



**JOURNAL OF  
THE INTERNATIONAL ASSOCIATION  
FOR SHELL AND SPATIAL  
STRUCTURES**

*FORMERLY BULLETIN OF THE INTERNATIONAL ASSOCIATION FOR SHELL AND SPATIAL STRUCTURES*

Prof. D. h-C Eng .E. TORROJA, founder



SPECIAL ISSUE FOR THE  
**CENTENARY OF THE BIRTH OF FÉLIX CANDELA**

*Guest Editors: J. F. Abel and J. G. Oliva*

**Vol. 51 (2010) No. 1**

March n. 163

ISSN: 1028-365X



# Journal

VOL. 51 (2010) No. 1  
n. 163 March

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Printed by SODEGRAF

ISSN:1028-365X

Depósito legal: M. 1444-1960

# OPTIMALITY OF CANDELA'S CONCRETE SHELLS: A STUDY OF HIS POSTHUMOUS DESIGN

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**Editor's Note:** Manuscript submitted 15 February 2010; revision received 23 February; accepted 7 March. This paper is open for written discussion, which should be submitted to the IASS Secretariat no later than September 2010.

## ABSTRACT

*In the field of concrete shells it is possible to distinguish two periods, an initial constructive period that includes many structures erected until the 1970s, and the second phase directed mainly at analysis, due to the development in computing processors and the increasing use of methods of computational mechanics. This paper offers a brief review of the constructive time, focusing on the most emblematic works of Félix Candela, the centenary of whose birth is celebrated in 2010. Moreover, optimization techniques are applied to find some optimum geometric designs of an actual concrete shell – the posthumous work of Candela – presenting an aesthetic appearance similar to that which the designer had initially planned. The results confirm that improvements in the structural behaviour may be achieved with slight geometric changes.*

**Keywords:** Candela, concrete shells, shape optimization, structural response

## 1. INTRODUCTION

The structural behaviour of shells, compared to that of other types of structures, is characterized by a higher mechanical efficiency. Concrete shells depend on their configuration, not on their mass, for stability. If appropriate designs are carried out, thin shells can support high loads and cover important spaces using little material. In addition, shells present an attractive lightness and elegance from an aesthetic point of view, leading some authors to refer to them as the “prima donna” of structures [1] or that structure in which one discovers what is resisted only by contemplating its shape [2].

In the field of concrete shells it is possible to distinguish two periods, an initial constructive period that includes numerous structures erected until the 1970s, and the second devoted to analysis, when the computing processors were developing and methods of computational mechanics were beginning to prevail.

In this contribution, a brief review of the constructive period is made, focusing on the most emblematic work of Félix Candela, the centenary of

whose birth is celebrated in 2010. Then, optimization techniques are applied to find some optimum geometric designs of an actual concrete shell, the posthumous work of Candela, the results being close to his preconceived design. The final geometries should have an aesthetic shape similar to the form of the structure designed initially, which is a hyperbolic paraboloid (hyper). Slight changes in the form of this type of structures introduce improvements in their mechanical behaviour.

## 2. CONSTRUCTIVE PERIOD: BRIEF REVIEW OF FÉLIX CANDELA'S WORK

The evolution of concrete shells until the end of the 1970s covers from the historical approaches by trial and error until those times when certain events (new building techniques, improvements in materials and analysis with the help of computers) revolutionized the design. Medwadowski [3] describes this first period from the viewpoint of the relationship between theory and the form of shells. Two decades later, the same author further elaborates a review under the framework of the end of the twentieth century [4].

There have been great designers in the history of concrete shells construction [5], among others, in order of birth, Pier Luigi Nervi [6-8], Eduardo Torroja [9-12], Anton Tedesco [13, 14], Félix Candela [15-20] and Heinz Isler [21].

If not the inventor, Félix Candela is considered one of the masters of concrete shells. The mathematical complexity of these shell structures contrasts with the beauty and simplicity of their forms, the economy, and the high strength and lightness despite extreme thinness. In his work, there are all types of shell structures (cylindrical forms, domes and hyperbolic paraboloids or hypars). A graphical analysis of his main works may be consulted in [22].



**Figure 1.** Church San Antonio de las Huertas at Tacuba, Mexico, Candela, 1956 [Photo: Marcos Ontiveros]

One of his early shells was built in 1955 for the Stock Market at Mexico City. During the erecting, Candela began to think of using the free edge, i.e., a shell without edge beams. It would be in the Church San Antonio de las Huertas at Tacuba, Mexico (Figure 1), where he already uses the concepts meditated in the Stock Market experience and proposes not to use edge beams. Only the hyperbolic-parabolic shape facilitates achieving this result.

In a free-edge element, the normal stress is zero due to the equilibrium conditions; however, the shear stress varies with the stiffness of the edge being analyzed. Sanz [23] distinguishes two cases:

- a) If the edge has sufficient stiffness, it is able to transmit shear stresses, i.e., to resist and transmit shear forces to the supports, relieving the rest of the shell. This becomes an arch subjected to forces in its directrix.

- b) If the stiffness of the edge is virtually zero, it is not capable of transmitting forces in the tangential direction, forcing the rest of the shell to absorb the increase of the forces through its generatrices.

Torroja [24] recommends that the edge must be quite stiff longitudinally, but lightweight to avoid distorting the membrane state. Anyway, the emergence of bending is inevitable, which becomes important when the dimensions are large.

From these early experiences, an enormous variety of vaults with free edge was built: triangular, square, pentagonal, hexagonal, and octagonal. Probably the most famous of these structures is the shell roof of the restaurant Los Manantiales at Xochimilco, Mexico (Figure 2). This structure, at full maturity of Candela's professional life, often means a constructive fantasy difficult to overcome. Billington [25] describes it as follows:

This roof is made up of eight hyperbolic paraboloidal vaults arranged on a circular ground plan of about 140 feet in diameter. Apart from deeply recessed glass wall, the paper-thin (1 5/8 inches) roof is the entire structure. Structure and form are one, and the thinness is expressed so powerfully that it is hard to believe the building is concrete. It is emphatically not a natural form; rather, it is artificial and the product of a disciplined mind.

The finite element analysis of this structure and its state of conservation may be consulted in [26].

A special case was the open chapel Lomas de Palmira at Cuernavaca, Mexico (Figure 3). This



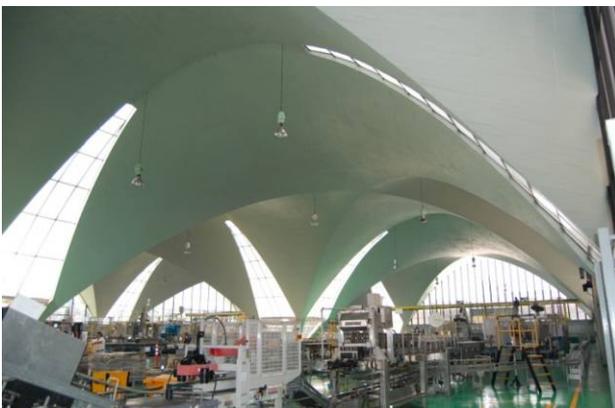
**Figure 2.** Los Manantiales Restaurant at Xochimilco, Mexico, Candela, 1958 [Photo: Marcos Ontiveros]



**Figure 3.** Chapel Lomas de Palmira at Cuernavaca, Mexico, Candela, 1958 [Photo: Princeton University Candela Archive]

chapel was built as a single equilateral hyperboloid, with 30 m span and 24 m height. In the project, there was some openwork in the upper zone forming a decorative image. In the erecting, while scaffolding was being removed, this part of the structure collapsed. It was decided to modify the project by: (i) removing any discontinuity in the shell, (ii) lowering its height by about 6 m, and (iii) increasing the thickness of the frontal zone, in order to raise the stiffness and achieve greater safety against buckling and the wind action. Bastera, Chamizo and Gutiérrez [27] argue that the accident had to be due to a constructive problem (too young concrete, scaffolding removed defectively, existence of local defects), since the analysis results of the original design showed admissible values.

Another example of thin shells applied to large open-plan constructions is the Bacardí Rum Factory at Cuautitlán, Mexico (Figure 4). The roof is composed by six vaults of square plan of 30 m side, the largest vaults built by the company Cubiertas



**Figure 4.** Bacardí Rum Factory at Cuautitlán, Mexico, Candela, 1960 [Photo: Marcos Ontiveros]

Ala, founded by Candela in 1950. A detailed explanation of the calculation of these structures can be found in [28].

The underwater restaurant at L'Oceanogràfic (The Oceanographic Park at Valencia, Spain) (Figure 5) and the access building to the same park (Figure 6) are the latest examples of such structures. These shells were designed by Candela shortly before his death in 1997, becoming his posthumous works.

The restaurant is a groined vault system composed of eight radially symmetrical lobes. Each lobe is part of a hyperboloid, where Z-axis is vertical and X and Y-axes are contained on a horizontal plane and form an angle of  $22.5^\circ$  between them. The free edge of a lobe reaches a height of 12.27 m. It is created by the intersection of the hyperboloid with a plane inclined  $60^\circ$ . This plane starts from the line that unites the bases of two consecutive ribs. These bases, which form the supports for the ruled surface, are situated on the vertices of an octagon with sides of 13.44 m. The distance between two opposite supports is 35.10 m. The shell is designed with a thickness of 0.06 m that gradually increases in a central zone of 4 m in diameter up to a maximum value of 0.225 m at the intersection of the ribs. The material is reinforced concrete ( $f_{ck} = 30 \text{ N/mm}^2$ ), reinforcing steel ( $f_{yk} = 500 \text{ N/mm}^2$ ), and an addition of  $40 \text{ kg/m}^3$  of steel fibres. A more detailed description of the restaurant design can be found in [29].

The geometry of the concrete shell structure designed for the entrance of the L'Oceanogràfic (Figure 6) comes from the intersection of three lobes that describe the shape of a hyperboloid [30]. The shell is based on the hand-drawn sketches by Candela, which inspired the subsequent building



**Figure 5.** Underwater Restaurant in L'Oceanogràfic at Valencia, Spain (Candela, 2000) [Source: www.cac.es]



(a)



(b)

**Figure 6.** Shell structure at the entrance of the L'Oceanogràfic at Valencia, Spain (Candela, 2001): (a) The shell under construction; (b) The shell today [Photos: A. Tomás (2001, 2005)]

project. It is interesting to emphasize the scaffolding that supports the formwork (Figure 6a), especially the perpendicular layout of the ledgers. If the hyper were not equilateral, these ledgers would form an angle of  $90^\circ$ . This angle would correspond with the director axes of the paraboloid.

This shell is used as the initial model of the shape optimization process that is presented in Section 4.

### 3. STRUCTURAL BEHAVIOUR AND SHAPE OPTIMIZATION OF CONCRETE SHELLS

Although shells can adopt any form, double-curvature shells present important advantages in their mechanical behaviour compared to other forms, since it is possible to avoid the appearance of bending moments in them [29, 31, 32]. Their particular behaviour is due to the arch-effect in two planes and, in contrast to the arch contained in only one plane, it allows supporting different load configurations, mainly by means of membrane internal forces, with a very low risk of bending.

Moreover, these surfaces have a practically inalterable form, and are in equilibrium whatever the type of distribution of the loads, within certain limits. This geometry involves enough mechanical features to be a very efficient structure, even without stiffening elements such as edge beams. This implies that shell structures designed to behave as membranes are, by themselves, optimum structures.

Unfortunately, as usually occurs in optimum systems, this high mechanical efficiency induces a structural behaviour that is extremely sensitive to imperfections (see how the buckling load of a shell decreases when slight geometric imperfections appear [33, 34]). Considering the normal construction procedures, the probability that constructive problems arise, or that local defects exist in the geometry or thickness, is relatively high. Some real examples associated to these problems may stand out, such as the frequent appearance of cracks when removing scaffolding under the shell. There are some more serious cases, such as the partial collapse of the Chapel Lomas de Palmira at Cuernavaca [27]. Other cases may be consulted in [35].

The structural behaviour of shells is developed essentially due to their form. It would be interesting to find, if possible, small modifications in their geometry without modifying their initial aesthetic configuration too much and still complying with the design conditions. These modifications would improve that mechanical behaviour still further. It could be attempted, for example, to reach a distribution of stresses in the thickness which is as uniform as possible, and this would imply to have shells free of bending or, at least, with some acceptable bending values [36]. Among different techniques used in form-finding of concrete shells, optimization techniques represent an effective means to achieve this purpose in the field of computational mechanics [37].

The approach to shape optimization of concrete shells depends on the objective function used in the problem. A shape of the shell with a predefined stress distribution can be obtained, e.g., a bending-free shape where it is not necessary to lay out the shell reinforcement. Apart from the standard objective functions, such as the weight or the surface, others can also be used, such as the strain energy and the stress levelling. In [30], the sizing and shape optimum design problem of concrete shells using several objective functions was investigated, and the buckling behaviour of the designs by using nonlinear stability studies and semi-empirical methods was analyzed.

#### 4. SHAPE AND SIZE OPTIMIZATION OF AN ACTUAL CANDELA'S HYPAR

The shape and size optimization of the concrete shell structure that is the access building to L'Oceanogràfic (Figure 6) is presented below. The basic parameters and design variables used in the definition of the geometry are shown in sections 4.2.2 and 4.2.3. A complete description of that definition may be consulted in [30]. After analysing the results of the initial design, several optimization processes, under predominant gravitational loads, have been carried out.

The material of the structure is concrete. The mid-surface of the shell was provided with reinforcement netting, which was used to account for time-dependent effects of the concrete, since these effects can have a considerable influence when the thickness of the shell is small with respect to other dimensions. Therefore, the contribution of the reinforcement was not considered in the analysis, except in the effects of its density. The specific weight of the material is  $25 \text{ kN/m}^3$  (a value commonly used for reinforced concrete). The mechanical properties are 30 MPa for the characteristic compressive strength of concrete ( $f_{ck}$ ), 20 MPa for the design compressive strength ( $f_{cd}$ ), 1.35 MPa for the design tensile strength ( $f_{ct,d}$ ), 0.20 for the Poisson's ratio, and 28576 MPa for the secant Young's modulus referred to the concrete age of 28 days.

The shell has been stiffened by two types of ribs with similar dimensions as used by Domingo, Lázaro and Serna [29]. One type spreads from the support to the centre of the structure (main ribs). The other surrounds a small central hole made in

the shell gauging a band of 200 mm wide (hole rib). It was necessary to create that small circular hole of 100 mm diameter at the intersection of the lobes, in order to avoid meshing problems arising from the distortion of the elements generated in the area surrounding the centre, which have very acute angles.

Because of the symmetry of geometry and loads, the different analyses were carried out on one sixth of the shell, applying symmetry boundary conditions to the nodes in the symmetry planes, and restricting the translations in the  $x$ ,  $y$ , and  $z$  directions of the nodes in the foundation plane. A convergence study using several mesh sizes was carried out to determine the mesh to use for this structure [30]. The most appropriate mesh, combining solution time and accuracy, was therefore employed in this study (Figure 7).

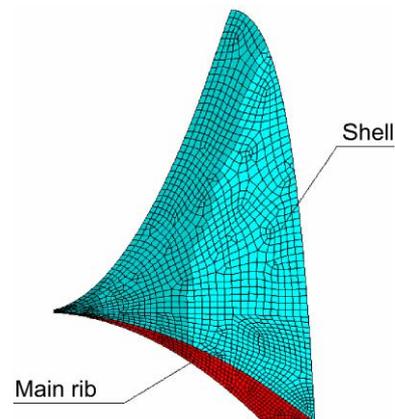


Figure 7. FE mesh of one sixth of the structure

The applied loads are the weight of the structure and the distributed load of  $1 \text{ kN/m}^2$ . The action of the wind was not considered because of its slight contribution to the whole load, only 5.87% of the gravitational loads. This percentage is a maximum value obtained by adopting a simplified and safe hypothesis for introducing wind into the analysis model [30].

#### 4.1. Formulation of the optimum design problem of the hypar shell

##### 4.1.1. Objective functions

The objective functions were the following:

- 1) Strain energy of the structure,

- 2) weight of the hypar, and
- 3) highest tensile stress at the nodes of the model.

#### 4.1.2. Design variables

The concrete shell structure under study consists of the intersection of three hyperbolic paraboloids. Each paraboloid contains two sets of generating lines, each of these sets parallel to a director plane. The intersection of the two director planes defines the  $Z$ -axis, forming together an angle  $\omega$ . The expression that defines the mid-surface of the shell in cylindrical coordinate system is given by:

$$z = kr^2 \left[ \cos\left(\frac{90+\omega}{2} - \alpha\right) - \frac{\sin\left(\frac{90+\omega}{2} - \alpha\right)}{\tan(\omega)} \right] \\ \left[ \cos\left(\alpha - \frac{90-\omega}{2}\right) - \frac{\sin\left(\alpha - \frac{90-\omega}{2}\right)}{\tan(\omega)} \right] \quad (1)$$

where  $k$  is the mid-surface constant,  $\omega$  the angle between the two director planes of the hypar and  $z$ ,  $r$ ,  $\alpha$  are the cylindrical coordinates.

The following design variables, whose initial values were proposed in sketches by Candela, were used for the design of the entrance to the Oceanographic Park:

- 1)  $k$  Constant of the mid-surface of the hypar. In the optimization processes, the initial value was  $0.14 \text{ m}^{-1}$ , with  $0.13 \text{ m}^{-1}$  and  $0.17 \text{ m}^{-1}$  being the minimum and maximum values, respectively.
- 2)  $\omega$  Angle between the two director planes of the hypar. Its initial value is  $90^\circ$  (equilateral hypar). The stated lower and upper limits are  $84^\circ$  and  $91^\circ$ , respectively.
- 3)  $\beta$  Angle of the inclined plane that defines the free edge of the hypar with respect to a horizontal plane. Its initial value was  $75^\circ$ , allowing for a variation interval between  $74^\circ$  and  $75^\circ$ , since the design is very sensitive to

this variable. With this interval, the structure cannot be lower than 19 m in height.

- 4)  $t_1$  Shell thickness. A minimum initial value of 60 mm for constructive conditions was chosen. In the optimization processes, the thickness was allowed to range from 60 to 80 mm.
- 5)  $t_2$  Hole rib thickness. The initial value was 80 mm, with a minimum of 60 mm.
- 6)  $t_3$  Main ribs thickness. The initial value was 350 mm, with a variation interval between 60 and 400 mm.

It is interesting to keep in mind that the retention of  $k$  as a design parameter implies that the hypar configuration will be maintained in the optimization process, thus retaining the straight-line generator advantage for forming and constructability.

#### 4.1.3. Constraints

The maximum extreme-fibre stresses at the outer surfaces of the shell were restricted depending on the design strength of the material of the shell:

$$\sigma_t \leq 0.85f_{ct,d} \quad (2)$$

$$\sigma_c \leq 0.85f_{cd} \quad (3)$$

where  $\sigma_t$  is the tensile stress and  $\sigma_c$  the compressive stress. Two shape parameters of the hypar (the height of the highest point of the free edge and the radius or distance from the  $Z$ -axis to the support) were also restricted. This was necessary because the values of these parameters tend to decrease during the optimization runs, distorting the geometry of the structure and thus departing significantly from the hyperbolic paraboloid. Furthermore, its appearance would not match the design criteria. The stated minimum values are 19 m for the height of the free edge and 11.5 m for the radius.

#### 4.2. Results

In the first stage, the analysis of the initial design allowed to obtain outstanding information, such as the stresses and displacements at the points of the structure and the buckling load.

In the second stage, several optimization runs were

executed with the purpose of improving the structural behaviour under the worst of several load combinations. The optimization runs of the initial model were classified into two groups depending on:

- 1) The objective function used (strain energy, weight or tensile stress), and
- 2) the minimum thickness allowed (60 or 80 mm).

For each objective function, two optimization runs were carried out depending on the minimum thickness allowed.

A buckling study of the structure was carried out for the initial model with three different thicknesses and for the optimum designs. Two types of

analysis have been done (linear and nonlinear), although, following the recommendations of Lee and Hinton [38] for this kind of shape optimization processes with linear analysis, the behaviour of the shell against instability has been studied using nonlinear analysis.

Moreover, it was considered of interest to calculate the final values of two geometric parameters: (i) the height of the free edge of the hyper and (ii) the radius or distance in ground plan from the centre of the structure to one of its supports. The comparison of these parameters in the different processes could help in visualizing and showing the changes that have taken place in the geometry of the initial model. The final values of the variables of geometry in the different optimization processes are shown in Table 1.

**Table 1.** Optimization processes. Final values of variables of geometry

Process	$t_1$	$t_2$	$t_3$	$k$	$\omega$	Radius	Height
	mm			$m^{-1}$	deg	m	
Initial model	60	80	350	0.140	90.0	13.63	24.39
$SE (t_{min} = 60 \text{ mm})$	61	60	362	0.158	85.4	11.87	19.04
$SE (t_{min} = 80 \text{ mm})$	80	95	400	0.165	85.0	11.54	19.00
$W (t_{min} = 60 \text{ mm})$	60	72	264	0.150	85.9	12.27	19.02
$W (t_{min} = 80 \text{ mm})$	80	81	333	0.141	86.5	12.79	19.05
$\sigma_t (t_{min} = 60 \text{ mm})$	159	81	385	0.139	86.5	12.91	19.01

$SE$  = strain energy;  $W$  = weight;  $\sigma_t$  = tensile stress;  $t_{min}$  = minimum thickness.

**Table 2.** Optimization processes. Final values of objective functions, shell thickness ( $t_1$ ), maximum compressive stress ( $\sigma_{c,max}$ ) and maximum vertical displacement ( $U_{z,max}$ )

Process	Objective functions					
	$SE$	$W$	$\sigma_t$	$t_1$	$\sigma_{c,max}$	$U_{z,max}$
	Nm	kN	MPa	mm	MPa	mm
Initial model	881.8	473.8	2.2	60	7.9	8.7
$SE (t_{min} = 60 \text{ mm})$	297.9	330.2	1.3	61	5.0	3.9
$SE (t_{min} = 80 \text{ mm})$	300.7	443.3	1.3	80	4.2	3.3
$W (t_{min} = 60 \text{ mm})$	318.3	309.5	1.3	60	4.9	3.9
$W (t_{min} = 80 \text{ mm})$	367.2	431.4	1.1	80	4.3	3.3
$\sigma_t (t_{min} = 60 \text{ mm})$	514.4	767.9	1.1	159	4.8	2.3

$SE$  = strain energy;  $W$  = weight;  $\sigma_t$  = tensile stress;  $t_{min}$  = minimum thickness.

It is observed that the angle  $\omega$  (angle between the director planes) decreases in all the optimization runs, implying that the hyper is no longer equilateral. On the other hand, the height of the initial model decreases in all runs tending to the stated minimum value of 19 m. Regarding the thickness of the shell, it can be seen that the allowed minimum value is reached when the strain energy and the weight are optimised. However, when the maximum tensile stress is optimised, the thickness of the shell is nearly 160 mm, indicating the high cost of a form having the membrane behaviour when geometric constraints are used.

The final values of the objective functions are shown in Table 2, together with three additional parameters whose analysis and comparison could be useful: the shell thickness  $e_1$ , the maximum compressive stress  $\sigma_{c,max}$  and the maximum vertical displacement  $U_{z,max}$ .

From the above results, it could be seen that in all the optimization processes, the maximum compressive stresses are below 5 MPa and the maximum tensile stresses are lower than the design tensile strength of the concrete. In addition, the maximum vertical displacement of the structure is lower than 4 mm, which concurs with the results obtained by Ortega and Arias [32] for this type of optimised structures, where vertical displacements are below 10% of the shell thickness.

When tensile stress is the objective function, the weight approximately doubles the values obtained in the other optimization processes. However, no substantial decrease in the tensile stress is achieved, which confirms the high cost of a form with a membrane behaviour using geometric constraints,

as mentioned previously.

The geometries of the initial design and of one of the final designs are compared in Figure 8. The final design was obtained using strain energy as the objective function and a minimum thickness of 60 mm as the constraint. Both geometries are intersected to provide a better perspective and show the slight differences between them.

Finally, Table 3 shows the results of a buckling analysis by using: eigenvalue (linear) analysis; nonlinear analysis considering geometric and material nonlinearity; and the latter analysis but modified by a factor ( $\alpha_2$ ) to consider concrete creep according to IASS Recommendations [39]. The buckling loads correspond to the final designs in each optimization process and to the initial model with a thickness of 60 mm.

From the stability study, the high buckling load obtained shows the high stiffness of this structure. This stiffness is also confirmed by the maximum vertical displacement obtained of 8.73 mm in the initial model, and less than 4 mm in the optimum designs (Table 2).

The influence of the thickness is decisive, because its increase implies a decrease of the shell slenderness and therefore an increment in the buckling load.

Secondly, the buckling load is approximately double in the optimum designs with respect to the initial models, with the same thickness, thus reaffirming that designing this type of concrete shell structures by using optimization techniques provides an added benefit.

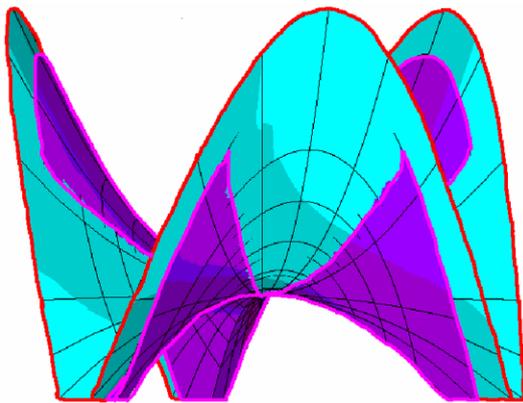
**Table 3.** Buckling load (shown as a factor of weight of the shell)

	Initial model	Optimum design				
		SE ( $t_{min} = 60$ )	SE ( $t_{min} = 80$ )	W ( $t_{min} = 60$ )	W ( $t_{min} = 80$ )	$\sigma_t$ ( $t_{min} = 60$ )
$t$ (mm)	60	61	80	60	80	159
L	8.7	17.4	25.6	15.8	22.4	53.4
NL	5.2	12.2	16.1	11.0	14.8	22.2
NL ( $\alpha_2$ )	2.6	6.0	7.9	5.4	7.2	10.9

SE = strain energy; W = weight;  $\sigma_t$  = tensile stress;  $t$  = final thickness (mm);  $t_{min}$  = minimum thickness (mm);

L = buckling load from linear analysis; NL = buckling load from nonlinear analysis;

NL ( $\alpha_2$ ) = buckling load from nonlinear analysis but modified by a concrete creep factor  $\alpha_2$ .



**Figure 8.** Intersection of initial model (light shaded) and a final design (dark shaded)

Finally, it is necessary to underline the importance of including in the study a significant phenomenon that affects the shell stability: the structural effect of long-term deformations. The buckling loads of the nonlinear analysis modified by a creep factor are between 66 and 80% less than the buckling load of the linear analysis. Nevertheless, this phenomenon has been studied more in depth numerically in [40], in which a reduction of about 10% in the factor of load-carrying capacity of a shell has been obtained.

## 5. CONCLUSIONS

In this contribution, a brief review of the constructive period in the field of concrete shells has been made, focusing on the most emblematic works of Félix Candela, the centenary of whose birth is celebrated in 2010.

Afterwards, optimization techniques have been applied to find some optimum geometric designs of an actual concrete shell -the access building to the Oceanographic Park at Valencia in Spain-, which is the posthumous work of Candela. From the results obtained in the present paper, the following interesting aspects can be highlighted:

- 1) Form finding of concrete shells by using shape optimization techniques leads to improvements in their mechanical behaviour. Slight changes in the shape are usually enough to achieve these improvements.
- 2) The shell structure studied in this work presents an excellent behaviour against

instability phenomena. The buckling load might be improved by increasing the geometric curvatures and the shell thickness.

- 3) It is convenient to underline the importance of including in the study a phenomenon that affects the shell stability: the structural effects of the long-term deformations in concrete. The buckling load obtained from the nonlinear analysis, modified by the concrete creep factor according to IAASS Recommendations, is within a third and a fifth of that obtained in the study of initial stability.

As a concluding remark, the work done by Félix Candela is admirable at a time when the present powerful processors in computers were not available. The architect, *builder and structural artist* [19] designed and analyzed multitude of concrete shells that constitute his legacy; structures that are optimum by themselves.

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