) for L-MAC protocol

the previous numerical examples: B = 800 bits, B_p = B'_p = 100 bits, B_l = B'_l = 400 bits, B_a = 400 bits, $C = 20$, P_{rx} = 35.4 mW and P_{tx_p} = 76.2 mW.

At this point, the previous results can be applied to a given MAC protocol just by adjusting properly the model parameters. The $\overline{s}$ parameter depends on the propagation model, the hardware used and the distribution of nodes (see section IV). The $\overline{\boldsymbol{\xi}}$ parameter basically depends on the traffic model and the protocol operation. That is, the parameters of eq. (22), $B_p, B'_p, B_l, B'_l, B, B_a, C$, take their value according to the MAC protocol operation. Therefore, someone interested in evaluating a MAC protocol with this method should analize the desired MAC protocol to determine the value of these parameters. The meaning of each one is provided in section IV-D.

In the remainder of this section we show how to do this evaluation. We apply the previous analysis to evaluate the benefits of TPC for two representative WSN protocols: a TDMA protocol (L-MAC) and a contention protocol (S-MAC). Since we keep the propagation model (path loss), hardware used (Mica2) and the distribution of nodes (normal bivariate) as in section IV, we focus on the protocol operation to determine $\overline{\boldsymbol{\xi}}$.

A. *TDMA example: L-MAC*

Lightweight Medium Access Protocol (L-MAC) [8] is a TDMA protocol proposed for sensor networks, based on a modification of the Eyes MAC (E-MAC) [7]. In L-MAC each node selects a unique timeslot by using the slot occupancy information from its one-hop neighbors. Once a node has selected a slot it always use it to transmit either a control message (preamble) or a control plus data message.

Our generic protocol model (see Section IV-D) can directly be applied to this protocol operation. L-MAC uses a pure TDMA access scheme, and directly notifies neighbors of transmissions in the preamble packet. Thus, the listening/notifying phase of our model (see Fig. 3) is not necessary (therefore $B_l = 0$). Additionally, the timeslot period is $C = 32$ and there are not acknowledgements, $B_a = 0$. Therefore, a L-MAC timeslot includes only a preamble and data (when corresponding).

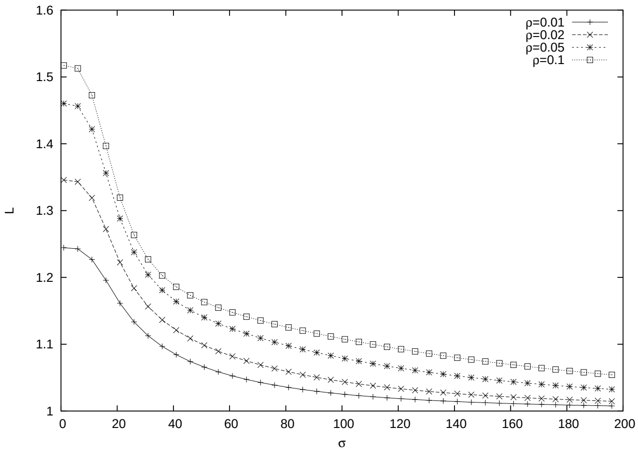

Fig. 10. L vs. σ for different Link loads (ρ) for S-MAC protocol

The TPC can be used with this protocol without changing the topology if nodes always transmit their preamble at nominal power. The following Data section would be sent at a controlled power. Thus, we set directly in eq. (22) the following parameters according to the L-MAC operation to evaluate L:

- n = 100 nodes
- Data size, B = 800 bits.
- Preamble, B_p = B'_p = 96 bits.
- Signaling, B_l = B'_l = 0 bits.
- Auxilary, B_a = 0 bits.
- Preamble period, C = 32.
- P_{rx} = 35.4 mW.
- P_{tx_p} = 76.2 mW.

Fig. 9 depicts the evaluation of L versus σ through eq. (28) for different link loads (ρ). Results show considerable savings, in the order of 10-20%, for mid-large values of σ. Moreover, in Fig. 9 savings are more noticeable for low σ and high values of ρ. This trend is sound, since if network nodes are close and transmit a high number of packets, the TPC is more effective.

B. *Contention example: S-MAC*

S-MAC [3] is a contention-based protocol proposed for WSN. It uses a timeslot structure, similar to Fig. 3. Nodes synchronize their active/sleep periods by means of the short SYNChronization (SYNC) packet. SYNCs are periodically broadcasted by stations in the preamble, which allows nodes to correct time drifts. Data transmission is performed by means of CSMA/CA, i.e the RTS/CTS/Data/ACK sequence. The RTS/CTS packets are transmitted in the listening/notifying stage indicated in Fig. 3. Finally, in the message exchange period the Data packet and the ACK are transmitted, corresponding respectively to the lengths B and B_a. Again, the analysis can be done just by setting these parameters according to the S-MAC operation in equations 15), (16) and (17). We have chosen the values employed in the reference implementation of S-MAC for TinyOS [23], which are:

- n = 100 nodes
- Data size, B = 800 bits.
- Preamble, B_p = 727 bits, B'_p = 100 bits.
- Signaling, B_l = 1226 bits, B'_l = 100 bits.
- Auxilary, B_a = 100 bits.
- Preamble period, C = 20.
- P_{rx} = 35.4 mW.
- P_{tx_p} = 76.2 mW.

Fig. 10 plots L versus σ for different link loads (ρ). Results show a worse behavior than in the L-MAC case studied in the previous section. The average saving is below 10% for mid-large values of σ. The reason of this worsening is twofold. On one hand, S-MAC nodes transmit more control packets (RTS, CTS) than L-MAC ones. Since these packets are always sent at nominal power, more energy is wasted. On the other hand, in L-MAC nodes go to sleep just after the preamble (if they are not the data receivers). In S-MAC all nodes have to wait *always* at least until the end of the listening period to sleep.

We must remark that, altough S-MAC is a contention protocol, we have assumed that no collisions are possible in our analysis. Therefore, the real values will be even worse that those predicted by Fig. 10. This yields to the conclusion that S-MAC is not a good candidate for TPC use.

C. Discussion

In the introduction we stated that we are interested in the maximum energy saving that an *ideal* TPC may provide. In this way, basically two strong assumptions have been made in this paper: (1) every node knows in advance the power necessary to reach its neighbors and (2) there are no collisions. Therefore, the results drawn in the previous sections should be interpreted accordingly: they show the trend of the energy saving obtained by the use of TPC, that is, an estimate of what may be achieved. Clearly, the conditions of the real scenario will determine the actual saving, either if collisions are likely or nodes implement some mechanism to decide the power needed to reach their neighbors.

In any case, our analytical method allows to quickly estimate the benefits of TPC in order to decide if it worths adding it to a MAC proposal. Let us take the case of L-MAC: since it is a pure TDMA protocol, the no-collision assumption applies. In this case, the saving may be close to the upper bound derived here, and so it worths developing an heuristic to gather information about the powers needed and performing simulations to test it. On the other hand, the results for S-MAC clearly suggest not to use TPC: the upper bound is already low and the energy saving will be further decreased by collisions. In addition, in S-MAC more than one transmission schedule may be adopted [3], increasing the overall listening time.

VIII. Conclusions

In this paper we have developed an analytical method to compute the energy savings provided by a general MAC TPC mechanism, which can be applied to most of the current WSN MAC proposals. We have shown that the average energy savings, measured through the L ratio, converges if the number of nodes is large (a very likely condition in WSN).

An upper bound of the energy saving achievable by TPC can be quickly obtained by adjusting the formula parameters according to the operation of the protocol under evaluation. It has been show how to apply this method with two representative protocols. The conclusion derived is that the TPC mechanism analyzed is worth being included in some proposals of WSN MAC layer. Energy savings up to 10-20% can be expected in TDMA access protocols like L-MAC, while contention ones, like S-MAC, achieve no significative improvements.

Future work will include the computation of L ratio by means of simulation to verify the results, and to extend our work to a broader set of WSN MAC protocols, traffic patterns and random distribution deployments. Furthermore, we aim to test also this type of TPC strategy in real test-beds using Mica2 motes.

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APPENDIX
1. RADIO CHANNEL MODEL

Let P_{tx} and P_{rx} be the transmission and reception signal power, respectively. Let d be the distance between peers. WSN media can be considered a time-invariant narrow-band channel, which can be modeled using a path-loss approximation [22], where:

$$P_{rx}(d) = P_{tx}(\frac{\lambda}{4\pi d_0})^2(\frac{d_0}{d})^\alpha \tag{29}$$

being α the path-loss coefficient (typically, $\alpha \in [2, 4]$) calculated at a reference distance d_0.

There are two main contributions to the noise in the transmission: (1) the channel noise is usually considered Additive White Gaussian Noise (AWGN) with spectral power density $N_0 = KT$, where K is the Boltzmann constant and T is the absolute temperature. And (2) the internal noise of the receiver, characterized by a noise figure ($F \approx$ 10-15 dB). Let E_b be the energy per bit, R be the bit rate and B be the transmission bandwidth. The Signal to Noise Ratio ($\frac{S}{N}$) is:

$$\frac{S}{N} = \frac{P_{rx}}{KTBF} = \frac{E_b}{N_0}\frac{R}{B} \tag{30}$$

Mica2 motes use the NCFSK modulation, then the bit probability error (b_e) is [24]:

$$b_e = \frac{1}{2}\exp(-\frac{1}{2}\frac{E_b}{N_0}) \tag{31}$$

And, consequently, the packet probability error (p_e) for a packet of n bits is:

$$p_e = 1 - (1 - b_e)^n \tag{32}$$

Given a target error packet probability ($\widehat{p_e}$), the necessary power transmission ($\widehat{P_{tx}}$) can be obtained from the previous equations. This objective error packet probability $\widehat{p_e}$ is the quality (figure of merit) we would like to have in our communications. First, notice that:

$$\widehat{\frac{E_b}{N_0}} = -2\ln(2\widehat{b_e}) = -2\ln(2[1 - (1 - \widehat{p_e})^{\frac{1}{n}}]) \tag{33}$$

And from eq. (30),

$$\begin{gathered}\frac{\widehat{P_{rx}}}{KTBF} = -2\ln(2[1 - (1 - \widehat{p_e})^{\frac{1}{n}}])\frac{R}{B} \Rightarrow \\ \widehat{P_{rx}} = -2KTFR\ln(2[1 - (1 - \widehat{p_e})^{\frac{1}{n}}])\end{gathered} \tag{34}$$

Then, from eq. (29), $\widehat{P_{tx}}$ can be expressed as:

$$\widehat{P_{tx}} = d^\alpha \Omega \tag{35}$$

where,

$$\Omega \triangleq -2KTRF\ln(2[1 - (1 - \widehat{p_e})^{\frac{1}{n}}])(\frac{4\pi d_0}{\lambda})^2(\frac{1}{d_0})^\alpha \tag{36}$$

APPENDIX
2. PROBABILITY DENSITY FUNCTION OF THE QUADRATIC DISTANCE WITH NORMAL DISTRIBUTION

Theorem 1 *The pdf of $\mathbf{d^2}$ for a Normal distribution is:*

$$f_{d^2}(x) = \frac{1}{4\sigma^2}\exp(-\frac{x}{4\sigma^2}) \tag{37}$$

Demonstration: First, notice that

$$\begin{aligned}\boldsymbol{\Delta X} = \boldsymbol{X_i} - \boldsymbol{X_j} &= N(0,\sigma) - N(0,\sigma) = \\ &= N(0, \sqrt{2}\sigma) \\ \boldsymbol{\Delta Y} = \boldsymbol{Y_i} - \boldsymbol{Y_j} &= N(0,\sigma) - N(0,\sigma) = \\ &= N(0, \sqrt{2}\sigma)\end{aligned} \tag{38}$$

Therefore,

$$(\frac{\boldsymbol{\Delta X}}{\sqrt{2}\sigma})^2 + (\frac{\boldsymbol{\Delta Y}}{\sqrt{2}\sigma})^2 = \chi_2^2 \tag{39}$$

Thus,

$$d^2 = \Delta X^2 + \Delta Y^2 = 2\sigma^2\chi_2^2 \tag{40}$$

Since $\chi_2^2 = \exp(\frac{1}{2})$, the PDF of d^2 is:

$$\begin{aligned}\Pr[d^2 \leq x] &= \Pr[2\sigma^2\chi_2^2 \leq x] = \Pr[\chi_2^2 \leq \tfrac{x}{2\sigma^2}] = \\ &= \int_0^{\frac{x}{2\sigma^2}} \frac{1}{2}\exp(-\frac{u}{2})du = \\ &= 1 - \exp(-\frac{x}{4\sigma^2})\end{aligned} \tag{41}$$

And, finally, the pdf of d^2 is the derivate of the PDF:

$$f_{d^2}(x) = \frac{d(1-\exp(-\frac{x}{4\sigma^2}))}{dx} = \frac{1}{4\sigma^2}\exp(-\frac{x}{4\sigma^2}) \tag{42}$$

Hence, proved.

Appendix

3. Average number of neighbors with Normal distribution

Let n be the total number of nodes in the network. Let v be the random variable "Number of neighbors of a node". Since nodes position is selected independently, the mpf of v is:

$$\Pr[v = i] = \binom{n-1}{i}\eta^i(1-\eta)^{(n-1-i)} \tag{43}$$

for i=1, $\ldots, n$-1

where η is the probability that two nodes independently selected are neighbors, that is the probability that distance between nodes (d) is less than d_S. From the pdf of d^2 obtained in Appendix 2, we know that $\eta = \Pr[d^2 \leq d_S^2] = 1 - \exp(-\frac{d_S^2}{4\sigma^2})$. Additionally, the first order moment of v can be obtained:

$$\overline{v} = \sum_{i=1}^{n-1} i\binom{n-1}{i}\eta^i(1-\eta)^{(n-1-i)} = \eta(n-1) \tag{44}$$